Solutions Manual

Fluid Mechanics With Engineering Applications

10th Edition

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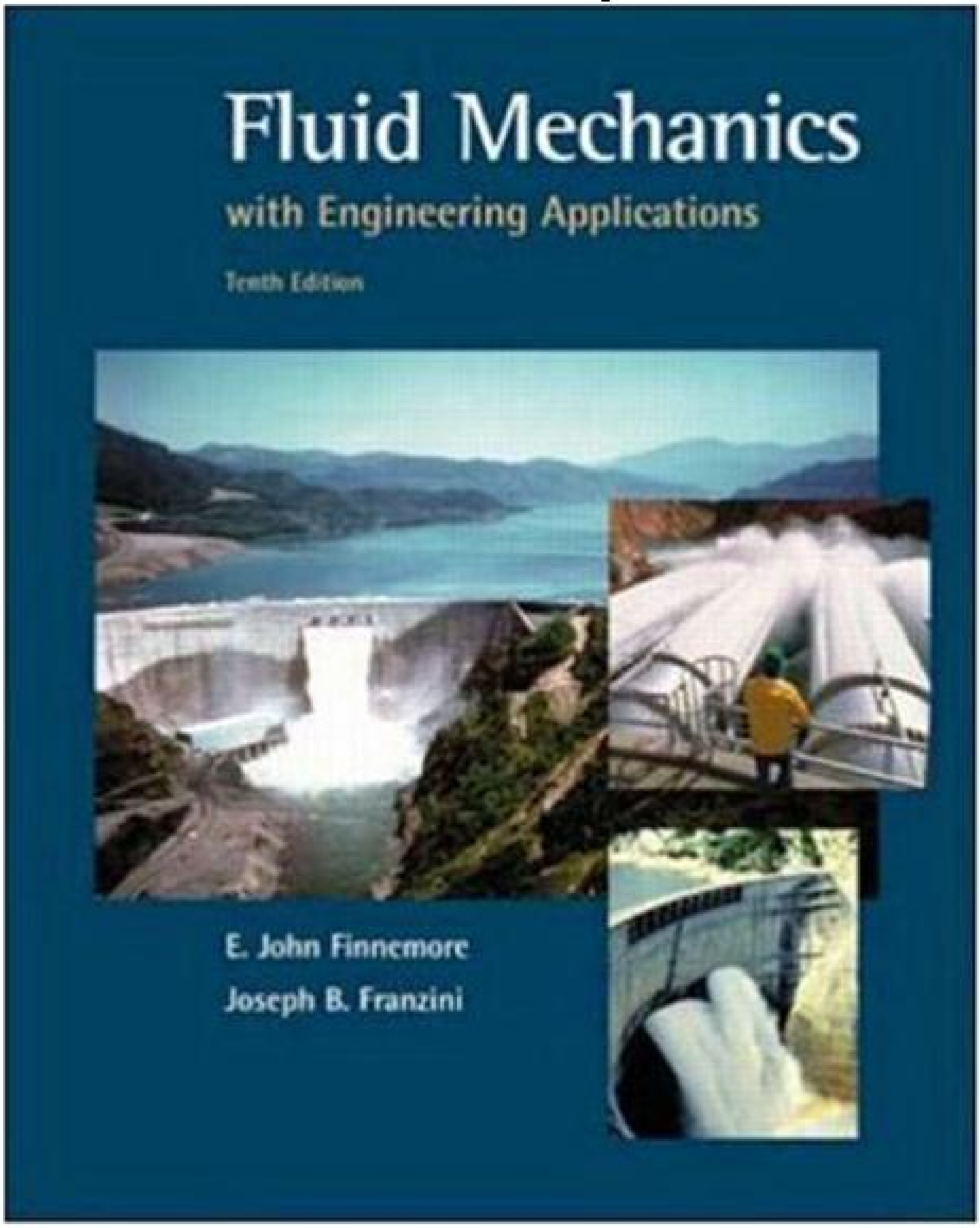


Chapter 2

Properties of Fluids

PROBLEM SELECTION GUIDE

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
2.3	***Density, Specific Weight, Specific Volume, and Specific Gravity***						
	X[1]2.3.1	BG	V Easy	V Short	1	2.3.2	
	2.3.2	SI	V Easy	V Short	1	2.3.1	
	2.3.3	BG	V Easy	V Short	1	2.3.4	
	2.3.4	SI	V Easy	V Short	1	2.3.3	
	2.3.5	SI	V Easy	V Short	1	P2.2	
	2.3.6	B	V Easy	V Short	1		
	2.3.7	BG	Easy	V Short	1	P2.3	
	P 2.1	SI	V Easy	V Short	1		
	2.2	BG	V Easy	V Short	1	X2.3.5	
	2.3	SI	Easy	V Short	1	X2.3.7	
	2.4	BG	Easy	Medium	3		
	2.5	SI	Medium	Short	1		
2.5	***Compressibility of Liquids***						
	X 2.5.1	B	Easy	V Short	2		2-D interpolation; unit conversions
	2.5.2	BG	Easy	V Short	1	2.5.4	
	2.5.3	BG	Easy	V Short	1		
	2.5.4	SI	Easy	V Short	1	2.5.2	
	2.5.5	SI	Easy	Short	1	P2.7	
	P 2.6	BG	Easy	Short	5		
	2.7	BG	Easy	Short	1	X2.5.5	
	2.8	SI	Medium	Medium	3		Interpolation in 2 directions
2.6	***Specific Weight of Liquids***						
	X 2.6.1	B	V Easy	V Short	2	2.6.2	†
	2.6.2	B	V Easy	V Short	2	2.6.1	†
	2.6.3	BG	Easy	Short	1		
	2.6.4	SI	Easy	Short	1		
	P 2.9	BG	Medium	Short	1		† Interpolation
	2.10	SI	Medium	Medium	1		

/cont...

[1] For all Exercises (identifed by "X"), answers are given in Appendix F of the textbook.
† Answers are sensitive to values that are read from graphs.
X = Exercise, P = (end-of-chapter) Problem, S = Sample Problem.

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
2.7	***Property Relations for Perfect Gases***						
	X 2.7.1	SI	Easy	V Short	1		
	2.7.2	BG	Easy	V Short	1		
	2.7.3	BG	Easy	Short	1	2.7.4	
	2.7.4	SI	Easy	Short	1	2.7.3	
	2.7.5	BG	Easy	Short	1		
	2.7.6	BG	Easy	Short	1		Partial pressures
	2.7.7	N	Medium	Short	1		Derivation
	P 2.11	SI	Easy	Short	2		
	2.12	BG	Easy	Short	3	2.13	
	2.13	SI	Easy	Short	3	2.12	
	2.14	BG	Easy	Short	3		Partial pressures
	2.15	SI	Medium	Medium	3		Partial pressures
	2.16	BG	Medium	Medium	5		Partial pressures
2.8	***Compressibility of Gases***						
	X 2.8.1	BG	V Easy	V Short	1	2.8.2	
	2.8.2	SI	V Easy	V Short	1	2.8.1	
	2.8.3	SI	Easy	Short	2		
	2.8.4	BG	Medium	Short	1	P2.19	
	P 2.17	BG	Easy	Short	2	2.18	
	2.18	SI	Easy	Short	2	2.17	
	2.19	SI	Medium	Short	1	X2.8.4	
2.11	***Viscosity***						
	X 2.11.1	B	V Easy	V Short	1		† Unit conversions (minor)
	2.11.2	BG	V Easy	V Short	1		†
	2.11.3	BG	V Easy	V Short	1		
	2.11.4	SI	Easy	V Short	1	2.11.5	
	2.11.5	BG	Easy	V Short	1	2.11.4	
	2.11.6	B	Easy	Short	1		Unit conversions
	2.11.7	B	Easy	Short	3		†
	2.11.8	SI	Easy	Short	2		†
	2.11.9	B	Easy	Short	1	P2.23	† Unit conversions
	2.11.10	N	Easy	Short	1		Integration
	2.11.11	BG	Easy	Medium	1		Unit conversion (minor)
	2.11.12						
	P 2.20	SI	Medium	Short	1		Unit conversion (minor)
	2.21	BG	Medium	Short	1	2.22	
	2.22	SI	Medium	Short	1	2.21	
	2.23	BG	Medium	Short	1	X2.11.9	†
	2.24	BG	Hard	Medium	1		† Integration
	2.25	SI	Medium	Short	1		Integration
	2.26	N	Hard	Medium	1		Integration
	2.27	SI			2		
	2.28	SI			3		

/cont...

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
2.12	*Surface Tension*						
	X 2.12.1	BG	V Easy	V Short	1	2.12.2	†
	2.12.2	SI	Easy	V Short	1	2.12.1	†
	2.12.3	BG	V Easy	V Short	1		
	2.12.4	SI	Easy	V Short	1		
	2.12.5	BG	Easy	Short	1		† Interpolation
	P 2.29	BG	Easy	Short	1		
	2.30	SI	Easy	Short	1		
	2.31	SI	Medium	Short	1	2.32	
	2.32	BG	Medium	Short	1	2.31	
2.13	*Vapor Pressure of Liquids*						
	X 2.13.1	SI	V Easy	V Short	1		
	2.13.2	BG	Easy	Short	1	P2.34	Interpolation twice
	P 2.33	BG	V Easy	V Short	1		
	2.34	SI	Easy	Short	1	X2.13.2	Interpolation twice

Chapter 2
PROPERTIES OF FLUIDS

Sec. 2.3: Density, Specific Weight, Specific Volume, and Specific Gravity – *Exercises (7)*

2.3.1 BG *If the specific weight of a liquid is 52 lb/ft³, what is its density?*

Eq. 2.1: $\rho = 52/32.2 = 1.615\ \text{slugs/ft}^3$ ◄

2.3.2 SI *If the specific weight of a liquid is 8.1 kN/m³, what is its density?*

Eq. 2.1: $\rho = 8100/9.81 = 826\ \text{kg/m}^3$ ◄

2.3.3 BG *If the specific volume of a gas is 375 ft³/slug, what is its specific weight in lb/ft³?*

Eqs. 2.1, 2.2: $\gamma = \rho g = \dfrac{g}{v} = \dfrac{32.2\ \text{ft/sec}^2}{375\ \text{ft}^3\text{/slug}}\left(\dfrac{\text{lb}}{\text{slug}\cdot\text{ft/sec}^2}\right) = 0.0859\ \text{lb/ft}^3$ ◄

2.3.4 SI *If the specific volume of a gas is 0.70 m³/kg, what is its specific weight in N/m³?*

Eqs. 2.1, 2.2: $\gamma = \rho g = \dfrac{g}{v} = \dfrac{9.81\ \text{m/s}^2}{0.70\ \text{m}^3\text{/kg}}\left(\dfrac{\text{N}}{\text{kg}\cdot\text{m/s}^2}\right) = 14.01\ \text{N/m}^3$ ◄

2.3.5 SI *A certain gas weighs 16.0 N/m³ at a certain temperature and pressure. What are the values of its density, specific volume, and specific gravity relative to air weighing 12.0 N/m³?*

Eq. 2.1: $\rho = 16/9.81 = 1.631\ \text{kg/m}^3$ ◄

Eq. 2.2: $v = 1/1.631 = 0.613\ \text{m}^3\text{/kg}$ ◄

$s = 16/12 = 1.333$ ◄

2.3.6 B *The specific weight of glycerin is 78.6 lb/ft³. Compute its density and specific gravity. What is its specific weight in kN/m³?*

Eq. 2.1: $\rho = 78.6/32.2 = 2.44\ \text{slugs/ft}^3$ ◄

$s = 78.6/62.4 = 1.260$ ◄ so $\rho = 1.260\ \text{Mg/m}^3$

Eq. 2.1: $\gamma = 9.81(1.260) = 12.36\ \text{kN/m}^3$ ◄

2.3.7 BG *If a certain gasoline weights 43 lb/ft³, what are the values of its density, specific volume, and specific gravity relative to water at 60°F? Use Appendix A.*

Eq 2.1: $\rho = 43/32.2 = 1.335\ \text{slugs/ft}^3$ ◄

Eq 2.2: $v = 1/1.335 = 0.749\ \text{ft}^3\text{/slug}$ ◄

Table A.1: ρ_{water} at 60°F = 1.938 slugs/ft³; $s = 1.335/1.938 = 0.689$ ◄

Sec 2.3: Density, Specific Weight, Specific Volume, and Specific Gravity -- *Problems 2.1–2.5*

2.1 *If the specific weight of a gas is 12.40 N/m³, what is its specific volume in m³/kg?*

SI

$$\text{Eq. 2.2:}\quad \upsilon = \frac{1}{\rho} = \frac{g}{\gamma} = \frac{9.81\ \text{m/s}^2}{12.40(\text{kg}\cdot\text{m/s}^2)/\text{m}^3} = 0.791\ \text{m}^3/\text{kg} \quad ◄$$

2.2 *A gas sample weighs 0.108 lb/ft³ at a certain temperature and pressure. What are the values of its density, specific volume, and specific gravity relative to air weighing 0.075 lb/ft³?*

BG

Eq. 2.1: $\rho = 0.108/32.2 = 0.003\,35\ \text{slugs/ft}^3$ ◄

Eq. 2.2: $\upsilon = 1/0.003\,35 = 298\ \text{ft}^3/\text{slug}$ ◄ $s = 0.108/0.075 = 1.440$ ◄

2.3 *If a certain liquid weighs 8600 N/m³, what are the values of its density, specific volume, and specific gravity relative to water at 15°C? Use Appendix A.*

SI

Eq. 2.1: $\rho = 8600/9.81 = 877\ \text{kg/m}^3$ ◄ Eq. 2.2: $\upsilon = 1/877 = 0.001\ 141\ \text{m}^3/\text{kg}$ ◄

Table A.1: ρ_{water} at 15°C = 999.1 kg/m³ ; $s = 877/999.1 = 0.877$ ◄

2.4 *Find the change in volume of 15.00 lb of water at ordinary atmospheric pressure for the following conditions: (a) reducing the temperature by 50°F from 200°F to 150°F, (b) reducing the temperature by 50°F from 150°F to 100°F, and (c) reducing the temperature by 50°F from 100°F to 50°F. Calculate each and note the trend in the changes in volume.*

BG

$W = \gamma V$, so $V = W/\gamma = 12.00/\gamma$

T °F	γ (Table A.1) lb/ft³	$V = 15.00/\gamma$ ft³
200	60.12	0.249 501
150	61.20	0.245 098
100	62.00	0.241 935
50	62.41	0.240 346

(*a*) $\Delta V_{200-150} = 0.004\ 40\ \text{ft}^3$ decrease in volume ◄

(*b*) $\Delta V_{150-100} = 0.003\ 16\ \text{ft}^3$ decrease in volume ◄

(*c*) $\Delta V_{100-50} = 0.001\ 589\ \text{ft}^3$ decrease in volume ◄

For $\Delta T = 50°$ F, the volume change increases with temperature ◄

2.5 *Initially when 1000.00 mL of water at 10°C are poured into a glass cylinder the height of the water column is 1000.0 mm. The water and its container are heated to 70°C. Assuming no evaporation, what then will be the depth of the water column if the coefficient of thermal expansion for the glass is 3.8×10⁻⁶ mm/mm per °C?*

SI

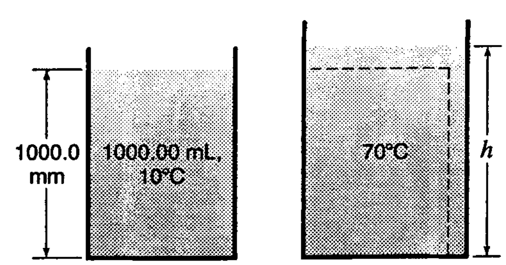


Figure P2.5

Table A.1 for water at 10°C: $\rho = 999.7\ \text{kg/m}^3$;

for water at 70°C: $\rho = 977.8\ \text{kg/m}^3$.

Mass of water $= \rho V = \rho_{10} V_{10} = \rho_{70} V_{70}$

$(999.7\ \text{kg/m}^3)(1000.0\ \text{mL}) = (977.8\ \text{kg/m}^3) V_{70}$; so $V_{70} = 1022.40\ \text{mL} = 1022\ 400\ \text{mm}^3$

$A_{10} = V_{10}/h_{10} = 1000\,000.00\ \text{mm}^3/1000.0\ \text{mm} = 1000.0\ \text{mm}^2$

$$A_{70} = A_{10}[1 + (70 - 10)3.8 \times 10^{-6}]^2 = 1000.5\ \text{mm}^2\ ; \qquad h_{70} = \frac{V_{70}}{A_{70}} = \frac{1022\,400}{1000.5} = 1021.9\ \text{mm} \quad ◄$$

Sec. 2.5: Compressibility of Liquids – *Exercises (5)*

2.5.1 *To two significant figures what is the bulk modulus of water in MN/m² at 50°C under a pressure of 30 MN/m²? Use Table 2.1.*

B

From inside cover: 50°C = 122°F; 30 MN/m² = 4351 psi

From Table 2.1, by linear interpolation in two directions: $E_v \approx 360{,}400$ psi

$E_v \approx 360{,}400(6895/10^6) = 2485\ \text{MN/m}^2 \approx 2500\ \text{MN/m}^2$ ◄

2.5.2 *At normal atmospheric conditions, approximately what pressure in psi must be applied to water to reduce its volume by 2% ? Use Table 2.1.*

BG

Table 2.1: At normal atmospheric conditions, $E_v \approx 320{,}000$ psi

From Eq. 2.3*a*: $\Delta v/v_1 = -0.02 = -\Delta p/320{,}000$; $\Delta p = 6400$ psi ◄

2.5.3 *Water in a hydraulic press is subjected to a pressure of 4500 psia at 68°F. If the initial pressure is 15 psia, approximately what will be the percentage decrease in specific volume? Use Table 2.1*

BG

From Table 2.1: $E_v \approx (320{,}000 + 348{,}000)/2 = 334{,}000$ psi

Eq. 2.3: $\dfrac{\Delta v}{v_1} = \dfrac{-(4500 - 15)}{365{,}000} = -0.013\,43$ or 1.34% decrease ◄

2.5.4 *At normal atmospheric conditions, approximately what pressure in MPa must be applied to water to reduce its volume by 3% ?*

SI

Table 2.1: At normal atmospheric conditions, $E_v \approx 320{,}000$ psi

$E_v \approx (320{,}000\ \text{psi})(6895) = 2.21 \times 10^9\ \text{Pa} = 2210\ \text{MPa}$

From Eq. 2.3*a*: $\dfrac{\Delta v}{v_1} = -0.03 = \dfrac{-\Delta p}{2210}$; $\Delta p = 66.2$ MPa ◄

2.5.5 *A rigid cylinder, inside diameter 15 mm, contains a column of water 500 mm long. What will the column length be if a force of 2 kN is applied to its end by a frictionless plunger? Assume no leakage.*

SI

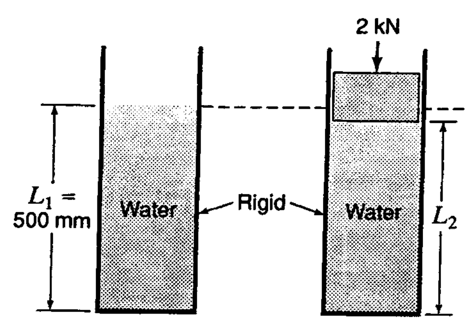


Figure X2.5.5

$p_1 = 0;\ p_2 = \dfrac{\text{Force}}{\text{Area}} = \dfrac{2\ \text{kN}}{\pi(0.0075\ \text{m})^2}$

$= 11\,320\ \text{kN/m}^2 = 11.32\ \text{MPa}$

$E_v \approx 320{,}000\ \text{psi}(6895) = 2.21 \times 10^9\ \text{Pa} = 2210\ \text{MPa}$

Since the tube is rigid, using Eq. 2.3*b*:

$$\frac{v_2 - v_1}{v_1} = \frac{L_2 - L_1}{L_1} \approx \frac{-(p_2 - p_1)}{E_v}$$

so $L_2 - 0.5 \approx -(0.5\ \text{m})\dfrac{11.32 - 0}{2210} = -0.00256$ m

$L_2 \approx 0.5 - 0.00256 = 0.497$ m or 497 mm ◄

Sec. 2.5: Compressibility of Liquids -- *Problems* 2.6–2.8

2.6 *At a depth of 4 miles in the ocean the pressure is 9,520 psi. Assume specific weight at the surface is 64.00 lb/ft³ and that the average volume modulus is 320,000 psi for that pressure range. (a) What will be the change in specific volume between that at the surface and at that depth? (b) What will be the specific volume at that depth? (c) What will be the specific weight at that depth? (d) What is the percentage change in the specific volume? (e) What is the percentage change in the specific weight?*

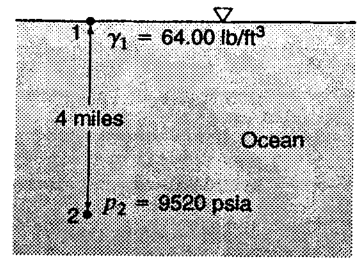


Figure P2.6

BG

(*a*) $v_1 = 1/\rho_1 = g/\gamma_1 = 32.2/64.00 = 0.503$ ft³/slug

Eq. 2.3: $\Delta v = \dfrac{-0.503(9520 - 0)}{320{,}000} = -0.014\,97$ ft³/slug ◄

(*b*) From Eq. 2.3: $v_2 = v_1 + \Delta v = 0.503 - 0.014\,97 = 0.488$ ft³/slug ◄

(*c*) $\gamma_2 = g/v_2 = 32.2/0.488 = 66.0$ lb/ft³ ◄

(*d*) $\Delta v/v_1 = 0.014\,97/0.503 = 2.98\%$ decrease ◄

(*e*) $\Delta\gamma/\gamma_1 = (66.0 - 64.00)/64.00 = 3.07\%$ increase ◄

2.7 *Water at 68°F is in a long rigid cylinder of inside diameter 0.600 in. A plunger applies pressure to the water. If, with zero force, the initial length of the column of water is 25.00 in, what will be its length be if a force of 420 lb is applied to the plunger. Assume no leakage and no friction.*

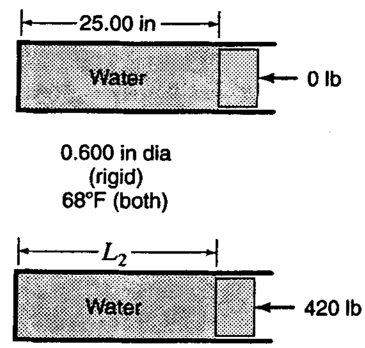


Figure P2.7

BG

$p_1 = 0$ psi gage $= 14.7$ psia

$p_2 =$ Force/Area $= 420/\pi(0.300)^2 = 1485$ lb/in² gage $= 1500$ psia

Since the tube is rigid, from Eq. 2.3:

$$\Delta v/v = \Delta L/L = (L_2 - L_1)/L_1 = -(p_2 - p_1)/E_v$$

Table 2.1 at 68°F: $(E_v)_1 = 320{,}000$ psi, $(E_v)_2 = 330{,}000$ psi,

∴ average for the range = 325,000 psi.

So $L_2 - 25.00 = -25.00(1485 - 0)/325{,}000 = -0.1143$ in ; $L_2 = 25.00 - 0.1143 = 24.9$ in ◄

2.8 *Find the change in volume of 10.00 m³ of water for the following situations: (a) a temperature increase from 60°C to 70°C with constant atmospheric pressure, (b) a pressure increase from zero to 10 MN/m² with temperature remaining constant at 60°C, and (c) a temperature decrease from 60°C to 50°C combined with a pressure increase from zero to 10 MN/m².*

SI

(*a*) Table A.1 (atmospheric pressure): $\gamma_{60} = 9.642$ kN/m³, $\gamma_{70} = 9.589$ kN/m³

$W = \gamma V = \gamma_{60}V_{60} = \gamma_{70}V_{70}$ so $9.642(10) = 9.589V_{70}$, from which $V_{70} = 10.055\,3$ m³

$\Delta V_{70-60} = 10.055\,3 - 10.000\,0 = 0.055\,3$ m³ increase in volume ◄

(*b*) 10 MN/m² ≡ $10 \times 10^6/6894 = 1450$ psi ; 60°C ≡ 32 + (9/5)60 = 140° F

From Table 2.1, by linear interpolation in 2 directions, for 140°F, 740 psia: $E_v = 331{,}400$ psi

$E_v \equiv 331{,}400(6895) = 2.28 \times 10^9$ Pa $= 2280$ MN/m²

Letting the constant mass of water be m, then $m = \rho V$ and so $1/\rho = V/m$

But from Eq. 2.2: $v = 1/\rho$ so $v = V/m$

/cont...

Using this relation to substitute for v into Eq. 2.3, remembering that m is constant:

$(v_2 - v_1)/v_1 = (\mathcal{V}_2 - \mathcal{V}_1)/\mathcal{V}_1 = -\Delta p/E_v$

So $\Delta\mathcal{V} = -\mathcal{V}\Delta p/E_v = -(10.00\ \text{m}^3)10/2280 = -0.043\,8\ \text{m}^3$ ◄

The minus sign indicates a reduction in volume.

(*c*) Table A.1: $\gamma_{60} = 9.642\ \text{kN/m}^3$, $\gamma_{50} = 9.689\ \text{kN/m}^3$. 55°C = 131°F.

Constant weight = $\gamma_{60}\mathcal{V}_{60} = \gamma_{50}\mathcal{V}_{50}$ so $9.642(10) = 9.689V_{50}$ from which $\mathcal{V}_{50} = 9.95\ \text{m}^3$

So due to temperature decrease, $\Delta\mathcal{V}_{50-60} = -0.048\,5\ \text{m}^3$ (decrease).

From Table 2.1, by linear interpolation in 2 directions, for 131°F, 740 psia: $E_v = 334{,}100$ psi

$E_v \equiv 334{,}100(6895) = 2.30 \times 10^9\ \text{Pa} = 2300\ \text{MN/m}^2$

From Eq. 2.3: $\Delta\mathcal{V} = -\mathcal{V}_1\Delta p/E_v = -10(10/2300) = -0.043\,4\ \text{m}^3$ (decrease)

Summing the changes for both causes: $\Delta\mathcal{V} = 0.048\,5 + 0.043\,4 = 0.091\,9\ \text{m}^3$ (decrease) ◄

Sec. 2.6: Specific Weight of Liquids -- ***Exercises*** *(4)*

2.6.1 *Use Fig. 2.1 to find the approximate specific weight of water in lb/ft³ under the following conditions: (a) at a temperature of 60°C under 101.4 kPa abs pressure; (b) at 60°C under a pressure of 13.79 MPa abs.*

B

(*a*) From Fig. 2.1 at 60°C: At 101.3 kPa abs, $\gamma \approx 61.4$ pcf ◄

(*b*) At 13.79 MPa abs, $\gamma \approx 61.8$ pcf ◄

2.6.2 *Use Fig. 2.1 to find the approximate specific weight of water in kN/m³ under the following conditions: (a) at a temperature of 160°F under normal atmospheric pressure; (b) at 160°F under a pressure of 2000 psia.*

B

(*a*) From Fig. 2.1 at 160°F: At 14.7 psia, $\gamma \approx 9.59\ \text{kN/m}^3$ ◄

(*b*) At 2000 psia, $\gamma \approx 9.65\ \text{kN/m}^3$ ◄

2.6.3 *A vessel contains 5.0 ft³ of water at 40°F and atmospheric pressure. If the water is heated to 80°F what will be the percentage change in its volume? What weight of water must be removed to maintain the volume at its original value? Use Appendix A.*

BG

Table A.1: $\gamma_{40} = 62.43$ pcf, $\gamma_{80} = 62.22$ pcf

Weight of water = $\gamma\mathcal{V} = \gamma_{40}\mathcal{V}_{40} = \gamma_{80}\mathcal{V}_{80}$; $62.43(5) = 62.22\mathcal{V}_{80}$; $\mathcal{V}_{80} = 5.016\,88\ \text{ft}^3$

$$\frac{\Delta\mathcal{V}}{\mathcal{V}_{40}} = \frac{0.016\,88}{5} = 0.338\%\ \text{increase}$$ ◄

Must remove $(0.016\,88\ \text{ft}^3)(62.22\ \text{lb/ft}^3) = 1.050$ lb ◄

2.6.4 *A cylindrical tank (diameter = 8.00 m and depth = 5.00 m) contains water at 15°C and is brimful. If the water is heated to 60°C, how much water will spill over the edge of the tank? Assume the tank does not expand with the change in temperature. Use Appendix A.*

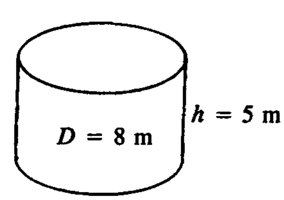
SI

Table A.1: $\gamma_{15} = 9.798\ \text{kN/m}^3$, $\gamma_{60} = 9.642\ \text{kN/m}^3$

Tank vol = $\mathcal{V}_{15} = \pi 4^2(5) = 251.33\ \text{m}^3$

Weight of water in tank = $9.798(251.33) = 2463$ kN

$\mathcal{V}_{60} = 2463/9.642 = 255.39\ \text{m}^3$; $\Delta\mathcal{V} = 255.39 - 251.33 = 4.07\ \text{m}^3$ ◄



Sec. 2.6: Specific Weight of Liquids -- *Problems 2.9–2.10*

2.9 *A heavy closed steel chamber is filled with water at 40°F and at atmospheric pressure. If the temperature of the water and the chamber is raised to 80°F, what will be the new pressure of the water? The coefficient of thermal expansion of the steel is 6.6 × 10⁻⁶ in/in per °F; Assume the chamber is unaffected by the water pressure. Use Table A.1 and Fig. 2.1.*

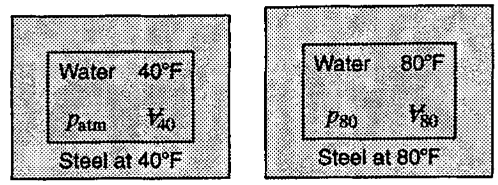


Figure P2.9

BG

$V_{80} = V_{40}[1 + (80 - 40)6.6 \times 10^{-6}]^3 = 1.000\,792 V_{40}$

Table A.1: $\gamma_{40} = 62.43$ lb/ft³ at p = atmos.

Wt of water = $\gamma V = \gamma_{40} V_{40} = \gamma_{80} V_{80}$; $62.43 V_{40} = \gamma_{80}(1.000\,792 V_{40})$; $\gamma_{80} = 62.38$ pcf

Fig. 2.1 for $\gamma = 62.38$ pcf, by linear interpolation: $p_{80} \approx 882$ psia ◄

2.10 *Repeat Exer. 2.6.4 for the case where the tank is made of a material which has a coefficient of thermal expansion of 4.6 × 10⁻⁶ mm/mm per °C.*

Exer. 2.6.4: A cylindrical tank (diameter = 8.00 m and depth = 5.00 m) contains water at 15°C and is brimful. If the water is heated to 60°C, how much water will spill over the edge of the tank? Use Appendix A.

SI

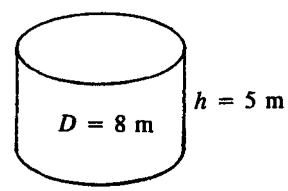


Table A.1: $\gamma_{15} = 9.798$ kN/m³, $\gamma_{60} = 9.642$ kN/m³

At 15°C, $V_{tank} = \pi 4^2(5) = 251.33$ m³; $\Delta T = 60 - 15 = 45°$ C

At 60°C, $V_{tank} = 251.33[1 + (\Delta T)4.6 \times 10^{-6}]^3 = 251.33(1.000\,621)$

$\Delta V_{tank} = 251.33(0.000\,621) = 0.1561$ m³ increase

Weight of water = $\gamma V = \gamma_{15} V_{15} = \gamma_{60} V_{60}$; $9.798(251.33) = 9.642 V_{60}$ from which $V_{60} = 255.39$ m³

$\Delta V_{water} = 255.39 - 251.33 = 4.07$ m³ increase ; spill $\Delta V = 4.07 - 0.1561 = 3.91$ m³ ◄

Sec. 2.7: Property Relations for Perfect Gases -- *Exercises (7)*

2.7.1 *A gas at 60°C under a pressure of 10 000 mb abs has a specific weight of 99 N/m³. What is the value of R for the gas? What gas might this be? Refer to Appendix A, Table A.5.*

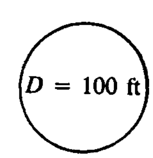
SI

Eq. 2.5: $R = \dfrac{gp}{\gamma T} = \dfrac{9.81\ \text{m/s}^2(10\,000 \times 100\ \text{N/m}^2)}{99\ \text{N/m}^3(273+60)\text{K}} = 298\ \text{m}^2/(\text{s}^2\cdot\text{K})$ ◄

Table A.5: This gas with $R = 298$ might be carbon monoxide or nitrogen. ◄

2.7.2 *A hydrogen-filled balloon of the type used in cosmic-ray studies is to be expanded to its full size, which is a 100-ft-diameter sphere, without stress in the wall at an altitude of 150,000 ft. If the pressure and temperature at this altitude are 0.14 psia and −67°F respectively, find the volume of hydrogen at 14.7 psia and 60°F which should be added on the ground. Neglect the balloon's weight.*

BG



Eq. 2.4: $pv = RT$; $v = V/m = Vg/W$; ∴ $pVg/W = RT$; ∴ pV/T = constant

$$\frac{14.7 V_1}{460 + 60} = \frac{0.14 \times (4/3)\pi 50^3}{460 - 67}; \quad V_1 = 6600\ \text{ft}^3$$ ◄

2.7.3 BG *Calculate the density, specific weight, and specific volume of air at 120°F and 50 psia.*

Eq. 2.5: $\gamma = \dfrac{gp}{RT} = \dfrac{32.2(50 \times 144)}{1715(460+120)} = 0.233 \text{ lb/ft}^3$ ◄

Eq. 2.1: $\rho = \gamma/g = 0.233/32.2 = 0.007\,24 \text{ slugs/ft}^3$ ◄

Eq. 2.2: $v = 1/\rho = 138.2 \text{ ft}^3\text{/slug}$ ◄

2.7.4 SI *Calculate the density, specific weight, and specific volume of air at 50°C and 3400 mb abs.*

Eq. 2.5: $\gamma = \dfrac{gp}{RT} = \dfrac{9.81(3400 \times 100)}{287(273+50)} = 36.0 \text{ N/m}^3$ ◄

Eq. 2.1: $\rho = \gamma/g = 36.0/9.81 = 3.67 \text{ kg/m}^3$ ◄

Eq. 2.2: $v = 1/\rho = 0.273 \text{ m}^3\text{/kg}$ ◄

2.7.5 BG *If natural gas has a specific gravity of 0.6 relative to air at 14.7 psia and 68°F, what are its specific weight and specific volume at that same pressure and temperature. What is the value of R for the gas? Solve without using Table A.2.*

Table A.5 for air: $R = 1715$ ft·lb/(slug·°R). Eq. 2.5: $\gamma_{air} = \dfrac{gp}{RT} = \dfrac{32.2(14.7)144}{1715(460+68)} = 0.0753 \text{ lb/ft}^3$

Sec. 2.3: $\gamma_{ngas} = s(\gamma_{air}) = 0.6(0.0753) = 0.0452 \text{ lb/ft}^3$ ◄

Eqs. 2.1 and 2.2: $v = 1/\rho = g/\gamma = 32.2/0.0452 = 713 \text{ ft}^3\text{/slug}$ ◄

Eq. 2.5: For a given T and p, gp/T is constant

$\therefore\ \gamma R = \gamma_{air}R_{air} = \gamma_{ngas}R_{ngas}$ and $R_{ngas} = R_{air}(\gamma_{air}/\gamma_{ngas}) = R_{air}/s_{ngas}$

or $R_{ngas} = 1715/0.6 = 2858$ ft·lb/(slug·°R) ◄

2.7.6 BG *Given that a sample of dry air at 40°F and 14.7 psia contains 21% oxygen and 78% nitrogen by volume. What is the partial pressure (psia) and specific weight of each gas?*

Table A.5: R(oxygen) = 1554 ft²/(sec²·°R); R(nitrogen) = 1773 ft²/(sec²·°R).

From Dalton's law, for O_2: $p = 0.21(14.7) = 3.09$ psia ◄

Eq. 2.5 for 21% O_2: $\gamma = 0.21\dfrac{(32.2 \text{ ft/sec}^2)(14.7 \times 144 \text{ lb/ft}^2)}{[1554 \text{ ft}^2/(\text{sec}^2\cdot°\text{R})](460+40)°\text{R}} = 0.018\,42 \text{ lb/ft}^3$ ◄

From Dalton's law, for N_2: $p = 0.78(14.7) = 11.47$ psia ◄

Eq. 2.5 for 78% N_2: $\gamma = 0.78\dfrac{(32.2)(14.7 \times 144)}{1773(500)} = 0.060\,0 \text{ lb/ft}^3$ ◄

2.7.7 N *Prove that Eq (2.7) follows from Eqs (2.4) and (2.6).*

Eq. 2.4: $v = RT/p$; Eq. 2.6: $p_1 v_1^n = p_2 v_2^n$

Eliminate v: $p_1\left(\dfrac{R_1T_1}{p_1}\right)^n = p_2\left(\dfrac{R_2T_2}{p_2}\right)^n$; but $R_1 = R_2$

Thus $\dfrac{T_1^n}{p_1^{n-1}} = \dfrac{T_2^n}{p_2^{n-1}}$ or $\left(\dfrac{T_2}{T_1}\right)^n = \left(\dfrac{p_2}{p_1}\right)^{n-1}$ Finally $\dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{(n-1)/n}$ Q.E.D. ◄

Sec. 2.7: Property Relations for Perfect Gases -- ***Problems*** *2.11–2.16*

2.11 *(a) Calculate the density, specific weight, and specific volume of oxygen at 20°C and 50 kN/m² abs. (b) If the oxygen is enclosed in a rigid container of constant volume, what will be the pressure if the temperature is reduced to −100°C?*

SI

(*a*) Table A.5 for oxygen: $R = 260\ \mathrm{m^2/(s^2\cdot K)}$

Eq. 2.5: $\gamma = \dfrac{gp}{RT} = \dfrac{(9.81\ \mathrm{m/s^2})(50\ \mathrm{kN/m^2})}{[260\ \mathrm{m^2/(s^2\cdot K)}](273 + 20)\mathrm{K}} = 0.006\,44\ \mathrm{kN/m^3} = 6.44\ \mathrm{N/m^3}$ ◄

Eq. 2.1: $\rho = \gamma/g = 6.44/9.81 = 0.656\ \mathrm{kg/m^3}$ ◄

Eq. 2.2: $v = 1/\rho = 1/0.656 = 1.524\ \mathrm{m^3/kg}$ ◄

(*b*) Eq. 2.4: $pv = RT$; $v = \text{const}$, $R = \text{const}$

$\therefore\ \dfrac{p}{T} = \text{constant} = \dfrac{50}{273 + 20} = \dfrac{p_2}{273 - 100}$; $p_2 = 29.5\ \mathrm{kN/m^2}$ ◄

2.12 *(a) If water vapor in the atmosphere has a partial pressure of 0.50 psia and the temperature is 90°F, what is its specific weight? (b) If the barometer reads 14.50 psia, what is the partial pressure of the (dry) air, and what is its specific weight? (c) What is the specific weight of the atmosphere (air plus the water vapor present)?*

BG

(*a*) Table A.5 for water vapor: $R = 2760\ \mathrm{ft^2/(sec^2\cdot{}^\circ R)}$

Eq. 2.5 for water vapor: $\gamma = \dfrac{32.2(0.50 \times 144)}{2760(460+90)} = 0.001\,527\ \mathrm{lb/ft^3}$ ◄

(*b*) $p_{\text{air}} = 14.50 - 0.50 = 14.00$ psia ◄

Table A.5 for air: $R = 1715\ \mathrm{ft^2/(sec^2\cdot{}^\circ R)}$

Eq. 2.5: Air, $\gamma = \dfrac{32.2(14.00 \times 144)}{1715(460+90)} = 0.0688\ \mathrm{lb/ft^3}$ ◄

(*c*) Atmos, $\gamma = \gamma_{\text{air}} + \gamma_{\text{wvapor}} = 0.0688 + 0.001\,527 = 0.0703\ \mathrm{lb/ft^3}$ ◄

2.13 *(a) If water vapor in the atmosphere has a partial pressure of 3500 Pa and the temperature is 30°C, what is its specific weight? (b) If the barometer reads 102 kPa abs, what is the partial pressure of the (dry) air, and what is its specific weight? (c) What is the specific weight of the atmosphere (air plus the water vapor present)?*

SI

(*a*) Table A.5 for water vapor: $R = 462\ \mathrm{m^2/(s^2\cdot K)}$

Eq. 2.5: $\gamma = \dfrac{9.81\ \mathrm{m/s^2} \times 3500\ \mathrm{N/m^2}}{[462\ \mathrm{m^2/(s^2\cdot K)}](273+30)\mathrm{K}} = 0.245\ \mathrm{N/m^3}$ ◄

(*b*) $p_{\text{air}} = 102 - (3500/1000) = 98.5$ kPa abs ◄

Table A.5 for air: $R = 287\ \mathrm{m^2/(s^2\cdot K)}$

Eq. 2.5: $\gamma_{\text{air}} = \dfrac{gp}{RT} = \dfrac{(9.81\ \mathrm{m/s^2})(98\,500\ \mathrm{N/m^2})}{[287\ \mathrm{m^2/(s^2\cdot K)}](273+30)\mathrm{K}} = 11.11\ \mathrm{N/m^3}$ ◄

(*c*) $\gamma_{\text{atmos}} = \gamma_{\text{air}} + \gamma_{\text{wvapor}} = 11.11 + 0.245 = 11.36\ \mathrm{N/m^3}$ ◄

2.14 *If the specific weight of water vapor in the atmosphere is 0.0065 lb/ft³ and of the (dry) air is 0.074 lb/ft³ when the temperature is 70°F, (a) what are the partial pressures of the water vapor and the dry air in psia, (b) what is the specific weight of the atmosphere (air and water vapor), and (c) what is the barometric pressure in psia?*

BG

(*a*) Eq. 2.5: $\gamma_{\text{wvapor}} = gp/RT$; Table A.5 for water vapor: $R = 2760\ \text{ft}^2/(\text{sec}^2\cdot{}^\circ\text{R})$

$$\therefore\ p = \frac{\gamma RT}{g} = \frac{(0.000\,65\ \text{lb/ft}^3)[2760\ \text{ft}^2/(\text{sec}^2\cdot{}^\circ\text{R})](460 + 70)^\circ\text{R}}{32.2\ \text{ft/sec}^2} = 29.5\ \text{psfa} = 0.205\ \text{psia} \quad ◀$$

Eq. 2.5: $\gamma_{\text{air}} = gp/RT$; Table A.5 for air: $R = 1715\ \text{ft}^2/(\text{sec}^2\cdot{}^\circ\text{R})$

$$\therefore\ p = \frac{\gamma RT}{g} = \frac{(0.074\ \text{lb/ft}^3)[1715\ \text{ft}^2/(\text{sec}^2\cdot{}^\circ\text{R})](460 + 70)^\circ\text{R}}{32.2\ \text{ft/sec}^2} = 2090\ \text{psfa} = 14.51\ \text{psia} \quad ◀$$

(*b*) $\gamma_{\text{atmos}} = \gamma_{\text{air}} + \gamma_{\text{wv}} = 0.0740 + 0.000\,65 = 0.747\ \text{lb/ft}^3$ ◀

(*c*) $p_{\text{atm}} = p_{\text{air}} + p_{\text{wv}} = 14.51\ \text{psia} + 0.205\ \text{psia} = 14.72\ \text{psia}$ ◀

2.15 *If an artificial atmosphere consists of 20% oxygen and 80% nitrogen by volume, at 101.32 kN/m² abs and 20°C, what are (a) the specific weight and partial pressure of the oxygen, (b) the specific weight and partial pressure of the nitrogen, and (c) the specific weight of the mixture?*

SI

Table A.5: $R(\text{oxygen}) = 260\ \text{m}^2/(\text{s}^2\cdot\text{K})$; $R(\text{nitrogen}) = 297\ \text{m}^2/(\text{s}^2\cdot\text{K})$

Eq. 2.5: 100% O_2: $$\gamma = \frac{(9.81\ \text{m/s}^2)(101.32\ \text{kN/m}^2)}{[260\ \text{m}^2/(\text{s}^2\cdot\text{K})](273 + 20)\text{K}} = 13.05\ \text{N/m}^3$$

100% N_2: $$\gamma = \frac{(9.81\ \text{m/s}^2)(101.32\ \text{kN/m}^2)}{[297\ \text{m}^2/(\text{s}^2\cdot\text{K})](273 + 20)\text{K}} = 11.42\ \text{N/m}^3$$

(*a*) Each m³ of mixture contains 0.2 m³ of O_2 and 0.8 m³ of N_2.

So for 20% O_2: $\gamma = 0.20(13.05) = 2.61\ \text{N/m}^3$ ◀

From Eq. 2.5 for 20% O_2: $$p = \frac{\gamma RT}{g} = \frac{2.61(260)293}{9.81} = 20{,}300\ \text{Pa} = 20.3\ \text{kPa} \quad ◀$$

(*b*) For 80% N_2: $\gamma = 0.80(11.42) = 9.14\ \text{N/m}^3$ ◀

$$p = \frac{\gamma RT}{g} = \frac{9.14(297)293}{9.81} = 81{,}060\ \text{Pa} = 81.1\ \text{kPa} \quad ◀$$

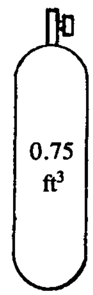
(*c*) Mixture: $\gamma = 2.61 + 9.14 = 11.75\ \text{N/m}^3$ ◀

2.16 *When the ambient air is at 70°F, 14.7 psia, and contains 21% oxygen by volume, 4.5 lb of air are pumped into a scuba tank, capacity 0.75 ft³. (a) What volume of ambient air was compressed? (b) When the filled tank has cooled to ambient conditions, what is the (gage) pressure of the air in the tank? (c) What is the partial pressure (psia) and specific weight of the ambient oxygen? (d) What weight of oxygen was put in the tank? (e) What is the partial pressure (psia) and specific weight of the oxygen in the tank?*

BG

(*a*) From Table A.2: $\gamma_1 = 0.074\,95 \text{ lb/ft}^3$. $\therefore\ V_{\text{air}} = \dfrac{W}{\gamma_1} = \dfrac{4.5 \text{ lb}}{0.074\,95 \text{ lb/ft}^3} = 60.0 \text{ ft}^3$ ◄

(*b*) $\gamma_2 = \dfrac{W}{V} = \dfrac{4.5 \text{ lb}}{0.75 \text{ ft}^3} = 6.00 \text{ lb/ft}^3$



From Section 2.7 $n = 1$ for isothermal conditions, so from Eq. 2.6:

$$\frac{p_1}{\rho_1} = \frac{p_2}{\rho_2}; \quad \therefore\ p_2 = \left(\frac{\rho_2}{\rho_1}\right)p_1 = \left(\frac{\gamma_2}{\gamma_1}\right)p_1 = \left(\frac{6.00 \text{ lb/ft}^3}{0.074\,95 \text{ lb/ft}^3}\right)14.7 \text{ psia} = 1177 \text{ psia}$$

$p_2 = 1177 - 14.7 = 1162 \text{ psig}$ ◄

(*c*) Table A.5 for oxygen: $R = 1554 \text{ ft}^2/(\text{sec}^2\cdot{}^\circ\text{R})$

Eq. 2.5 for 100% O_2: $\gamma = \dfrac{(32.2 \text{ ft/sec}^2)(14.7 \text{ lb/in}^2)(144 \text{ in}^2/\text{ft}^2)}{[1554 \text{ ft}^2/(\text{sec}^2\cdot{}^\circ\text{R})](460+70)^\circ\text{R}} = 0.0828 \text{ lb/ft}^3$

$\therefore$ for 21% O_2 by volume: $\gamma = 0.21(0.0828) = 0.017\,38 \text{ lb/ft}^3$ ◄

From Eq. 2.5 with 21% O_2: $p = \dfrac{\gamma RT}{g} = \dfrac{(0.017\,38 \text{ lb/ft}^3)[1554 \text{ ft}^2/(\text{sec}^2\cdot{}^\circ\text{R})](530^\circ\text{R})}{32.2 \text{ ft/sec}^2}$

$p = 445 \text{ psfa} = 3.09 \text{ psia}$ ◄

(*d*) $W_{O_2} = \gamma_1 V_{\text{air}} = (0.017\,38 \text{ lb/ft}^3)60 \text{ ft}^3 = 1.043 \text{ lb}$ ◄

(*e*) $\gamma_2 = \dfrac{W}{V} = \dfrac{1.043 \text{ lb}}{0.75 \text{ ft}^3} = 1.391 \text{ lb/ft}^3$ ◄

$$p = \frac{\gamma_2 RT}{g} = \frac{1.391(1554)530}{32.2} = 35{,}579 \text{ psfa} = 247 \text{ psia}$$ ◄

Sec. 2.8: Compressibility of Gases -- *Exercises (4)*

2.8.1 *Methane at 22 psia is compressed isothermally, and nitrogen at 16 psia is compressed isentropically. What is the modulus of elasticity of each gas? Which is the more compressible?*

BG

Methane, isothermal: $n = 1$; $E_v = np = (1)22 = 22 \text{ psi}$ ◄

N_2, isentropic: $n = k = 1.40$; $E_v = 1.40(16) = 22.4 \text{ psi}$ ◄

$E_v(\text{methane}) < E_v(N_2)$, methane is more compressible ◄

2.8.2 *Methane at 140 kPa abs is compressed isothermally, and nitrogen at 100 kPa abs is compressed isentropically. What is the modulus of elasticity of each gas? Which is the more compressible?*

SI

Methane, isothermal: $n = 1$; $E_v = np = (1)140 = 140 \text{ kPa}$ ◄

N_2, isentropic: $n = k = 1.40$; $E_v = 1.40(100) = 140 \text{ kPa}$ ◄

$E_v(\text{methane}) = E_v(N_2)$, the compressibilities are equal ◄

2.8.3 *(a) If 10 m^3 of nitrogen at 30°C and 125 kPa are expanded isothermally to 25 m^3, what is the resulting pressure? (b) What would the pressure and temperature have been if the process had been isentropic? The adiabatic exponent k for nitrogen is 1.40.*

SI

(*a*) Isothermal: $p_1 v_1 = p_2 v_2$ = constant ; 125(10/mass) = p_2(25/mass); p_2 = 50 kPa abs ◄

(*b*) Isentropic: $p_1 v_1^{1.40} = p_2 v_2^{1.40}$

$125(10/\text{mass})^{1.40} = p_2(25/\text{mass})^{1.40}$; $p_2 = 125(10/25)^{1.40}$ = 34.7 kPa abs ◄

Eq. 2.7: $T_2 = (273+30)(34.7/125)^{0.40/1.40}$ = 210.0 K = −63.0°C ◄

2.8.4 *Helium at 25 psia and 65°F is isentropically compressed to one-fifth of its original volume. What is its final pressure?*

BG

Isentropic process: $n = k$; Table A.5 for helium: R = 12,420 ft·lb/(slug·°R), k = 1.66

Eq. 2.4: $v = \dfrac{RT}{p} = \dfrac{12{,}420(460+65)}{25(144)}$ = 1811 ft³/slug

Eq. 2.6: $pv^{1.66}$ = constant ; $25(1811)^{1.66} = p_2(1811/5)^{1.66}$; p_2 = 362 psia ◄

Sec. 2.8: Compressibility of Gases -- ***Problems*** *2.17−2.19*

2.17 *(a) If 10 ft^3 of carbon dioxide at 50°F and 15 psia is compressed isothermally to 2 ft^3, what is the resulting pressure? (b) What would the pressure and temperature have been if the process had been isentropic? The adiabatic exponent k for carbon dioxide is 1.28.*

BG

(*a*) Isothermal: $n = 1$; Eq. 2.6: $p\,v$ = constant; 15(10/mass) = p_2(2/mass); p_2 = 75.0 psia ◄

(*b*) Isentropic: $n = k = 1.28$; Eq. 2.6: $pv^n = pv^{1.28}$ = constant; $15(10/\text{mass})^{1.28} = p_2(2/\text{mass})^{1.28}$

$p_2 = 15(10/2)^{1.28}$ = 117.7 psia ◄

Eq. 2.7: $T_2 = T_1(p_2/p_1)^{0.28/1.28} = (460 + 50)(117.7/15)^{0.219}$ = 800°R = 340°F ◄

2.18 *(a) If 350 L of carbon dioxide at 20°C and 120 kN/m² abs is compressed isothermally to 50 L, what is the resulting pressure? (b) What would the pressure and temperature have been if the process had been isentropic? The isentropic exponent k for carbon dioxide is 1.28.*

SI

(*a*) Isothermal, Eq. 2.6: $p\,v$ = constant; 120(0.35/mass) = p_2(0.05/mass); p_2 = 840 kN/m² abs ◄

(*b*) Isentropic: $n = k = 1.28$; Eq. 2.6: $pv^n = pv^{1.28}$ = constant;

$120(0.35/\text{mass})^{1.28} = p_2(0.05/\text{mass})^{1.28}$; $p_2 = 120(0.35/0.05)^{1.28}$ = 1448 kN/m² abs ◄

Eq. 2.7: $T_2 = (273 + 20)(1448/120)^{0.28/1.28}$ = 505 K = 232°C ◄

2.19 *Helium at 180 kN/m² abs and 20°C is isentropically compressed to one-fifth of its original volume. What is its final pressure?*

SI

Isentropic process: $n = k$; Table A.5 for helium: R = 2077 N·m/(kg·K), k = 1.66

Eq. 2.4: $v = \dfrac{RT}{p} = \dfrac{2077(273+20)}{180(1000)}$ = 3.38 m³/kg

Eq. 2.6: $180(3.38)^{1.66} = p_2(3.38/5)^{1.66}$; p_2 = 2600 kPa abs ◄

Sec. 2.11: Viscosity – ***Exercises*** *(12)*

2.11.1 *At 60°F what is the kinematic viscosity of the gasoline in Fig. A.2, the specific gravity of which is 0.680? Give the answer in both BG and SI units.*

B

Fig. A.2: 4.8×10^{-6} ft²/sec ◄

Fig. A.2: 4.5×10^{-7} m²/s = 0.0045 cm²/s = 0.45 centistokes ◄

2.11.2 *To what temperature must the fuel oil with the higher specific gravity in Fig. A.2 be heated in order that its kinematic viscosity may be reduced to three times that of water at 40°F?*

BG

Table A.1 for water at 40°F: kinematic viscosity $\nu = 16.64 \times 10^{-6}$ ft²/sec.

Fuel oil: $\nu = 3(16.64 \times 10^{-6}) = 4.99 \times 10^{-5}$ ft²/sec.

Fig. A.2 for fuel oil ($s = 0.968$) with $\nu = 4.99 \times 10^{-5}$ ft²/sec: $T = 375$°F ◄

2.11.3 *Compare the ratio of the absolute viscosities of air and water at 70°F with the ratio of their kinematic viscosities at the same temperature and at 14.7 psia.*

BG

Table A.1 for water at 70°F and 14.7 psia: $\mu = 20.50 \times 10^{-6}$ lb·sec/ft², $\nu = 10.59 \times 10^{-6}$ ft²/sec

Table A.2 for air at 70°F and 14.7 psia: $\mu = 0.382 \times 10^{-6}$ lb·sec/ft², $\nu = 0.164 \times 10^{-3}$ ft²/sec

Absolute ratio = μ_{air}/μ_{water} = 0.382/20.50 = 1:53.7 ◄

Kinematic ratio = ν_{air}/ν_{water} = 164/10.59 = 15.5:1 ◄

2.11.4 *A flat plate 200 mm × 750 mm slides on oil (μ = 0.85 N·s/m²) over a large plane surface (Fig. X2.11.4). What force F is required to drag the plate at a velocity v of 1.2 m/s, if the thickness t of the separating oil film is 0.6 mm?*

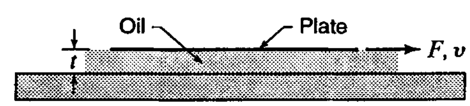


Figure X2.11.4

SI

$$\text{Eq. 2.9:}\quad \tau = \mu\frac{dv}{dy} = 0.85\frac{1.2}{0.000\,6} = 1700\ \text{N/m}^2$$

From Eq. 2.9: $F = \tau A = 1700(0.20 \times 0.75) = 255$ N ◄

2.11.5 *Refer to Fig. X2.11.4. A flat plate 2 ft × 3 ft slides on oil (μ = 0.024 lb·sec/ft²) over a large plane surface. What force F is required to drag the plate at a velocity v of 4 fps, if the thickness t of the separating oil film is 0.025 in?*

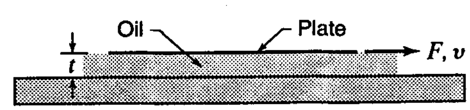


Figure X2.11.4

BG

$$\text{Eq. 2.9:}\quad \tau = \mu\frac{dV}{dy} = 0.024\frac{4}{0.025/12} = 46.1\ \text{lb/ft}^2$$

From Eq. 2.9: $F = \tau A = (46.1\ \text{lb/ft}^2)(2\ \text{ft} \times 3\ \text{ft}) = 276$ lb ◄

2.11.6 *A liquid has an absolute viscosity of 3.2×10^{-4} lb·sec/ft². It weighs 56 lb/ft³. What are its absolute and kinematic viscosities in SI units?*

B

$$\mu = 3.2 \times 10^{-4}\frac{\text{lb·sec}}{\text{ft}^2}\left(\frac{\text{ft}^2}{144\ \text{in}^2}\right)\frac{6.895\ \text{kN/m}^2}{\text{lb/in}^2} = 15.32 \times 10^{-6}\ \text{kN·s/m}^2$$

$$= 15.32\ \text{mN·s/m}^2 = 15.32\ \text{centipoise}$$ ◄

$$\text{Eq. 2.11:}\quad \nu = \frac{\mu}{\rho} = \frac{15.32\ \text{mN·s/m}^2}{(56/62.4)\text{g/cm}^3} = \frac{15.32\ \text{m(kg·m/s}^2\text{)s/m}^2}{897\ \text{kg/m}^3}$$

$$= 17.07 \times 10^{-6}\ \text{m}^2\text{/s} = 0.1707\ \text{stokes}$$ ◄

2.11.7 *(a) What is the ratio of the absolute viscosity of water at a temperature of 70°F to that of water at 200°F? (b) What is the ratio of the absolute viscosity of the crude oil in Fig. A.1 (s = 0.925) to that of the gasoline (s = 0.680), both being at a temperature of 60°F? (c) In cooling from 300 to 80°F, what is the ratio of the change of the absolute viscosity of the SAE 30 Western oil to that of the SAE 30 Eastern oil? Refer to Appendix A.*

B

(*a*) Table A.1: $20.50 \times 10^{-6}/(6.37 \times 10^{-6}) = 3.21$ ◄

(*b*) Fig. A.1: $9.0 \times 10^{-2}/(3.1 \times 10^{-4}) = 290$ ◄

(*c*) Fig. A.1: $\dfrac{3.0 \times 10^{-1} - 3.2 \times 10^{-3}}{1.7 \times 10^{-1} - 3.4 \times 10^{-3}} = 1.782$ ◄

Note: Readings from the figure may vary somewhat.

2.11.8 *A space 16 mm wide between two large plane surfaces is filled with SAE 30 Western lubricating oil at 35°C (Fig. X2.11.8). What force F is required to drag a very thin plate of 0.4-m² area between the surfaces at a speed v = 0.25 m/s (a) if this plate is equally spaced between the two surfaces? (b) If t = 5 mm? Refer to Appendix A.*

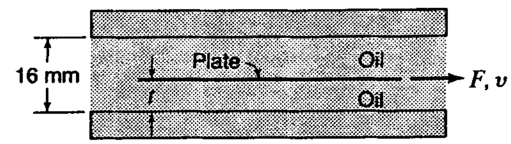


Figure X2.11.8

SI

Fig. A.1 for SAE Western lubricating oil at 35°C:

$\mu = 0.18 \text{ N·s/m}^2$

(*a*) Eq. 2.9: $\tau = 0.18\left(\dfrac{0.25}{8/1000}\right) = 5.63 \text{ N/m}^2$; From Eq. 2.9: Force = 5.63(2)0.4 = 4.50 N ◄

(*b*) Eq. 2.9: $\tau_1 = 0.18\left(\dfrac{0.25}{5/1000}\right) = 9.00 \text{ N/m}^2$; $\tau_2 = 0.18\left(\dfrac{0.25}{11/1000}\right) = 4.09 \text{ N/m}^2$

From Eq. 2.9: $F_1 = \tau_1 A = 9.00(0.4) = 3.60$ N ; $F_2 = \tau_2 A = 4.09(0.4) = 1.636$ N

∴ Force $= F_1 + F_2 = 5.24$ N ◄

2.11.9 *A journal bearing consists of an 80-mm shaft in an 80.4-mm sleeve 120 mm long, the clearance space (assumed to be uniform) being filled with SAE 30 Western lubricating oil at 40°C (Fig. X2.11.9). Calculate the rate at which heat is generated at the bearing when the shaft turns at 150 rpm. Express the answer in kN·m/s, J/s, Btu/hr, ft·lb/sec, and hp. Refer to Appendix A.*

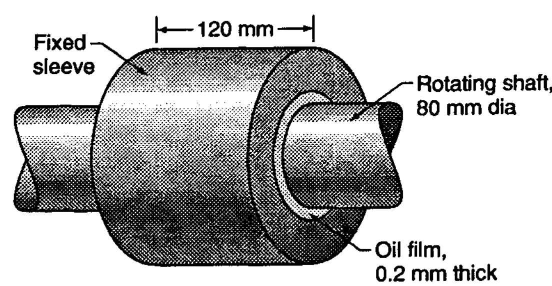


Figure X2.11.9

B

Fig. A.1: $\mu = 1.1 \times 10^{-1} = 0.11 \text{ N·s/m}^2$

Eq. 2.9: $\tau = \mu\dfrac{dv}{dy} = \mu\dfrac{r\omega}{\Delta D/2}$

$= 0.11\dfrac{40(150 \times 2\pi/60)}{0.4/2} = 346 \text{ N/m}^2$

Torque $= (\tau A)r = 346(\pi \times 0.08 \times 0.12)0.04 = 0.417$ N·m

Rate of heat generation $= T\omega = 0.417\left(\dfrac{2\pi \times 150}{60}\right) = 6.55$ N·m/s

= 6.55 J/s = 0.006 55 kN·m/s = 22.4 Btu/hr = 0.008 78 hp = 4.83 ft·lb/sec ◄◄

(using conversion factors from inside the back cover).

2.11.10 *In using a rotating-cylinder viscometer, a bottom correction must be applied to account for the drag on the flat bottom of the inner cylinder. Calculate the theoretical amount of this torque correction, neglecting centrifugal effects, for a cylinder of diameter d, rotated at a constant angular velocity ω, in a liquid of absolute viscosity μ, with a clearance Δh between the bottom of the inner cylinder and the floor of the outer one.*

N

Let r = variable radius. $dA = 2\pi r\,dr,\ \tau = \mu r\omega/\Delta h$

$$\text{Torque} = \int r \times \tau dA = \frac{2\pi\mu\omega}{\Delta h}\int_0^{d/2} r^3 dr = \frac{\pi\mu\omega}{32\Delta h}d^4 \quad ◀$$

2.11.11 *Assuming a velocity distribution as shown in Fig. X2.11.11, which is a parabola having its vertex 12 in from the boundary, calculate the velocity gradients for y = 0, 3, 6, 9, and 12 in. Also calculate the shear stresses in lb/ft² at these points if the fluid's absolute viscosity is 600 cP.*

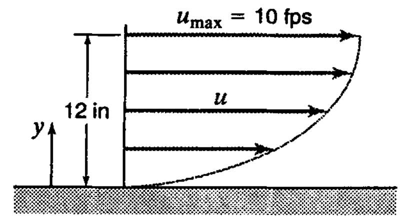


Figure X2.11.11

BG

Back cover: μ = 600 cP = 6 P = 0.6 N·s/m²

μ = 0.6(0.020 885) = 0.012 53 lb·sec/ft². Parabola: $Y = aX^2$

For u in in/sec and y in inches: $120 - u = a(12 - y)^2$

$u = 0$ at $y = 0 \rightarrow a = 120/12^2 = 5/6$; $u = 120 - (5/6)(12 - y)^2$; $du/dy = (5/3)(12 - y)$

Eq. 2.9: $\tau = 0.012\,53\ du/dy$

y (in)	0	3	6	9	12	
du/dy (sec⁻¹)	20	15	10	5	0	◀◀
τ (lb/ft²)	0.251	0.1880	0.1253	0.0627	0	◀◀

2.11.12 *Air at 50 psia and 60°F is flowing through a pipe. Table A.2 indicates that its kinetic viscosity ν is 0.158 × 10⁻³ ft²/sec. (a) Why is this ν value incorrect? (b) What is the correct value?*

BG

(*a*) From Sec. 2.11: This ν is incorrect because it varies strongly with pressure (due to ρ changes). ◀

Table A.2 is for p = 14.7 psia, our sample is at p = 50 psia.

(*b*) Sec. 2.11: μ is virtually pressure independent. ∴ $\mu_{50} = \mu_{14.7}$.

Table A.2 for air at 60°F and 14.7 psia: $\mu_{14.7} = 0.374 \times 10^{-6}$ lb·sec/ft², $\rho_{14.7}$ = 0.002 374 slug/ft³

Table A.5 for air: R = 1715 ft²/(sec²·°R).

$$\text{From Eq. 2.4: } \rho_{50} = \frac{p_{50}}{RT} = \frac{50 \times 144\ \text{lb/ft}^2}{[1715\ \text{ft}^2/(\text{sec}^2\cdot{}^\circ\text{R})](460+60)^\circ\text{R}} = 0.008\,07\ \text{lb}\cdot\text{sec}^2/\text{ft}^4 \text{ (or slug/ft}^3)$$

$$\text{Eq. 2.11: } \nu_{50} = \frac{\mu_{50}}{\rho_{50}} = \frac{\mu_{14.7}}{\rho_{50}} = \frac{0.374\times10^{-6}\ \text{lb}\cdot\text{sec/ft}^2}{0.008\,07\ \text{lb}\cdot\text{sec}^2/\text{ft}^4} = 46.3 \times 10^{-6}\ \text{ft}^2/\text{sec} \quad ◀$$

Sec. 2.11: Viscosity -- *Problems* 2.20–2.28

2.20 *The absolute viscosity of a certain gas is 0.0234 cP while its kinematic viscosity is 181 cSt, both measured at 1013 mb abs and 100°C. Calculate its approximate molar mass and suggest what gas it may be.*

SI

$$\text{Eq. 2.11: } \rho = \frac{\mu}{\nu} = \frac{0.0234\ \text{cP}}{181\ \text{cSt}} = \frac{2.34\times10^{-5}\ \text{N}\cdot\text{s/m}^2}{181\times10^{-6}\ \text{m}^2/\text{s}} = 0.1293\ \text{kg/m}^3$$

$$\text{From Eqs. 2.1 and 2.5: } R = \frac{p}{\rho T} = \frac{101.3\ \text{kN/m}^3}{(0.1298\ \text{kg/m}^3)(273+100)\text{K}} = 2100\ \text{N}\cdot\text{m/(kg}\cdot\text{K)}$$

Sec. 2.7: $M = R_0/R$ = 8312/2100 = 3.96 ◀ ; Table A.5: The gas may be helium ◀

2.21 *A hydraulic lift of the type commonly used for greasing automobiles consists of a 10.000-in-diameter ram which slides in a 10.006-in-diameter cylinder (Fig. P2.21), the annular space being filled with oil having a kinematic viscosity of 0.0038 ft²/sec and specific gravity of 0.83. If the rate of travel of the ram v is 0.5 fps, find the frictional resistance F when 6 ft of the ram is engaged in the cylinder.*

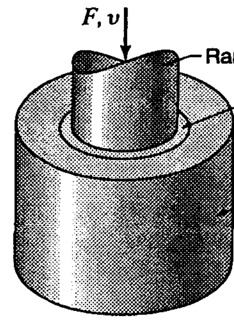


Figure P2.21

BG

$\gamma = 0.83 \times 62.4 = 51.8\ \text{lb/ft}^3,\ \rho = \dfrac{51.8}{32.2} = 1.608\ \text{slug/ft}^3$

Eq. 2.11: $\mu = \nu\rho = 0.0038(1.608) = 0.006\ 11\ \text{lb·sec/ft}^2$

Eq. 2.9: $\tau = \mu\dfrac{dv}{dy} = 0.006\ 11\left(\dfrac{0.5}{0.003/12}\right) = 12.22\ \text{lb/ft}^2$

From Eq. 2.9: Friction force $F = \tau A = 12.22(6 \times \pi \times 10/12) = 192.0\ \text{lb}$ ◄

2.22 *A hydraulic lift of the type commonly used for greasing automobiles consists of a 280.00-mm-diameter ram which slides in a 280.18-mm-diameter cylinder (similar to Fig. P2.21), the annular space being filled with oil having a kinematic viscosity of 0.000 42 m²/s and specific gravity of 0.86. If the rate of travel of the ram is 0.22 m/s, find the frictional resistance when 2 m of the ram is engaged in the cylinder.*

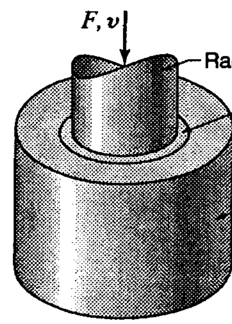


Figure P2.21

SI

$\gamma = 0.86(9810) = 8440\ \text{N/m}^3,\ \rho = 8440/9.81 = 860\ \text{kg/m}^3$

Eq. 2.11: $\mu = \nu\rho = 0.000\ 42(860) = 0.361\ \text{N/m}^2$

Eq. 2.9: $\tau = \mu\dfrac{du}{dy} = 0.361\left(\dfrac{0.22}{0.09/1000}\right) = 883\ \text{N/m}^2$

From Eq. 2.9: Friction force $= \tau A = 883(2\pi 280/1000) = 1553\ \text{N}$ ◄

2.23 *A journal bearing consists of a 8.00-in shaft in a 8.01-in sleeve 10 in long, the clearance space (assumed to be uniform) being filled with SAE 30 Eastern lubricating oil at 100°F. Calculate the rate at which heat is generated at the bearing when the shaft turns at 100 rpm. Refer to Appendix A. Express the answer in Btu/hr.*

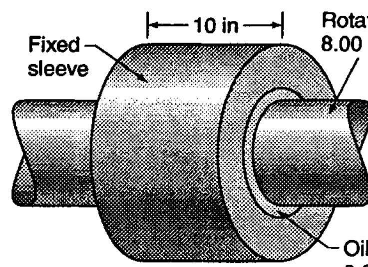


Figure P2.23

BG

Fig. A.1 at 100°F: $\mu = 0.0021\ \text{lb·sec/ft}^2$

$u = \pi\left(\dfrac{8}{12}\right)\left(\dfrac{100}{60}\right) = 3.49\ \text{fps}$

Eq. 2.9: $\tau = (0.0021)\dfrac{3.49}{0.005/12} = 17.59\ \text{lb/ft}^2$

From Eq. 2.9: Friction $= \tau A = 17.59\left(\dfrac{10}{12}\pi\dfrac{8}{12}\right) = 30.7\ \text{lb}$

Per Sample Prob. 2.9: Energy loss rate $= T\omega = Fu = 30.7(3.49) = 107.2\ \text{ft·lb/sec}$

Heat generation rate $= \dfrac{107.2\ \text{ft·lb}}{\text{sec}}\left(\dfrac{\text{Btu}}{778\ \text{ft·lb}}\right)\dfrac{3600\ \text{sec}}{\text{hr}} = 496\ \text{Btu/hr}$ ◄

2.24 *Repeat Prob. 2.23 for the case where the sleeve has a diameter of 8.50 in. Compute as accurately as possible the velocity gradient in the fluid at the shaft and sleeve.*

Prob. 2.23: A journal bearing consists of a 8.00-in shaft in a sleeve 10 in long, the clearance space (assumed to be uniform) being filled with SAE 30 Eastern lubricating oil at 100°F. Calculate the rate at which heat is generated at the bearing when the shaft turns at 100 rpm. Refer to Appendix A. Express the answer in Btu/hr.

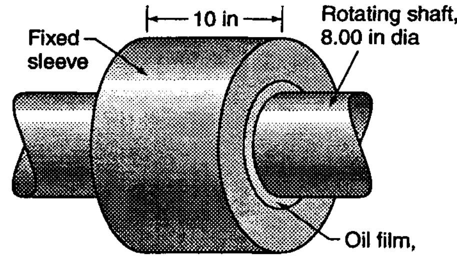


Figure P2.23

BG

Fig. A.1 at 100°F: $\mu = 0.0021$ lb·sec/ft^2

Fig. 2.6*a*: $u_1 = \pi\left(\frac{8.0}{12}\right)\left(\frac{100}{60}\right) = 3.49$ fps

8.00 in = 0.667 ft diameter = 0.333 ft radius ; 8.50 in = 0.708 ft diameter = 0.354 ft radius

$$T = \tau A r = \tau\left(2\pi r\frac{10}{12}\right)r, \quad \text{i.e.,} \quad \tau = \frac{T}{5.24r^2}\ \text{lb/ft}^2$$

$$\text{Eq. 2.9:} \quad \tau = 0.0021\frac{du}{dy} = -0.0021\frac{du}{dr} = \frac{T}{5.24r^2}$$

$$-du = \frac{90.9T}{r^2}dr\ ; \quad -\int_{u_1}^{0} du = 90.9T\int_{0.333}^{0.354} r^{-2}dr = 90.9T\left[\frac{r^{-1}}{-1}\right]_{0.333}^{0.354}$$

$$0 + 3.49 = 90.9T\left(-\frac{1}{0.354} + \frac{1}{0.333}\right) = 90.9T(0.1765); \quad T = \frac{3.49}{16.05} = 0.217\ \text{ft·lb}$$

$$\tau_1 = \frac{0.217}{5.24\times 0.333^2} = 0.374\ \text{lb/ft}^2 = 0.0021\left[\frac{du}{dy}\right]_1 ; \quad \left[\frac{du}{dy}\right]_1 = 178.0\ \text{fps/ft} \quad ◄$$

$$\left[\frac{du}{dy}\right]_2 = 178.0\left[\frac{0.333}{0.354}\right]^2 = 157.7\ \text{fps/ft} \quad ◄$$

Rate of energy loss = $T\omega$ = (0.217) 100(2π/60) = 2.28 ft·lb/sec

Rate of heat generation = $2.28\frac{3600}{778}$ = 10.54 Btu/hr ◄

2.25 *A disk spins within an oil-filled enclosure, having 2.4-mm clearance from flat surfaces each side of the disk. The disk surface extends from radius 12 to 86 mm. What torque is required to drive the disk at 660 rpm if the oil's absolute viscosity is 0.12 N·s/m^2?*

SI

Let r = variable radius. $dA = 2\pi r\,dr$, $\tau = \mu r\omega/\Delta h$

$$\text{Torque} = 2\int r \times \tau dA = \frac{4\pi\mu\omega}{\Delta h}\int_{r_1}^{r_2} r^3 dr = \frac{4\pi\mu\omega}{\Delta h}\frac{(r_2^4 - r_1^4)}{4} = \frac{\pi\mu\omega}{\Delta h}(r_2^4 - r_1^4)$$

$$= \frac{\pi \times 0.12\frac{\text{N·s}}{\text{m}^2}\left(660 \times 2\pi\frac{\text{rev/min}}{60\ \text{s/min}}\right)\frac{(86^4 - 12^4)\text{mm}^4}{(10^3\ \text{mm/m})^4}}{\frac{2.4\ \text{mm}}{1000\ \text{mm/m}}} = 0.594\ \text{N·m} \quad ◄$$

2.26 *It is desired to apply the general case of Sample Prob. 2.9 to the extreme cases of a journal bearing (α = 0) and an end bearing (α = 90°). But when α = 0, $r = \tan\alpha = 0$, so T = 0; when α = 90°, contact area = ∞ due to b, so T = ∞. Therefore, devise an alternative general derivation which will also provide solutions to these two extreme cases.*

Sample Prob. 2.9: Oil of absolute viscosity μ fills the gap of thickness Y. Obtain an expression for the torque T required to rotate the truncated cone at constant speed ω rad/sec.

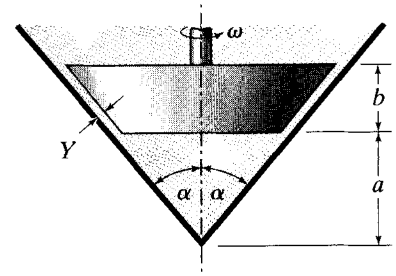
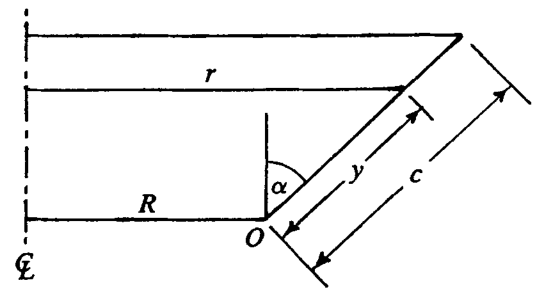


Figure S2.9

N

(*i*) Prevent r from going to zero by prescribing $r_{\min} = R$ (see sketch);

(*ii*) Prevent contact area from becoming infinite by prescribing the sloping dimension c ($=b/\cos\alpha$).



Define y to be the distance along the sloping surface to radius r, with y = 0 at point O (and dispense with a and b). Then $r = R + y\sin\alpha$

with (per Fig. S2.9) rotation velocity

$$U = \omega r = \omega(R + y\sin\alpha)$$

Eq. 2.9: $\tau = \mu\dfrac{du}{dy} = \mu\dfrac{U}{Y} = \dfrac{\mu\omega}{Y}(R + y\sin\alpha)$; $dA = 2\pi r\,dy = 2\pi(R + y\sin\alpha)dy$; $dF + \tau dA$

$dT = r\,dF = r\tau dA = (2\pi\mu\omega/Y)(R + y\sin\alpha)^3 dy$; $T = \int dT = (2\pi\mu\omega/Y)\int_0^c (R + y\sin\alpha)^3 dy$

Expanding and integrating $T = \dfrac{2\pi\mu\omega}{Y}\left(R^3c + \dfrac{3}{2}R^2c^2\sin\alpha + Rc^3\sin^2\alpha + \dfrac{c^4}{4}\sin^3\alpha\right)$ ◄

(*a*) Journal bearing: $\alpha = 0$, $\sin\alpha = 0$, so $T = \dfrac{2\pi\mu\omega}{Y}R^3c$ ◄

(*b*) Flat end bearing: $\alpha = 90°$, $R = 0$, so $\sin\alpha = 1$, let $D = 2c$,

then $T = \dfrac{2\pi\mu\omega}{Y}\dfrac{(D/2)^4}{4} = \dfrac{\pi\mu\omega}{32Y}D^4$ ◄ (This agrees with the solution to Exer. 2.11.10.)

2.27 *Some free air at standard sea-level pressure (101.33 kPa abs) and 20°C has been compressed. Its pressure is now 200 kPa abs and its temperature is 20°C. Table A.2 indicates that its kinetic viscosity ν is 15 × 10^{-6} m²/s. (a) Why is this ν incorrect? (b) What is the correct value?*

SI

(*a*) From Sec. 2.11: This ν is incorrect because it varies strongly with pressure (due to ρ changes). ◄

Table A.2 is for p_1 = 101.33 kPa abs, but we need a value for p_2 = 200 kPa abs.

(*b*) Sec. 2.11: μ is virtually independent of pressure. ∴ $\mu_2 = \mu_1$.

Table A.2 for air at 20°C and 101.33 kPa abs: $\mu_1 = 18.1 \times 10^{-6}$ N·s/m², ρ_1 = 1.205 kg/m³.

Table A.5 for air: R = 287 m²/(s²·K).

From Eq. 2.4: $\rho_2 = \dfrac{p_2}{RT_2} = \dfrac{200 \times 1000 \text{ N/m}^2}{[287 \text{ m}^2/(\text{s}^2\cdot\text{K})](273 + 20)\text{K}} = 2.38 \text{ N}\cdot\text{s}^2/\text{m}^4$ (or kg/m³)

Eq. 2.11: $\nu_2 = \dfrac{\mu_2}{\rho_2} = \dfrac{\mu_1}{\rho_2} = \dfrac{18.1 \times 10^{-6} \text{ N}\cdot\text{s/m}^2}{2.38 \text{ N}\cdot\text{s}^2/\text{m}^4} = 7.61 \times 10^{-6}$ m²/s ◄

2.28 *Some free air at standard sea-level pressure (101.33 kPa abs) and 20°C has been compressed isentropically. Its pressure is now 194.5 kPa abs and its temperature is 80°C. Table A.2 indicates that its kinetic viscosity ν is 20.9 × 10^{-6} m²/s. (a) Why is this ν incorrect? (b) What is the correct value? (c) What would the correct value be if the compression were isothermal instead?*

SI

(*a*) From Sec. 2.11: This ν is incorrect because it varies strongly with pressure (due to ρ changes). ◄

Table A.2 is for p_1 = 101.33 kPa abs, but we need a value for p_2 = 194.5 kPa abs.

(*b*) Table A.2 for air at 20°C and 101.33 kPa abs: $\mu_1 = 18.1 \times 10^{-6}\ \text{N·s/m}^2$, $\rho_1 = 1.205\ \text{kg/m}^3$.

Sec. 2.11: μ is virtually independent of pressure. ∴ at 80°C (Table A.2), $\mu_2 = 20.9 \times 10^{-6}\ \text{N·s/m}^2$

Table A.5 for air: $R = 287\ \text{m}^2/(\text{s}^2\text{·K})$.

From Eq. 2.4: $\rho_2 = \dfrac{p_2}{RT_2} = \dfrac{194.5 \times 1000\ \text{N/m}^2}{[287\ \text{m}^2/(\text{s}^2\text{·K})](273 + 80)\text{K}} = 1.920\ \text{N·s}^2/\text{m}^4$ (or kg/m³)

Eq. 2.11: $\nu_2 = \dfrac{\mu_2}{\rho_2} = \dfrac{20.9 \times 10^{-6}\ \text{N·s/m}^2}{1.920\ \text{N·s}^2/\text{m}^4} = 10.89 \times 10^{-6}\ \text{m}^2/\text{s}$ ◄

(*c*) $T_2 = T_1 = 20°\text{C}$. μ is virtually independent of pressure, so $\mu_2 = \mu_1 = 18.1 \times 10^{-6}\ \text{N·s/m}^2$

From Eq. 2.4: $\rho_2 = \dfrac{p_2}{RT_2} = \dfrac{194.5 \times 1000\ \text{N/m}^2}{[287\ \text{m}^2/(\text{s}^2\text{·K})](273 + 20)\text{K}} = 2.31\ \text{N·s}^2/\text{m}^4$ (or kg/m³)

Eq. 2.11: $\nu_2 = \dfrac{\mu_2}{\rho_2} = \dfrac{\mu_1}{\rho_2} = \dfrac{18.1 \times 10^{-6}\ \text{N·s/m}^2}{2.31\ \text{N·s}^2/\text{m}^4} = 7.83 \times 10^{-6}\ \text{m}^2/\text{s}$ ◄

Sec. 2.12: Surface Tension -- *Exercises (5)*

2.12.1 *Tap water at 68°F stands in a glass tube of 0.32-in diameter at a height of 4.50 in. What is the true static height?*

BG

Fig. 2.7: Capillary rise ≈ 0.058 in. True static height ≈ 4.50 − 0.058, say 4.44 in ◄

2.12.2 *Distilled water at 20°C stands in a glass tube of 6.0-mm diameter at a height of 18.0 mm. What is the true static height?*

SI

20°C = 68°F; tube dia = 6.0 mm = 0.236 in. Fig. 2.7: Capillary rise ≈ 0.162 in = 4.11 mm

True static height ≈ 18.00 − 4.11 = 13.89 mm ◄

2.12.3 *Use Eq. (2.12) to compute the capillary depression of mercury at 68°F (θ = 140°) to be expected in a 0.05-in-diameter tube.*

BG

Table A.4 for mercury at 68°F: $s = 13.56$, $\sigma = 0.032$ lb/ft.

Eq. 2.12: $h = \dfrac{2\sigma\cos 140°}{\gamma r} = \dfrac{2(0.032)0.766}{13.56 \times 62.4(0.025/12)} = 0.0278\ \text{ft} = 0.334\ \text{in}$ ◄

2.12.4 *Compute the capillary rise in mm of pure water at 10°C expected in an 0.8-mm-diameter tube.*

SI

Table A.1 at 10°C: $\sigma_{water} = 0.0742$ N/m, $\gamma = 9.804$ kN/m³

Eq. 2.12 with $\theta = 0$: $h = \dfrac{2\sigma}{\gamma r} = \dfrac{2(0.0742\ \text{N/m})}{(9804\ \text{N/m}^3)(0.0004\ \text{m})} = 0.0378\ \text{m} = 37.8\ \text{mm}$ ◄

2.12.5 *Use Eq. (2.12) to compute the capillary rise of water to be expected in an 0.28-in-diameter tube. Assume pure water at 68°F. Compare the result with Fig. 2.7.*

BG

Interpolating from Table A.1 for 68°F: σ = 0.004 99 lb/ft.

Eq. 2.12: $h = \dfrac{2\sigma}{\gamma r} = \dfrac{2(0.004\,99 \text{ lb/ft})(12 \text{ in/ft})}{(62.4 \text{ lb/ft}^3)(0.14/12 \text{ ft})} = 0.1645 \text{ in}$ ◄

Fig. 2.7 shows a capillary rise of ≈ 0.130 in ◄

Sec. 2.12: Surface Tension -- ***Problems*** *2.29−2.32*

2.29 *Pure water at 50°F stands in a glass tube of 0.04 in diameter at a height of 6.78 in. Compute the true static height.*

BG

Table A.1 at 50°F: γ = 62.41 lb/ft³, σ = 0.005 09 lb/ft

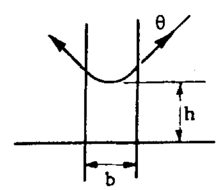
Eq. 2.12 with $\theta = 0°$: $h = \dfrac{2\sigma}{\gamma r} = \dfrac{2(0.005\,09 \text{ lb/ft})}{62.41 \text{ lb/ft}^3(0.02/12 \text{ ft})} = 0.0979 \text{ ft} = 1.174 \text{ in}$

True static height = 6.78 in − 1.174 in = 5.61 in ◄

2.30 *(a) Derive an expression for capillary rise (or depression) between two vertical parallel plates. (b) How much would you expect 10°C water to rise (in mm) if the clean glass plates are separated by 1.2 mm?*

SI

(*a*) $(\cos\theta)2\sigma Z = \gamma bhZ$, so $h = 2\sigma\cos\theta/(b\gamma)$ ◄



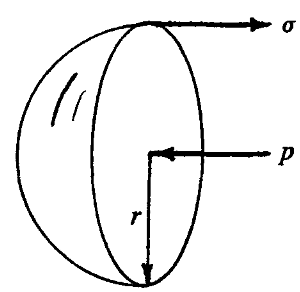
(*b*) For water, $\theta = 0°$. Table A.1 at 10°C: γ = 9804 N/m³, σ = 0.0742 N/m

$\therefore\ h = \dfrac{2(0.0742 \text{ N/m})\cos 0°}{(1.2/1000 \text{ m})(9804 \text{ N/m}^3)} = 0.012\,61 \text{ m} = 12.61 \text{ mm}$ ◄

2.31 *By how much does the pressure inside a 2-mm-diameter air bubble in 15°C water exceed the pressure in the surrounding water?*

SI

Table A.1 at 15°C: σ = 0.0735 N/m.



Cut the bubble on a plane through its center, consider force equilibrium.

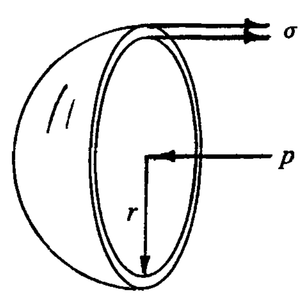
$\sigma \times$ circumference = $p \times$ area; $\sigma(2\pi r) = p(\pi r^2)$

$p = \dfrac{2\pi r\sigma}{\pi r^2} = \dfrac{2\sigma}{r} = \dfrac{2(0.0735 \text{ N/m})}{0.001 \text{ m}} = 147 \text{ N/m}^2 = 147 \text{ Pa}$ ◄

2.32 *Determine the excess pressure inside an 0.5-in-diameter soap bubble floating in air, given the surface tension of the soap solution is 0.0035 lb/ft.*

SI

Cut the bubble on a plane through its center, and consider force equilibrium, noting that surface tension acts on both the inside and outside surfaces.



$\sigma \times 2 \times$ circumference = $p \times$ area; $2\sigma(2\pi r) = p(\pi r^2)$

$p = \dfrac{4\pi r\sigma}{\pi r^2} = \dfrac{4\sigma}{r} = \dfrac{4(0.0035 \text{ lb/ft})}{0.25/12 \text{ ft}} = 0.672 \text{ lb/ft}^2$

p = 0.004 67 psi ◄

Sec. 2.13: Vapor Pressure of Liquids – *Exercises (2)*

2.13.1 *At what pressure in millibars absolute will 70°C water boil?*
SI

Table A.1 at 70°C: p_v = 31.16 kN/m² abs. Inside cover: 10 mb = 1 kN/m²

The water will boil at 31.16 kN/m² abs = 311.6 mb abs ◄

2.13.2 *At approximately what temperature will water boil in Mexico City (elevation 7400 ft)? Refer to Appendix A.*
BG

Table A.3, by interpolation: p_{at} = 11.21 psia at 7400 ft elevation

Table A.1, by interpolation: p_v water = 11.21 psia at about 198.6°F.

∴ Water boils at 198.6°F ◄

Sec. 2.13: Vapor Pressure of Liquids -- *Problems 2.33–2.34*

2.33 *Water at 170°F in a beaker is placed within an airtight container. Air is gradually pumped out of the container. What reduction below standard atmospheric pressure of 14.7 psia must be achieved before the water boils?*
BG

Table A.1 at 170°F: p_v = 5.99 psia

14.7 − 5.99 = 8.71 psi; the pressure must be reduced by 8.71 psi ◄

2.34 *At approximately what temperature will water boil on top of Mount Kilimanjaro (elevation 5895 m)? Refer to Appendix A.*
SI

Table A.3, by interpolation: p_{at} = 47.934 kPa abs at 5895 m elevation.

Table A.1, by interpolation: p_v of water = 47.934 kPa abs at about 80.26°C.

So water will boil there at about 80.3°C ◄

Chapter 3

Fluid Statics

PROBLEM SELECTION GUIDE

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
3.2	***Variation of Pressure in a Static Fluid***						
	X[1]3.2.1	BG	V Easy	V Short	1	3.2.2	
	3.2.2	SI	V Easy	V Short	1	3.2.1	
	3.2.3	BG	V Easy	V Short	1	P3.1	Uses Sec. 2.3
	3.2.4	BG	V Easy	V Short	1		
	P 3.1	SI	V Easy	V Short	1	3.2.3	Uses Sec. 2.3
	3.2	BG	Medium	Medium	1		
	3.3	BG	Hard	Medium	1		Differ'n, integr'n. Uses Secs 2.3 & 2.5
3.3	***Pressure Expressed in Height of Fluid***						
	X 3.3.1	SI	V Easy	V Short	2	3.3.2	
	3.3.2	BG	V Easy	V Short	2	3.3.1	
	3.3.3	SI	V Easy	V Short	1		
	P 3.4	BG	Easy	Short	1		Integration
	3.5	BG	Medium	Long	1		Trial & error; uses Secs. 2.3, 2.7, 2.12
3.4	***Absolute and Gage Pressures***						
	X 3.4.1	BG	Easy	V Short	1	3.4.2	
	3.4.2	SI	Easy	V Short	1	3.4.1	
	3.4.3	SI	Easy	Short	1	3.4.4-5	
	3.4.4	BG	Easy	Short	1	3.4.3-5	
	3.4.5	SI	Easy	Short	1	3.4.3-4	
	3.4.6	BG	Easy	Short	2		
	P 3.6	B	Easy	Medium	1		Unit conversions
	3.7	BG	Medium	Medium	1	3.8	Uses $pv = RT$ (Sec. 2.7)
	3.8	SI	Medium	Medium	2	3.7	Uses $pv = RT$ (Sec. 2.7)

/cont...

[1] For all Exercises (identified by "X"), answers are given in Appendix F of the textbook.
X = Exercise, P = (end-of-chapter) Problem, S = Sample Problem.

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
3.5	***Measurement of Pressure***						
	X 3.5.1	BG	V Easy	V Short	1		
	3.5.2	SI	V Easy	V Short	1		
	3.5.3	SI	V Easy	Short	1		Unit conversions
	3.5.4	BG	Easy	Short	1		
	3.5.5	BG	Easy	Short	1	3.5.6	Uses $pv = RT$ (Sec. 2.7)
	3.5.6	SI	Easy	Short	1	3.5.5	Uses $pv = RT$ (Sec. 2.7)
	3.5.7	BG	Easy	Short	2		Interpolation
	3.5.8	BG	Easy	Short	1		
	3.5.9	BG	Easy	Short	2		
	3.5.10	BG	Easy	Short	2	3.5.11	
	3.5.11	SI	Easy	Short	2	3.5.10	
	P 3.9	SI	Easy	Medium	2		
	3.10	N	Medium	Short	1		Symbols only
	3.11	SI	Medium	Medium	1		
	3.12	BG	Medium	Long	1		
3.7	***Center of Pressure***						
	X 3.7.1	N	Easy	V Short	1		
	3.7.2	N	Easy	V Short	1		
	3.7.3	N	Medium	Short	1		Optional integration
	3.7.4	N	Medium	Short	1		
	3.7.5	N	Medium	Medium	2		Integration
	3.7.6	SI	Easy	Short	1		
	3.7.7	SI	Easy	Short	1		
	3.7.8	BG	Easy	Short	2		
	3.7.9	SI	Easy	Short	2		Moments
	3.7.10	BG	Easy	Medium	1		Moments
	3.7.11	BG	Medium	Short	1		Moments
	3.7.12	SI	Easy	Medium	3		
	3.7.13	BG	Easy	Medium	1	3.7.14	
	3.7.14	SI	Easy	Medium	1	3.7.13	
	3.7.15	SI	Easy	Medium	1		
	3.7.16	BG	Easy	Medium	2		
	P 3.13	N	Medium	Medium	1		
	3.14	SI	Medium	Medium	1		
	3.15	BG	Medium	Medium	2		
	3.16	SI	Medium	Medium	1		Moments
	3.17	BG	Medium	Medium	2		Moments
	3.18	BG	Medium	Long	3		
	3.19	BG	Medium	Long	2		

/cont...

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
3.8	*Force on Curved Surface*						
	X 3.8.1	BG	V Easy	V Short	1		
	3.8.2	BG	Easy	Short	1	3.8.3	
	3.8.3	SI	Easy	Short	1	3.8.2	
	3.8.4	B	Easy	Short	1		Unit conversions
	3.8.5	BG	Medium	Medium	1		
	3.8.6	BG	Easy	Medium	4		
	3.8.7	BG	Medium	Medium	1	3.8.8	
	3.8.8	SI	Medium	Medium	1	3.8.7	
	3.8.9	SI	Medium	Medium	3	3.8.10	
	3.8.10	BG	Medium	Medium	3	3.8.9	
	P 3.20	BG	Medium	Medium	3		
	3.21	SI	Medium	Medium	3		Integration; moments
	3.22	SI	Medium	Long	1		Requires planimetry; moments or cutout
	3.23	BG	Medium	Long	1		Moments or cutout
3.9	*Buoyancy and Stability of Submerged and Floating Bodies*						
	X 3.9.1	BG	V Easy	V Short	1		
	3.9.2	SI	V Easy	V Short	1		
	3.9.3	BG	Easy	Short	1		
	3.9.4	SI	Easy	Short	1		Simultaneous equations
	3.9.5	BG	Easy	Short	1		Simultaneous equations
	3.9.6	BG	Easy	Short	1		
	3.9.7	SI	Easy	Short	2		
	3.9.8	SI	Easy	Medium	1		Differentiation
	3.9.9	SI	Easy	Medium	2		
	3.9.10	BG	Medium	Medium	2		Moments
	3.9.11	BG	Medium	Medium	3	3.9.12	Moments
	3.9.12	SI	Medium	Medium	3	3.9.11	Moments
	P 3.24	SI	Medium	Medium	1		
	3.25	BG	Medium	Medium	1	3.26	Uses $pv = RT$ (Sec. 2.7)
	3.26	BG	Medium	Medium	1	3.25	Uses $pv = RT$ (Sec. 2.7)
	3.27	BG	Medium	Medium	1		Moments
	3.28	BG	Medium	Medium	2	3.29	Moments
	3.29	SI	Medium	Medium	2	3.28	Moments
	3.30	BG	Medium	Long	2		Moments
3.10	*Fluid Masses Subjected to Acceleration*						
	X 3.10.1	BG	Easy	Short	1	3.10.2	
	3.10.2	SI	Easy	Short	1	3.10.1	
	3.10.3	BG	Easy	Short	1	3.10.4	
	3.10.4	SI	Easy	Short	1	3.10.3	
	3.10.5	BG	Easy	Short	1	3.10.6	
	3.10.6	SI	Easy	Short	1	3.10.5	
	3.10.7	BG	Easy	Medium	1	3.10.8	Sketch
	3.10.8	SI	Easy	Medium	1	3.10.7	Sketch
	P 3.31	SI	Medium	Medium	1	S3.10	
	3.32	BG	Medium	Medium	1		

Chapter 3
FLUID STATICS

Sec 3.2: Variation of Pressure in a Static Fluid -- *Exercises (4)*

3.2.1 *Neglecting the pressure on the surface and the compressibility of water, what is the pressure in pounds per square inch on the ocean floor at a depth of 15,500 ft? The specific weight of ocean water under ordinary conditions is 64.0 lb/ft³.*

BG

Eq. 3.4: $p = \gamma h = 64.0(15{,}500)/144 = 6890$ psi ◄

3.2.2 *Neglecting the pressure on the surface and the compressibility of water, what is the pressure in kPa at a depth of an instrument 4600 m below the surface of the ocean? The specific weight of ocean water under ordinary conditions is 10.05 kN/m³.*

SI

Eq. 3.4: $p = \gamma h = 10.05(4600) = 46\ 700\ \text{kN/m}^2$ ◄

3.2.3 *A pressure gage at elevation 18.0 ft on the side of an industrial tank containing a liquid reads 11.4 psi. Another gauge at elevation 12.0 ft reads 13.7 psi. Compute the specific weight, density, and specific gravity of the liquid.*

BG

From Eq. 3.3: $\Delta p = \gamma(\Delta h)$; $(13.7 - 11.4)144 = \gamma(18 - 12)$; $\gamma = 55.2\ \text{lb/ft}^3$ ◄

Eq. 2.1: $\rho = \dfrac{\gamma}{g} = \dfrac{55.2}{32.2} = 1.714\ \text{slugs/ft}^3$ ◄ $s\ \dfrac{55.2}{62.4} = 0.885$ ◄

3.2.4 *Where an underground oil pipeline crosses under a stream in a gully, it is 68 ft deeper than on either side. When the oil (s = 0.88) is not flowing, what is the oil pressure in the line under the stream, if it is 32 psi at each side of the gully?*

BG

$p_s = 32$ psi. Using Eq. 3.4: $p_b = p_s + \gamma_o h = p_s + s\gamma_w h = 32.0 + 0.88(62.4)68/144 = 57.9$ psi ◄

Sec. 3.2: Variation of Pressure in a Static Fluid -- *Problems 3.1–3.3*

3.1 *A pressure gage at elevation 4.8 m on the side of a storage tank containing oil reads 34.7 kPa. Another gage at elevation 2.2 m reads 57.5 kPa. Compute the specific weight, density, and specific gravity of the liquid.*

SI

From Eq. 3.3: $\gamma = \dfrac{\Delta p}{\Delta h} = \dfrac{(57.5 - 34.7\ \text{kN/m}^2)}{(4.8 - 2.2\ \text{m})} = 8.77\ \text{kN/m}^3$ ◄

Eq. 2.1: $\rho = \dfrac{\gamma}{g} = \dfrac{8770\ \text{N/m}^3}{9.81\ \text{m/s}^2} = \dfrac{8770\ (\text{kg}\cdot\text{m/s}^2)/\text{m}^3}{9.81\ \text{m/s}^2} = 792\ \text{kg/m}^3$ ◄

Sec. 2.3: $s = \dfrac{\gamma}{\gamma_w} = \dfrac{8.77}{9.81} = 0.894$ ◄

3.2 *On a certain day the barometric pressure at sea level is 30.0 inHg and the temperature is 60°F. The pressure gage on an airplane flying overhead indicates that the atmospheric pressure at that point is 9.7 psia and that the temperature is 42°F. Calculate as accurately as you can the height of the airplane above sea level. Assume a linear decrease of temperature with elevation.*

BG

At sea level: p_o = 30.0 inHg(14.696 psia/29.92 inHg) = 14.74 psia

Per Sample Prob. 3.1(*d*) with linear lapse rate *b*: $T = a + bz$ and so

$$\frac{p_z}{p_o} = \left(\frac{a+bz}{a}\right)^{-g/Rb} = \left(\frac{T}{T_o}\right)^{-g/Rb} ; \quad \frac{9.7}{14.74} = \left(\frac{42+460}{60+460}\right)^{-g/Rb} ; \quad -g/Rb = \log 0.658 / \log 0.965 = 11.87$$

Table A.5 for air: $R = 1715$ ft²/(sec²·°R). ∴ $-32.2/(1715b) = 11.87$; $b = -0.001\,582$°F/ft

$T = a + bz$: $42 + 460 = 60 + 460 - 0.001582z$; $z = 11{,}380$ ft ◄

3.3 *Repeat Exer. 3.2.1, but consider the effects of compressibility (E_v = 330,000 psi). Neglect changes in density caused by temperature variations. (Hint: As a starting point, express Eq. (2.3) in terms of γ and integrate to determine γ as a function of z.)*

Exer. 3.2.1: Find the pressure (psi) at a depth of 15,500 ft in the ocean (γ = 64.0 pcf at the surface). Neglect the pressure on the surface.

BG

From Eqs. 2.2 and 2.1: $v = 1/\rho = g/\gamma$

Differentiating, $\dfrac{dv}{d\gamma} = \dfrac{-g}{\gamma^2} = \dfrac{-v}{\gamma}$ or $\dfrac{dv}{v} = \dfrac{-d\gamma}{\gamma}$; Also, from Eq. 3.2: $dp = -\gamma\, dz$

∴ from Eq. 2.3*a*: $E_v = -\dfrac{dp}{dv/v} = -\dfrac{\gamma\, dz}{d\gamma/\gamma}$; $dz = -\dfrac{E_v\, d\gamma}{\gamma^2}$; integrate both sides

$$z = \frac{E_v}{\gamma} + c \; ; \quad \text{at } z = 0,\ \gamma = 64, \text{ so } c = -\frac{E_v}{64} \; ; \quad z = \frac{E_v}{\gamma} - \frac{E_v}{64} \; ; \quad \text{so } \gamma = \frac{64}{1 + \dfrac{64}{E_v}z}$$

But also (Eq. 3.2): $\gamma = -dp/dz$ (Note: p increases as elevation z decreases.)

Equating γ expressions, separating variables, and integrating:

$$\int_p^0 dp = -64\int_{-15{,}500}^{0} \frac{dz}{1 + (64/E_v)z} \; ; \quad 0 - p = -64\left(\frac{E_v}{64}\right)\left[\ln\left(1 + \frac{64}{E_v}z\right)\right]_{-15{,}500}^{0}$$

$$p = (330{,}000 \text{ psi})\left[\ln 1 - \ln\left(1 + \frac{(64 \text{ pcf})(-15{,}500 \text{ ft})}{(330{,}000 \text{ lb/in}^2)(144 \text{ in}^2/\text{ft}^2)}\right)\right] = 6960 \text{ psi}$$ ◄

Sec 3.3: Pressure Expressed in Height of Fluid -- *Exercises (3)*

3.3.1 *An open tank contains 5.0 m of water covered with 2 m of oil (γ = 8.0 kN/m³). Find the gage pressure (a) at the interface between the liquids and (b) at the bottom of the tank.*

SI

(*a*) Eq. 3.4: $p = \gamma h$ = (8 kN/m³)2 m = 16 kN/m² = 16 kPa at interface ◄

(*b*) p_b = 16 + (9.81)5 = 65.1 kN/m² = 65.1 kPa at tank bottom ◄

3.3.2 BG *An open tank contains 7 ft of water covered with 2.2 ft of oil (s = 0.88). Find the gage pressure (a) at the interface between the liquids and (b) at the bottom of the tank.*

(*a*) Eq. 3.4: $p = \gamma h = 0.88(62.4)2.2/144 = 0.839$ psi at interface ◀

(*b*) $p_b = 0.839 + 62.4(7)/144 = 3.87$ psi at tank bottom ◀

3.3.3 SI *If air had a constant specific weight of 12 N/m³ and were incompressible, what would be the height of air surrounding the earth to produce a pressure at the surface of 101.3 kPa abs?*

Eq. 3.5: $h = p/\gamma = 101.3/12.00 = 8.44$ km ◀

Sec. 3.3: Pressure Expressed in Height of Fluid -- ***Problems*** *3.4–3.5*

3.4 BG *If the specific weight of a sludge can be expressed as $\gamma = 64.0 + 0.22h$, determine the pressure in psi at a depth of 14 ft below the surface. γ is in lb/ft³, and h is in feet below the surface.*

Eq. 3.2: $dp = \gamma dh = (64 + 0.22h)dh$; integrating both sides: $p = 64h + 0.11h^2$

For $h = 14$ ft: $p = 64(14)/144 + 0.11(14)^2/144 = 6.37$ psi ◀

3.5 BG *A bubble 4 in below the water surface contains 2×10^{-7} lb of air. If the temperature is 60°F and the barometric pressure is 14.7 psia, calculate the diameter of the bubble. Refer to Secs. 2.7 and 2.12, and ignore the partial pressure of water vapor inside the bubble.*

Appendix A, Table A.1 for water at 60°F: $\sigma = 0.005\ 04$ lb/ft

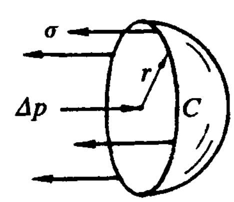
Table A.5 for air: $R = 1715$ ft²/(sec²·°R). Let bubble radius = r ft.

From Eq. 2.5: Inside bubble $p_{\text{air}} = \dfrac{\gamma RT}{g}$ where $\gamma = \dfrac{W}{V} = \dfrac{2 \times 10^{-7}}{(4/3)\pi r^3}$ pcf

$$\therefore\ p_{\text{air}} = \frac{2 \times 10^{-7}}{(4/3)\pi r^3} \times \frac{1715(460 + 60)}{32.2} = \frac{0.001\,322}{r^3}$$

Also, inside bubble $p_{\text{air}} = p_{\text{atm}} + p_{\text{water}} + \Delta p_{\text{surf tension}}$

Considering eqilibrium of half-bubble (single surface):



$$C\sigma = A\Delta p\ ,\ \text{i.e.,}\ \ 2\pi r\sigma = \pi r^2 \Delta p\ ,\ \text{so}\ \ \Delta p = \frac{2\sigma}{r}$$

$$\therefore\ p_{\text{air}} = p_{\text{atm}} + \gamma h + \frac{2\sigma}{r}$$

$$\text{So:}\ \ \frac{0.001\,322}{r^3} = 14.7(144) + 62.4\left(\frac{4}{12}\right) = \frac{2(0.00504)}{r}\ \text{psf}$$

This can be rearranged into a cubic equation in r.

By successive trials, etc. (see Sample Prob. 3.5), we find $r = 0.008\ 52$ ft

(Using Eq. B.9 we can demonstrate that the other two roots involve imaginary numbers.)

Bubble diameter = $2r = 0.017\ 04$ ft = 0.204 in ◀

Sec 3.4: Absolute and Gage Pressures -- ***Exercises*** *(6)*

3.4.1 *A gage is connected to a tank in which the pressure of the fluid is 42 psi above atmospheric (Fig. X3.4.1a). If the absolute pressure of the fluid remains unchanged but the gage is in a chamber where the air pressure is reduced to a vacuum of 25 inHg (Fig. X3.4.1b), what reading in psi will then be observed?*

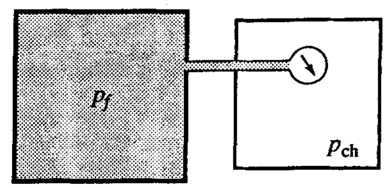


Figure X3.4.1*b*

BG

From Sec. 3.4: $p_{reading} = p_{fluid} - p_{surrounding} = 42 \text{ psi} - p_{surr}$

[Normally $p_{surr} = 0$ (atmospheric), so $p_{rdg} = 42$ psi]

When $p_{surr} = 25$ inHg vacuum $= 25\left(\dfrac{14.696}{29.92}\right)$ psi vac $= 12.28$ psi vac

Then $p_{rdg} = 42 - (-12.28) = 54.3$ psi ◄

3.4.2 *A gage is connected to a tank in which the pressure of the fluid is 305 kPa above atmospheric (Fig. X3.4.1a). If the absolute pressure of the fluid remains unchanged but the gage is in a chamber where the air pressure is reduced to a vacuum of 648 mmHg (Fig. X3.4.1b), what reading in kPa will then be observed?*

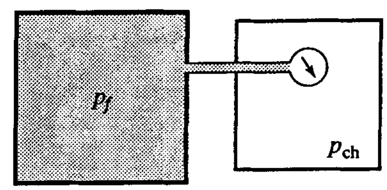


Figure X3.4.1*b*

SI

From Sec. 3.4: $p_{reading} = p_{fluid} - p_{surrounding} = 305 \text{ kPa} - p_{surr}$

[Normally $p_{surr} = 0$ (atmospheric), so $p_{rdg} = 305$ kPa]

When $p_{surr} = 648$ mmHg vacuum $= 648\left(\dfrac{101.325}{760}\right)$ kPa vac $= 86.4$ psi vac

Then $p_{rdg} = 305 - (-86.4) = 391$ kPa ◄

3.4.3 *If the atmospheric pressure is 780 mb abs and a gage attached to a tank reads 330 mmHg vacuum, what is the absolute pressure within the tank?*

SI

Eq. 3.4: $p_{gage} = \gamma h = (13.56 \times 9.81 \text{ kN/m}^3)(0.33 \text{ m}) = 43.9 \text{ kN/m}^2 \text{ vac.} = -43.9 \text{ kPa}$

Inside front cover: $p_{atm} = (780 \text{ mb abs})(0.1 \text{ kPa/mb}) = 78.0$ kPa abs

Eq. 3.7: $p_{abs} = p_{atm} + p_{gage} = 78.0 - 43.9 = 34.1$ kPa ◄

3.4.4 *If the atmospheric pressure is 14.20 psia and a gage attached to a tank reads 12.5 inHg vacuum, what is the absolute pressure within the tank?*

BG

$\dfrac{\text{inHg (actual)}}{\text{psi (actual)}} = \dfrac{\text{inHg (standard)}}{\text{psi (standard)}}$; $\dfrac{-12.5}{x} = \dfrac{29.92}{14.696}$; $x = -6.14$ psi pressure within the tank

Eq. 3.7: $p_{abs} = p_{atm} + p_{gage} = 14.20 - 6.14 = 8.06$ psia ◄

3.4.5 *If the atmospheric pressure is 955 mb abs and a gage attached to a tank reads 190 mmHg vacuum, what is the absolute pressure within the tank?*

SI

$\dfrac{-190 \text{ mm}}{x} = \dfrac{760 \text{ mmHg}}{101.325 \text{ kN/m}^2}$; $x = -25.3$ kPa = pressure in tank

Inside front cover: $p_{atm} = (955 \text{ mb})(0.1 \text{ kPa/mb}) = 95.5$ kPa

Eq. 3.7: $p_{atm} = p_{atm} + p_{gage} = 95.5 - 25.3 = 70.2$ kPa abs ◄

3.4.6 *If the atmospheric pressure is 29.92 inHg, what will be the height of water in a water barometer if the temperature of the water is (a) 70°F; (b) 120°F? Be as precise as possible.*

BG

Standard atmosphere: 29.92 inHg = 14.696 psi

(*a*) At 70°F, from Table A.1: p_{vapor} = 0.36 psia, γ = 62.30 lb/ft^3

Water barometer height = (14.696 − 0.36)144/62.30 = 33.14 ft ◄

(*b*) At 120°F, from Table A.1: p_{vapor} = 1.69 psia, γ = 61.71 lb/ft^3

Water barometer height = (14.696 −1.69)144/61.71 = 30.35 ft ◄

Sec. 3.4: Absolute and Gage Pressures -- *Problems 3.6−3.8*

3.6 *The absolute pressure on a gas is 41 psia and the atmospheric pressure is 965 mb abs. Find the gage pressure in psi, kPa, and mb.*

B

Inside front cover: p_{abs} = 41 psia(6.894 76 kPa/psi) = 282.7 kPa abs

Inside front cover: p_{atm} = 965 mb abs = (965 mb abs)(0.1 kPa/mb) = 96.5 kPa abs

Eq. 3.7: Gage pressure = $p_{abs} - p_{atm}$ = 282.7 − 96.5 = 186.2 kPa ◄

Gage pressure = (186.2 kPa)(10 mb/kPa) = 1862 mb ◄

Gage pressure = $\dfrac{186.2\text{ kPa}}{6.894\,76\text{ kPa/psi}}$ = 27.0 psi ◄

3.7 *The tire of an airplane is inflated at sea level to 60 psi. Assuming the tire does not expand, what is the pressure within the tire at elevation 40,000 ft? Assume standard atmosphere. Express the answer in psig and psia.*

BG

Let subscript 1 indicate sea level, subscript 2 indicate elevation 40,000 ft.

Eq. 3.7: $(p_1)_{abs} = p_{atm} + p_{gage}$ = 14.696 + 60 = 74.7 psia

Table A.3 for standard atmosphere: T_1 = 59°F = 519°R, T_2 = −69.7°F = 390.3°R

Eq. 2.4: $R = \dfrac{p_1 v_1}{T_1} = \dfrac{p_2 v_2}{T_2}$; $v_1 = v_2$; $\dfrac{74.7}{519} = \dfrac{p_2}{394}$; $(p_2)_{abs}$ = 56.2 psia ◄

From Table A.3 at 40,000 ft: $(p_2)_{atm}$ = 2.73 psia

Eq. 3.7: $(p_2)_{gage} = (p_2)_{abs} - (p_2)_{atm}$ = 56.2 − 2.73 = 53.4 psi ◄

3.8 *The tire of an airplane is inflated at sea level to 350 kPa. Assuming the tire does not expand, what is the pressure within the tire at elevation (a) 10 000 m and (b) 20 000 m? Assume standard atmosphere. Express answers in both kPa gage and kPa abs.*

SI

At sea level (subscript 1):

Table A.3 for standard atmosphere: $(p_1)_{atm}$ = 101.325 kPa, T_1 = 15°C = 15 + 273 K = 288 K

Eq. 3.7: $(p_1)_{abs} = (p_1)_{atm} + (p_1)_{gage}$ = 101 + 350 = 451 kPa abs

(*a*) At 10,000 m (subscript 2):

Table A.3: $(p_2)_{atm}$ = 26.499 kPa, T_2 = −49.898°C = 273 − 49.9 K = 223.1 K

Eq. 2.4: $R = \dfrac{p_1 v_1}{T_1} = \dfrac{p_2 v_2}{T_2}$; $v_1 = v_2$; $\dfrac{451}{288} = \dfrac{p_2}{223.1}$; $(p_2)_{abs}$ = 350 kPa abs ◄

/cont...

Eq. 3.7: $(p_2)_{gage} = (p_2)_{abs} - (p_2)_{atm} = 350 - 26.5 = 323$ kPa ◄

(*b*) At 20,000 m (subscript 3):

Table A.3: $(p_3)_{atm} = 5.529$ kPa, $T_3 = -56.5°C = 273 - 56.5$ K $= 216.5$ K

$\frac{451}{288} = \frac{p_3}{216.5}$; $(p_3)_{abs} = 339$ kPa abs ◄ Eq. 3.7: $(p_3)_{gage} = 339 - 5.53 = 334$ kPa ◄

The gage pressure in the tire will gradually decrease with elevation up to about 12,000 m. Above that elevation the gage pressure will increase because of the temperature variation in the atmosphere.

Sec 3.5: Measurement of Pressure – *Exercises (11)*

3.5.1 *If the atmospheric pressure is equivalent to 33.40 ft of water, what must be the reading (to 0.01 ft) on a barometer containing an alcohol (s = 0.78) if the vapor pressure of the alcohol at the temperature of observation is 2.09 psia.*

BG

Sec. 3.5: (33.40 ft of water)(14.696 psia/33.91 ft water) = 14.47 psia

From Eq. 3.8: Barometer reading (Fig. 3.6*a*) $= y = \frac{p_{atm} - p_{vapor}}{\gamma} = \frac{(14.47 - 2.09)144}{(0.78)62.4} = 36.64$ ft ◄

3.5.2 *A scientist plans to build a water barometer. When the atmospheric pressure is 990 mb abs and the water temperature is 70°C, what would you expect the barometer reading (water rise) to be?*

SI

Table A.1 at 70°C: $\gamma = 9.589$ kN/m³ ; $p_v/\gamma = 3.25$ m ; $p_{atm} = 990$ mb abs $= 99$ kN/m² abs

From Eq. 3.8: Barometer reading (Fig. 3.6*a*) $= y = \frac{p_{atm}}{\gamma} - \frac{p_v}{\gamma} = \frac{99}{9.589} - 3.25 = 7.07$ m ◄

3.5.3 *In Sample Prob. 3.4 suppose the atmospheric pressure is 1028 mb abs. What must be the absolute pressure at A? Express in mb abs and in mHg.*
Sample Prob. 3.4: $(p_A)_{gage} = 112.0$ *kPa.*

SI

Given: $p_{atm} = 1028$ mb abs; using the inside cover: $(p_A)_{gage} = 112.0$ kN/m² $= 1120$ mb

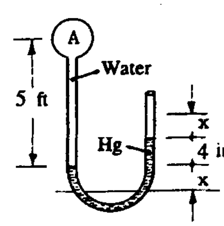
Eq. 3.7: $(p_A)_{abs} = p_{atm} + p_{gage} = 1028 + 1120 = 2148$ mb abs ◄

$$\left(\frac{p_A}{\gamma}\right)_{abs} = \frac{214.8 \text{ kN/m}^2}{13.56 \times 9.81 \text{ kN/m}^3} = 1.615 \text{ mHg abs}$$ ◄

3.5.4 *In Fig. X3.5.4, originally the manometer reading $R_m = 4$ in when $h = 5$ ft. Atmospheric pressure is 14.70 psia. If the absolute pressure at A is doubled, what will be the manometer reading?*

BG

$p_{atm} = 14.70$ psia (given) $= 14.70\left(\frac{33.91}{14.696}\right) = 33.92$ ft of water



Eq. 3.9: $(p_A/\gamma)_{abs} = 33.92 + (4/12)(13.56) - 5 = 33.4$ ft

$2(32.4) = 66.9 = 33.92 + \left(\frac{4 + 2x}{12}\right)13.56 - \left(5 + \frac{x}{12}\right)$; $x = 15.36$ in.

So new manometer reading $= 4 + 2(15.36) = 34.7$ in ◄

3.5.5 *Gas confined in a rigid container exerts a pressure of 25 psi when its temperature is 40°F. What pressure would the gas exert if the temperature were raised to 165°F? Barometric pressure remains constant at 29.0 inHg.*

BG

Sec. 3.5: $p_{atm} = (\text{inHg})_{atm}\dfrac{\text{lb/in}^2\ (\text{standard})}{\text{inHg (standard)}} = 29.0\left(\dfrac{14.696}{29.92}\right) = 14.25\ \text{psi}$

Eq. 3.7: $p_{abs} = p_{atm} + p_{gage} = 14.25 + 25 = 39.2$ psia at 40°F

Eq. 2.4: $R = \dfrac{p\,v}{T} = \dfrac{39.2v}{460 + 40} = p'_{abs}\dfrac{v}{460+165}$; $p'_{abs} = 39.2\dfrac{625}{500} = 49.1$ psia

$p'_{gage} = 49.1 - 14.25 = 34.8$ psi ◀

3.5.6 *Gas confined in a rigid container exerts a pressure of 200 kPa when its temperature is 5°C. What pressure would the gas exert if the temperature were raised to 80°C? Barometric pressure remains constant at 29.0 inHg.*

SI

29 inHg = 737 mmHg ; Sec. 3.5: $p_{atm} = 737$ mm(101.325 kPa/760 mm) = 98.2 kN/m^2

Eq. 3.7: $p_{abs} = p_{atm} + p_{gage} = 98.2 + 200 = 298.2$ kPa

Eq. 2.4: $R = \dfrac{p\,v}{T} = \dfrac{298.2v}{273+5} = p'_{abs}\dfrac{v}{273+80}$

$p'_{abs} = 378.7$ kN/m^2 abs ; $p'_{gage} = 378.7 - 98.2 = 280$ kPa ◀

3.5.7 *(a) A mercury manometer (Fig. 3.11) is connected to a pipeline carrying water at 150°F and located in a room where the temperature is also 150°F. If the elevation of point B is 6 ft above A and the mercury manometer reading is 48 in, what is the pressure in the pipe in psi? Be as precise as possible, and note the effect of temperature. Note that at 150°F the specific gravity of mercury is 13.45. (b) Repeat, assuming all temperatures are 68°F.*

BG

(*a*) Fig. 3.10: $p_A = \gamma_{water} z + \gamma_{Hg} y$

Table A.1 at 150°F: $\gamma_{water} = 61.20$ lb/ft^3

$p_A = 61.20(6) + (62.4 \times 13.45)\dfrac{48}{12} = 3724$ psf = 25.9 psi ◀

(*b*) Table A.1 at 68°F: $\gamma_{water} = 62.31$ lb/ft^3 (by interpolation)

$p_A = 62.31(6) + (62.4 \times 13.56)\dfrac{48}{12} = 3758$ psf = 26.1 psi ◀

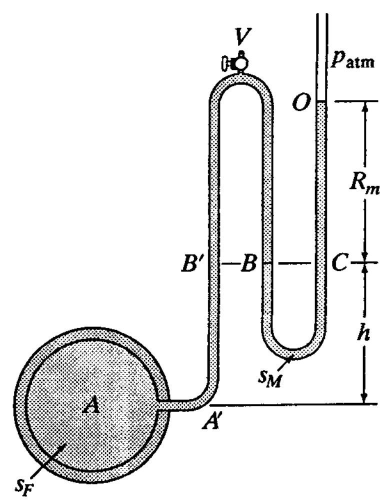


Figure 3.11

3.5.8 *In Fig. X3.5.8, atmospheric pressure is 14.80 psia; the gage reading at A is 3.7 psi; the vapor pressure of the alcohol is 1.4 psia. Compute x and y.*

BG

Working in terms of absolute pressure heads,

$$\frac{1.4(144)}{0.90(62.4)} + x = \frac{(14.80+3.7)144}{0.90(62.4)}; \quad x = 43.8 \text{ ft} \quad ◀$$

$$0 + \frac{45(13.56)}{12(0.9)} - \left(\frac{45}{12} + y\right) = \frac{(14.80+3.7)144}{0.90(62.4)};$$

$y = 5.31$ feet ◀

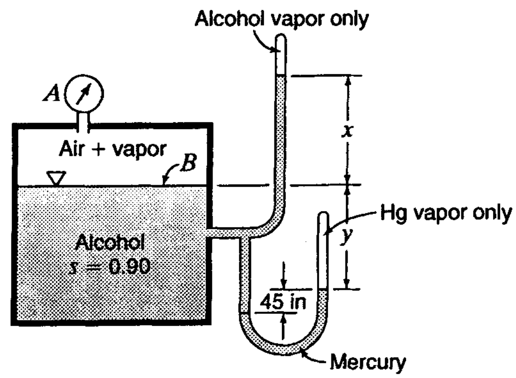


Figure X3.5.8

3.5.9 *Refer to the manometer of Fig. 3.14b. A and B are at the same elevation. Water is contained in A and rises in the tube to a level 52 in above A. Kerosene is contained in B. The inverted U-tube is filled with air at 11 psi and 70°F. Atmospheric pressure is 14.70 psia. (a) Determine the difference in pressure between A and B if the manometer reading is 10 in. Express the answer in psi. (b) What is the absolute pressure in B in inches of mercury, and feet of kerosene?*

BG

(*a*) $\gamma_{air} = 0$; Table A.4 for kerosene: $s = 0.81$

Eq. 3.12, modified: $p_A/\gamma - 52/12 + 0.81(52 - 10)/12 = p_B/\gamma$

$p_A/\gamma - p_B/\gamma = 4.33 - 2.84 = +1.498$ ft water

$14.70(1.498)/33.91 = 0.650$ psi (*A* is at the higher pressure) ◀

(*b*) $(p_B/\gamma)_{abs} = 29.92 + (29.92)11/14.70 + (52 - 10)0.81/13.56$

$= 54.8$ inHg ◀

$$= \frac{(54.8)13.56}{(0.81)12} = 76.5 \text{ ft kerosene} \quad ◀$$

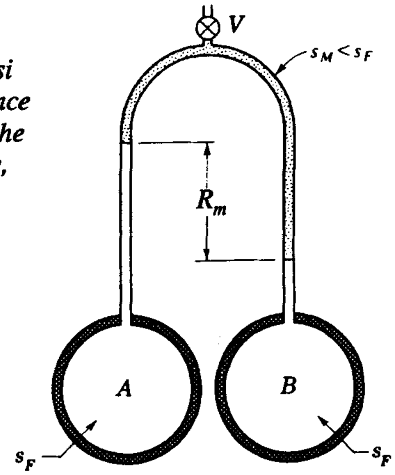


Figure 3.14*b* with $z_A = z_B$

3.5.10 *(a) Two vessels are connected to a differential manometer using mercury (s = 13.56), the connecting tubing being filled with water. The higher-pressure vessel is 5 ft lower in elevation than the other. Room temperature prevails. If the mercury reading is 4.0 in, what is the pressure difference in feet of water, and in psi? (b) If carbon tetrachloride (s = 1.59) were used instead of mercury, what would the manometer reading be for the same pressure difference?*

BG

(*a*) Given $z_B - z_A = 5$ ft.

Eq. 3.12*a*: $p_A/\gamma - p_B/\gamma = 5 + (13.56 - 1.0)4/12$

$p_A/\gamma - p_B/\gamma = 5 + 4.19 = 9.19$ ft of water ◄

$\Delta p = 9.19(14.696/33.91) = 3.98$ psi ◄

(*b*) Eq. 3.12*a*: $p_A/\gamma - p_B/\gamma = 5 + (1.59 - 1.0)R_m/12$

9.19 ft of water $= 5 + (1.59 - 1.0)R_m/12$

$R_m = 4.19 \times 12/0.59 = 85.2$ in of CCl_4 ◄

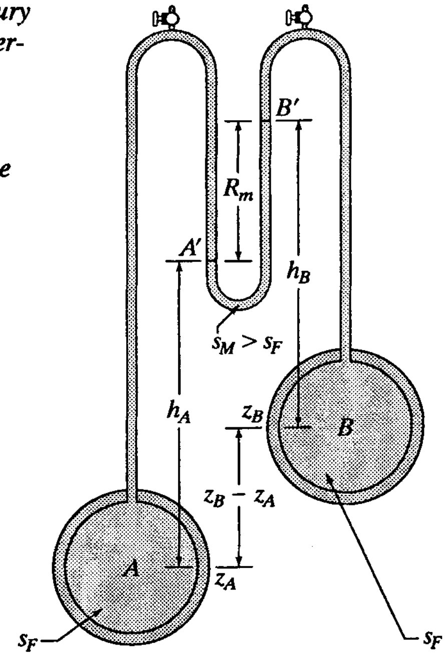


Figure 3.14*a*

3.5.11 *(a) Two vessels are connected to a differential manometer using mercury (s = 13.56), the connecting tubing being filled with water. The higher-pressure vessel is 1.5 m lower in elevation than the other. Room temperature prevails. If the mercury reading is 100 mm, what is the pressure difference in m of water, and in kPa? (b) If carbon tetrachloride (s = 1.59) were used instead of mercury, what would the manometer reading be for the same pressure difference?*

SI

(*a*) Given $z_B - z_A = 1.5$ m.

Eq. 3.12*a*: $p_A/\gamma - p_B/\gamma = 1.5 + (13.56 - 1.0)0.100$

$= 1.5 + 1.256 = 2.76$ m ◄

$\Delta p = (2.76 \text{ m})(9.81 \text{ kN/m}^3) = 27.0$ kPa ◄

(*b*) Eq. 3.12*a*: $(p_A/\gamma - p_B/\gamma) = 1.5 + (1.59 - 1.0)R_m$

$\therefore 2.76 - 1.5 = (0.59)R_m$; $R_m = 2.13$ m of CCl_4 ◄

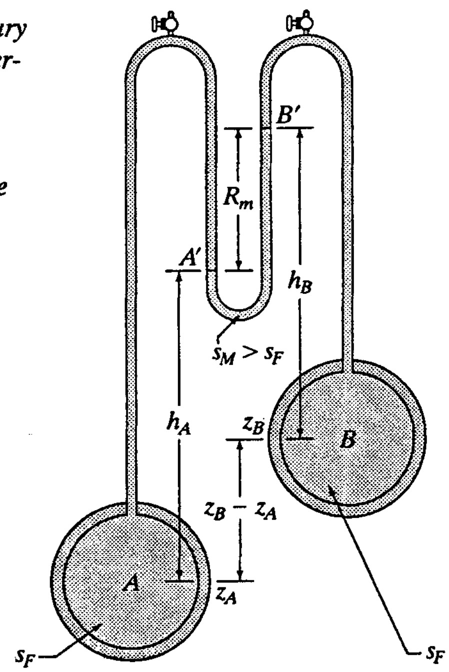


Figure 3.14*a*

Sec. 3.5: Measurement of Pressure – ***Problems*** *3.9–3.12*

3.9 *In Fig. X3.5.8 assume the following: atmospheric pressure = 930 mb abs; vapor pressure of the alcohol = 110 mb abs; x = 3.30 m and y = 1.60 m. Compute the reading (a) on the pressure gage and (b) on the manometer.*

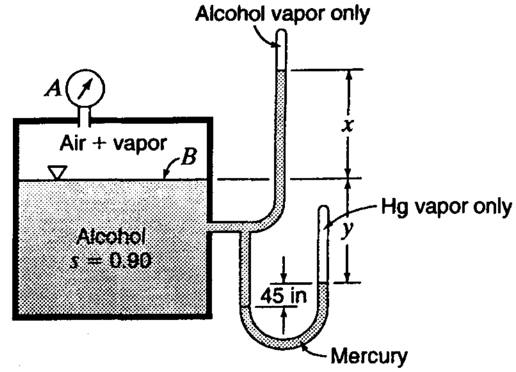


Figure X3.5.8

SI

(*a*) $11\text{ kPa} + (0.90)9.81(3.3) = 40.1\text{ kN/m}^2 = (p_B)_{abs}$

Eq. 3.7: $(p_A)_{gage} = (p_A)_{abs} - p_{atm} = 40.1 - 93$

$= -52.9\text{ kPa}$ ◄

$= 52.9\text{ kPa vac}\left(\dfrac{760\text{ mmHg}}{101.325\text{ kPa abs}}\right)$

$= 397\text{ mmHg vac}$ ◄

(*b*) $0 + R_m\dfrac{13.56}{0.90} - R_m - 1.6 = \left(\dfrac{p_B}{\gamma}\right)_{abs}$; however, $\left(\dfrac{p_B}{\gamma}\right)_{abs} = \dfrac{40.1}{0.90\times 9.81} = 4.55\text{ m}$

$(15.07)R_m - R_m = 1.6 + 4.55\text{ m}$; $R_m = 0.437\text{ m} = 437\text{ mm}$ ◄

3.10 *The diameter of tube C in Fig. 3.11 is d_1, and that of tube B is d_2. Let z_0 be the elevation of the mercury above A when both mercury columns are at the same level. R is the distance the right-hand column of mercury rises above z_0 when the fluid in A is under pressure. Let γ' be the specific weight of the mercury (or any other measuring fluid), while γ is the specific weight of the fluid in A and the connecting tubing. Prove that $p_A = \gamma z_0 + [\gamma' + (\gamma' - \gamma)(d_1/d_2)^2]R = M + NR$ where M and N are constants. Note that this equation involves only one variable, which is the reading R on the scale for column C. It also shows the significance of having d_2 large compared with d_1.*

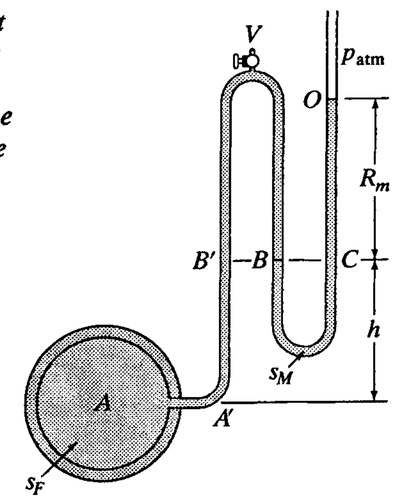


Figure 3.11

N

Let x = depression of Hg in tube B below equilibrium level z_0

From 0 to A: $0 + (R + x)\gamma' + (z_0 - x)\gamma = p_A$

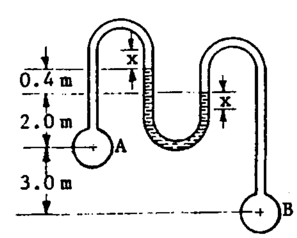
Since Hg is incompressible, $Rd_1^2(\pi/4) = xd_2^2(\pi/4)$; $x = R(d_1/d_2)^2$

$\therefore\ p_A = [R + R(d_1/d_2)^2]\gamma + z_0\gamma - \gamma R(d_1/d_2)^2$

$= \gamma z_0 + [\gamma' + (\gamma' - \gamma)(d_1/d_2)^2]R$ Q.E.D. ◄

3.11 *What would be the manometer reading in Sample Prob. 3.4 if $p_B - p_A = 145$ kPa?*

Sample Prob. 3.4: Manometer fluid = Hg. $\gamma_A = 8.4\text{ kN/m}^3$, $\gamma_B = 12.4\text{ kN/m}^3$, $p_B = 207$ kPa, find p_A. Express all pressure heads in terms of liquid B.



SI

$\dfrac{p_B}{\gamma_B} - \dfrac{p_A}{\gamma_B} = \dfrac{145\text{ kN/m}^2}{12.4\text{ kN/m}^3} = 11.69\text{ m (of liquid B)}$

$\dfrac{p_A}{\gamma} - (2.4 + x)\dfrac{8.4}{12.4} + (2x + 0.4)\dfrac{13.56(9.81)}{12.4} + (5 - x) = \dfrac{p_B}{\gamma}$

$-1.626 - 0.677x + 21.46x + 4.29 + 5.0 - x = 11.69$; $x = 0.204$ m

Manometer reading $= 0.4 + 2x = 0.807$ mHg ◄

3.12 *At a certain point the gage pressure in a pipeline containing gas (γ = 0.05 lb/ft³) is 5.6 in of water. The gas is not flowing, and all temperatures are 60°F. What is the gage pressure in inches of water at another point in the line whose elevation is 650 ft greater than the first point? Make and state clearly any necessary assumptions.*

BG

Assume γ_{gas} = 0.05 lb/ft³ = constant

Note that the change in pressure in the atmosphere must be considered.

Assume γ_{air} = 0.076 lb/ft³ = constant ; Let A be the lower point.

$(p_A/\gamma)_{abs} = (p_A/\gamma)_{atm} + 5.6/12$ ft of water (1)

$(p_B/\gamma)_{abs} = (p_B/\gamma)_{atm} + x/12$ ft of water (2)

$(p_A/\gamma)_{atm} - (p_B/\gamma)_{atm} = 650$ ft of air $= 650(0.076/62.4) = 0.792$ ft of water

$(p_A/\gamma)_{abs} - (p_B/\gamma)_{abs} = 650$ ft of gas $= 650(0.05/62.4) = 0.521$ ft of water

Subtract (2) from (1) and substitute the other relationships.

$(p_A/\gamma)_{abs} - (p_B/\gamma)_{abs} = (p_A/\gamma)_{atm} - (p_B/\gamma)_{atm} + (5.6/12) - (x/12)$

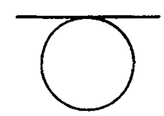
$0.521 = 0.792 + (5.6/12) - (x/12)$; $x = 8.85$ inches of water ◄

Sec 3.7: Center of Pressure -- *Exercises (16)*

3.7.1 *A circular area of diameter d is vertical and submerged in a liquid. Its upper edge is coincident with the liquid surface. Derive an expression for the depth to its center of pressure.*

N

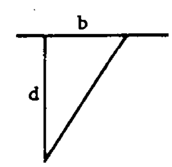
Eq. 3.18: $h_p = d/2 + \dfrac{\pi d^4/64}{(d/2)\,\pi d^2/4} = d/2 + d/8 = 5d/8$ ◄



3.7.2 *If a triangle of height d and base b is vertical and submerged in a liquid with its base at the liquid surface, derive an expression for the depth to its center of pressure.*

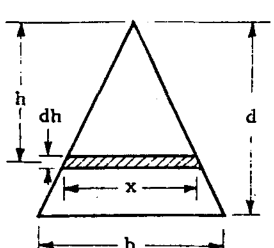
N

Eq. 3.18: $h_p = d/3 + \dfrac{bd^3/36}{(d/3)(bd/2)} = d/3 + d/6 = d/2$ ◄



3.7.3 *If a triangle of height d and base b is vertical and submerged in liquid with its vertex at the liquid surface, derive an expression for the depth to its center of pressure.*

N



Eq. 3.14: $F = \int p\,dA = \int \gamma h\,dA = \int \gamma h x\,dh$

But $x = h(b/d)$ so $F = \gamma \dfrac{b}{d}\displaystyle\int_0^d h^2\,dh$

As $h_p F = \int h\,dF = \gamma \dfrac{b}{d}\displaystyle\int_0^d h^3\,dh$

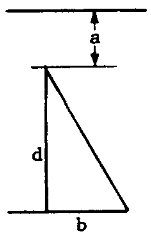
$$h_p = \frac{\int_0^d h^3\,dh}{\int_0^d h^2\,dh} = \left.\frac{h^4/4}{h^3/3}\right|_0^d = \frac{3}{4}d \quad ◄$$

Alternatively, Eq. 3.18: $h_p = h_c + \dfrac{I_c}{h_c A} = \dfrac{2}{3}d + \dfrac{(1/36)bd^3}{(2/3)d(bd/2)} = \dfrac{2}{3}d + \dfrac{d}{12} = (3/4)d$ ◄

3.7.4 *Repeat Exer. 3.7.3 for the same triangle but with its vertex a distance a below the liquid surface.*

Exer. 3.7.3: The submerged triangle of height d and base b is vertical. Derive an expression for the depth to its center of pressure.

N

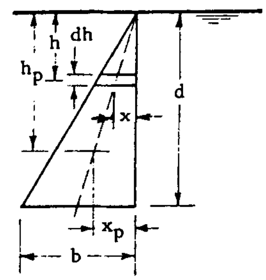


Eq. (3.18): $$h_p = (a + 2d/3) + \frac{bd^3/36}{(a+2d/3)(bd/2)} = (a+2d/3) + \frac{d^2}{(a+2d/3)\,18}$$

$$= \frac{18(a^2 + 4ad/3 + 4d^2/9) + d^2}{18(a + 2d/3)} = \frac{6a^2 + 8ad + 3d^2}{6a + 4d} \quad ◀$$

3.7.5 *A vertical right-triangle of height d and base b submerged in liquid has its vertex at the liquid surface. Find the distance from the vertical side to the center of pressure by (a) inspection; (b) calculus.*

N



(*a*) By inspection: $h_p = (3/4)d$ from Exer. 3.7.3

On median: $x_p = (3/4)(b/2) = (3/8)b$ ◀

(*b*) By calculus: with $x = \frac{1}{2}\frac{hb}{d}$ (to center of each strip) and $dA = \frac{hb}{d}dh$

$$x_p = \frac{\int xh\,dA}{\int h\,dA} = \frac{\int_0^d \frac{hb}{2d}(h)\frac{hb}{d}\,dh}{\int_0^d (h)\frac{hb}{d}\,dh} = \frac{\frac{b^2}{2d^2}\int_0^d h^3 dh}{\frac{b}{d}\int_0^d h^2 dh} = \frac{(b)\frac{d^4}{4}}{(2d)\frac{d^3}{3}} = \frac{3}{8}b \quad ◀$$

3.7.6 *A plane surface is circular with a diameter of 2 m. If it is vertical and the top edge is 0.5 m below the water surface, find the magnitude of the force on one side and the depth to the center of pressure.*

SI

$h_c = 0.5 + d/2 = 1.50$ m ; Eq. 3.16: $F = \gamma h_c A = 9.81(1.50)\pi(1.00)^2 = 46.2$ kN ◀

From Eq. 3.18 and Table A.7: $$y_p - y_c = \frac{I_c}{y_c A} = \frac{\pi d^4}{64}\left(\frac{1}{h_c}\right)\frac{4}{\pi d^2} = \frac{d^2}{16h_c} = \frac{2.00^2}{16(1.50)} = 0.1667 \text{ m}$$

$h_p = h_c + (y_p - y_c) = 1.50 + 0.1667 = 1.667$ m ◀

3.7.7 *Find the magnitude and depth of the point of application of the force on the circular gate shown in Fig. X3.7.7 if h = 5 ft and D = 4 ft dia.*

BG

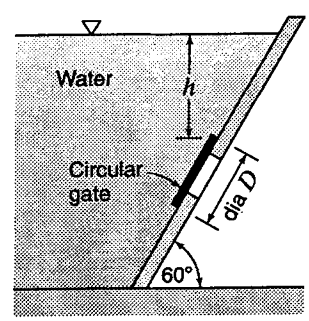


Figure X3.7.7

Eq. 3.16: $F = \gamma h_c A = 62.4(5 + 2\sin 60°)\pi(2)^2 = 5280$ lb ◀

$$\frac{I_c}{y_c A} = \frac{(\pi/64)(4)^4}{(5/\cos 30° + 2)\pi(2)^2} = 0.1286 \text{ ft}$$

Eq. 3.18: $y_p = y_c + \frac{I_c}{y_c A}$ (slope distance)

$y_p = 5/\cos 30° + 2 + 0.1286 = 7.90$ ft

$h_p = y_p \cos 30° = 7.90\cos 30° = 6.84$ ft ◀

3.7.8 *A rectangular plate 5 ft by 4 ft is at an angle of 30° with the horizontal, and the 5-ft side is horizontal. Find the magnitude of the force on one side of the plate and the depth of its center of pressure when the top edge is (a) at the water surface; (b) 1 ft below the water surface.*

BG

(*a*) $h_c = 0 + 2\sin 30° = 1.0$ ft ; Eq. 3.16: $F = \gamma h_c A = 62.4(1.00)(5\times 4) = 1248$ lb ◀

$y_o = d/2 = 2$ ft ; Eq. 3.18: $y_p = y_c + \dfrac{bd^3/12}{y_c(bd)} = y_c + \dfrac{d^2}{12y_c} = 2 + \dfrac{4^2}{12(2)} = 2.67$ ft

$h_p = y_p \sin 30° = 2.67\sin 30° = 1.333$ ft ◀

(*b*) $h_c = 1 + 2\sin 30° = 2.0$ ft ; $F = 62.4(2.0)(5\times 4) = 2500$ lb ◀

Per Fig. 3.16: $y_c = h_c/\sin 30° = 2/\sin 30° = 4$ ft

Eq. 3.18: $y_p = y_c + \dfrac{d^2}{12y_c} = 4 + \dfrac{4^2}{12(4)} = 4.33$ ft

$h_p = y_p \sin 30° = 4.33\sin 30° = 2.17$ ft ◀

3.7.9 *In Fig. X3.7.9 the rectangular flashboard MN shown in cross-section (a = 5.4 m) is pivoted at B. (a) What must be the maximum height of B above N if the flashboard is on the verge of tipping when the water surface rises to M? (b) If the flashboard is pivoted at the location determined in (a) and the water surface is 1 m below M, what are the reactions at B and N per m length of board perpendicular to the figure?*

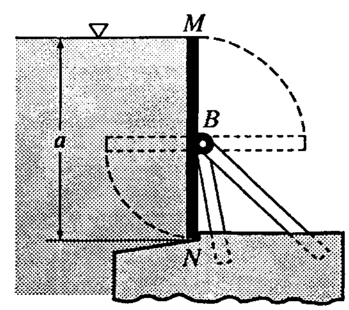


Figure X3.7.9

SI

(*a*) For critical stability, center of pressure must be at *B*.

Thus *B* will be 5.4/3 = 1.800 m above *N*. ◀

(*b*) For 4.4 m depth of water, Eq. 3.16: $F = 9.81(4.4/2)4.4$

$= 95.0$ kN/m

$\Sigma M_B = 0 = 95.0(1.8 - 4.4) - 1.8N_x$; $N_x = 17.59$ kN/m ◀

$\therefore\ B_x = 95.0 - 17.59 = 77.4$ kN/m ◀ $B_y = 0$ ◀

3.7.10 *The gate MN in Fig. X3.7.10 rotates about an axis through N. If a = 3.3 ft, b = 1.3 ft, d = 2 ft, and the width perpendicular to the plane of the figure is 3 ft, what torque applied to the shaft through N is required to hold the gate closed?*

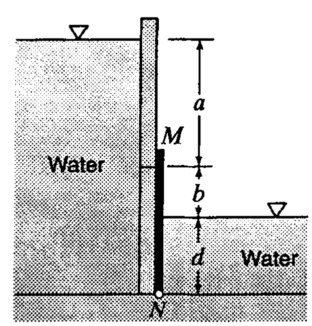


Figure X3.7.10

BG

$(h_c)_L = 3.3 + \dfrac{3.3}{2} = 4.95$ ft

Eq. 3.16: $F_L = \gamma(h_c)_L A = 62.4(4.95)(3.3\times 3) = 3060$ lb

$(y_p - y_c)_L = \dfrac{I_c}{y_c A} = \dfrac{bh^3/12}{h_c(bh)} = \dfrac{h^2}{12h_c} = \dfrac{3.3^2}{12(4.95)} = 0.1833$ ft

$F_R = \gamma h_c A = (62.4)\dfrac{2.0}{2}(2\times 3) = 374$ lb ; $(y_p)_R = (2/3)2 = 1.333$ ft

$\Sigma M_N = 3060(6.6 - 4.95 - 0.1833) - 374(2 - 1.333) - \text{Torque} = 0$;

Torque = 4240 ft·lb ◀

3.7.11 *What minimum value of b in Fig. X3.7.11 is necessary to keep the rectangular masonry wall from sliding if it weighs 160 lb/ft³, a = 14 ft, c = 16 ft, and the coefficient of friction is 0.45? With this minimum b value, will it also be safe against overturning? Assume that water does not get underneath the block.*

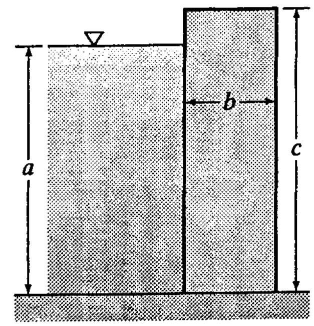


Figure X3.7.11

BG

$F_x = \gamma h_c A = 62.4(\frac{14}{2})14 = 6120$ lb/ft

$W = (160)16b = 2560b$ lb/ft ; $\quad R_V = W$

$\Sigma F_x = 6120 - 0.45(2560b) = 0$; $\quad b = 5.31$ ft ◄

Consider moments about toe of F_x, W, R_V (distance x_t from toe):

For equilibrium, $\Sigma M_{\text{toe}} = 0 = 6120\left(\frac{14}{3}\right) - (2560)5.31\left(\frac{5.31}{2}\right) + (2560)5.31x_t$

$$x_t = \frac{36{,}100 - 28{,}500}{(2560)5.31} = 0.554 \text{ ft}$$

As $x_t > 0$, the dam is safe against overturning, assuming water does not get under the block. ◄

3.7.12 *A rectangular plate submerged in water is 5 by 4 m, the 5-m side being horizontal and the 4-m side being vertical. Determine the magnitude of the force on one side of the plate and the depth to its center of pressure if the top edge is (a) at the water surface; (b) 1 m below the water surface; (c) 100 m below the water surface.*

SI

Eq. 3.16: $F = \gamma h_c A = (9.81)h_c(4 \times 5 \text{ m}) = 196.2h_c$

(*a*) $h_c = 2$ m ; $F = 196.2(2) = 392$ kN ◄

(*b*) $h_c = 3$ m ; $F = 196.2(3) = 589$ kN ◄

(*c*) $h_c = 102$ m ; $F = 196.2(102) = 20\ 000$ kn ◄

Eq. 3.18: $h_p = y_c + \frac{I_c}{y_c A} = y_c + \frac{bh^3}{12y_c bh} = y_c + \frac{h^2}{12y_c}$

(*a*) $y_c = 2$ m ; $h_p = 2 + \frac{4^2}{(12)2} = 2.67$ m ◄

(*b*) $y_c = 3$ m ; $h_p = 3 + \frac{4^2}{(12)3} = 3.44$ m ◄

(*c*) $y_c = 102$ m ; $h_p = 102 + \frac{4^2}{12(102)} = 102.0$ m ◄

3.7.13 *The right-triangular plate shown in Fig. X3.7.13 is submerged in a vertical plane with its base horizontal. Determine the depth and horizontal position of the center of pressure when a = 1 ft, b = 3 ft, and d = 4.5 ft.*

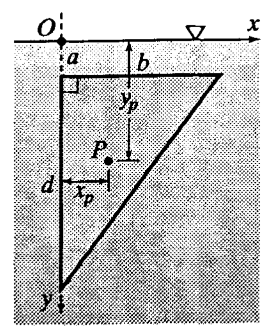


Figure X3.7.13

BG

Eq. 3.18: $y_p = y_c + \frac{I_c}{y_c A}$; $\quad$ Table A.7 : $I_c = \frac{bh^3}{36}$

where $A = \frac{3(4.5)}{2} = 6.75$ ft²; $y_c = \frac{h}{3} + 1 = 2.5$ ft; $I_c = \frac{bh^3}{36} = 7.59$ ft⁴

$\therefore\ y_p = y_c + \frac{I_c}{y_c A} = 2.5 + \frac{7.59}{2.5(6.75)} = 2.95$ ft ◄

Below P: $y' = 1 + 4.5 - y_p = 2.55$ ft

To median, $x_p = \frac{1}{2}(2.55)\frac{3}{4.5} = 0.850$ ft ◄

3.7.14 *Repeat Exer. 3.7.13, but with a = 0.2 m, b = 1.0 m, and d = 2.0 m.*
Exercise 3.7.13: For the submerged, vertical right triangular plate of Fig. X3.7.13, find the depth and horizontal position of the center of pressure.

SI

Eq. 3.18: $y_p = y_c + \frac{I_c}{y_c A}$; Table A.7 : $I_c = \frac{bh^3}{36}$;

$A = (1.0)2.0/2 = 1.000 \text{ m}^2$

$y_c = \frac{h}{3} + 0.2 = 0.867 \text{ m}$; $I_c = \frac{1.0(2.0)^3}{36} = 0.222 \text{ m}^4$;

$\therefore\ y_p = 0.867 + \frac{0.222}{0.867(1.000)} = 1.123 \text{ m}$ ◄

Below *P*: $y' = 0.2 + 2.0 - y_p = 1.077 \text{ m}$;

to median, $x_p = 0.5(1.077)1.0/2.0 = 0.269 \text{ m}$ ◄

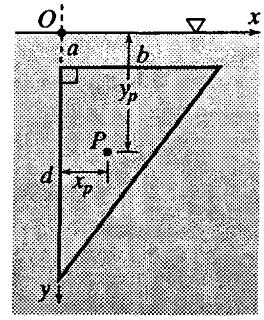


Figure X3.7.13

3.7.15 *A rectangular area is 5 by 6 m, with the 5 m side horizontal. It is placed with its centroid 4 m below a water surface and rotated about a horizontal axis in the plane area and through its centroid. Find the magnitude of the force on one side and the distance between the center of pressure and the centroid of the plane when the angle with the horizontal, θ = 90, 60, 30, and 0°.*

SI

Eq. 3.16: $F = \gamma h_c A = (9.81)4(5\times 6) = 1177 \text{ kN}$ for any angle θ ◄

From Eq. 3.18: $y_p - y_c = \frac{I_c}{y_c A} = \frac{bd^3/12}{y_c(bd)} = \frac{d^2}{12y_c} = \frac{6^2}{12y_c} = \frac{3}{y_c} = \frac{3\sin\theta}{h_c} = \frac{3}{4}\sin\theta$

From which

θ	$y_p - y_c$	
90°	0.750 m	◄
60°	0.650 m	◄
30°	0.375 m	◄
0°	0 m	◄

3.7.16 *Figure X3.7.16 shows a cylindrical tank with 0.25-in-thick walls, containing water. What is the force on the bottom? What is the force on the annular surface MM? What is the weight of the water? Find the longitudinal (vertical) tensile stress in the sidewalls BB if (a) the tank is suspended from the top; (b) it is supported from the bottom. Neglect the weight of the tank.*

BG

Eq. 3.16: Force on bottom $F_B = \gamma h_c A = 62.4(2)\pi = 392 \text{ lb}$ ◄

Force on surface MM, $F_S = \gamma h_c A = 62.4(1)(\pi/4)(2^2 - 1^2) = 147.0 \text{ lb}$ ◄

Weight of water $= 62.4\,\pi[1^2 + (1/2)^2] = 245 \text{ lb}$ ◄

(*a*) Suspended from top, $\sigma = \frac{F_B}{A} = \frac{392}{\pi(r_1^2 - r_2^2)} = \frac{392}{\pi[12^2 - (23.5/2)^2]}$

$= 392/18.65 = 21.0 \text{ lb/in}^2$ ◄

(*b*) Supported from bottom, $\sigma = F_S/A = 147/18.65 = 7.88 \text{ lb/in}^2$ ◄

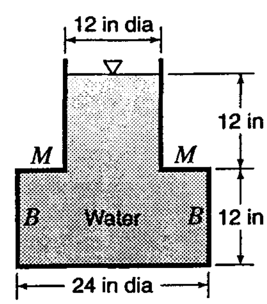
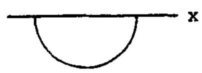


Figure X3.7.16

Sec. 3.7: Center of Pressure -- *Problems 3.13–3.19*

3.13 *A vertical semicircular area has its diameter in a liquid surface. Derive an expression for the depth to its center of pressure.*

N



Eq. 3.18 and Table A.7: $h_p = \frac{4r}{3\pi} + \frac{I_c}{(4r/3\pi)(\pi r^2/2)} = \frac{4r}{3\pi} + \frac{I_c}{2r^3/3}$

$$I_x = (1/2)\frac{\pi d^4}{64} = (\pi/2)\frac{(2r)^4}{64} = (1/2)\frac{\pi r^4}{4} = \frac{\pi r^4}{8}$$

$$I_c = \frac{\pi r^4}{8} - (1/2)\pi r^2\left(\frac{4r}{3\pi}\right)^2 = \frac{\pi r^4}{8} - \frac{16r^4}{18\pi} = r^4\left(\frac{\pi}{8} - \frac{8}{9\pi}\right) = r^4\left(\frac{9\pi^2 - 64}{72\pi}\right)$$

$$h_p = \frac{4r}{3\pi} + \frac{r^4(9\pi^2 - 64)/72\pi}{(2/3)r^3} = \frac{4r}{3\pi} + \frac{3}{2}r\left(\frac{25}{226}\right) = 0.424r + 0.1659r = 0.590r \quad ◄$$

3.14 *The Utah-shaped plate shown in Fig. P3.14 is submerged in oil (s = 0.94) and lies in a vertical plane. Find the magnitude and location of the hydrostatic force acting on one side of the plate.*

SI

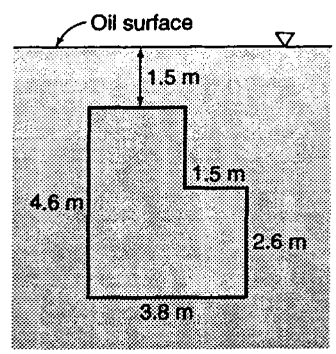


Figure P3.14

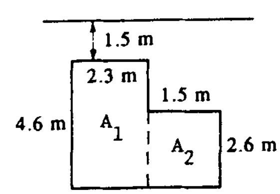
Eq. 3.16: $F = \gamma h_c A$

$F_1 = (9.81 \times 0.94)(1.5 + 2.3)(2.3 \times 4.6) = 371 \text{ kN}$

Eq. 3.18: $h_p = y_c + \frac{I_c}{y_c A}$

$(h_p)_1$

$F_2 = (9.81 \times 0.94)(1.5 + 2 + 1.3)(1.5 \times 2.6) = 173 \text{ kN}$



$(h_p)_2 = 4.8 + \frac{(1/12)1.5 \times 2.6^3}{4.8(1.5 \times 2.6)} = 4.92 \text{ m}$

$F = F_1 + F_2 = 543 \text{ kN}$ ◄

$F(h_p) = F_1(h_p)_1 + F_2(h_p)_2$;

$h_p = (371 \times 4.26 + 173 \times 4.92)/543 = 4.47$ m below oil surface. ◄

3.15 *The commom type of irrigation head gate shown in Fig. P3.15 is a plate that slides over the opening to a culvert. The coefficient of friction between the gate and its sliding ways is 0.6. Find the force required to slide open this 600-lb gate if it is set (a) vertically; (b) on a 2:1 slope (n = 2), as is common.*

BG

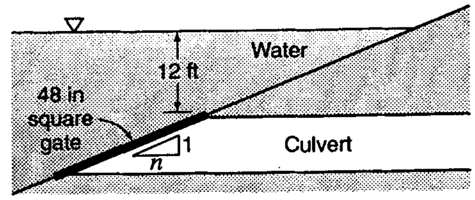


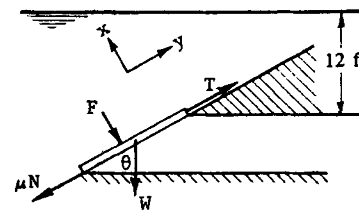
Figure P3.15

(*a*) Gate vertical: Eq. 3.16: $h_c = 12 + 4/2 = 14$;

$F_x = \gamma h_c A = 62.4(14)4^2 = 13{,}980 \text{ lb}$

Let T = force parallel to gate required to open gate:

$\Sigma F_y = 0$; $T - 600 - 0.6(13{,}980) = 0$; $T = 8990 \text{ lb}$ ◄



(*b*) On 2:1 slope: Eq. 3.16: $h_c = [12 + (4/2)\sin\theta]$

$F = \gamma h_c A = 62.4[12 + (4/2)\sin\theta]4^2 = 12{,}870 \text{ lb}$

$\Sigma F_x = 0$; $N = F + W\cos\theta$

$\Sigma F_y = 0$; $T = \mu N + W\sin\theta = \mu(F + W\cos\theta) + W\sin\theta$

$$T = 0.6\left(12{,}870 + 600\frac{2}{\sqrt{5}}\right) + \frac{600}{\sqrt{5}} = 8310 \text{ lb}$$ ◄

3.16 *In the drainage of irrigated lands it is frequently desirable to install automatic flap gates to prevent a flood from backing up into the lateral drains from a river. Suppose a square flap gate, of side b = 1.5 m and weight 8 kN, is hinged 1 m above its center (a = 0.25 m), as shown in Fig. P3.16, and the face is sloped 4° from the vertical. To what depth will water rise behind the gate before it will open?*

SI

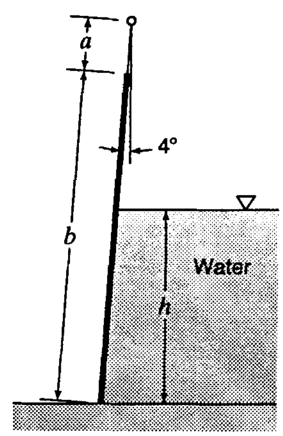


Figure P3.16

Closing moment of gate about hinge = $8000(1.0)\sin 4° = 558 \text{ N·m}$

Find the depth h which produces an opening moment of 558 N·m

Eq. 3.16: $$F = \gamma h_c A = 9810\left(\frac{h}{2}\right)\frac{1.5h}{\cos 4°} = 7380h^2$$

Moment arm $$= (1.5 + 0.25) - \frac{h}{3\cos 4°} = 1.75 - 0.334h$$

Then $7380h^2(1.75 - 0.334h) = 558$; $h^3 - 5.24h^2 + 0.226 = 0$

This cubic equation may be solved by trial and error, etc. (see Sample Prob. 3.5), yielding $h = 0.212$ m or 5.23 m (impossible) or -0.204 m (meaningless). So $h = 0.212$ m ◄

3.17 *Find the minimum value of z for which the gate in Fig. P3.17 will rotate counterclockwise if the gate is (a) rectangular, 5 ft high by 4 ft wide; (b) triangular, 4 ft base as axis, height 5 ft. Neglect friction in bearings.*

BG

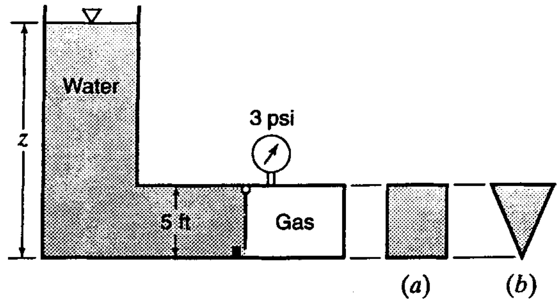


Figure P3.17

(*a*) Closing moment due to gas:

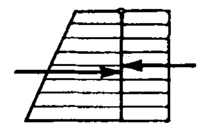
$(5\times4)(3\times144)2.5 = 21{,}600$ ft·lb

Opening moment due to water:

$$F_{water}[h_p-(z-5)],\quad \text{where } h_p = (z-2.5)+\frac{I_c}{h_cA},\quad I_c=\frac{bh^3}{12}$$

$$= 62.4(z-2.5)(5\times4)\left[(z-2.5)+\frac{(1/12)4(5)^3}{(z-2.5)20}-(z-5)\right]$$

$$= 1248(z-2.5)\left[2.5+\frac{25}{12(z-2.5)}\right] = 104(30z-50) = 3120z-5200$$



Equate moments and solve for z: $3120z-5200 = 21{,}600$; $z = 8.59$ ft ◄

(*b*) Moment due to water:

$$F_{water}[h_p-(z-5)],\quad \text{where } h_p = \left(z-\frac{5\times2}{3}\right)+\frac{I_c}{h_cA},\qquad I_c=\frac{bh^3}{36}$$

$$= 62.4\left(z-\frac{10}{3}\right)\left(\frac{5\times4}{2}\right)\left[\left(z-\frac{10}{3}\right)+\frac{(1/36)4(5)^3}{\left(z-\frac{10}{3}\right)10}-(z-5)\right]$$

$$= 624\left(z-\frac{10}{3}\right)\left[\frac{5}{3}+\frac{25}{18(z-10/3)}\right] = 624(5/3)(z-2.5) = 1040z-2600$$

Moment due to gas:

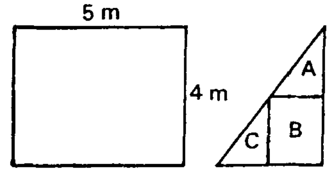
$(3\times144)(2\times5)(5/3) = 7200$ ft·lb ; equating moments, $z = 9.42$ ft ◄

3.18 *Repeat Exer. 3.7.12 for the case where a 2-m-thick layer of oil (s = 0.8) is resting on the water, and replace "water surface" by "oil surface."*

Exer. 3.7.13: A rectangular plate, 5 m H by 4 m V, is submerged in water. Determine the magnitude of the force on one side of the plate and the location of its center of pressure if the top edge is (a) at the water surface; (b) 1 m below the water surface; (c) 100 m below the water surface.

SI

Eq. 3.18: $y_p = y_c+\dfrac{I_c}{y_cA} = y_c+\dfrac{bh^3/12}{y_c(bh)} = y_c+\dfrac{h^2}{12y_c}$



(*a*) $F_A = \gamma h_cA = 0.8(9.81)1(2\times5) = 78.5$ kN at $y_p = 1.333$ m

$F_B = 0.8(9.81)2(2\times5) = 157.0$ kN

at $y_p+2 = \frac{2}{2}+2 = 3$ m below the oil surface

$F_C = 9.81(1/2\times2)(2\times5) = 98.1$ kN at $y_p+2 = \frac{2}{3}(2)+2 = 3.33$ m

$\Sigma F = 78.5+157.0+98.1 = 334$ kN ◄

$334h_p = 78.5(1.333)+157.0(3)+98.1(3.33)$; $h_p = 2.71$ m ◄ (See alternate solution below) (*b*) $F_A = 0.8(9.81)1.5(1\times5) = 58.9$ kN at $y_p = 1.5 + \dfrac{1^2}{12(1.5)} = 1.556$ m

$F_B = 0.8(9.81)2(3\times5) = 235$ kN at $y_p + 2 = \frac{3}{2} + 2 = 3.50$ m

$F_C = 9.81(0.5\times3)(3\times5) = 221$ kN at $y_p + 2 = \frac{2}{3}(3) + 2 = 4.00$ ft

$\Sigma F = 58.9 + 235 + 221 = 515$ kN ◄

$515h_p = 58.9(1.556) + 235(3.50) + 221(4.00)$; $h_p = 3.49$ m ◄

(*c*) $F_A = 0$

$F_B = 0.8(9.81)2(4\times5) = 314$ kN at $y_p + 2 = 100 + 2 = 102$ m

$F_C = 9.81(\frac{4}{2} + 98)(4\times5) = 19620$ kN at $y_p + 2 = 100 + \dfrac{4^2}{12(100)} + 2 = 102.01$ m

$\Sigma F = 0 + 314 + 19\,620 = 19\,930$ kN ◄

$19\,930h_p = 0 + 314(102) + 19\,620(102.01)$; $h_p = 102.0$ m ◄

Alternate solution (*a*):

$F_{oil} = 0.8(9.81)2(4\times5) = 314$ kN at $y_p = \frac{2}{3}(4) = 2.67$ m

$F_{water-oil} = 0.2(9.81)1(2\times5) = 19.62$ kN at $y_p + 2 = \frac{2}{3}(2) + 2 = 3.33$ m

$\Sigma F = 314 + 19.62 = 334$ kN ◄

$334h_p = 314(2.67) + 19.62(3.33)$; $h_p = 2.71$ m ◄

3.19 *Refer to Sample Problem 3.7. If the oil depth were 2 ft rather than 1.5 ft, (a) compute the total force and (b) determine the depth of its center of pressure.*

Sample Prob. 3.7 (refer to it): For component D, $h_c = y_c = 0.424$ ft below the water top surface; for the semicircular end, $D = 2$ ft, $I = 0.393$ ft^4, $I_c = 0.1098$ ft^4.

BG

(*a*) For *A* (oil): $h_c = y_c = 0.5(2 \text{ ft}) = 1.00$ ft below the free oil surface, so

$F_A = \gamma_{oil} h_c A_{oil} = (0.8 \times 62.4)1.00(2 \times 2) = 199.7$ lb

For *B*: $F_B = pA = \gamma hA = (0.8 \times 62.4)2(\pi 1^2/2) = 156.8$ lb

For *D*: $h_c = y_c = 0.424$ ft below the water top surface (as before)

$F_D = \gamma h_c A = 62.4(0.424)\pi 1^2/2 = 41.6$ lb (as before)

Total force $= F_A + F_B + F_D = 398$ lb ◄

(*b*) For semicircular end: $D = 2$ ft, $I = 0.393$ ft^4, $I_c = 0.1098$ ft^4

Below the oil surface: $(y_p)_A = \frac{2}{3}(2.00) = 1.333$ ft; $(y_p)_B = 2 + 0.424 = 2.42$ ft;

Eq. 3.18: $(y_p)_D = 2.00 + 0.424 + \dfrac{0.1098}{(0.424)\pi 1^2/2} = 2.59$ ft

Finally: $Fy_p = F_A(y_p)_A + F_B(y_p)_B + F_D(y_p)_D$

$398y_p = 199.7(1.333) + 156.8(2.42) + 41.6(2.59)$; $y_p = 1.894$ ft ◄

Sec 3.8: Force on a Curved Surface -- *Exercises (10)*

3.8.1 *A vertical-thrust bearing for a large hydraulic gate consists of a 9-in-radius bronze hemisphere mating into a steel hemispherical shell in the gate bottom. What minimum oil pressure will maintain a complete oil film if the vertical thrust on the bearing is 600,000 lb?*

BG

Projected area $= \pi r^2 = 81\pi$ in^2 ; $p = F/A = 600{,}000/(81\pi) = 2360$ psi ◄

3.8.2 *The cross section of a tank is as shown in Fig. X3.8.2. BC is a cylindrical surface with $r = 6$ ft, and $h = 10$ ft. If the tank contains gas at a pressure of 8 psi, determine the magnitude and location of the horizontal- and vertical-force components acting on unit width of tank wall ABC.*

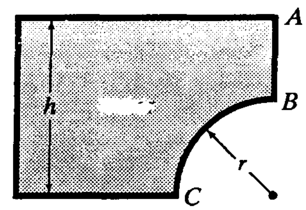


Figure X3.8.2

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Eq. 3.14: $F_x = pA = (8\times144)10 = 11{,}520$ lb/ft ◄

located 5 ft below the top. ◄

$F_z = (8\times144)6 = 6910$ lb/ft ◄ located 3.0 ft to the left of *AB* ◄

3.8.3 *Find the answers called for in Exer. 3.8.2 if $r = 2$ m, $h = 3.5$ m, and the tank contains gas at a pressure of 50 kPa.*

Exer. 3.8.2: In Fig. X3.8.2 BC is a cylindrical surface. Find the magnitude and location of the horizontal- and vertical-force components acting on unit width of tank wall ABC.

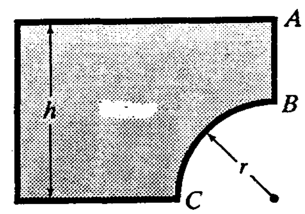

Figure X3.8.2

SI

$F_x = 50(3.5) = 175$ kN/m ◄ at 1.75 m below the top ◄

$F_z = 50(2) = 100$ kN/m ◄ at 1.0 m to the left of *AB* ◄

3.8.4 *A spherical steel tank of 15-m diameter contains gas under a pressure of 350 kPa. The tank consists of two half-spheres joined together with a weld. What will be the tensile force across the weld in kN/m? If the steel is 20.0 mm thick, what is the tensile stress in the steel? Express in kPa and in psi. Neglect the effects of cross-bracing and stiffeners.*

B

Eq. 3.14: Force pulling on weld $= pA = (350\ \text{kN/m}^2)(\pi 15^2/4\ \text{m}^2) = 61\,900$ kN

Force/length $= 61\,900/(\pi D) = 61\,900/(15\pi) = 1313$ kN/m ◄

$$\sigma = \frac{\text{Force/length}}{\text{thickness}} = \frac{1313\ \text{kN/m}}{0.022\ \text{m}} = 65\,600\ \text{kPa}$$ ◄

From inside back cover: (65 600 kPa)(1000 Pa/kPa)(0.000 1450 psi/Pa) = 9520 psi ◄

3.8.5 *Determine the force F required to hold the cone in the position shown in Fig. X3.8.5. Assume the cone is weightless.*

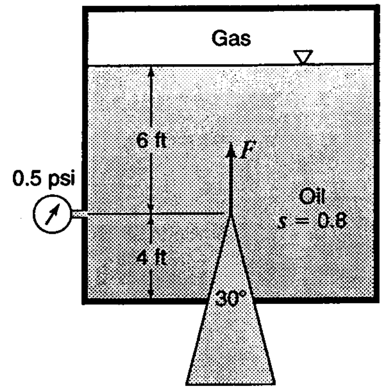

Figure X3.8.5

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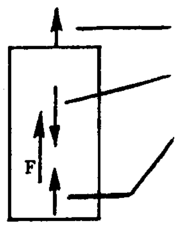
Sketch shows vertical projection of 1/2 cone above opening.

Opening $r = 4\tan 15° = 1.072$ ft

$p_{gas} = 0.5\text{ psi} - 0.8(62.4)6/144 = 0.5\text{ psi} - 2.08\text{ psi} = -1.58\text{ psi}$



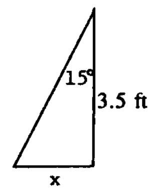
$1.58(144)\pi(1.072)^2 = 821$ lb

$62.4(0.8)\pi(1.072)^2 10 = 1802$ lb

$(62.4)0.8(\pi/3)1.072^2(4) = 240$ lb

$\Sigma F_z = 821 - 1802 + 240 + F = 0$

$F = 740$ lb ◄



3.8.6 *The hemispherical body shown in Fig. X3.8.6 (r = 2 ft) projects into a tank. Find the horizontal and vertical forces acting on the hemispherical projection for the following cases: (a) the tank is full of water with the free surface 5 ft above A; (b) the tank contains CCl_4 (s = 1.59) to the level of A overlain with water having its free surface 5 ft above A; (c) the tank is closed and contains only gas at a pressure of 6 psi; (d) the tank is closed and contains water to the level of A overlain with gas at a pressure of 2 psi. Assume the gas weighs 0.075 lb/ft³.*

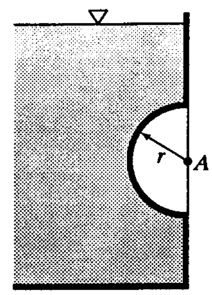


Figure X3.8.6

BG

Projected area on vertical plane $= \pi r^2 = \pi 2^2 = 12.57$ ft²

Table A.8: Volume of hemisphere $= \frac{2}{3}\pi r^3 = \frac{2}{3}\pi 2^3 = 16.76$ ft³

(*a*) Eq. 3.16: $F_x = \gamma h_c A = 62.4(5)(12.57) = 3920$ lb ◄ $F_z = 62.4(16.76) = 1046$ lb ◄

(*b*) $F_x = 3920 + (1.59 - 1.00)62.4\left(\frac{4\times 2}{3\pi}\right)12.57 = 4120$ lb ◄

$F_z = \frac{1046}{2} + \frac{1046}{2}(1.59) = 1354$ lb ◄

(*c*) $F_x = 6(144)12.57 = 10{,}860$ lb ◄ $F_z = 0.075(1/2)(4/3)\pi 2^3 = 1.257$ lb ◄

(*d*) $F_x = 2(144)12.57 + 62.4\left(\frac{4\times 2}{3\pi}\right)\frac{12.57}{2} = 3950$ lb ◄

$F_z = (1046/2) + (16.76/2)0.075 = 523$ lb ◄

3.8.7 *Repeat Exer. 3.8.2 where the tank is open at the top and contains water to a depth h = 10 ft.*

Exer. 3.8.2: In Fig. X3.8.2 BC is a cylindrical surface with r = 6 ft. Find the magnitude and location of the horizontal- and vertical-force components acting on unit width of tank wall ABC.

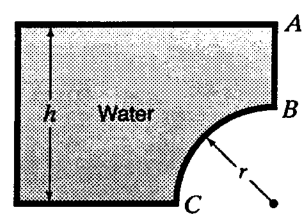
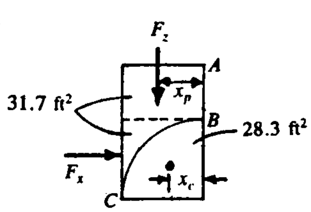


Figure X3.8.2

BG

Eqs. 3.20, 3.16: $F_x = \gamma h_c A = 62.4(1/2\times 10)10 = 3120$ lb/ft ◄

F_x acts at: $y_p = (2/3)10 = 6.67$ ft below surface ◄



$(\pi/4)6^2 = 28.3$ ft² ; $10(6) - 28.3 = 31.7$ ft²

Table A.7: $x_c = 4r/(3\pi) = 2.55$ ft

Eq. 3.21: $F_z = W = 62.4(31.7) = 1980$ lb/ft ◄

Moments of areas about *AB*: $(31.7)x_p + (28.3)2.55 = (10\times 6)3$

F_z acts at: $x_p = 3.40$ ft to left of *AB* ◄

3.8.8 *Repeat Exer. 3.8.2 where r = 2 m, and the tank is open at the top and contains water to a depth h = 3.5 m.*

Exer. 3.8.7: In Fig. X3.8.2 BC is a cylindrical surface. Find the magnitude and location of the horizontal- and vertical-force components acting on unit width of tank wall ABC.

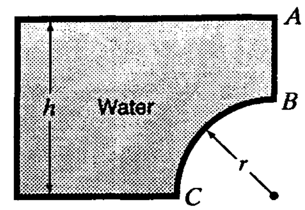
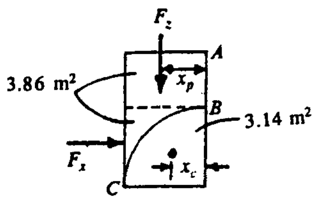


Figure X3.8.2

SI

Eqs. 3.20, 3.16: $F_x = \gamma h_c A = 9.81(1/2 \times 3.5)3.5 = 60.1$ kN/m ◄

F_x acts at: $y_p = (2/3)3.5 = 2.33$ m below surface



$(\pi/4)2^2 = \pi$ m² ; $3.5(2) - \pi = 3.86$ m²

Table A.7: $x_c = 4r/(3\pi) = 0.849$ m

Eq. 3.21: $F_z = W = 9.81(3.86) = 37.9$ kN/m ◄

Moments of areas about AB: $(3.86)x_p + (\pi)0.849 = (3.5 \times 2)1$

F_z acts at: $x_p = 1.123$ m to the left of AB ◄

3.8.9 *A tank with vertical ends contains water and is 6 m long normal to the plane of Fig. X3.8.9. The sketch shows a portion of its cross-section where MN is one-quarter of an ellipse with semiaxes b and d. If a = 1.0 m, b = 2.5 m, and d = 4 m, find, for the surface represented by MN, the magnitude and position of the line of action of (a) the horizontal component of force; (b) the vertical component of the force; (c) the resultant force and its direction relative to the horizontal.*

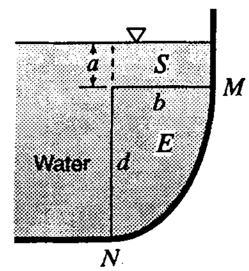


Figure X3.8.9

SI

(*a*) $h_c = a + d/2 = 1 + 4/2 = 3.00$ m

Eqs. 3.20, 3.16: $F_x = \gamma h_c A = (9.81)3.00(4 \times 6) = 706$ kN ◄ Acts at:

$$h_p = y_c + \frac{I_c}{y_c A} = y_c + \frac{h^2}{12 y_c} = 3.00 + \frac{4^2}{12(3.0)} = 3.44 \text{ m below surface} \blacktriangleleft$$

(*b*) Using Table A.7: $A_E = \dfrac{2.5\pi 4}{4} = 7.85$ m² ; $A_S = 1.0 \times 2.5 = 2.5$ m²

Eq. 3.21: $F_y = W = 6(7.85 + 2.5)9.81 = 609$ kN ◄

For (E), quarter ellipse (Table A.7): $x_c = \dfrac{4b}{3\pi} = \dfrac{4(2.5\text{ m})}{3\pi} = 1.061$ m to right of N

For (S), water above b: $x_c = b/2 = 2.5/2 = 1.25$ m

For both together: $x_c = \dfrac{(Ax_c)_E + (Ax_c)_S}{A_E + A_S} = \dfrac{7.85(1.061) + 2.5(1.25)}{7.85 + 2.5} = 1.107$ m

∴ Vertical component acts at 1.107 m to the right of N ◄

(*c*) $R = \sqrt{F_x^2 + F_y^2} = \sqrt{706^2 + 609^2} = 933$ kN ◄

Acts through intersection of F_x and F_y. ◄ $\theta = \tan^{-1}(F_y/F_x) = \tan^{-1}(609/706) = 40.8°$ ◄

3.8.10 *Find the answers called for in Exer. 3.8.9 if a = 2 ft, b = 6 ft, d = 9 ft, the tank is 12 ft long, and MN represents a parabola with vertex at N.*

Exer. 3.8.9: The tank length is perpendicular to Fig. X3.8.9, it has vertical ends, and contains water. Find, for the surface represented by MN, the magnitude and position of the line of action of (a) the horizontal component of force; (b) the vertical component of the force; (c) the resultant force and its direction with the horizontal.

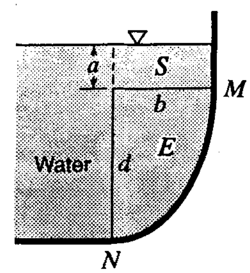


Figure X3.8.9

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(*a*) $h_c = a + d/2 = 2 + 9/2 = 6.50$ ft

Eqs. 3.20, 3.16: $F_x = \gamma h_c A = (62.4)6.50(9\times 12) = 43{,}800$ lb ◀

Acts at: $h_p = y_c + \dfrac{I_c}{y_c A} = y_c + \dfrac{h^2}{12 y_c} = 6.50 + \dfrac{9^2}{12(6.50)} = 7.54$ ft below surface ◀

(*b*) Using Table A.7: $A_E = \frac{2}{3}(9\times 6) = 36\ \text{ft}^2$; $A_S = 2\times 6 = 12\ \text{ft}^2$

Eq. 3.21: $F_y = W = 12(36+12)62.4 = 35{,}900$ lb ◀

For (*E*), semi-parabola (Table A.7): $x_c = \frac{3}{8}b = \frac{3}{8}(6) = 2.25$ ft to right of *N*

For (*S*), water above *b*: $x_c = b/2 = 6/2 = 3.00$ ft

For both together: $x_c = \dfrac{(Ax_c)_E + (Ax_c)_S}{A_E + A_S} = \dfrac{36(2.25)+12(3.00)}{36+12} = 2.44$ ft

∴ Vertical component acts at 2.44 ft to the right of *N* ◀

(*c*) $R = \sqrt{F_x^2 + F_y^2} = \sqrt{43{,}800^2 + 35{,}900^2} = 56{,}700$ lb ◀

Acts through intersection of F_x and F_y. ◀ $\theta = \tan^{-1}(F_y/F_x) = \tan^{-1}(35{,}900/43{,}800) = 39.4°$ ◀

Sec. 3.8: Force on Curved Surface -- *Problems* 3.20–3.23

3.20 *(a) Find the horizontal and vertical forces per foot of width acting on the Tainter gate shown in Fig. P3.20. (b) Locate the horizontal force and indicate the line of action of the vertical force without actually computing its location. (c) Locate the vertical force (hint: consider the resultant).*

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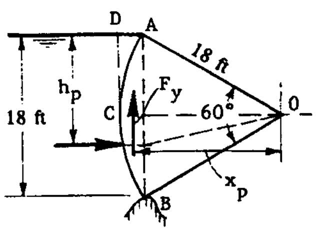
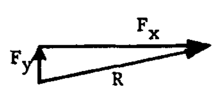




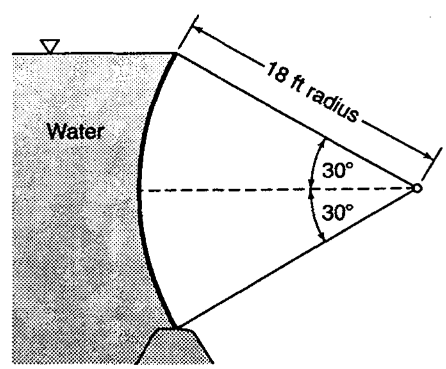


Figure P3.20

(*a*) Eq. 3.16: $F_x = \gamma h_c A = 62.4(18/2)18 = 10{,}110$ lb/ft ◀

Eq. 3.21: $F_y = W = \gamma(\text{Area } ABCD - \text{Area } ACD) = 62.4(\text{Area of segment } ABC)$

$$= 62.4\left[\pi(18)^2\frac{60}{360} - \frac{18\times 18\cos 30°}{2}\right] = 1831\ \text{lb/ft}$$ ◀

(*b*) $h_p = (2/3)18 = 12$ ft ◀ F_y acts through the centroid of segment *ABC* ◀

(*c*) Since all forces are normal to the circular arc, the resultant *R* must pass through *O*, and x_p may be determined from similarity of force and space triangles. Thus

$\dfrac{x_p}{F_x} = \dfrac{(12-18/2)}{F_y}$; $x_p = 3\left(\dfrac{10{,}110}{1831}\right) = 16.56$ ft ◀

3.21 *The cross-section of a gate is shown in Fig. P3.21. Its dimension normal to the plane of the paper is 8 m, and its shape is such that $x = 0.2y^2$. The gate is pivoted about O. Develop analytic expressions in terms of the water depth y upstream of the gate for the following: (a) horizontal force; (b) vertical force; (c) clockwise moment acting on the gate. Compute (a), (b), and (c) for the case where the water depth is 2.5 m.*

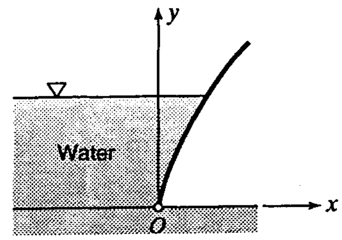


Figure P3.21

SI

(*a*) For 8-m width: $F_x = \gamma h_c A = \gamma(y/2)10y = 4\gamma y^2$ ◀

For $y = 2.5$ m, $F_x = 4(9.81)2.5^2 = 245$ kN ◀

(*b*) $F_z = W = \gamma(\text{side area})8 = 10\gamma\int_0^y x\,dy = 8\gamma\int_0^y 0.2y^2 dy = \gamma(0.2y^3/3)8 = 0.533\gamma y^3$ ◀

For y = 2.5 m, $F_z = 0.533(9.81)2.5^3 = 81.8$ kN ◀

(*c*) Clockwise $M = 4\gamma y^2(1/3)y + 8\gamma\int x(x/2)\,dy = (4/3)\gamma y^3 + 4\gamma(0.04y^5/5) = (4/3)\gamma y^3 + 0.0320\gamma y^5$ ◀

For $y = 2.5$ m, $M = (4/3)(9.81)2.5^3 + 0.0320(9.81)2.5^5 = 235$ kN·m clockwise ◀

3.22 *A tank has an irregular cross section as shown in Fig. P3.22. Determine as accurately as possible the magnitude and location of the horizontal- and vertical-force components on a one-meter length of the wall ABCD when the tank contains water to a depth of 2 m. To determine areas, use a planimeter or count squares (0.25 m grid); make a cardboard cutout, or take approximate moments of the squares, to locate the centroid.*

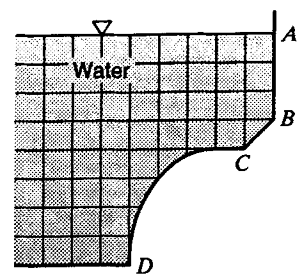


Figure P3.22

SI

Eqs. 3.20, 3.16: $F_x = \gamma h_c A = 9.81(1/2\times 2)2 = 19.62$ kN/m ◀

F_x acts at: $y_p = (2/3)2 = 1.333$ m below the surface ◀

Planimeter or count squares to determine water column area A_s above *BCD*.

$A_s = 26.1$ squares $= 1.630\ \text{m}^2$ (by calculation!).

Eq. 3.21: $F_z = W = \gamma A_s = 9.81(1.630) = 15.99$ kN/m ◀

Make a cardboard cutout or take moments of squares to locate the centroid of A_s:
$x_p = 0.690$ m (by calculation!) F_z acts about 0.690 m to left of *AB* ◀

3.23 *Repeat Exer. 3.8.2 where the tank contains 4 ft of water overlain with a gas that is under a pressure of 0.8 psi.*

Exer. 3.8.2: In Fig. X3.8.2 h = 10 ft and BC is a cylindrical surface with r = 6 ft. Find the magnitude and location of the horizontal- and vertical-force components acting on unit width of tank wall ABC.

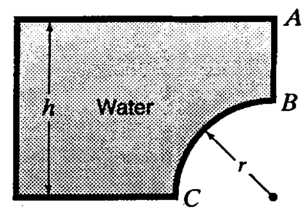


Figure X3.8.2

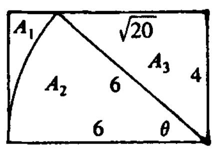
BG

Effect of gas pressure is transmitted throughout the water.

$F_x = F_{gas} + F_{water} = (0.8\times 144)10 + 62.4(1/2\times 4)4$

$= 1152 + 499 = 1651$ lb/ft ◄

Moments about the top: $(1152 + 499)z_p = 1152(5) + 499(10-4/2)$; $z_p = 5.91$ ft below top of tank ◄



$\sin\theta = 4/6$; $\theta = 41.8°$

$A_2 = \pi 6^2(41.8/360) = 13.14$ ft² ; $A_3 = 0.5(4)\sqrt{20} = 8.94$ ft²

$A_1 + A_2 + A_3 = A$; $A_1 + 13.14 + 8.94 = 6(4)$

$A_1 = 1.921$ ft² = water area vertically above curved surface.

$F_z = (0.8\times 144)6 + 62.4(1.921) = 691 + 119.8 = 811$ lb/ft ◄

Take moments about *AB* (or use cardboard cutouts) to find centroid of F_z:

Using Table A.7: $x_{c2} = \dfrac{4(6)180}{3(41.8)\pi}\sin\dfrac{41.8°}{2}\cos\dfrac{41.8°}{2} = 3.65$, $x_{c3} = \dfrac{\sqrt{20}}{3} = 1.491$ ft from AB

$1.921x_{c1} + 13.14(3.65) + 8.94(1.491) = 24(3)$; from which $x_{c1} = 5.55$ ft from *AB*

$(691 + 119.8)x_p = (691)3 + (119.8)5.55$; $x_p = 3.38$ ft from *AB* ◄

Sec 3.9: Buoyancy and Stability of Submerged and Floating Bodies -- *Exercises (12)*

3.9.1 *A balloon weighs 160 lb and has a volume of 7200 ft³. It is filled with helium, which weighs 0.0112 lb/ft³ at the temperature and pressure of the air, which in turn weighs 0.0807 lb/ft³. What load will the balloon support, or what force in a cable would be required to keep it from rising?*

BG

(Buoyant force) (Wt of helium) (Balloon)

$\Sigma F_z = 0$; $7200(0.0807) - 7200(0.0112) - 160 - T = 0$

Required force in the cable, $T = 340$ lb ◄

3.9.2 *For the conditions shown in Figure X3.9.2, find the force F required to lift the concrete-block gate if the concrete weighs 23.6 kN/m³. Neglect friction.*

SI

$\Sigma F_z = 0$; F + (salt w.) − (fresh w.) − (conc. block) = 0

$F + 9.81(1.025)1.8\pi(0.3)^2 - 9.81(3)\pi 0.3^2 - 23.6(0.3)\pi 0.3^2 = 0$

$F + 5.12 - 8.32 - 2.00 = 0$; $F = 5.21$ kN ◄

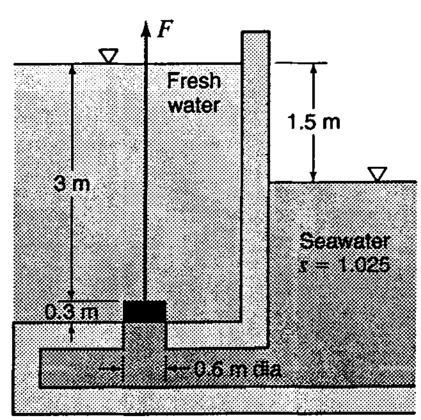


Figure X3.9.2

3.9.3 *An iceberg in the ocean floats with one-eighth of its volume above the surface. What is its specific gravity relative to ocean water, which weight 64 lb/ft³. What portion of its volume would be above the surface if the ice were floating in pure water?*

BG

Let $\forall$ = total volume of iceberg ; volume submerged = $\forall_1 = (7/8)\forall$

$W = B$; $\gamma'\forall = (7/8)\gamma\forall$; $\therefore\ s' = (\gamma'/\gamma) = 7/8 = 0.875$ relative to ocean water ◄

and $\gamma' = (7/8)64 = 56.0$ lb/ft³, $s = 56.0/62.4 = 0.897$, and so

$(1 - 0.897)100\% = 10.26\%$ of its volume would be above the water surface in fresh water. ◄

3.9.4 *Determine the volume of an object that weighs 200 N in water and 300 N in oil (s = 0.88). What is the specific weight of the object?*

SI

$W - \forall(9810) = 200$ N (1)

$W - \forall(0.88 \times 9810) = 300$ N (2)

Solve (1) and (2) simultaneously to find $W = 1033$ N and $\forall = 0.0849$ m³ ◄

Specific weight = 1033/0.0849 = 12 160 N/m³ ◄

3.9.5 *An 8-in diameter solid cylinder 3 in high weighing 3.4 lb is immersed in liquid (γ = 52 lb/ft³) contained in a tall, upright metal cylinder of 9 in diameter (Fig. X3.9.5). Before immersion, the liquid was 3 in deep (= x + z). At what level will the solid cylinder float? Find the distance z between the bottoms of the two cylinders.*

BG

Volume A = Volume B ;

so $\pi 4^2 x = \pi(4.5^2 - 4^2)y$; $16x = 4.25y$ or $x = 0.266y$

Weight $= F_B$ i.e. $3.4 = 52\pi(4/12)^2(x+y)/12$ so $x + y = 2.25$ in

Solving, $y = 1.776$ in, $x = 0.472$ in, $\therefore\ z = 3 - x = 2.53$ in ◄

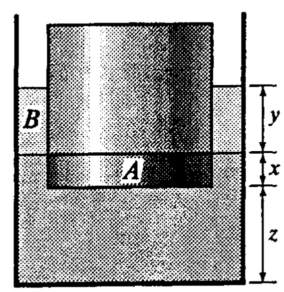


Figure X3.9.5

3.9.6 *A metal block 1.5 ft square and 1 ft deep is floated on a body of liquid which consists of a 10-in layer of water above a layer of mercury. The block metal weighs 120 lb/ft³. (a) What is the position of the bottom of the block? (b) If a downward vertical force of 600 lb now acts on the center of this block, what is the new position of the bottom of the block? Assume that the tank containing the fluid is of infinite dimensions.*

BG

(*a*) Block weighs $(1 \times 1.5^2)120 = 270$ lb

$270 = [(x/12)(13.6 \times 62.4) + (10/12)62.4]1.5^2$; $x = 0.962$ inches into the mercury ◄

(*b*) In this case the top of the block is below the water surface.

Thus, $600 + 270 = \left[\frac{x}{12}(13.6 \times 62.4) + \frac{12-x}{12}62.4\right]1.5^2$; $x = 4.95$ inches into the mercury ◄

3.9.7 *Two spheres, each of 1.5 m diameter, weigh 8 and 24 kN respectively. They are connected with a short rope and placed in water. (a) What is the tension in the rope and what portion of the lighter sphere's volume portrudes from the water? (b) What should be the weight of the heavier sphere in order for the lighter sphere to float halfway out of the water? Assume that the sphere volumes remain constant.*

SI

(*a*) Lower sphere: $F_B = \gamma(\text{vol. displaced}) = 9.81(\pi/6)1.5^3 = 17.34$ kN

$T = 24 - 17.34 = 6.66$ kN ◄

Upper sphere: $F_B = 8\text{ kN} + T = 14.66$ kN

% above water = (17.34 − 14.66)/17.34 = 15.41% of volume ◄

(*b*) Upper sphere: $F_B = (1/2)\gamma(\text{vol. sphere}) = (1/2)9.81(\pi/6)1.5^3 = 8.67$ kN

$W = B$; $W_L + 8 = 17.34 + 8.67$; $W_L = 18.00$ kN ◄

3.9.8 *A hydrometer (Fig. 3.22a) consists of a 6-mm-diameter cylinder of length 180 mm attached to a 20-mm-diameter weighted sphere. The cylinder has a mass of 0.6 g and the mass of the sphere is 6.4 g. At what level will this device float in liquids having specific gravities 0.8, 1.0, and 1.2? Is the scale spacing on the cylindrical stem uniform? Why or why not?*

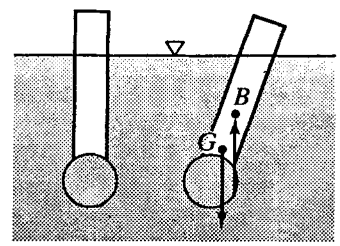


Figure 3.22(a)

SI

Vol of sphere = $(\pi/6)(20)^3 = 4190\text{ mm}^3$

Volume of submerged cylinder = $\pi 3^2 y = 28.3y\text{ mm}^3$, where y is the submerged length of the cylinder in mm.

Mass of hydrometer = 7 grams = $s(4190 + 28.3y)/1000$ grams ; $y = [(7000/s) - 4190]/28.3$ mm

If $s = 0.8$, $y = 161.3$ mm ◄

diff = 61.9 mm

If $s = 1.0$, $y = 99.4$ mm ◄

diff = 41.2 mm

If $s = 1.2$, $y = 58.2$ mm ◄

Scale spacing is <u>not</u> uniform because submergence is not directly proportional to specific gravity, as from the above equation: $y = 7000/(28.3s) - 4180/28.3$ and $dy/ds = -7000/(28.3s^2)$ ◄

3.9.9 *A cylindrical bucket of 250 mm diameter and 400 mm high weighing 20.0 N contains oil (s = 0.80) to a depth of 180 mm. (a) When placed to float in water, what will be the immersion depth to the the bottom of the bucket? (b) What is the maximum volume of oil that the bucket can hold and still float?*

SI

(*a*) $\Sigma F_z = W_B + W_{oil} - \gamma_{water} h\pi r^2 = 0$; $0.020\text{ kN} + (0.8)9.81\pi(0.125)^2 0.18 - 9.81h\pi(0.125)^2 = 0$

$0.020 + 0.0693 - 0.482h = 0$; $h = 0.893/0.482 = 0.1855\text{ m} = 185.5$ mm ◄

(*b*) When immersed 400 mm, the oil depth h is given by

$0.020 + (0.8)9.81\pi(0.125)^2 h - 9.81(0.40)\pi(0.125)^2 = 0$

$0.020 + 0.385h - 0.1926 = 0$; $h = 0.1726/0.385 = 0.448\text{ m} = 448$ mm

448 mm > 400 mm, therefore the bucket will float when full of oil

Max vol = $\pi(0.125)^2 0.40 = 0.01963\text{m}^3 = 19.63$ L ◄

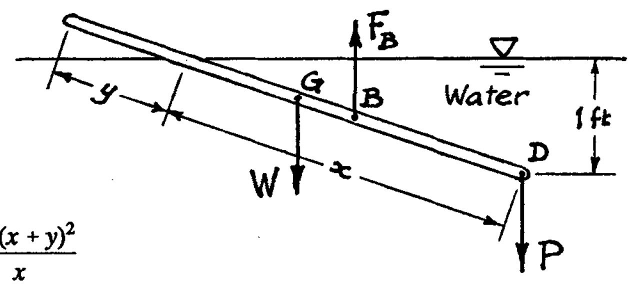
3.9.10 *End D of an 8-ft-long, uniformly thin wooden rod (s = 0.7) is held 1 ft below the surface of still water. (a) How much of the rod remains above the water surface? (b) If the rod diameter is 1 inch, what force at D is required to hold it in place?*

BG

(*a*) Let A = cross-sectional area of rod.

Let γ = specific weight of water.

$$W = A(x+y)0.7\gamma \ ; \quad F_B = Ax\gamma$$



$$\Sigma \mathrm{Mts}_D = 0: \quad F_B\left(\frac{x}{2}\right) = W\left(\frac{x+y}{2}\right)$$

$$\therefore \quad x = \frac{W}{F_B}(x+y) = \frac{A(x+y)0.7\gamma}{Ax\gamma}(x+y) = \frac{0.7(x+y)^2}{x}$$

$$x^2 = 0.7(x+y)^2 \ ; \quad x = \sqrt{0.7}(x+y) = \sqrt{0.7}(8 \text{ ft}) = 6.69 \text{ ft}$$

$$y = 8 - x = 8 - 6.69 = 1.307 \text{ ft} \quad ◀$$

(*b*) $A = (\pi/4)1^2 = 0.785 \text{ in}^2 = 0.0654 \text{ ft}^2$

$$F_B = \gamma\forall = \gamma Ax = 62.4(0.0654)6.69 = 27.3 \text{ lb}$$

$$W = 0.7\gamma\forall' = 0.7(62.4)0.0654(8) = 22.9 \text{ lb}$$

$$\Sigma F_y = 0: \quad P = F_B - W = 27.3 - 22.9 = 4.47 \text{ lb} \quad ◀$$

3.9.11 *A solid, half-cylinder-shaped log, of 1.50 ft radius and 10 ft long, floats in water with the flat face up (Fig. X3.9.11). (a) If the draft (immersion depth of the lowest point) is 0.90 ft, what is the uniform specific weight of the log? (b) The log tilts about its axis (zero net applied force) by less than 23°. Is it in stable equilibrium? Justify your answer with a sketch and logic. (c) If the log tilts by 20° (right side down; zero net applied force), what is the magnitude and sense of any moment that results?*

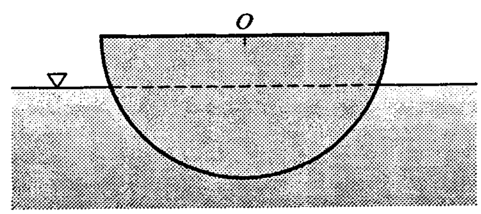
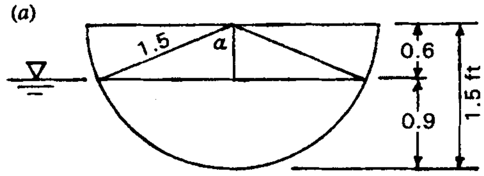


Figure X3.9.11

BG

(*a*)



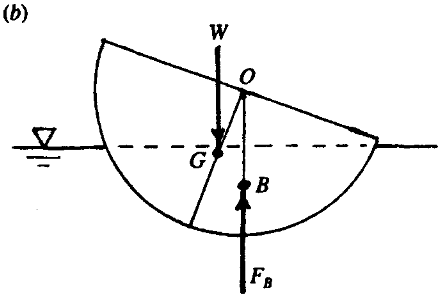
(*a*) $$\gamma_s \pi r^2 L/2 = 62.4\left[\frac{2\alpha}{360}\pi r^2 - 2\left(\frac{1}{2}\right)0.6\sqrt{1.5^2 - 0.6^2}\right]L$$

$$\cos\alpha = 0.6/1.5 = 0.4 \ ; \qquad \alpha = 66.4°$$

$$\therefore \qquad \gamma = 62.4(2.61 - 0.825)/3.53 = 31.5 \text{ lb/ft}^3 \quad ◀$$

(*b*) When the body tilts per the sketch, B remains below axis O, but G moves to the left. So W and F_B create a righting moment. Therefore, yes the log is in stable equilibrium. ◀

(*b*)



(*c*) From sketch (*b*) above and Table A.7:

$$OG = 4r/(3\pi) = 4(1.5)/(3\pi) = 0.637 \text{ ft}$$

$$\text{Moment} = Wa = W(OG \sin 20°)$$

$$= 31.5(10\pi 1.5^2/2)(0.637 \sin 20°)$$

$$= 242 \text{ ft·lb counterclockwise} \quad ◀$$

3.9.12 *A solid, half-cylinder-shaped log, of 0.48 m radius and 2.5 m long, floats in water with the flat face up (Fig. X3.9.11). (a) If the draft (immersion depth of the lowest point) is 0.30 m, what is the uniform specific weight of the log? (b) The log tilts about its axis (zero net applied force) by less than 22°. Is it in stable equilibrium? Justify your answer with a sketch and logic. (c) If the log tilts by 18° (left side down; zero net applied force), what is the magnitude and sense of any moment that results?*

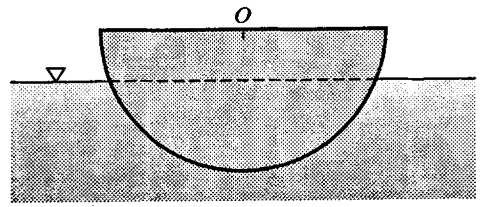
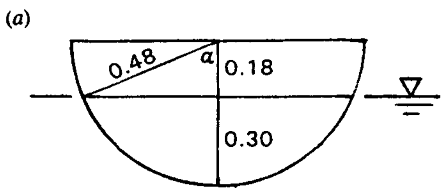


Figure X3.9.11

SI

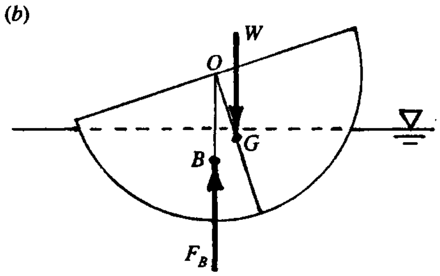


(a) $\gamma\pi r^2 L/2 =$

$$(9810\,\text{N/m}^3)\left[\frac{2\alpha}{360}\pi r^2 - 2\left(\frac{1}{2}\right)0.18\sqrt{0.48^2-0.18^2}\right]L$$

$\cos\alpha = 0.18/0.48 = 0.375, \quad \alpha = 68.0°$

$\therefore \quad \gamma = 9810(0.273-0.0801)/0.3619 = 5240\,\text{N/m}^3$ ◀



(b) As the body tilts as per sketch, B remains below axis O, but G moves to the right. So W and F_B create a righting moment. ∴ Yes, the log is in stable equilibrium. ◀

(c) From sketch (b) above and Table A.7:

$OG = 4r/(3\pi) = 0.204$ m

Moment $= Wa = W(OG \sin 18°)$

$= 5240(2.5\pi 0.48^2/2)(0.204 \sin 18°)$

$= 298$ N·m clockwise ◀

Sec. 3.9: Buoyancy and Stability of Submerged and Floating Bodies -- *Problems* 3.24−3.30

3.24 *Find the approximate value of the maximum specific gravity of liquid for which the device of Exer. 3.9.8 will be stable.*

Exer. 3.9.8: The hydrometer in Fig. 3.22a consists of a 6-mm-diameter cylinder 180 mm long attached to a 20-mm-diameter weighted sphere. The cylinder has a mass of 0.6 g and mass of the sphere is 6.4 g.

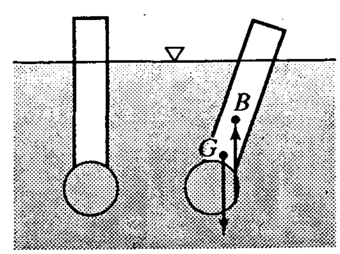


Figure 3.22(a)

SI

The device is stable if its center of buoyancy is above its center of gravity.
The critical point of stability occurs when the line of action of the weight and the buoyancy forces are coincident when the device is tilted.
This critical stability occurs when the center of gravity = the center of buoyancy, i.e., $\bar{y}_g = \bar{y}_b$.
Use the center of the sphere as the point of reference.

$$\bar{y}_g = \frac{6.4(0) + 0.6(10 + 90)}{6.4 + 0.6} = 8.57 \text{ mm above center of sphere}$$

V displaced by sphere $= \dfrac{\pi D^3}{6} = 4190\text{ mm}^3$, V displaced by cylinder $= \left(\dfrac{\pi}{4}\right)6^2 y = 28.3y\text{ mm}^3$

$$\bar{y}_b = \frac{4190(0) + 28.3y(10 + y/2)}{4190 + 28.3y}$$

So for critical stability: $8.57 = \dfrac{28.3y(10 + y/2)}{4190 + 28.3y}$

From this quadratic equation, $y = 49.0$ mm or -51.8 mm (meaningless). ∴ $y = 49.0$ mm

Also, mass of hydrometer = mass of fluid displaced, i.e., $6.4 + 0.6 = s(4190 + 28.3y)/1000$

Thus for $y = 49.0$ mm, the maximum measurable specific gravity, $s = 1.256$ ◀

3.25 *A 2.0-ft³ object weighing 650 lb is attached to a balloon of negligible weight and released in the ocean (γ = 64 lb/ft³). The balloon was originally inflated with 5.0 lb of air to a pressure of 20 psi. To what depth will the balloon sink? Assume that air temperature within the balloon stays constant at 50°F.*

BG

Find the volume occupied by 5 lb of air. Table A.5 for air: $R = 1715\ \text{ft}^2/(\text{sec}^2\cdot°R)$

Eq. 2.5: $\gamma = \dfrac{(14.7+20)144(32.2)}{1715(460+50)} = 0.1840$ pcf. Thus 5 lb of air occupies $5/0.1840 = 27.2\ \text{ft}^3 = \mathcal{V}_1$

At equilibrium depth: $\Sigma F_z = B - W = 0 = 64(2.0 + \mathcal{V}_2) - (650 + 5)$

so new volume of balloon, $\mathcal{V}_2 = 8.23\ \text{ft}^3$

From Eq. 2.4 with constant temperature: $pv = \text{const} = p\mathcal{V}/m$

and so for constant mass of air, $p\mathcal{V} = \text{const}$, i.e. $p_1\mathcal{V}_1 = p_2\mathcal{V}_2$

$(14.7 + 20)144(27.2) = (14.7 + p_2)144(8.23)$; $p_2 = 99.8$ psi

Depth $h = p_2/\gamma = (99.8)144/64 = 225$ ft ◄

3.26 *Work Prob. 3.25 with all data the same except assume the balloon was originally inflated with 5.0 lb of air to a pressure of 10 psi. In this latter case the balloon is more elastic because a lower pressure is obtained with the same amount of air.*

Prob. 3.25: A 2.0-ft³ object weighing 650 lb is attached to a balloon of negligible weight and released in the ocean (γ = 64 lb/ft³). To what depth will the balloon sink? Assume that air temperature within the balloon stays constant at 50°F.

BG

Find the volume occupied by 5 lb of air. Table A.5 for air: $R = 1715\ \text{ft}^2/(\text{sec}^2\cdot°R)$

Eq. 2.5: $\gamma = \dfrac{(14.7+10)144(32.2)}{1715(460+50)} = 0.1309$ pcf. This 5 lb of air occupies $5/0.1309 = 38.2\ \text{ft}^3 = \mathcal{V}_1$

At equilibrium depth: $\Sigma F_z = B - W = 0 = 64(2 + \mathcal{V}_2) - (650 + 5)$

so new volume of balloon, $\mathcal{V}_2 = 8.23\ \text{ft}^3$

From Eq. 2.4 with constant temperature: $pv = \text{const} = p\mathcal{V}/m$

and so for constant mass of air, $p\mathcal{V} = \text{const}$, i.e. $p_1\mathcal{V}_1 = p_2\mathcal{V}_2$

$(14.7 + 10)144(38.2) = (14.7 + p_2)144(8.23)$; $p_2 = 99.8$ psi

Depth $h = p_2/\gamma = (99.8)144/64 = 225$ ft ◄

3.27 *A wooden pole weighing 2 lb/ft has a cross-sectional area of 6.7 in² and is supported as shown in Fig. P3.27. The hinge is frictionless. Find θ.*

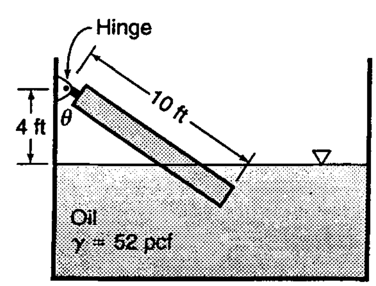


Figure P3.27

BG

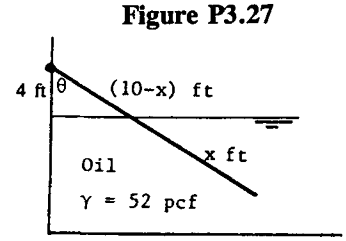
$W = 2(10) = 20$ lb at $(1/2)10\sin\theta = 5\sin\theta$ from hinge

$M_P = (20)5\sin\theta = 100\sin\theta$ ft·lb clockwise

Let x = immersed length of pole.

$F_B = \gamma_{\text{oil}}Ax = 52(6.7/144)x = 2.42x$ at $(10 - x/2)\sin\theta$

$M = 2.42x(10 - x/2)\sin\theta$ counterclockwise



Since $\Sigma M = 0$, we have $100 = 2.42x(10 - x/2) = 24.2x - 1.210x^2$

Solving quadratic equation, $x = 14.16$ ft (impossible) or 5.84 ft

$\cos\theta = \dfrac{4}{10-x} = \dfrac{4}{10-5.84} = 0.961$; $\theta = 16.12°$ ◄

3.28 *A rectangular block of uniform material and length L = 3 ft, width b = 1.25 ft, and depth d = 0.20 ft, is floating in a liquid. It assumes the position shown in Fig. P3.28 when a uniform vertical load of 1.30 lb/ft is applied at P. (a) Find the weight of the block. (b) If the load is suddenly removed, what is the righting moment before the block starts to move? (Hint: Refer also to Fig. 3.19.)*

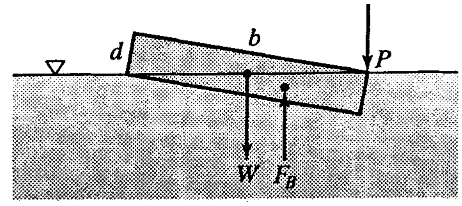


Figure P3.28

BG

$OP = \sqrt{1.25^2 + 0.20^2} = 1.266$ ft ; $BK = (2/3)d/2 = d/3$

$BK_x = BK\sin\alpha = (0.2/3)(0.2/1.266) = 0.01053$ ft

$GB_x = GK - BK_x = 1.266/6 - 0.01053 = a = 0.200$ ft

$BP_x = KP + BK_x = 1.266/3 + 0.01053 = 0.432$ ft

(*a*) Moments about *B*: $W(GB_x) = 1.3(3)BP_x$

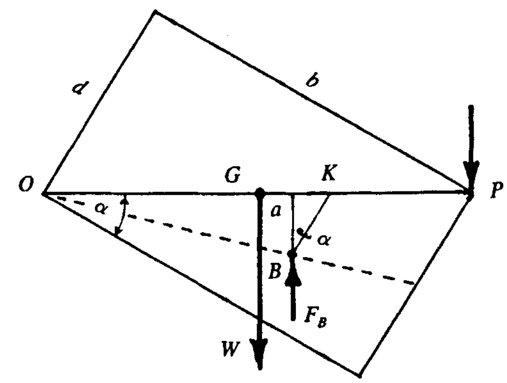
$W(0.200) = 3.9(0.432)$; $W = 8.41$ lb ◄

(*b*) ΣF_y: $F_B = W + 3.9$; $\therefore W < F_B$

Fig 3.19 for $W < F_B$:

Righting Moment $= Wa = 8.41(0.200)$

$= 1.687$ lb·ft counterclockwise ◄



3.29 *A rectangular block of uniform material and length L = 800 mm, width b = 300 mm, and depth d = 50 mm, is floating in a liquid. It assumes the position shown in Fig. P3.28 when a uniform vertical load of 20 N/m is applied at P. (a) Find the weight of the block. (b) If the load is suddenly removed, what is the righting moment before the block starts to move? (Hint: Refer also to Fig. 3.19.).*

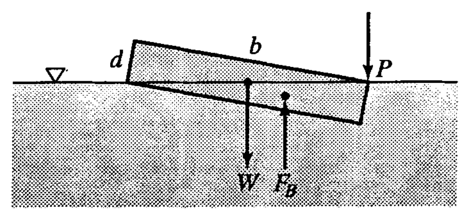


Figure P3.28

SI

$OP = \sqrt{300^2 + 50^2} = 304$ mm ; $BK = (2/3)d/2 = d/3$

$BK_x = BK\sin\alpha = (50/3)(50/304) = 2.74$ mm

$GB_x = GK - BK_x = 304/6 - 2.74 = 47.9$ mm $= a$

$BP_x = KP + BK_x = 304/3 + 2.74 = 104.1$ mm

(*a*) Moments about *B*: $W(GB_x) = 20(0.8)BP_x$

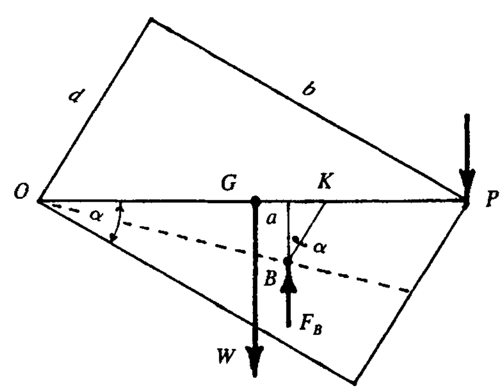
$W(47.9) = 16(104.1)$; $W = 34.7$ N ◄

(*b*) ΣF_y: $F_B = W + 16$ N , $\therefore W < F_B$

Fig 3.19 for $W < F_B$:

Righting Moment $= Wa = 34.7(0.0479$ m$)$

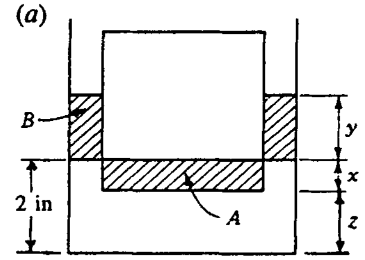
$= 1.666$ N·m counterclockwise ◄



3.30 *A solid block, 4 in wide by 4 in deep and 3 in high weighs 0.90 lb. It floats in liquid (γ = 55 lb/ft³) inside a cubic container of side 5 in. Before immersion the liquid was 2 in deep. (a) At what level will the block float? Find the distance z from the bottom of the block to the bottom of the container. (b) If the block is tilted by a couple (no net force) to an angle of 15° so that two sides remain vertical, what will be the righting moment in lb·in?*

BG

(a)



Volume A = Volume B

$x(4^2) = y(5^2 - 4^2)$

$x = 0.563y$

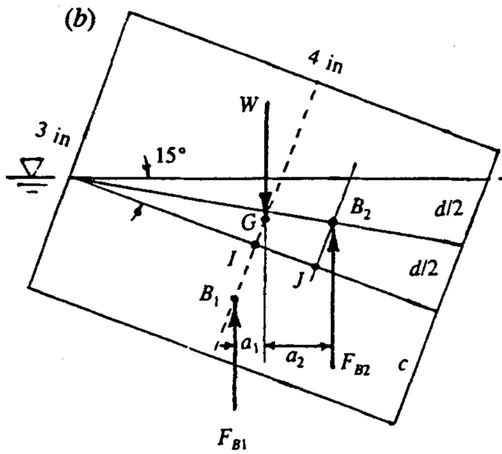
$F_B = W: \quad \frac{(x+y)4^2}{12^3}(55) = 0.9$

$x + y = 1.767$

Solving 2 equations with 2 unknowns: $y = 1.131$ in ; $x = 0.636$ in

$x + z = 2$ in (given). $\therefore z = 2 - x = 2 - 0.636 = 1.364$ in ◄

(b)



$d = 4 \tan 15° = 1.072$ in

Displaced volume = $(1/2)d(4^2) + c(4^2) = (x+y)4^2$

$c = (x+y) - (d/2) = 1.767 - (1.072/2) = 1.231$ in

Divide buoyancy into two components, F_{B1} and F_{B2} at B_1 and B_2 respectively. Since $W = \Sigma F_B = 0.90$ lb, either can be used to calculate the righting moment.

$F_{B1} = \gamma(Vol) = 55(4^2)c/12^3 = 0.627$ lb

$F_{B2} = 55(4^2)(d/2)/12^3 = 0.273$ lb

$IJ = 4/6 = 0.667$ in, $\quad 1.5 - c = 1.5 - 1.231 = 0.269$ in

$B_2J = (2/3)(d/2) = d/3 = 1.072/3 = 0.357$ in

$B_1G = 1.5 - c/2 = 0.884$ in ; $\quad a_1 = BG_1 \sin 15° = 0.229$ in

$a_2 = IJ\cos 15° + (B_2J - GI)\sin 15° = 0.667$ in

Moments about G: Righting moment = $F_2a_2 - F_1a_1$

$= 0.273(0.667) - 0.627(0.229) = 0.0385$ lb·in counterclockwise ◄

Sec 3.10: Fluid Masses Subjected to Acceleration – *Exercises (8)*

3.10.1 *What must be the hydrostatic gage pressure at a depth of 8 inches in a bucket of oil (s = 0.86) that is in an elevator being accelerated upward at 15 ft/sec²?*

BG

Eq. 3.24: $\partial p/\partial z = -\rho(g + a_z)$; $\quad \Delta p = -\rho(g + a_z)\Delta z = -(\gamma/g)(g + a_z)\Delta z$

$$\Delta p = -\frac{62.4 \times 0.86}{32.2}(32.2 + 15)\left(\frac{-8}{12}\right) = 52.4 \text{ psf} = 0.364 \text{ psi}$$ ◄

3.10.2 *What must be the hydrostatic gage pressure at a depth of 250 mm in a bucket of oil (s = 0.88) that is in an elevator being accelerated upward at 4 m/s²?*

SI

Eq. 3.24: $\partial p/\partial z = -\rho(g + a_z)$; $\quad \Delta p = -\rho(g + a_z)\Delta z = -(\gamma/g)(g + a_z)\Delta z$

$$\Delta p = -0.88\frac{\text{gm}}{\text{cm}^3}\left(\frac{9807 \text{ N/m}^3}{9.807 \text{ m/s}^2}\right)(9.807 + 4.00)\frac{\text{m}}{\text{sec}^2}(-0.250 \text{ m}) = 3040 \text{ Pa}$$ ◄

3.10.3 *A tank containing water to a depth of 5 ft is accelerated upward at 8 ft/s². Calculate the pressure on the bottom of the tank.*

BG

From Eq. 3.24: $\Delta p = -(\gamma/g)(a_z + g)\Delta z = -(62.4/32.2)(8 + 32.2)(-5) = 390$ psf

$\Delta p = 390/144 = 2.70$ psi ◀

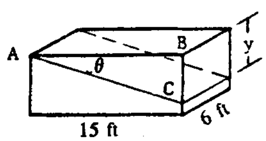
3.10.4 *A tank containing water to a depth of 2.5 m is accelerated upward at 3.6 m/s². Calculate the pressure on the bottom of the tank.*

SI

From Eq. 3.24: $\Delta p = -\dfrac{\gamma}{g}(a_z + g)\Delta z = -\left(\dfrac{9807\ \text{N/m}^3}{9.807\ \text{m/s}^2}\right)(3.60 + 9.81)(-2.5) = 33\,500\ \text{Pa} = 33.5\ \text{kPa}$ ◀

3.10.5 *Suppose the tank shown in Fig. 3.24 is rectangular and completely open at the top. It is 15 ft long, 6 ft wide, and 4 ft deep. If it is initially filled to the top, how much liquid will be spilled if it is given a horizontal acceleration $a_x = 0.2g$ in the direction of its length?*

BG



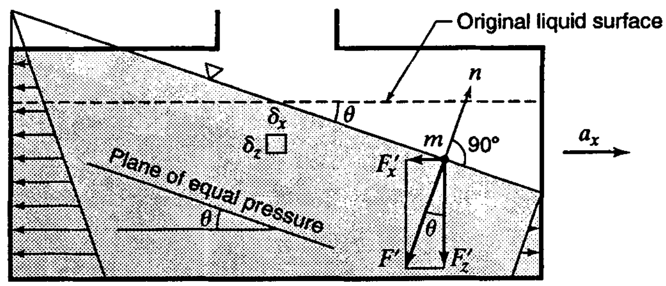


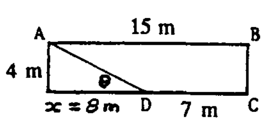
Figure 3.24a

Sec 3.11: $\tan\theta = -a_x/g = -0.2$; $y = -0.2(15) = -3.0$ ft

As the tank is more than 3 ft deep, the spill will be wedge *ABC* or $(1/2)15(3)6 = 135$ ft³ ◀

3.10.6 *Suppose the tank shown in Fig. 3.24 is rectangular and completely open at the top. It is 15 m long, 5 m wide, and 4 m deep. If it is initially filled to the top, how much liquid will be spilled if it is given a horizontal acceleration $a_x = 0.5g$ in the direction of its length?*

SI



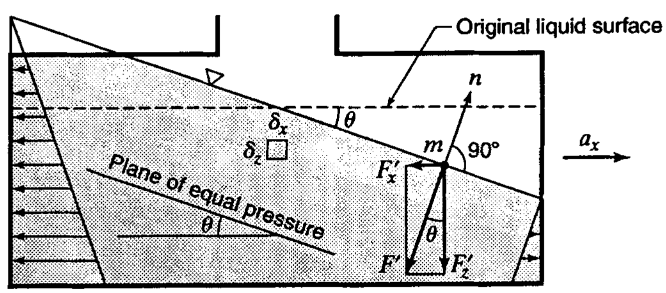


Figure 3.24a

Sec 3.11: $\tan\theta = -a_x/g = -0.5$; $x = 4/\tan\theta = 4/0.5 = 8$ m

Spill = volume *ABCD* = $[7(4) + (0.5)8(4)]5 = 220$ m³ ◀

3.10.7 *If the tank of Exer 3.10.5 is closed at the top and is completely filled, what must be the pressure difference between the left-hand end at the top and the right-hand end at the top if the liquid has a specific weight of 50 lb/ft^3 and the horizontal acceleration is $a_x = 0.3g$? Sketch planes of equal pressure, indicating their magnitude; assume zero pressure in the upper right-hand corner.*

Exer. 3.10.5: The rectangular tank is 15 ft long, 6 ft wide, and 4 ft deep.

BG

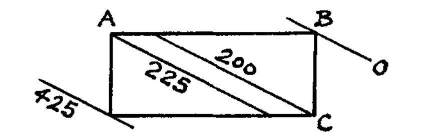
Eq. 3.23: $\partial p/\partial x = -\rho a_x = -(50/32.2)0.3(32.2) = -15$ psf per ft

$p_A - p_B = \Delta p = (\partial p/\partial x)\Delta x = 15(15) = 225$ psf ◄

Eq. 3.24 with $a_z = 0$: $\partial p/\partial z = -\rho g = -\gamma = -50$ psf per ft

$p_C - p_B = \Delta p = (\partial p/\partial z)\Delta z = 50(4) = 200$ psf

Lines of equal pressure are as follows:



◄

3.10.8 *If the tank of Exer 3.10.6 is closed at the top and is completely filled, what must be the pressure difference between the left-hand end at the top and the right-hand end at the top if the liquid has a specific weight of 8.0 kN/m^3 and the horizontal acceleration is $a_x = 0.3g$? Sketch planes of equal pressure, indicating their magnitude; assume zero pressure in the upper right-hand corner.*

Exer. 3.10.6: The rectangular tank is 15 m long, 5 m wide, and 4 m deep.

SI

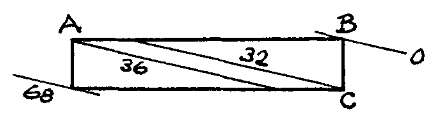
Eq. 3.23: $\partial p/\partial x = -\rho a_x = -(\gamma/g)a_x = -(8.0/9.81)0.3(9.81) = -2.4$ kPa per m

$p_A - p_B = \Delta p = (\partial p/\partial x)\Delta x = 2.4(15) = 36.0$ kPa ◄

Eq. 3.24 with $a_z = 0$: $\partial p/\partial z = -\rho g = -\gamma = -8.0$ kPa per m

$p_C - p_B = \Delta p = (\partial p/\partial z)\Delta z = 8.0(4) = 32.0$ kPa

Lines of equal pressure are as follows:



◄

Sec. 3.10: Fluid Masses Subjected to Acceleration -- *Problems* *3.31–3.32*

3.31 *Refer to Sample Prob. 3.10. Suppose the velocity of the airplane is 225 m/s, with all other data unchanged. What then would be the slope of the liquid surface in the tank?*

Sample Prob 3.10: At a particular instant an airplane is travelling upward at 40° to the horizontal, and losing speed at 4 m/s^2. Also it is moving on a concave-upward circular path of radius 2600 m.

SI

$a_n = v^2/r = 225^2/2600 = 19.47$ m/s^2

$a_t = -4$ m/s^2 (given)

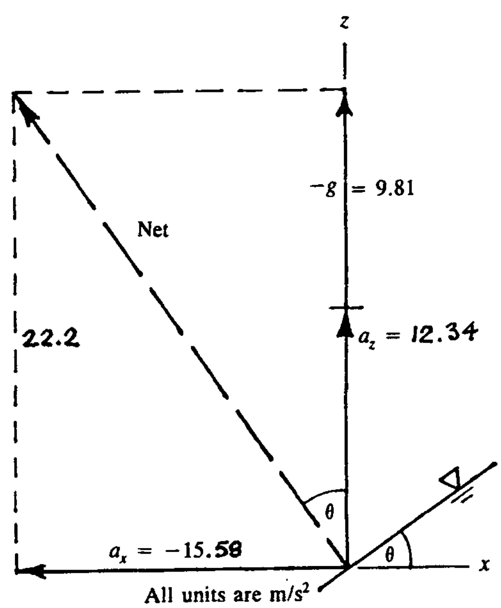
$a_x = -4\cos 40° - 19.47\sin 40° = -15.58$ m/s^2

$a_z = -4\sin 40° + 19.47\cos 40° = +12.34$ m/s^2

Eq. 3.26: Surface slope $= \tan\theta$

$$= \frac{dz}{dx} = -\frac{(-15.58)}{12.34 + 9.81} = 0.703$$ ◄

or $\theta = \tan^{-1}0.703 = 35.1°$ from the horizontal. ◄



3.32 *At a particular instant an airplane is traveling upward at a velocity of 180 mph in a direction that makes an angle of 30° with the horizontal. At this instant the airplane is losing speed at the rate of 3.6 mph/sec. Also, it is moving on a concave-upward circular path of radius 5000 ft. Determine the slope of the free liquid surface in the airplane's fuel tank.*

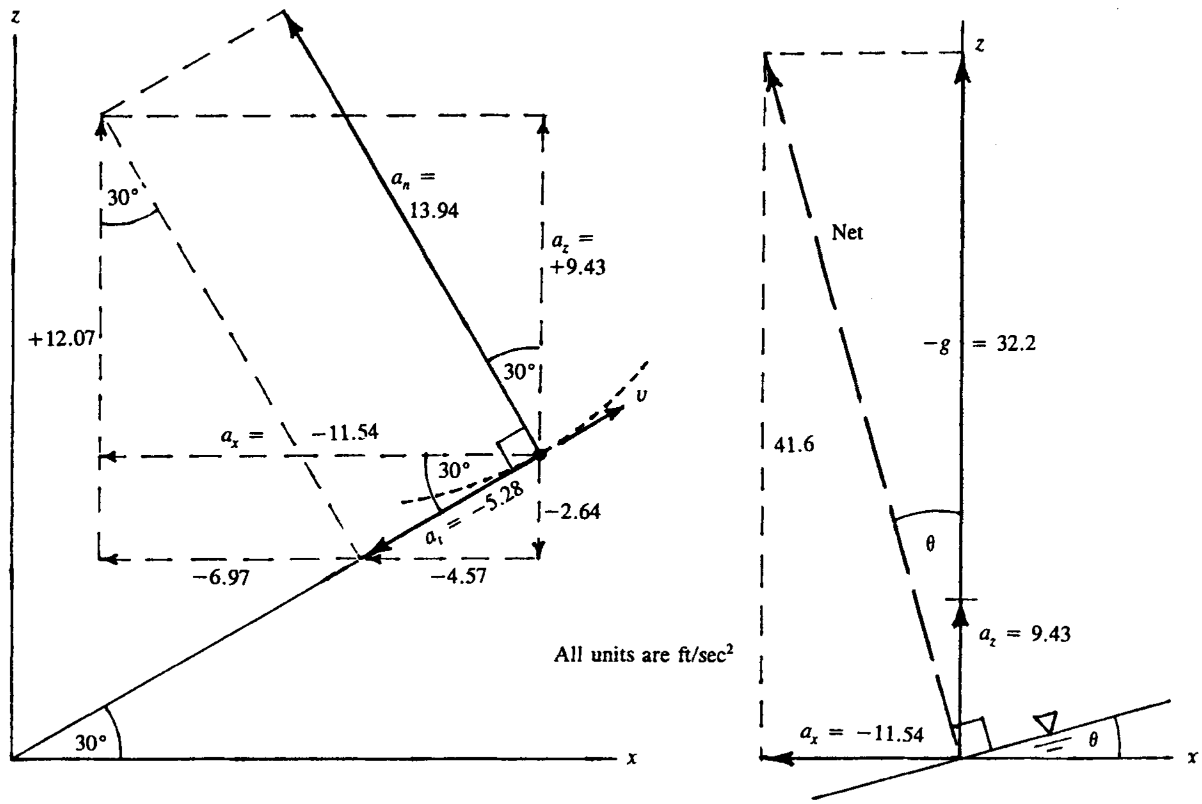
BG

180 mph = 180(5280 ft/mi)/(3600 sec/hr) = 264 fps ;   3.6 mph = 5.28 fps

$a_n = v^2/r = 264^2/5000 = 13.94$ ft/sec^2 ; $a_t = -5.28$ ft/sec^2.

$a_x = -5.28\cos 30° - 13.94\sin 30° = -11.54$ ft/sec^2

$a_z = -5.28\sin 30° + 13.94\cos 30° = +9.43$ ft/sec^2



Liquid surface is perpendicular to net force, i.e., to net acceleration.

Eq. 3.26: Surface slope $= \tan\theta = \dfrac{dz}{dx} = -\dfrac{(-11.54)}{(9.43 + 32.2)} = 0.277$ ◄

or $\theta = \tan^{-1}0.277 = 15.50°$ from the horizontal ◄

Chapter 4
Basics of Fluid Flow

PROBLEM SELECTION GUIDE

Sec.	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
4.3	***Steady Flow and Uniform Flow***						
	X[1]4.3.1	N	Easy	V Short	6		
4.5	***Flow Rate and Mean Velocity***						
	X 4.5.1	N	Easy	Short	1		Integration
	4.5.2	BG	V Easy	V Short	1		
	4.5.3	SI	Easy	Short	1	P4.1	
	4.5.4	BG	Easy	Short	1	4.5.5	
	4.5.5	SI	Easy	Short	1	4.5.4	
	P 4.1	BG	Easy	Short	1	4.5.3	
	4.2	BG	Easy	Short	1	4.3	Uses $pv = RT$ (Sec. 2.7)
	4.3	SI	Easy	Short	1	4.2, S4.1	Uses $pv = RT$ (Sec. 2.7)
	4.4	BG	Medium	Long	1	4.5	Graphical solution
	4.5	SI	Medium	Long	1	4.4	Graphical solution
4.7	***Equation of Continuity***						
	X 4.7.1	BG	V Easy	V Short	1	4.7.2	
	4.7.2	SI	V Easy	V Short	1	4.7.1	
	4.7.3	BG	Easy	Short	1		
	P 4.6	SI	Easy	Short	2		
	4.7	BG	Medium	Medium	1		Numerical differentiation
4.10	***Use and Limitations of Flow Net***						
	X 4.10.1	SI	Medium	Short	1		
	4.10.2	SI	Easy	Short	1		† Measure Fig X4.10.2
	4.10.3	BG	Easy	Medium	2		† Measure Fig 4.10
	4.10.4	SI	Easy	Medium	2		† Measure Fig 4.12
	P 4.8	BG	Medium	Medium	1	4.9	† Measure Fig X4.10.2
	4.9	SI	Medium	Medium	1	4.8	† Measure Fig X4.10.2
	4.10	N	Medium	Medium	1		† Measure Fig 4.10; plot
	4.11	BG	Hard	Long	1		Sketch flow net; plot

/cont...

[1] For all Exercises (identified by "X"), answers are given in Appendix F of the textbook.
† Answers are sensitive to values that are measured from figures.
X = Exercise, P = (end-of-chapter) Problem, S = Sample Problem.

Sec.	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
4.12	*Velocity and Acceleration in Steady Flow*						
	X 4.12.1	N	V Easy	V Short	1		
	4.12.2	N	Medium	Short	1		Differentiation
	4.12.3	N	Medium	Medium	1	4.12.4	Partial differentiation
	4.12.4	N	Medium	Medium	1	4.12.3	Partial differentiation
	4.12.5	N	Medium	Medium	1		Partial differentiation
	4.12.6	BG	V Easy	V Short	1	4.12.7	
	4.12.7	SI	V Easy	V Short	1	4.12.6	
	4.12.8	BG	Medium	Medium	1	P4.12	Differentiation
	P 4.12	SI	Medium	Short	1	4.12.8	Differentiation
	4.13	N	Medium	Medium	1		Partial differentiation
	4.14	N	Medium	Medium	1		Sketch flow field
	4.15	N	Medium	Medium	1		Sketch flow field
	4.16	N	Medium	Long	1		† Sketch flow field; numerical differ'n
	4.17	BG	Medium	Medium	1		Numerical differentiation
	4.18	SI	Medium	Medium	1		† Measure Fig P4.18; numerical differ'n
4.13	*Velocity and Acceleration in Unsteady Flow*						
	X 4.13.1	N	Medium	Short	1		Differentiation
	4.13.2	N	Medium	Short	1		Partial differentiation
	4.13.3	BG	Easy	Short	1	4.13.4	Differentiation
	4.13.4	SI	Easy	Short	1	4.13.3	Differentiation
	P 4.19	BG	Medium	Medium	1	4.20	Partial differentiation
	4.20	SI	Medium	Medium	1	4.19	Partial differentiation
	4.21	SI	Medium	Medium	1		† Measure Fig P4.18; numerical differ'n
Ch.4	*Miscellaneous*						
	P 4.22	N	Medium	Medium	1		Plot
	4.23	BG	Medium	Medium	1		Plot
	4.24	SI	Medium	Long	3		Plot

Chapter 4
BASICS OF FLUID FLOW

Sec 4.3: Steady Flow and Uniform Flow – *Exercise* (1)

4.3.1 *Classify the following cases of flow as to whether they are steady or unsteady, uniform of nonuniform: (a) water flowing from a tilted pail; (b) flow from a rotating lawn sprikler; (c) flow through the hose leading to the sprinkler; (d) a natural stream during dry-weather flow; (e) a natural stream during flood; (f) flow in a city water-distribution main through a straight section of constant diameter with no side connections. (Note: There is room for legitimate argument in some of the above cases, which should stimulate independent thought.)*

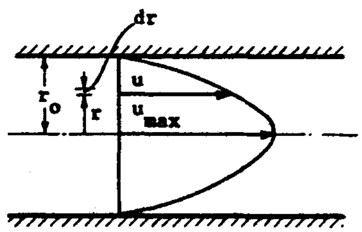
N

(*a*) Unsteady, nonuniform; (*b*) Unsteady (although steady with respect to rotating frame of reference), nonuniform; (*c*) Steady, uniform; (*d*) Almost steady, nonuniform; (*e*) Unsteady, nonuniform; (*f*) Unsteady, uniform. ◄◄

Sec 4.5: Flow Rate and Mean Velocity – *Exercises* (5)

4.5.1 *In the laminar flow of a fluid through a pipe of circular cross section the velocity profile is exactly a true parabola. The volume of the paraboloid represents the rate of discharge. Prove that for this case the ratio of the mean velocity to the maximum velocity is 0.5.*

N



For paraboloid, $u = u_{max}[1 - (r/r_o)^2]$

Eq. 4.3: $Q = \int u\,dA = 2\pi u_{max}\int_r^{r_o}[1 - (r/r_o)]^2 r\,dr$

$$= u_{max}\frac{\pi r_o^2}{2}; \quad V_{mean} = V = \frac{Q}{A} = \frac{u_{max}(\pi r_o^2)}{2(\pi r_o^2)} = \frac{u_{max}}{2}$$

Thus $\dfrac{V_{mean}}{u_{max}} = 0.5$ Q.E.D. ◄

4.5.2 *A gas (γ = 0.05 lb/ft^3) flows at a rate of 0.8 lb/sec past section A through a long rectangular duct of uniform cross section 1.2 by 1.8 ft. At section B some distance along the duct the gas weighs 0.08 lb/ft^3. What is the average velocity of flow at sections A and B?*

BG

From Eq. 4.5: $V_A = \dfrac{G}{\gamma A} = \dfrac{0.8}{0.05(1.2\times 1.8)} = 7.41$ fps ◄

$V_B = \dfrac{0.8}{0.08(1.2\times 1.8)} = 4.63$ fps ◄

4.5.3 *The velocity of a liquid (s = 1.4) in a 150-mm pipeline is 0.8 m/s. Calculate the rate of flow in L/s, m^3/s, kg/s, and kN/s.*

SI

Eq. 4.3: $Q = AV = \pi(0.075)^2\times 0.8 = 0.014\,14$ m^3/s ◄

$= 0.014\,14$ (m^3/s) $\times 10^3$ (L/m^3) $= 14.14$ L/s ◄

From Eqs. 4.4 and 4.3: $\dot{m} = \rho Q = (1.4\times 1000)0.014\,14 = 19.79$ kg/s ◄

From Eqs. 4.5 and 4.3: $G = \gamma Q = (1.4\times 9.81)0.014\,14 = 0.1942$ kN/s ◄

4.5.4 *Water flows at 4 gal/min through a small circular hole in the bottom of a large tank. Assuming the water in the tank approaches the hole radially, what is the velocity in the tank at 2, 4, and 8 in from the hole?*

BG

Flow passes through a hemispherical surface area, with $A = \frac{1}{2}\pi D^2 = \frac{1}{2}\pi(2r)^2 = 2\pi r^2$ (Table A.8)

Q = 4 gal/min × (0.1337 ft³/g)/(60 sec/min) = 0.00891 cfs

r (in)	A (in²)	A (ft²)	$V = Q/A$
2	25.1	0.174 5	0.051 1
4	100.5	0.698	0.012 77
8	402.1	2.793	0.003 19

▲

4.5.5 *Water flows at 0.25 L/s through a small circular hole in the bottom of a large tank. Assuming the water in the tank approaches the hole radially, what is the velocity in the tank at 50, 100, and 200 mm from the hole?*

SI

Flow passes through a hemispherical surface area, with $A = \frac{1}{2}\pi D^2 = \frac{1}{2}\pi(2r)^2 = 2\pi r^2$ (Table A.8)

Q = 0.25 L/s = 0.000 25 m³/s

r (mm)	A (m²)	$V = Q/A$
50	0.015 71	0.015 92
100	0.062 83	0.003 98
200	0.251 33	0.000 995

▲

Sec 4.5: Flow Rate and Mean Velocity – *Problems* 4.1–4.5

4.1 *The velocity of a liquid (s = 1.26) in a 3-in pipeline is 2.4 fps. Calculate the rate of flow in cfs, gal/min, slugs/sec, and lb/sec.*

BG

Eq. 4.3: $Q = AV = \pi(1.5/12)^2 \times 2.4 = 0.1178$ cfs ◄

= 0.1178(448.8 gpm/cfs) = 52.9 gpm ◄

From Eqs. 4.4 and 4.3: $\dot{m} = \rho Q = (1.26 \times 1.940)0.1178 = 0.288$ slug/sec ◄

From Eqs. 4.5 and 4.3: $G = \gamma Q = (1.26 \times 62.4)0.1178 = 9.26$ lb/sec ◄

4.2 *Carbon dioxide flows in a 2-in by 3-in duct at a pressure of 46 psi and a temperature of 80°F. If the atmospheric pressure is 13.8 psia and the flow velocity is 10 fps, calculate the weight flow rate.*

BG

Table A.5 for carbon dioxide: R = 1123 ft·lb/(slug·R°)

Eq. 2.5: $\gamma = \frac{gp}{RT} = \frac{32.2(46+13.8)144}{1123(460+80)} = 0.457$ pcf

Eq. 4.5: $G = \gamma AV = 0.457(2/12)(3/12)10 = 0.1905$ lb/sec ◄

4.3 *Nitrogen at 40°C and under a pressure of 3000 mb abs flows in a 350-mm-diameter conduit at a mean velocity of 8 m/s. Find the mass flow rate.*

SI

Table A.5 for nitrogen: $R = 297\ \text{m}^2/(\text{s}^2{\cdot}\text{K})$

From Eq. 2.4: $\rho = \dfrac{p}{RT} = \dfrac{3000\ \text{mb} \times (1\ \text{KN/m}^2)/10\ \text{mb} \times (1000\ \text{kg·m/s}^2)/\text{kN}}{297\ \text{m}^2\text{s}^{-2}\text{K}^{-1}(273+40)\ \text{K}} = 3.23\ \text{kg/m}^3$

From Eq. 4.4: $\dot{m} = \rho AV = 3.23(\pi \times 0.175^2)8 = 2.48\ \text{kg/s}$ ◄

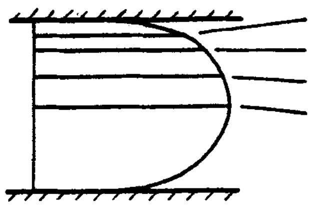
4.4 *Velocities in an 8-in-diameter circular conduit, measured at radii of 0, 1.44, 2.60, and 3.48 in, were 20.3, 19.7, 17.7, and 14.5 fps respectively. Find approximate values (graphically) of the volume flow rate and the mean velocity. Also determine the ratio of the mean velocity to the maximum velocity.*

BG

Note: Answers may vary somewhat due to graphical solution.

Eq. 4.3: $Q = \int u dA = u_1 dA_1 + u_2 dA_2 + \ldots$

Plot the velocity profile and divide the area into a number of equal (annular) parts.



For four equal parts (say),

$$\frac{A}{4} = \frac{\pi r^2}{4} = \pi r_1^2 = \pi(r_2^2 - r_1^2) = \pi(r_3^2 - r_2^2);\ dA = \frac{A}{4} = \frac{\pi 4^2}{4} = 4\pi\ \text{in}^2$$

r = 4 inches, so $r_0 = 0$, $r_1 = \sqrt{4} = 2.00$ inches, $r_2 = \sqrt{8} = 2.83$ inches, $r_3 = \sqrt{12} = 3.46$ inches

From the sketched profile, $u_1 \approx 7.8$ fps, $u_2 \approx 15.4$ fps, $u_3 \approx 17.7$ fps, $u_4 \approx 19.4$ fps

$Q = \Sigma(u_i dA_i) \approx (7.8 + 15.4 + 17.7 + 19.4)dA = (60.3\ \text{fps})(4\pi/144\ \text{ft}^2) = 5.26\ \text{cfs}$

$Q \approx 5.3$ cfs ◄

$V = Q/A \approx 5.26/(16\pi/144) = 15.075$ fps $\qquad V \approx 15.1$ fps ◄

$V/V_{max} \approx 15.075/20.3 = 0.743 \qquad V/V_{max} \approx 0.74$ ◄

If we had broken down the area into more parts, the accuracy would have been improved.

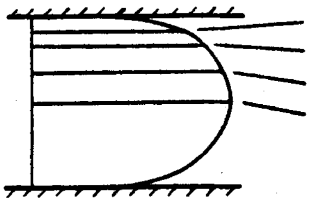
4.5 *Velocities in a 200-mm-diameter circular conduit, measured at radii of 0, 36, 65, and 87 mm, were 7.0, 6.8, 6.1, and 5.0 m/s respectively. Find approximate values (graphically) of the volume flow rate and the mean velocity. Also determine the ratio of the mean velocity to the maximum velocity.*

SI

Note: Answers may vary somewhat due to graphical solution.

Eq. 4.3: $Q = \int u dA = u_1 dA_1 + u_2 dA_2 + \ldots$

Plot the velocity profile and divide the area into a number of equal (annular) parts.



5.0 m/s at r = 87 mm
6.1 m/s at r = 65 mm
6.8 m/s at r = 36 mm
7.0 m/s at r = 0

For four equal parts (say).

$A/4 = \pi r^2/4 = \pi r_1^2 = \pi(r_2^2 - r_1^2) = \pi(r_3^2 - r_2^2)$

$dA = A/4 = \pi 100^2/4 = 2500\pi \text{ mm}^2$

r = 100 mm, so $r_0 = 0$, $r_1 = 50.0$ mm, $r_2 = \sqrt{5000} = 70.7$ mm, $r_3 = \sqrt{7500} = 86.6$ mm

From the sketched profile, $u_1 \approx 6.7$ m/s, $u_2 \approx 6.1$ m/s, $u_3 \approx 5.3$ m/s, $u_4 \approx 2.7$ m/s

$Q = \Sigma(u_i dA_i) \approx (6.7 + 6.1 + 5.3 + 2.7)dA = 20.8(2500\pi)$

$= 163\,400$ m(mm²/s) $= 0.1633$ m³/s $\qquad Q \approx 0.163$ m³/s ◄

$V = Q/A \approx 163\,400/(4 \times 2500\pi) = 5.20$ m/s $\qquad V \approx 5.2$ m/s ◄

$V/V_{max} \approx 5.20/7.0 = 0.743$ $\qquad V/V_{max} \approx 0.74$ ◄

If the area had been broken down into more parts, the accaracy would have been improved.

Sec 4.7: Equation of Continuity -- *Exercises* (3)

4.7.1 *Water flows in a river. At 8 A.M. the flow past bridge 1 is 2150 cfs. At the same instant the flow past bridge 2 is 1800 cfs. At what rate is water being stored in the river between the two bridges at this instant? Assume zero seepage and negligible evaporation.*

BG

Eq. 4.18: $Q_1 - Q_2 = d\forall/dt$; (2150 − 1800) ft³/sec $= d\forall/dt$; $d\forall/dt = 350$ ft³/sec ◄

4.7.2 *Water flows in a river. At 9 A.M. the flow past bridge 1 is 37.2 m³/s. At the same instant the flow past bridge 2 is 26.9 m³/s. At what rate is water being stored in the river between the two bridges at this instant? Assume zero seepage and negligible evaporation.*

SI

Eq. 4.18: $Q_1 - Q_2 = d\forall/dt$; (37.2 − 26.9) m³/s $= d\forall/dt$; $d\forall/dt = 10.30$ m³/sec ◄

4.7.3 *Gas is flowing in a long 9-in-diameter pipe from A to B. At section A the flow is 0.65 lb/sec while at the same instant at section B the flow is 0.72 lb/sec. The distance between A and B is 800 ft. Find the mean value of the time rate of change of the specific weight of the gas between sections A and B at that instant.*

BG

Eq. 4.15 × g: $\gamma_1 A_1 V_1 - \gamma_2 A_2 V_2 = (\partial\gamma/\partial t) \times \forall$; Eq. 4.16b: $\gamma A V = G$

Substituting: $G_1 - G_2 = (\partial\gamma/\partial t) \times \forall$

(0.65 − 0.72) lb/sec $= \partial\gamma/\partial t \times 800\pi(4.5/12)^2$ ft³; $\partial\gamma/\partial t = -0.000\,1981$ lb/ft³ per sec ◄

Sec 4.7: Equation of Continuity – *Problems* 4.6–4.7

4.6 *Gas flows at a steady rate in a 120-mm-diameter pipe that enlarges to a 180-mm-diameter pipe. (a) At a certain section of the 120-mm pipe the density of the gas is 165 kg/m³ and the velocity is 15 m/s. At a certain section of the 180-mm pipe the velocity is 10 m/s. What must be the density of the gas at that same section? (b) If these same data were given for the case of unsteady flow at a certain instant, could the problem be solved? Discuss.*

SI

(*a*) Eq. 4.16*a*: $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$; as $A \propto (\text{diam})^2$, we therefore obtain $\rho_1 d_1^2 V_1 = \rho_2 d_2^2 V_2$

$165(120)^2 15 = \rho_2 (180)^2 10$ from which $\rho_2 = 110.0 \text{ kg/m}^3$ ◄

(*b*) If the flow were unsteady, the problem could not be solved because no information is given on $\partial\rho/\partial t$. Also, the volume between the two sections is unknown. ◄

4.7 *A compressible fluid flows in a 20-in-diameter leaky pipe. Measurements are made simultaneously at two points A and B along the pipe that are 32,000 ft apart. Two sets of measurements are taken with an interval of exactly 45 min between them. The data are as follows:*

Time	ρ_1 *(slug/ft³)*	V_1 *(ft/s)*	ρ_2 *(slug/ft³)*	V_2 *(ft/s)*
0	*0.520*	*65*	*0.608*	*54*
45 min	*0.616*	*51*	*0.727*	*40*

Assuming ρ varies linearly with respect to time and distance, compute the approximate average mass rate of leakage between A and B.

BG

Modifying Eq. 4.15 to include for leakage, at any instant

$$\rho_1 A_1 V_1 - \rho_2 A_2 V_2 - \text{leakage rate} = (\partial\rho/\partial t) \times \mathcal{V}$$

$$\partial\rho/\partial t \text{ at Section 1} = \frac{\Delta\rho_1}{\Delta t} \approx \frac{0.616 - 0.520}{45 \times 60} = 3.56 \times 10^{-5} \text{ slug/(ft}^3\text{·sec)}$$

$$\partial\rho/\partial t \text{ at Section 2} = \frac{\Delta\rho_2}{\Delta t} \approx \frac{0.727 - 0.608}{45 \times 60} = 4.41 \times 10^{-5} \text{ slug/(ft}^3\text{·sec)}$$

During this time interval, and through the 32,000-ft length *AB*,

$$\text{average } \partial\rho/\partial t \approx \tfrac{1}{2}(3.56 + 4.41) \times 10^{-5} = 3.98 \times 10^{-5} \text{ slug/(ft}^3\text{·sec)}$$

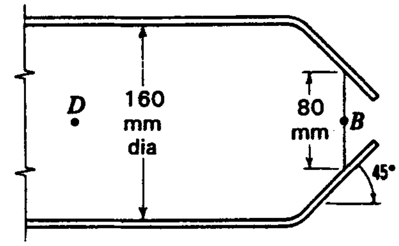
During any time interval, $\bar{\rho}_1 A_1 \bar{V}_1 - \bar{\rho}_2 A_2 \bar{V}_2 - \text{avg leak rate} \approx \overline{(\partial\rho/\partial t)} \times \mathcal{V}$

$$(0.5680)\frac{\pi}{4}\left(\frac{20}{12}\right)^2 58 - (0.6675)\frac{\pi}{4}\left(\frac{20}{12}\right)^2 47 - \text{avg leak rate} \approx 3.98 \times 10^{-5}\left[32{,}000 \times \frac{\pi}{4}\left(\frac{20}{12}\right)^2\right]$$

71.873 − 68.444 − avg leak rate ≈ 2.780 slug/sec; Avg leakage rate ≈ 0.649 slug/sec ◄

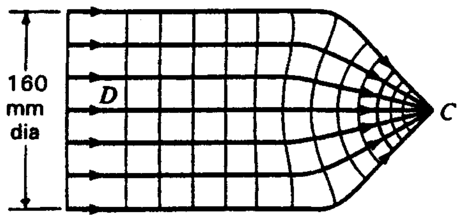
Sec 4.10: Use and Limitations of Flow New – *Exercises* (4)

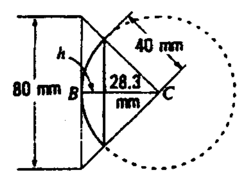
4.10.1 *An incompressible ideal fluid flows at 12 L/s through a circular 160-mm-diameter pipe into a conically converging nozzle like that of Sample Prob. 4.4 (diameter at B is 80 mm). Determine the average velocity of flow at sections D and B.*



SI

Consider the sketched flow net of Sample Prob 4.4. At section D the streamlines are parallel, so the area normal to the velocity vectors is a plane circle.





Thus, $V_D = \dfrac{Q}{A_D} = \dfrac{12 \times 10^{-3}\ \text{m}^3/\text{s}}{(\pi/4)(0.16\ \text{m})^2} = 0.597\ \text{m/s}$ ◀

At section B, however, the area normal to the streamlines is best approximated by the portion of the surface of a sphere of radius 40 mm which is inside the nozzle and passes through B (see right-hand solution figure of Sample Prob. 4.4). By table lookup, by integration, or from Sample Prob 4.4, $A_B = 2\pi rh$

$$h = 40 - 40\cos 45° = 11.72\ \text{mm}$$

$$V_B = \frac{Q}{A_B} = \frac{Q}{2\pi rh} = \frac{12 \times 10^{-3}\ \text{m}^3/\text{s}}{2\pi(0.040)0.011\,72\ \text{m}^2} = 4.08\ \text{m/s}$$ ◀

4.10.2 *Figure X4.10.2 shows the flow net for two-dimensional flow from a rounded, long-slotted exit from a tank. If U_0 = 1.8 m/s, what is the approximate flow velocity at A?*

SI

At B: $\Delta n_o \approx 12.05/6 = 2.01$ mm

At A: $\Delta n = \dfrac{c}{12} = \dfrac{\pi D}{12}$

$\approx \dfrac{\pi(85.3)}{12} = 22.3$ mm

From Eq. 4.19: $V_A = \dfrac{\Delta n_o}{\Delta n} U_o$

$\approx \dfrac{2.01}{22.3}(1.8) = 0.1619$

≈ 0.16 m/s ◀

Note: Answers may vary somewhat due to scaling.

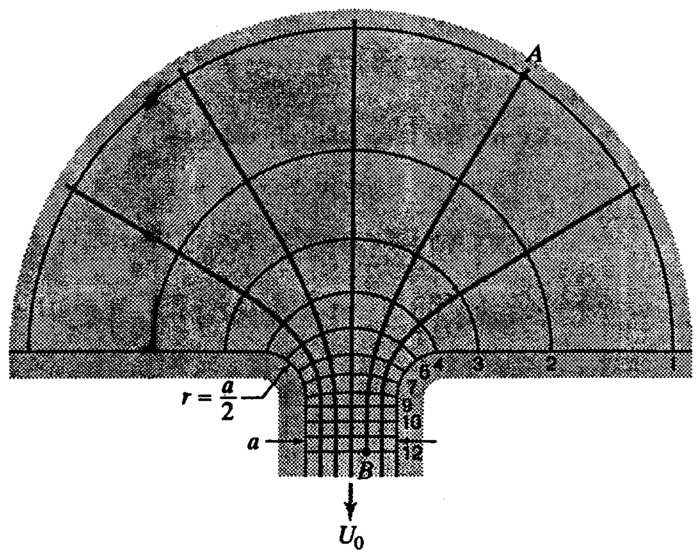


Figure X4.10.2

4.10.3 *Given that U_0 in Fig. 4.10 is 6.0 fps, find approximately (a) the maximum velocity in the bend, and (b) the uniform velocity in the downstream section.*

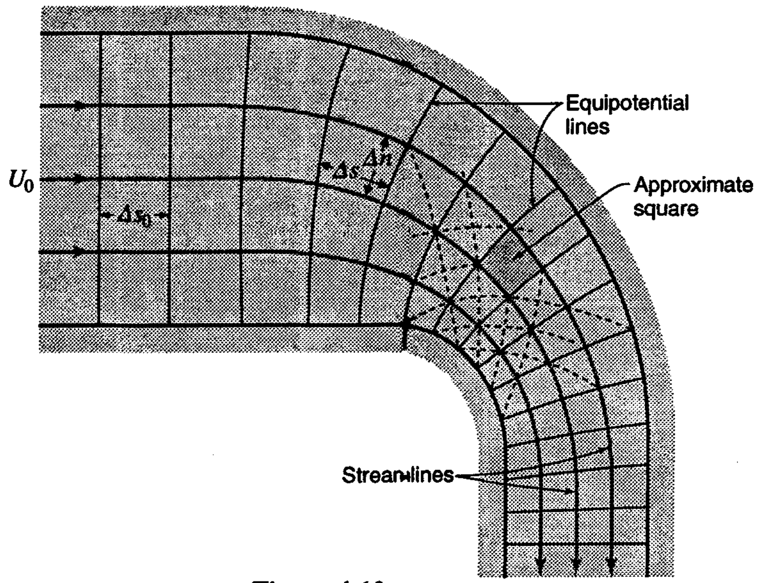


Figure 4.10

BG

(All values approximate)	Δn, mm	$\Delta n_o / \Delta n$	$V = (\Delta n_o / \Delta n) U_o$
In uniform entrance:	38.8/4 = 9.70	1.00	6.00 fps (given)
(*a*) In bend (smallest square)	3.50	2.77	16.6 fps ◄
(*b*) In uniform exit:	18.7/4 = 4.68	2.07	12.4 fps ◄

Note: Answers may vary somewhat due to scaling.

4.10.4 *Given that U_0 in Fig. 4.12 is 4 m/s, find approximately (a) the maximum and (b) the minimum velocity on the body surface.*

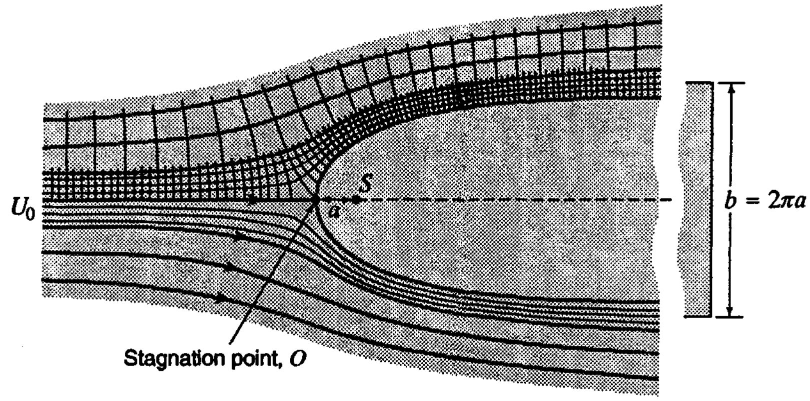


Figure 4.12

SI

(*a*) Measuring four of the small squares, on average:

At uniform entrance: $$\Delta n_o = \frac{1}{2}\left(\frac{L}{4} + \frac{W}{4}\right) \approx \frac{1}{2}\left(\frac{4.0 + 3.7}{4}\right) = 0.962 \text{ mm}$$

At the smallest square, about 0.6*b* or 3.8*a* downstream of the stagnation point:

$$\Delta n = \frac{1}{2}\left(\frac{L}{4} + \frac{W}{4}\right) \approx \frac{1}{2}\left(\frac{2.6 + 3.0}{4}\right) = 0.700 \text{ mm}$$

$$V_{max} = \frac{\Delta n_0}{\Delta n} U_0 \approx \frac{0.962}{0.700} 4 = 5.5 \text{ m/s} \quad ◄$$

Note: Answers may vary somewhat due to scaling.

(*b*) At stagnation point, $V_{min} = 0$ ◄

Sec 4.10: Use and Limitations of Flow Net -- *Problems* 4.8 -- 4.11

4.8 *Refer to Fig. X4.10.2. If a is 3 in and U_0 is 10 fps, approximately how long will it take a particle to move from point A to point B on the same streamline? (Note: Between each pair of equipotential lines, measure Δs, and then compute the average velocity and time increment.)*

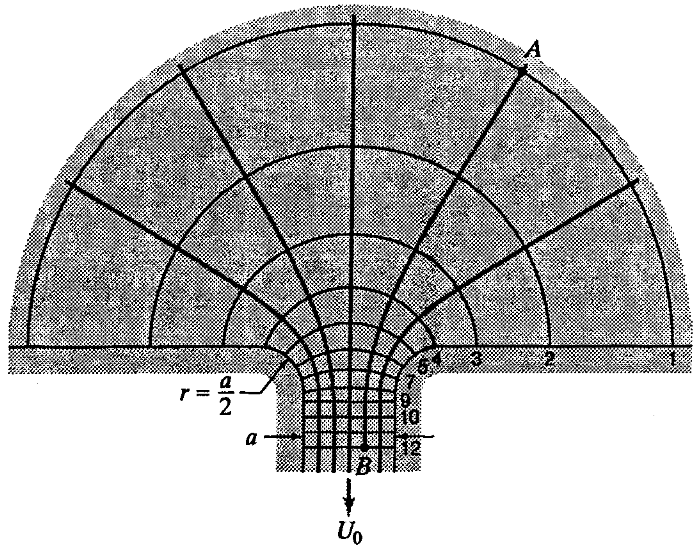


Figure X4.10.2

BG

U_o = 10 fps and a = 3 in. $\therefore\ \Delta s_0 = \frac{a}{6} = \frac{1}{2}$ in $= \frac{1}{24}$ ft

Equipotential space	$\Delta s / \Delta s_0$	$\bar{V} = \frac{10}{\Delta s/\Delta s_0}$	$\Delta s = \frac{1}{24}\frac{\Delta s}{\Delta s_0}$	$\Delta t = \Delta s/\bar{V}$
1-2	7.5	1.33 fps	0.31 ft	0.23 sec
2-3	5.5	1.82 fps	0.23 ft	0.13 sec
3-4	3.4	2.94 fps	0.14 ft	0.048 sec

and similarly on through space 11 - 12, yields $\Sigma \Delta t \approx 0.47$ sec ◄

Note: Answers may vary somewhat due to scaling.

4.9 *Repeat Prob. 4.8 using the following data: a = 150 mm and U_0 = 0.5 m/s. Find also the approximate velocity where the flow crosses equipotential line 3.*

Prob. 4.8: Refer to Fig. X4.10.2. Approximately how long will it take a particle to move from point A to point B on the same streamline? (Note: Between each pair of equipotential lines, measure Δs, and then compute the average velocity and time increment.)

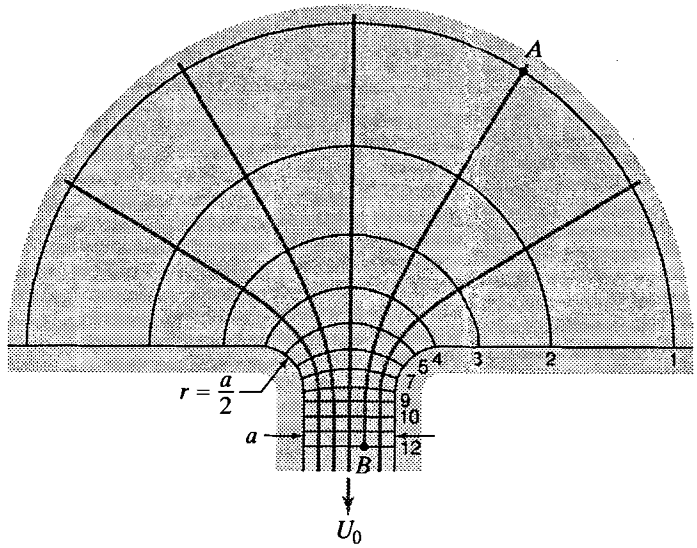


Figure X4.10.2

SI

U_0 = 0.5 m/s and a = 0.15 m. $\therefore\ \Delta s_0 = \frac{a}{6} = \frac{0.15 \text{ m}}{6} = 0.025$ m

Equipotential space	$\Delta s / \Delta s_0$	$\bar{V} = \frac{0.5}{\Delta s/\Delta s_0}$	$\Delta s = 0.025 \frac{\Delta s}{\Delta s_0}$	$\Delta t = \Delta s/\bar{V}$
1-2	7.5	0.067 m/s	0.188 m	2.80 sec
2-3	5.5	0.091 m/s	0.138 m	1.52 sec
3-4	3.4	0.147 m/s	0.085 m	0.58 sec

and similarly on through space 11 - 12, yields $\Sigma \Delta t \approx 5.7$ sec ◄

Area at equipotential line 3: arc length $\approx \left[\frac{34}{12.05}\right] 0.150 \left[\frac{\pi}{2}\right] = 0.665$ m.

$\therefore\ V_3 \approx \frac{0.5}{0.665/0.15} = 0.11$ m/s ◄

Note: Answers may vary somewhat due to scaling.

4.10 *Make an approximate plot of the frictionless velocity (relative to U_0) along both the inner and the outer boundaries of Fig. 4.10. By what percent is the ideal maximum inner velocity greater than the ideal minimum outer velocity?*

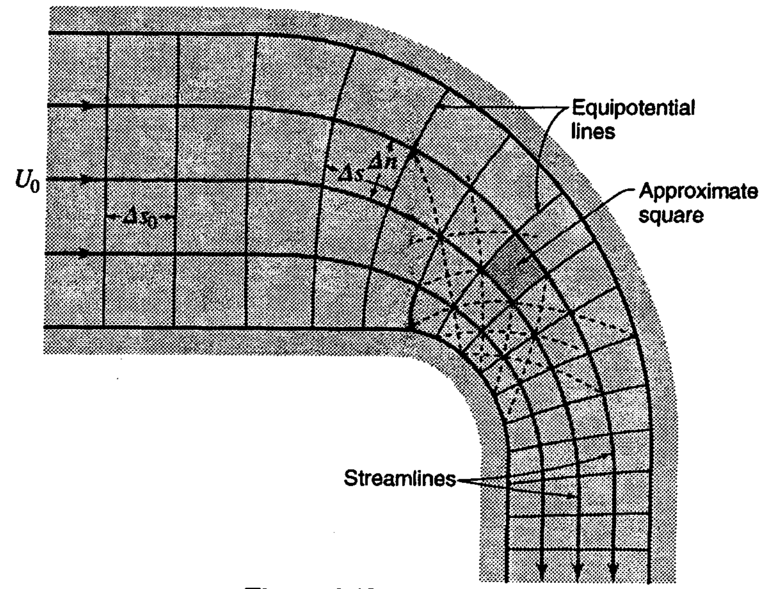
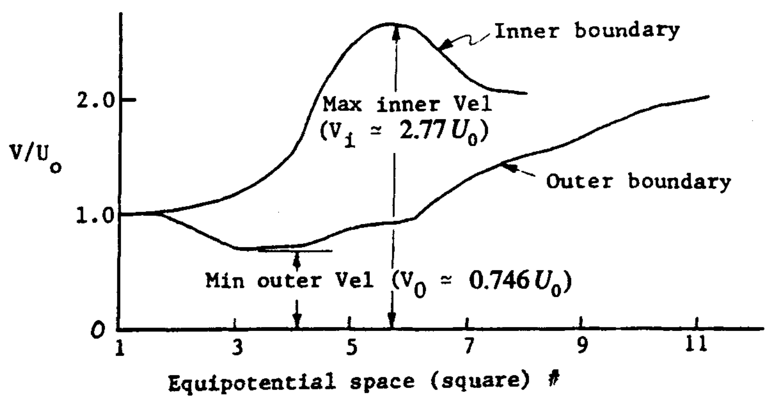


Figure 4.10

N

In stream tube, $q = \text{const} = AV \propto \Delta s V = \Delta s_0 U_0$, so $\dfrac{V}{U_0} = \dfrac{1}{(\Delta s/\Delta s_0)}$



$$\frac{9.77\text{ mm}}{3.50\text{ mm}} = 2.77$$

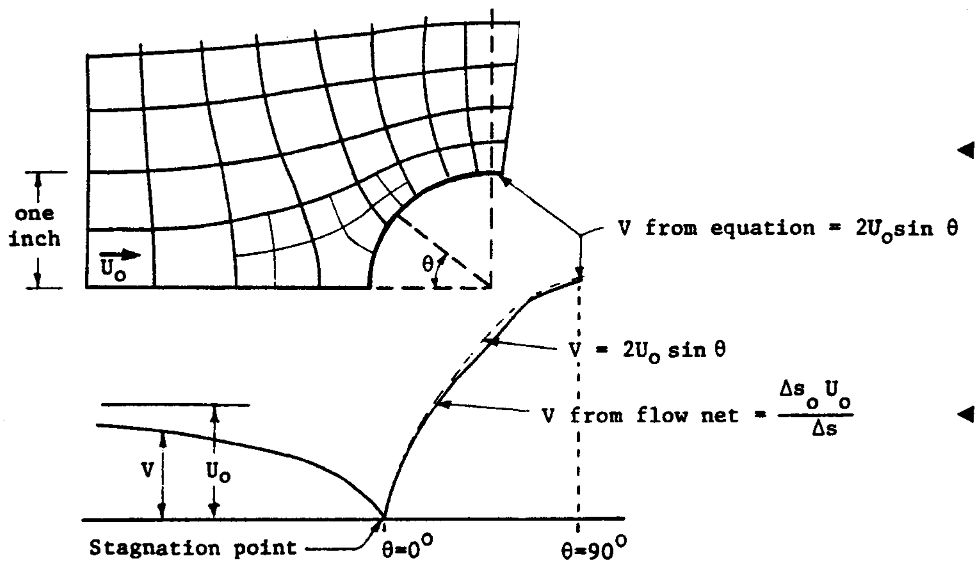
$$\frac{9.7\text{ mm}}{13.0\text{ mm}} = 0.746$$

Ratio $\dfrac{\text{max. inner vel.}}{\text{min. outer vel.}} = \dfrac{V_i}{V_o} \approx \dfrac{2.77U_0}{0.764U_0} = 3.74$, or V_i is about 274% greater than V_0 ◄

Note: Answers may vary somewhat due to scaling.

4.11 *Consider the two-dimensional flow about a 2-in-diameter cylinder. Sketch the flow net for the ideal flow around one quarter of the cylinder. Start with a uniform net of ½-in squares, and fill in with ¼-in squares where desirable. (Note: We can prove by classical hydrodynamics that the velocity tangent to the cylinder at a point 90° from the stagnation point is twice the uniform velocity.) From the flow net, determine the velocities (relative to U_0) along the center streamline from a point upstream where the velocity is uniform to the stagnation point, and then along the boundary of the cylinder from the stagnation point to the 90° point; plot them vs distance. By plotting a second curve on the same graph, compare the result thus obtained with the values given by the equation $V = 2U_0 \sin\theta$, where U_0 is the undisturbed stream velocity and θ is the angle subtended by the arc from the stagnation point to any point on the cylinder where V is desired.*

BG



Sec 4.12: Velocity and Acceleration in Steady Flow -- *Exercises* (8)

4.12.1 *A flow field is defined by u = 2, v = 3, w = 4. What is the velocity of flow? Specify units in terms of L and T.*

N

Sec. 4.12: $V = (2^2 + 3^2 + 4^2)^{1/2} = 5.39\ L/T$ ◄

4.12.2 *The velocity along a streamline lying on the x axis is given by $u = 6 + x^{0.4}$. What is the convective acceleration at x = 5? Specify units in terms of L and T. Assuming the fluid is incompressible, is the flow converging or diverging?*

N

$u = 6 + x^{0.4}$ ∴ $\partial u/\partial x = 0.4\,x^{-0.6}$

Eq. 4.23*a*: Convective accel. $= u(\partial u/\partial x) = 2.4x^{-0.6} + 0\ 4x^{-0.2}$

At $x = 5$: Convective accel. $= 2.4(5)^{-0.6} + 0.4(5)^{-0.2} = 1.204\ L/T^2$ ◄

If the fluid is incompressible, because the velocity increases the flow must be converging. ◄

4.12.3 *A flow field is defined by $u = 2x$, $v = y$. Derive expressions for the x and y components of acceleration. Find the magnitude of the velocity and acceleration at the point (3,2). Specify units in terms of L and T.*

N

$$\frac{\partial u}{\partial x} = 2;\ \frac{\partial u}{\partial y} = 0.\ \text{Eq. 4.28}a\text{:}\ a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = 2u + 0 = 4x \quad ◀$$

$$\frac{\partial v}{\partial x} = 0;\ \frac{\partial v}{\partial y} = 1.\ \text{Eq. 4.28}b\text{:}\ a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = 0 + v = y \quad ◀$$

At (3,2): $u = 2x = 6$; $v = y = 2$; $V = \sqrt{6^2+2^2} = 6.32\ L/T$ ◀

and $a_x = 4x = 12$; $a_y = y = 2$; $a = \sqrt{12^2+2^2} = 12.17\ L/T^2$ ◀

4.12.4 *A flow field is defined by $u = 2y$, $v = x$. Derive expressions for the x and y components of acceleration. Find the magnitude of the velocity and acceleration at the point (3,1). Specify units in terms of L and T.*

N

$$\frac{\partial u}{\partial x} = 0;\ \frac{\partial u}{\partial y} = 2.\ \text{Eq. 4.28}a\text{:}\ a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = 0 + 2v = 2x \quad ◀$$

$$\frac{\partial v}{\partial x} = 1;\ \frac{\partial v}{\partial y} = 0.\ \text{Eq. 4.28}b\text{:}\ a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = u + 0 = 2y \quad ◀$$

At (3,1): $u = 2y = 2$; $v = x = 3$; $V = \sqrt{2^2+3^2} = 3.61\ L/T$ ◀

and $a_x = 2x = 6$; $a_y = 2y = 2$; $a = \sqrt{6^2+2^2} = 6.32\ L/T^2$ ◀

4.12.5 *A flow field is defined by $u = 2y$, $v = xy$. Derive expressions for the x and y components of acceleration. Find the magnitude of the velocity and acceleration at the point (2,3). Specify units in terms of L and T.*

N

$$\frac{\partial u}{\partial x} = 0;\ \frac{\partial u}{\partial y} = 2.\ \text{Eq. 4.28}a\text{:}\ a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = 0 + 2xy = 2xy \quad ◀$$

$$\frac{\partial v}{\partial x} = y;\ \frac{\partial v}{\partial y} = x.\ \text{Eq. 4.28}b\text{:}\ a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = 2y^2 + x^2y \quad ◀$$

At (2,3): $u = 2y = 6$; $v = xy = 6$; $V = \sqrt{6^2+6^2} = 8.49\ L/T$ ◀

and $a_x = 2xy = 12$; $a_y = 2y^2 + x^2y = 2(3)^2 + (2)^2(3) = 18 + 12 = 30$

$a = \sqrt{12^2+30^2} = 32.3\ L/T^2$ ◀

4.12.6 *The velocity along a circular streamline of radius 4 ft is 2.2 fps. Find the normal and tangential components of the acceleration if the flow is steady.*

BG

$$\text{Eq. 4.19:}\ a_n = \frac{V^2}{r} = \frac{2.2^2}{4} = 1.210\ \text{ft/sec}^2 \quad ◀$$

Flow is steady (given), so $V_t = \text{const}$, $a_t = 0$ ◀

4.12.7 *The velocity along a circular streamline of radius 1.5 m is 0.75 m/s. Find the normal and tangential components of the acceleration if the flow is steady.*

SI

Eq. 4.19: $a_n = \dfrac{V^2}{r} = \dfrac{0.75^2}{1.5} = 0.375\ \text{m/s}^2$ ◄

Flow is steady (given), so V_t = const, $a_t = 0$ ◄

4.12.8 *A large tank contains an ideal liquid which flows out of the bottom of the tank through a 4-in-diameter hole. The rate of steady outflow is 5 cfs. Assume that the liquid approaches the center of the hole radially. Find the velocities and convective accelerations at points that are 2.5 and 5 ft from the center of the hole.*

BG

Area through which (steady) flow occurs is a hemispherical surface, of area $2\pi r^2$ (Table A.8)

Eq. 4.6: $V_r = \dfrac{Q}{A} = \dfrac{5}{2\pi r^2}; \quad \dfrac{\partial V}{\partial r} = -\dfrac{5}{\pi r^3}$

At $r_2 = 2.5$ ft: $V_2 = 0.1273$ fps ◄ ; At $r_3 = 5$ ft: $V_3 = 0.0318$ fps ◄

$$a_r = V\left(\frac{\partial V}{\partial s}\right) = V\left(-\frac{\partial V}{\partial r}\right) = \left[\frac{5}{2\pi r^2}\right]\left[\frac{+5}{\pi r^3}\right] = +\frac{12.5}{\pi^2 r^5}$$

$a_2 = \dfrac{12.5}{\pi^2 2.5^5} = 0.01297\ \text{ft/sec}^2$ ◄; $a_3 = 0.000\,405\ \text{ft/sec}^2$ ◄

Sec 4.12: Velocity and Acceleration in Steady Flow -- *Problems* 4.12 – 4.18

4.12 *An ideal liquid flows out the bottom of a large tank though a 100-mm-diameter hole at a steady rate of 0.80 m³/s. Assume the liquid approaches the center of the hole radially. Find the velocities and convective accelerations at points 0.75 and 1.5 m from the center of the hole.*

SI

Area through which (steady) flow occurs is a hemispherical surface, of area $2\pi r^2$ (Table A.8)

Eq. 4.6: $V = \dfrac{Q}{A} = \dfrac{0.80}{2\pi r^2}$, directed only along r.

∴ From Eq. 4.23: Convective $a = V\left(-\dfrac{\partial V}{\partial r}\right) = \dfrac{Q}{2\pi r^2} \times \dfrac{Q}{\pi r^3} = \dfrac{0.80^2}{2\pi^2 r^5}$

At $r = 0.75$ m: $V = 0.226$ m/s ◄; $a = 0.1366\ \text{m/s}^2$ ◄

At $r = 1.5$ m: $V = 0.0566$ m/s ◄; $a = 0.004\,27\ \text{m/s}^2$ ◄

4.13 *A flow field is defined by u = 3y, v = 2xy, w = 5z. Derive expressions for the x, y, and z components of acceleration. Find the magnitude of the velocity and acceleration at the point (1,2,1). Specify units in terms of L and T.*

N

$\frac{\partial u}{\partial x} = 0;\ \frac{\partial u}{\partial y} = 3;\ \frac{\partial u}{\partial z} = 0.$ Eq. 4.28*a*: $a_x = 0 + 3v + 0 = 6xy$ ◄

$\frac{\partial v}{\partial x} = 2y;\ \frac{\partial v}{\partial y} = 2x;\ \frac{\partial v}{\partial z} = 0.$ Eq. 4.28*b*: $a_y = 2xu + 2xv + 0 = 6y^2 + 4x^2y$ ◄

$\frac{\partial w}{\partial x} = 0;\ \frac{\partial w}{\partial y} = 0;\ \frac{\partial w}{\partial z} = 5.$ Eq. 4.28*c*: $a_z = 0 + 0 + 5(5z) = 25z$ ◄

At (1,2,1): $u = 3y = 6;\ v = 2xy = 4;\ w = 5z = 5;\ V = \sqrt{6^2+4^2+5^2} = 8.77\ L/T$ ◄

and $a_x = 6xy = 12;\ a_y = 6y^2 + 4x^2y = 24 + 8 = 32;\ a_z = 25;$

$a = \sqrt{12^2+32^2+25^2} = 42.3\ L/T^2$ ◄

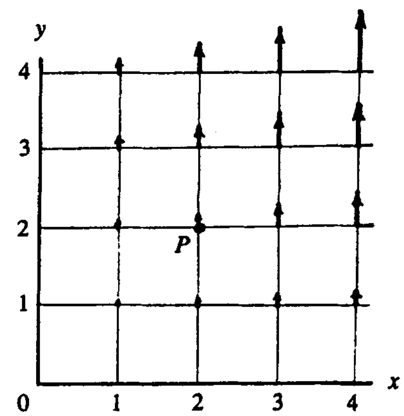
4.14 *Sketch the flow field defined by u = 0, v = 3xy, and derive expressions for the x and y components of acceleration. Find the acceleration at the point (2,2). Specify units in terms of L and T.*

N

Eq. 4.23*a*: $a_x = 0(0) + 3xy(0) = 0$ ◄

Eq. 4.23*b*: $a_y = 0(3y) + 3xy(3x) = 9x^2y$ ◄

At (2,2): $a = 9(2)^2(2) = 72\ L/T^2$ ◄



◄

4.15 *Sketch the flow field defined by u = 3y, v = 2, and derive expressions for the x and y components of acceleration. Find the magnitude of the velocity and acceleration for the point having the coordinates (3,4). Specify units in terms of L and T.*

N

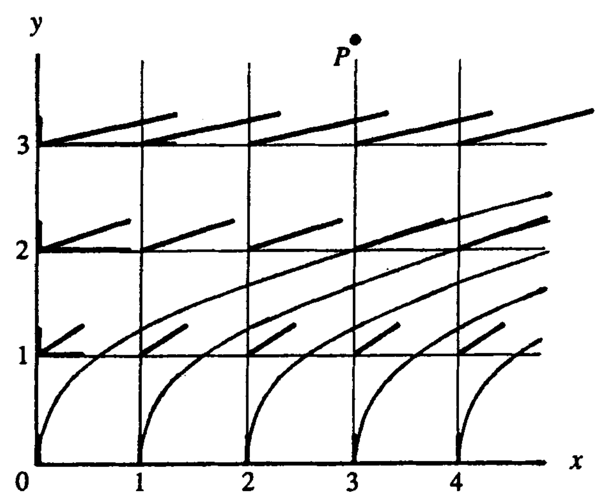
Eq. 4.23*a*: $a_x = 3y(0) + 2(3) = 6\ L/T^2$ ◄

Eq. 4.23*b*: $a_y = 3y(0) + 2(0) = 0$ ◄

At (3,4): $V = \sqrt{(3y)^2+2^2}$

$= \sqrt{12^2+2^2} = 12.2\ L/T$ ◄

$a = \sqrt{6^2+0^2} = 6\ L/T^2$ ◄



◄

4.16 *(a) Sketch the flow field defined by $u = -2y$, $v = 3x$, and derive expressions for the x and y components of acceleration. (b) As in Sample Prob. 4.3, find approximate values of the normal and tangential accelerations of the particle at the point (2,3). Specify units in terms of L and T. (c) Compare the values of $(a_n^2 + a_t^2)^{1/2}$ with the computed value $(a_x^2 + a_y^2)^{1/2}$.*

N

(*a*) Eq. 4.23*a*: $a_x = -2y(0) + 3x(-2) = -6x$ ◀

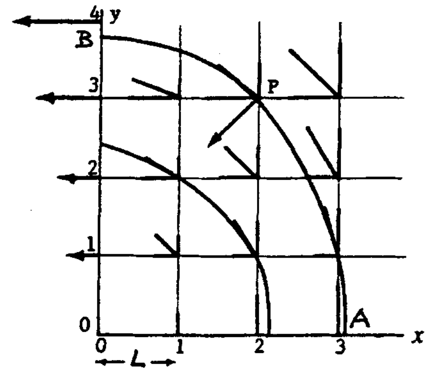
Eq. 4.23*b*: $a_y = -2y(3) + 3x(0) = -6y$ ◀

$V = \sqrt{(2y)^2 + (3x)^2} = \sqrt{4y^2 + 9x^2}$

At $P(2,3)$:

$V = \sqrt{4(3)^2 + 9(2)^2} = 8.49\ L/T$

$a = \sqrt{(-6\times 2)^2 + (-6\times 3)^2} = 21.6\ L/T^2$



◀

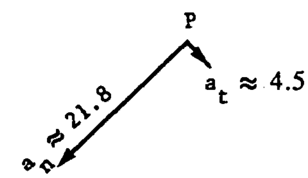
(*b*) Per Sample Prob. 4.3: From sketch of this flowfield, $y_A \approx 3.75\ L$, $x_B \approx 3.08\ L$. $s_B - s_A \approx 5.4\ L$ and at $P(2,3)$: $r \approx 3.3\ L$.

At A, $u_A = y = 0$, so $V_A = v_A = 3x \approx 3(3.08\ L) = +9.24\ L = 9.24\ L$ ↑

At B, $v_B = x = 0$, so $V_B = u_B = -2y \approx -2(3.75\ L) = -7.50\ L = 7.50\ L$ ←

$$\frac{\partial V}{\partial s} \approx \frac{V_B - V_A}{s_B - s_A} \approx \frac{7.50\,L - 9.24\,L}{+5.4\,L} = -0.527\ T^{-1}$$

Eq. 4.25: $a_n = \dfrac{V^2}{r} = \dfrac{8.49^2}{3.3} = 21.8\ L/T^2$ ◀



$a_t = V(\partial V/\partial s) \approx 8.49(-0.527) = -4.47 \approx -4.5\ L/T^2$ ◀

(*c*) $a = \sqrt{a_n^2 + a_t^2} \approx \sqrt{21.8^2 + 4.47^2} = 22.3 \approx 22\ L/T^2$

Cf. $a = 21.6\ L/T^2$, these two accelerations compare well ◀

Note: Answers may vary somewhat due to scaling.

4.17 *The steady flow rate in each of the four stream tubes of Fig. 4.10 is 15 cfs per foot perpendicular to the plane of the figure. By scaling, the dimensions of the shaded "square" have been found to be 1.65 ft wide on the upstream face, 1.53 ft wide on the downstream face, and 1.67 ft along the flow line through its center; the radius of that flow line measures 11.1 ft. Find the normal, tangential, and resultant accelerations of a fluid particle at the center of the shaded area.*

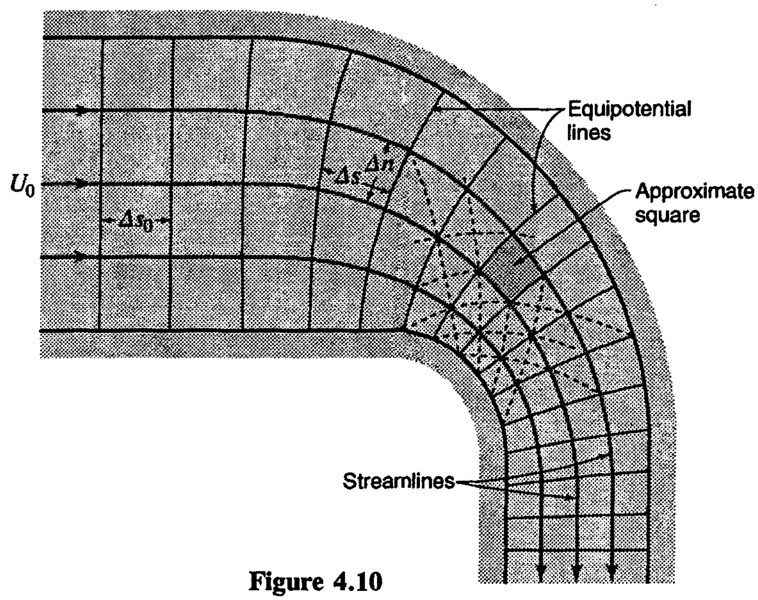


Figure 4.10

BG

At the center of the shaded "square": Width ≈ (1.65 + 1.53)/2 = 1.59 ft

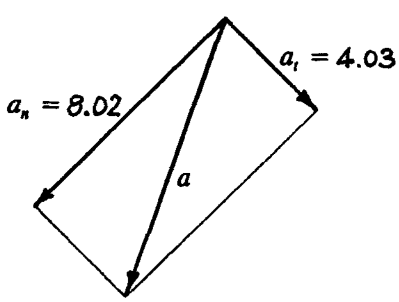
Eq. 4.6: $V = Q/A \approx 15/1.59 = 9.43$ fps

Eq. 4.25: $a_n = \dfrac{V^2}{r} \approx \dfrac{9.43^2}{11.1} = 8.02 \approx 8.0$ ft/sec² radially inward ◄

In the shaded "square", the tube width changes from 1.65 ft to 1.53 ft over a distance of 1.67 ft, so

$$\frac{\partial V}{\partial s} \approx \frac{(15/1.53) - (15/1.65)}{1.67} = 0.427 \text{ ft/sec}^2$$

Eq. 4.24: $a_t = V\dfrac{\partial V}{\partial s} \approx 9.43(0.427) = 4.03 \approx 4.0$ ft/sec² in the flow direction ◄



$a \approx \sqrt{8.02^2 + 4.03^2} = 8.97 \approx 9.0$ ft/sec² (downward) ◄

4.18 *Figure P4.18 represents a two-dimensional stream tube drawn to scale. If the flow rate is 25 m³/s per meter perpendicular to the plane of the sketch, determine approximate values of the normal and tangential accelerations of a fluid particle at C. What is the resultant acceleration of a particle at C?*

SI

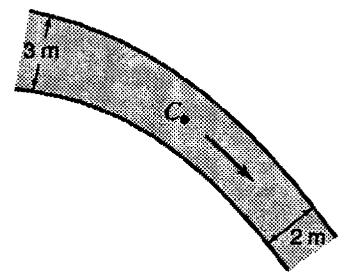


Figure P4.18

Note to instructor: Answers will vary depending on the measured dimensions. The variability of the results can be used to demonstrate the sensitivity to the accuracy of the measurements.

Measure Fig. 4.18: At C, width $\approx$ 2.3 m, $r \approx$ 14.7 m.

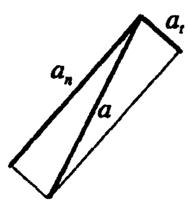
Eq. 4.6: $V_C = \dfrac{Q}{A} \approx \dfrac{25}{2.3} = 10.87$ m/s

Eq. 4.25: $a_n = \dfrac{V_C^2}{r} \approx \dfrac{10.87^2}{14.7} = 8.04 \approx 8.0$ m/s² radially inward ◄

In the 2 m surrounding C the width changes from about 2.35 m to 2.27 m, so

$$\frac{\partial V}{\partial s} \approx \frac{(25/2.27) - (25/2.35)}{2} = 0.1875 \text{ m/s per m}$$

Eq. 4.24: $a_t = V\dfrac{\partial V}{\partial s} \approx 10.87(0.1875) = 2.04 \approx 2.0$ m/s² tangentially downward to the right ◄



$a \approx \sqrt{8.04^2 + 2.04^2} = 8.29 \approx 8.3$ m/s² ◄

Sec 4.13: Velocity and Acceleration in Unsteady Flow -- *Exercises* (4)

4.13.1 *A flow is defined by $u = 2(1+t)$, $v = 3(1+t)$, $w = 4(1+t)$. What is the velocity of flow at the point (3,2,4) at $t = 2$? What is the acceleration at that point at $t = 2$? Specify units in terms of L and T.*

N

At t = 2 sec: $u = 6$ L/T, $v = 9$ L/T, $w = 12$ L/T

$V_{3,2,4} = \sqrt{6^2 + 9^2 + 12^2} = 16.16$ L/T ◄

u, v and w are all not functions of x, y, or z, so all convective accelerations (see Eq. 4.28) are zero

$\therefore\ a_x = \dfrac{\partial u}{\partial t} = 2\ L/T;\ \ a_y = \dfrac{\partial v}{\partial t} = 3\ L/T^2;\ \ a_z = \dfrac{\partial w}{\partial t} = 4\ L/T^2$

and $a = \sqrt{2^2 + 3^2 + 4^2} = 5.39$ L/T^2 ◄

4.13.2 *A two-dimensional flow field is given by $u = 2 + xy + 3t^2$, $v = 2xy^2 + t$. Find the velocity and acceleration of a particle of fluid at point (2,3) at $t = 4$. Specify units in terms of L and T.*

N

At point (2,3) with t = 4:

$u = 2 + 6 + 48 = 56$ L/T, $v = 36 + 4 = 40$ L/T; $V = \sqrt{u^2 + v^2} = 68.8$ L/T ◄

Eq. 4.28*a*: $a_x = (2 + xy + 3t^2)y + (2xy^2 + t)x + 6t = 272$ L/T^2

Eq. 4.28*b*: $a_y = (2 + xy + 3t^2)2y^2 + (2xy^2 + t)4xy + 1 = 1969$ L/T^2

Finally, $a = \sqrt{272^2 + 1969^2} = 1988$ L/T^2 ◄

4.13.3 *The flow velocity in fps along a circular streamline of radius 3 ft is 0.6 + 1.2t. Find the normal and tangential components of the acceleration when t = 1.5 sec.*

BG

At t =1.3 sec: $V = 0.6 + 1.2(1.5) = 2.40$ fps ; at any t: $\partial V/\partial t = 1.2$ ft/sec^2

Eq. 4.25: $a_n = V^2/r = 2.40^2/3 = 1.920$ ft/sec^2 ◄

Eq. 4.29: $a_t = V\dfrac{\partial V}{\partial s} + \dfrac{\partial V}{\partial t} = V(0) + 1.2 = 1.200$ ft/sec^2 ◄

4.13.4 *The flow velocity in m/s along a circular streamline of radius 1.5 m is 0.4 + 0.6t. Find the normal and tangential components of the acceleration when t = 1.2 s.*

SI

At t =1.2 sec: $V = 0.4 + 0.6(1.2) = 1.120$ m/s

Eq. 4.25: $a_n = V^2/r = 1.120^2/1.5 = 0.836$ m/s^2 ◄

Eq. 4.29: $a_t = V\dfrac{\partial V}{\partial s} + \dfrac{\partial V}{\partial t} = V(0) + 0.6 = 0.600$ m/s^2 ◄

Sec 4.13: Velocity and Acceleration in Unsteady Flow -- *Problems* 4.19 – 4.21

4.19 *A large tank contains an ideal liquid which flows out of the bottom through a 4-in-diameter hole. The outflow rate Q = 8 − 0.5t, where Q is in cfs and t is in sec. Assume the liquid approaches the center of the hole radially. Find the local acceleration at a point 2 ft from the center of the hole at times t = 5 sec and 10 sec. What is the total acceleration at a point 3 ft from the center of the hole at t = 10 sec?*

BG

Area through which flow occurs is a hemispherical surface, of area $2\pi r^2$ (Table A.8)

Eq. 4.6: $V = \dfrac{Q}{A} = \dfrac{8-0.5t}{2\pi r^2}$; local acceleration $= \dfrac{\partial V}{\partial t} = -\dfrac{0.5}{2\pi r^2}$

At r = 2 ft, local accel $= \left(\dfrac{\partial V}{\partial t}\right)_2 = -\dfrac{0.5}{2\pi 2^2} = -0.019\,89$ ft/sec^2 ◄

Local accel is indep of t, ∴ at r = 2, local accel = const.

∴ At r = 2 ft, t = 5 sec, local accel = − 0.019 89 ft/sec^2 ◄

and at r = 2 ft, t = 10 sec, local accel = − 0.019 89 ft/sec^2 ◄

Since s = const − r, $\dfrac{\partial V}{\partial s} = -\dfrac{\partial V}{\partial r} = \dfrac{+(8-0.5t)}{\pi r^3}$

Eq. 4.29 at t = 10 sec, r = 3 ft: Total $a = V\dfrac{\partial V}{\partial s} + \dfrac{\partial V}{\partial t}$

$$= \frac{8-0.5(10)}{2\pi 3^2} \times \frac{8-0.5(10)}{\pi 3^3} - \frac{0.5}{2\pi 3^2} = 0.001\,876 - 0.008\,84 = -0.006\,97 \text{ ft/sec}^2$$ ◄

4.20 *An ideal liquid flows out of the bottom of a large tank through an 80-mm-diameter hole. The outflow rate $Q = 0.4 - 0.02t^{0.5}$, where Q is in m^3/s and t is in s. Assume the liquid approaches the center of the hole radially. Find the local, convective, and total accelerations at a point 0.5 m from the center of the hole at time t = 12 s.*

SI

Area through which flow occurs is a hemispherical surface, of area $2\pi r^2$ (Table A.8)

Eq. 4.6: $V = \dfrac{Q}{A} = \dfrac{0.4 - 0.02t^{0.5}}{2\pi r^2}$, directed only along r

$$\left(\frac{\partial V}{\partial r}\right) = -\frac{0.4 - 0.02t^{0.5}}{\pi r^3};\quad \frac{\partial V}{\partial t} = -\frac{0.01t^{-0.5}}{2\pi r^2}$$

∴ From Sec. 4.13, at $r = 0.5$ m, $t = 12$ s: Local accel $= \partial V/\partial t = -0.001\ 838\ \text{m/s}^2$ ◄

and since $s = \text{const} - r$, convective accel $= V(\partial V/\partial s) = -V(\partial V/\partial r) = 0.1773\ \text{m/s}^2$ ◄

and, from Eq. 4.29: Total accel $= 0.1773 - 0.0018 = 0.1755\ \text{m/s}^2$ ◄

4.21 *Refer to the two-dimensional stream tube drawn to scale in Fig. P4.18. If the flow rate is $(18 - 4t)$ m^3/s per meter perpendicular to the plane of the sketch, with t in s, find approximate values of the normal tangential, and total accelerations of a fluid particle at C when t = 3 s.*

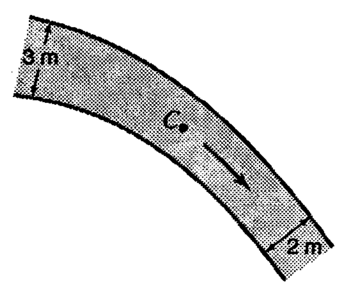


Figure P4.18

SI

Note to instructor: Answers will vary depending on the measured dimensions. The variability of the results can be used to demonstrate the sensitivity to the accuracy of the measurements.

Measure Fig. 4.18: At C, width ≈ 2.31 m, $r \approx 14.67$ m.

Eq. 4.6: $V_C = \dfrac{Q}{A} \approx \dfrac{18 - 4t}{2.31};\quad \dfrac{\partial V_C}{\partial t} \approx \dfrac{-4}{2.31} = -1.732$ m/s (at any t)

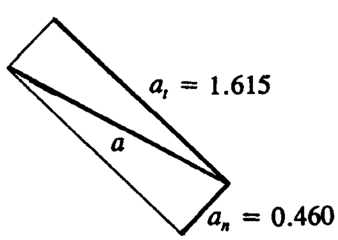
In the 2 m surrounding C, by measurement, the width changes from about 2.35 m to 2.27 m.

$$\frac{\partial V_C}{\partial s} \approx \frac{1}{2}\left(\frac{Q}{2.27} - \frac{Q}{2.35}\right)$$

When $t = 3$ s: $Q = 6\ \text{m}^3/\text{s}$, $V_C \approx 2.60$ m/s, $\partial V_C/\partial s \approx 0.0450$ m/s per m.

Eq. 4.25: $(a_C)_n = \dfrac{V_C^2}{r} \approx \dfrac{2.60^2}{14.67} = 0.460 \approx 0.46\ \text{m/s}^2$ radially inward ◄

Eq. 4.29: $(a_C)_t = V\left(\dfrac{\partial V}{\partial s}\right) + \dfrac{\partial V}{\partial t} \approx 2.60(0.0450) + (-1.732) = -1.615 \approx -1.6\ \text{m/s}^2$ ◄



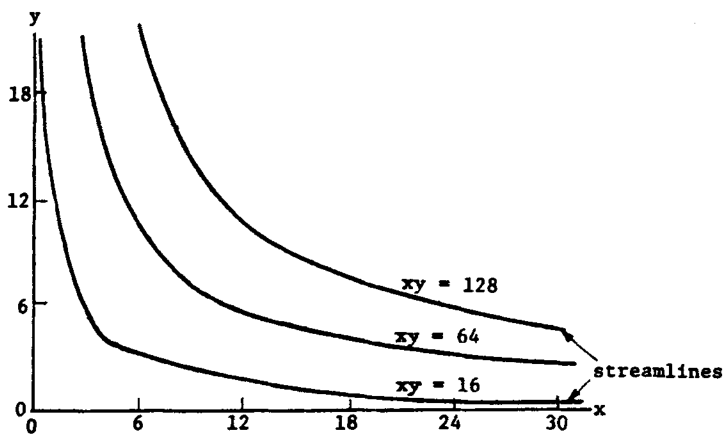
$a \approx \sqrt{1.615^2 + 0.460^2} = 1.679 \approx 1.68\ \text{m/s}^2$ ◄

Note: Answers may vary somewhat due to scaling.

Chapter 4: Miscellaneous – *Problems* 4.22 – 4.24

4.22 *Assume that the streamlines for a two-dimensional flow of a fricitonless incompressible fluid against a flat plate normal to the initial velocity may be represented by the equation xy = constant and that the flow is symmetrical about the plane through x = 0. A different streamline may be plotted for each value of the constant. Plot streamlines for values of the constant of 16, 64, and 128.*

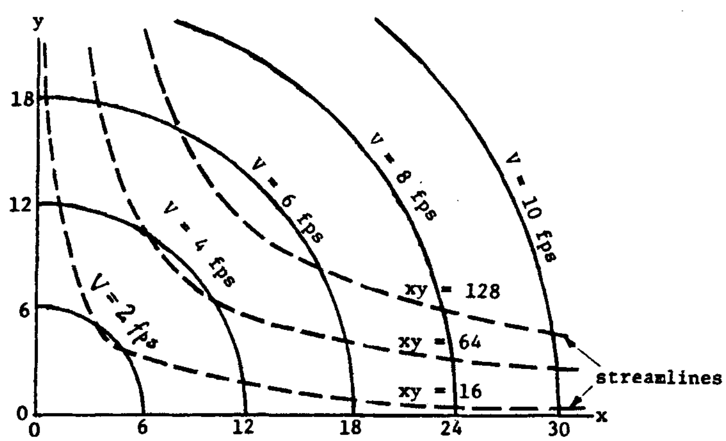
N



◄

4.23 *For the case in Prob. 4.22, we can show that the velocity components at any point are $u = ax$ and $v = -ay$, where a is a constant. Thus the actual velocity is $V = a(x^2+y^2)^{1/2} = ar$, where r is the radius to the origin. Let a = 1/3. Draw curves of equal velocity for values of 2, 4, 6, 8, and 10 fps. How does the velocity vary along the surface of the plate?*

BG



◄

Along the surface of the plate $V = x/3$ fps, where x ft = the distance from the origin. ◄

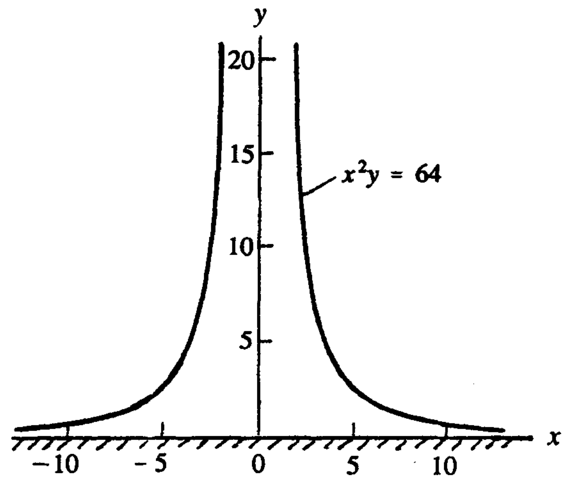
4.24 *For three-dimensional flow with the y axis as the centerline, assume that the equation for the bounding streamline of a jet impinging vertically downward on a flat plate is $x^2y = 64$. (a) Plot the flow showing the centerline and bounding streamlines of the jet. (b) What is the approximate average velocity in the vertical jet at y = 10 if the average velocity in the vertical jet is 5.0 m/s at y = 16? (c) For the above conditions find the approximate velocity along the plate at r = 12, 24, 36.*

SI

$x^2y = 64$

(*a*)

y	x
20	1.79
15	2.07
10	2.53
5	3.58
3	4.62
2	5.66
1	8
0.444	12



◄

(*b*) At $y = 10$, $x = \sqrt{6.4} = 2.53$

At $y = 16$, $x = 2.00$

$Q = AV = \pi R_{10}^2 V_{10} = \pi R_{16}^2 V_{16}$

$Q = \pi(6.4)V_{10} = \pi(2)^2 5 = 20\pi$

$V_{10} = 20/6.4 = 3.125$ m/s ◄

(*c*) At $x = 12$, $y = 64/144 = 0.444$; At $x = 24$, $y = 0.111$; At $x = 36$, $y = 0.0494$

$Q = 20\pi = 2\pi r y V = 2\pi(12)(0.444)V_{12} = 2\pi(24)(0.111)V_{24} = 2\pi(36)(0.0494)V_{36}$

$V_{12} = 1.875$ m/s ◄ $V_{24} = 3.75$ m/s ◄ $V_{36} = 5.63$ m/s ◄

Chapter 5

Energy Considerations in Steady Flow

PROBLEM SELECTION GUIDE

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
5.1	*Kinetic Energy of a Flowing Fluid*						
	X[1]5.1.1	N	Medium	Medium	1		Integration
	5.1.2	N	Medium	Medium	1		Integration
	5.1.3	N	Medium	Medium	1		Integration
	P 5.1	N	Medium	Medium	1		Integration
	5.2	N	Hard	Long	1		Integration
5.2	*Equation for Steady Motion of an Ideal Fluid Along a Streamline, and Bernoulli's Theorem*						
	X 5.2.1	BG	Easy	Short	1		
	5.2.2	BG	Easy	Short	1	5.2.3	
	5.2.3	SI	Easy	Short	1	5.2.2	
	5.2.4	SI	Easy	Short	1		
	5.2.5	BG	Medium	Short	1		
	X 5.3	SI	Easy	Short	1		
	5.4	BG	Easy	Short	2	5.5	
	5.5	SI	Easy	Short	2	5.4	
	5.6	BG	Medium	Medium	2		
	5.7	BG	Easy	Short	1	5.8	
	5.8	SI	Easy	Short	1	5.7	
	5.9	BG	Medium	Medium	1		Uses $pv = RT$ (Sec. 2.7). Interpolation.
	5.10	BG	Medium	Medium	1		
5.3	*Equation for Steady Motion of a Real Fluid Along a Streamline*						
	5.3.1	BG	Easy	Short	2	5.3.2	
	5.3.2	SI	Easy	Short	2	5.3.1	
	5.3.3	BG	Easy	Short	2		
	5.3.4	BG	Easy	Short	2	5.3.5	
	5.3.5	SI	Easy	Short	1	5.3.4	
	5.3.6	BG	Easy	Medium	1		
	5.3.7	SI	Easy	Medium	1		
	P 5.11	BG	Easy	Medium	1	5.12	
	5.12	SI	Easy	Medium	1	5.11	
	5.13	BG	Medium	Medium	2		
	5.14	SI	Medium	Medium	1		
	5.15	BG	Medium	Long	1		

/cont...

[1] For all Exercises (identified by "X"), answers are given in Appendix F of the textbook.
X = Exercise, P = (end-of-chapter) Problem, S = Sample Problem.

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
5.4	***Pressure in Fluid Flow***						
	X 5.4.1	BG	Easy	Short	1	5.4.2-3	
	5.4.2	SI	Easy	Short	1	5.4.1-3	
	5.4.3	BG	Easy	Short	1	5.4.1-2	
	P 5.16	SI	Medium	Short	1		
	5.17	SI	Medium	Medium	1		
	5.18	BG	Hard	Medium	2		Integration
	5.19	BG	Medium	Long	1		
5.6	***Energy Equation for Steady Flow of Incompressible Fluids, Bernoulli's Theorem***						
	X 5.6.1	BG	V Easy	Short	2	5.6.2	
	5.6.2	SI	V Easy	Short	2	5.6.1	
	5.6.3	BG	Medium	Short	1	5.6.4	
	5.6.4	SI	Medium	Short	1	5.6.3	
	P 5.20	SI	Medium	Short	1		Uses Sec. 4.5
	5.21	BG	Medium	Medium	2		
	5.22	BG	Medium	Medium	1	5.23	
	5.23	SI	Medium	Medium	1	5.22	
5.7	***Energy Equation for Steady Flow of Compressible Fluids***						
	X 5.7.1	BG	Easy	Short	2	5.7.2	Uses $pv = RT$ (Sec. 2.7)
	5.7.2	SI	Easy	Short	2	5.7.1	Uses $pv = RT$ (Sec. 2.7)
	5.7.3	BG	Medium	Medium	2	5.7.4	Uses $pv = RT$ (Sec. 2.7)
	5.7.4	SI	Medium	Medium	2	5.7.3	Uses $pv = RT$ (Sec. 2.7)
5.9	***Power Considerations in Fluid Flow***						
	X 5.9.1	BG	Easy	Short	1	5.9.2	
	5.9.2	SI	Easy	Short	1	5.9.1	
	5.9.3	BG	Easy	Medium	1	5.9.4	
	5.9.4	SI	Easy	Medium	1	5.9.3	
	5.9.5	BG	Easy	Medium	1	5.9.6	
	5.9.6	SI	Easy	Medium	1	5.9.5	
	P 5.24	BG	Easy	Medium	1	5.25	
	5.25	SI	Easy	Medium	1	5.24	
5.10	***Cavitation***						
	X 5.10.1	BG	Easy	Medium	1	5.10.2	Uses Sec. 3.5
	5.10.2	SI	Easy	Medium	1	5.10.1	Uses Sec. 3.5
	5.10.3	BG	Easy	Medium	1	5.10.4	Uses Sec. 3.5
	5.10.4	SI	Easy	Medium	1	5.10.3	Uses Sec. 3.5
	5.10.5	SI	Medium	Medium	1		Uses Sec. 3.5

/cont...

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
5.12	***Loss of Head at Submerged Discharge***						
	P 5.26	BG	Medium	Medium	1	5.27	
	5.27	SI	Medium	Medium	1	5.26	
	5.28	BG	Medium	Medium	1	5.29	
	5.29	SI	Medium	Medium	1	5.28	
	5.30	BG	Hard	Medium	1	5.31	Confirm assumed T°; interp; cavitation.
	5.31	SI	Hard	Medium	1	5.30	Confirm assumed T°; interp; cavitation.
	5.32	BG	Medium	Medium	1		Cavitation.
5.13	***Applications of Hydraulic Grade Line and Energy Line***						
	X 5.13.1	SI	Easy	Short	1	5.13.2	
	5.13.2	BG	Easy	Short	1	5.13.1	
	5.13.3	SI	Easy	Short	1	5.13.4	
	5.13.4	BG	Easy	Short	1	5.13.3	
	5.13.5	BG	Easy	Medium	1	5.13.6	Sketch
	5.13.6	SI	Easy	Medium	1	5.13.5	Sketch
	5.13.7	BG	Easy	Medium	1	5.13.8	
	5.13.8	SI	Easy	Medium	1	5.13.7	
	P 5.33	SI	Easy	Medium	1		
	5.34	BG	Easy	Medium	2		
	5.35	SI	Easy	Medium	1		
	5.36	BG	Easy	Medium	2	5.37	Cavitation
	5.37	SI	Easy	Medium	2	5.36	Cavitation
	5.38	BG	Easy	Medium	1		Sketch
	5.39	BG	Medium	Long	1	S5.11	Plot
	5.40	BG	Medium	Medium	2	5.41	Uses manometry (Sec. 3.6)
	5.41	SI	Medium	Medium	2	5.40	Uses manometry (Sec. 3.6)
5.14	***Method of Solution of Flow Problems***						
	X 5.14.1	BG	Easy	Short	1	5.14.2	
	5.14.2	SI	Easy	Short	1	5.14.1	
	5.14.3	BG	Medium	Medium	1	5.14.4	T & E (Trial and Error)
	5.14.4	SI	Medium	Medium	1	5.14.3	T & E
5.15	***Jet Trajectory***						
	X 5.15.1	SI	Easy	Short	4		
	5.15.2	SI	Medium	Short	1		
	5.15.3	BG	Medium	Short	1		
	P 5.42	N	Easy	Medium	1		Proof
	5.43	BG	Medium	Medium	1		Differentiation

/cont...

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
5.16	***Flow in a Curved Path***						
	X 5.16.1	BG	Easy	Medium	1		Integration
	5.16.2	SI	Easy	Medium	1		Integration
	P 5.44	BG	Hard	Medium	1	5.45	Integration
	5.45	SI	Hard	Medium	1	5.44	Integration
5.17	***Forced or Rotational Vortex***						
	X 5.17.1	BG	Easy	Short	3		
	5.17.2	SI	Easy	Short	1		
	5.17.3	BG	Easy	Short	1		
	P 5.46	BG	Easy	Short	1	5.47	
	5.47	SI	Easy	Short	1	5.46	
5.18	***Free or Irrotational Vortex***						
	X 5.18.1	SI	Easy	Medium	1	S5.14, 5.18.2	
	5.18.2	BG	Easy	Medium	1	S5.14, 5.18.1	
	P 5.48	BG	Medium	Medium	1	5.49	Integration
	5.49	SI	Medium	Medium	1	5.48	Integration

Chapter 5
ENERGY CONSIDERATIONS IN STEADY FLOW

Sec. 5.1: Kinetic Energy of a Flowing Fluid -- *Exercises (3)*

5.1.1 *Assume the velocity profile for turbulent flow in a circular pipe to be approximated by a parabola from the axis to a point very close to the wall where the local velocity is u = $0.6u_m$, where u_m is the maximum velocity at the axis (Fig. X5.1.1). The equation for this parabola is $u = u_m[1 - 0.4(r/r_0)^2]$. Find α.*

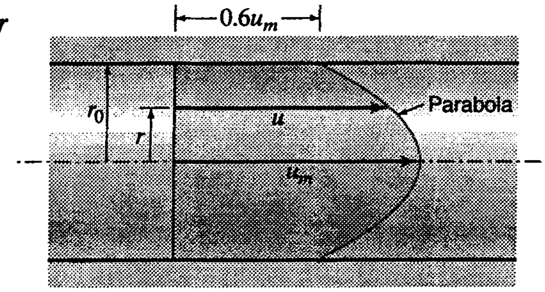


Figure X5.1.1

N

$u = u_m[1 - 0.4(r/r_0)^2]$; $dA = 2\pi r\,dr$

$$Q = AV = \int_0^{r_0} u\,dA = \int_0^{r_0} u_m\left[1 - 0.4\frac{r^2}{r_0^2}\right]2\pi r\,dr$$

$$= 2\pi u_m \int_0^{r_0}\left[r - \frac{0.4r^3}{r_0^2}\right]dr = 2\pi u_m\left[\frac{r^2}{2} - \frac{0.4r^4}{4r_0^2}\right]_0^{r_0} = 2\pi u_m\left[\frac{r_0^2}{2} - \frac{4}{40}r_0^2\right] = 2\pi u_m\left[\frac{16}{40}\right]r_0^2 = 0.8\pi u_m r_0^2$$

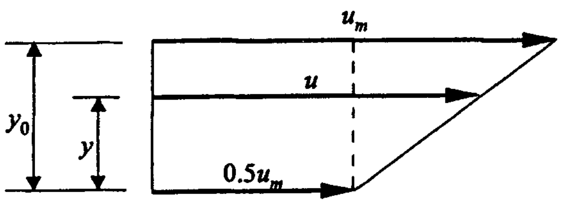
$$V = \frac{Q}{A} = \frac{0.8\pi u_m r_0^2}{\pi r_0^2} = 0.80u_m$$

$$\alpha = \frac{\int u^3 dA}{AV^3} = \frac{2\pi u_m^3\int_0^{r_0}[1 - 0.4(r/r_0)^2]^3 r\,dr}{\pi r_0^2(0.80u_m)^3} = \frac{3.91}{r_0^2}\int_0^{r_0}\left[r - 3r(0.4)\frac{r^2}{r_0^2} + 3r(0.4)^2\frac{r^4}{r_0^4} - r(0.4)^3\frac{r^6}{r_0^6}\right]dr$$

$$= \frac{3.91}{r_0^2}\left[\frac{1}{2} - \frac{1.2}{4} + \frac{0.48}{6} - \frac{0.064}{8}\right]r_0^2 = \frac{17}{16} = 1.063 \quad ◀$$

5.1.2 *Assume an open rectangular channel with the velocity at the surface twice that at the bottom, and with the velocity varying as a straight line from top to bottom. Find α.*

N



Similar Δ's: $\dfrac{u_m - 0.5u_m}{y_0} = \dfrac{u - 0.5u_m}{y}$

from which $u = \dfrac{u_m}{2}\left(\dfrac{y}{y_0} + 1\right)$

Mean velocity $V = (3/4)u_m$; so $u = \dfrac{2}{3}V\left(\dfrac{y}{y_0} + 1\right)$

Let channel width = B ; $dA = B\,dy$

$$\int u^3 dA = \int_0^{y_0}\left[\frac{u_m}{2}\left(\frac{y}{y_0} + 1\right)\right]^3 B\,dy = \frac{Bu_m^3}{8}\int_0^{y_0}\left(\frac{y}{y_0} + 1\right)^3 dy = \frac{Bu_m^3}{8}\int_0^{y_0}\left(\frac{y^3}{y_0^3} + 3\frac{y^2}{y_0^2} + 3\frac{y}{y_0} + 1\right)dy$$

$$= \frac{Bu_m^3}{8}\left[\frac{y^4}{4y_0^3} + \frac{3y^3}{3y_0^2} + \frac{3y^2}{2y_0} + y\right]_0^{y_0} = \frac{Bu_m^3}{8}\left[\frac{1}{4} + 1 + \frac{3}{2} + 1\right]y_0 = \frac{Bu_m^3}{8}[3.75]y_0 = \frac{15}{32}By_0u_m^3$$

Eq 5.4: $$\alpha = \frac{\int u^3 dA}{AV^3} = \frac{15By_0u_m^3}{32(By_0)(3u_m/4)^3} = 10/9 \quad ◀$$

5.1.3 *Find α for the case of a two-dimensional laminar flow, as between two flat plates, for which the velocity profile is parabolic.*

N

Let y_0 = half-width of the symmetric flow. Measuring y from the centerline:

when $y = 0$, $u = u_m$; when $y = y_0$, $u = 0$ $\Big\}$ $u = u_m[1 - (y/y_0)^2]$

Letting Q be the flow through the half-width:

$$Q = \int_0^{y_0} u\,dA = \int_0^{y_0} u_m\left[1 - \left(\frac{y}{y_0}\right)^2\right]B\,dy = Bu_m\int_0^{y_0}\left[1 - \frac{y^2}{y_0^2}\right]dy = Bu_m\left[y - \frac{y^3}{3y_0^2}\right]_0^{y_0} = Bu_m\left[1 - \frac{1}{3}\right]y_0$$

$$= \frac{2}{3}By_0u_m\ ;\quad V = \frac{Q}{A} = \frac{(2/3)By_0u_m}{By_0} = \frac{2}{3}u_m$$

$$\int u^3dA = Bu_m^3\int\left[1 - \left(\frac{y}{y_0}\right)^2\right]^3dy = Bu_m^3\int_0^{y_0}\left[1 - 3\frac{y^2}{y_0^2} + 3\frac{y^4}{y_0^4} - \frac{y^6}{y_0^6}\right]dy = Bu_m^3\left[y - \frac{3y^3}{3y_0^2} + \frac{3y^5}{5y_0^4} - \frac{y^7}{7y_0^6}\right]_0^{y_0}$$

$$= By_0u_m^3\left[1 - 1 + \frac{3}{5} - \frac{1}{7}\right] = \frac{16}{35}By_0u_m^3$$

Eq. 5.4: $$\alpha = \frac{\int u^3dA}{AV^3} = \frac{(16/35)By_0u_m^3}{(By_0)(2u_m/3)^3} = \frac{54}{35} = 1.543 \quad ◄$$

Sec. 5.1: Kinetic Energy of a Flowing Fluid – *Problems 5.1–5.2*

5.1 *Assume the seventh-root law [Eq. (8.49)] for a turbulent-velocity distribution between two smooth flat plates. Find α.*

N

From Eq. 8.38: $u = u_m\left(1 - \frac{r}{r_0}\right)^{1/7}$; $dA = Bdr$

For the half-flow: $$Q = AV = \int_0^{r_0} u\,dA = \int_0^{r_0} u_m\left(1 - \frac{r}{r_0}\right)^{1/7}B\,dr = Bu_m\int_0^{r_0}\left(1 - \frac{r}{r_0}\right)^{1/7}dr$$

$$= Bu_m\left(-\frac{7}{8}r_0\right)\left[\left(1 - \frac{r}{r_0}\right)^{8/7}\right]_0^{y_0} = -\frac{7}{8}Bu_mr_0[0 - 1] = \frac{7}{8}Bu_mr_0$$

But the half-area $A = Br_0$, so the mean velocity $V = \frac{Q}{A} = \frac{(7/8)Bu_mr_0}{Br_0} = \frac{7}{8}u_m$

$$\int u^3dA = \int u_m^3\left(1 - \frac{r}{r_0}\right)^{3/7}B\,dr = Bu_m^3\int\left(1 - \frac{r}{r_0}\right)^{3/7}dr$$

$$= Bu_m^3\left(-\frac{7}{10}r_0\right)\left[\left(1 - \frac{r}{r_0}\right)^{10/7}\right]_0^{r_0} = -\frac{7}{10}Bu_m^3r_0[0 - 1] = \frac{7}{10}Bu_m^3r_0$$

Eq. 5.4: $$\alpha = \frac{\int u^3dA}{AV^3} = \frac{(7/10)Bu_m^3r_0}{(Br_0)(7u_m/8)^3} = \frac{8^3}{10(7)^2} = 1.045 \quad ◄$$

5.2 *Assume the seventh-root law [Eq. (8.49)] for a turbulent-velocity distribution in smooth pipe flow. Find α.*

N

From Eq. 8.49: $u = u_m\left(1 - \dfrac{r}{r_0}\right)^{1/7}$; $dA = 2\pi r\,dr$

$$Q = AV = \int u\,dA = \int u_m\left(1 - \frac{r}{r_0}\right)^{1/7} 2\pi r\,dr = 2\pi u_m \int r\left(1 - \frac{r}{r_0}\right)^{1/7} dr$$

To integrate by parts, let $u = r$ and $dv = (1 - r/r_0)^{1/7}dr$

then $du = dr$ and $v = -(7/8)r_0(1 - r/r_0)^{8/7}$

$$\text{so that } \int_0^{r_0} r\left(1 - \frac{r}{r_0}\right)^{1/7} dr = \left[-\frac{7}{8}rr_0\left(1 - \frac{r}{r_0}\right)^{8/7}\right]_0^{r_0} - \int_c^{r_0}\left[-\frac{7}{8}r_0\left(1 - \frac{r}{r_0}\right)^{8/7}\right]dr$$

$$= -[0 - 0] + \frac{7}{8}r_0\int_0^{r_0}\left(1 - \frac{r}{r_0}\right)^{8/7} dr = \frac{7}{8}r_0\left(-\frac{7}{15}r_0\right)\left[\left(1 - \frac{r}{r_0}\right)^{15/7}\right]_0^{r_0} = -\frac{49}{8(15)}r_0^2[0 - 1] = \frac{49}{120}r_0^2$$

$$\text{so } AV = 2\pi u_m\frac{49}{120}r_0^2 = \frac{49}{60}\pi r_0^2 u_m\ ;\quad \text{but } A = \pi r_0^2\ ;\quad \therefore\ V = (49/60)u_m$$

$$\int u^3 dA = \int u_m^3\left(1 - \frac{r}{r_0}\right)^{3/7} 2\pi r\,dr = 2\pi u_m^3\int r\left(1 - \frac{r}{r_0}\right)^{3/7} dr$$

To integrate by parts, let $u = r$ and $dv = \left(1 - \dfrac{r}{r_0}\right)^{3/7} dr$; then $du = dr$ and $v = -\dfrac{7}{10}r_0\left(1 - \dfrac{r}{r_0}\right)^{10/7}$

$$\text{so that } \int_0^{r_0} r\left(1 - \frac{r}{r_0}\right)^{3/7} dr = \left[-\frac{7}{10}rr_0\left(1 - \frac{r}{r_0}\right)^{10/7}\right]_0^{r_0} - \int_0^{r_0}\left[-\frac{7}{10}r_0\left(1 - \frac{r}{r_0}\right)^{10/7}\right]dr$$

$$= [-0 + 0] + \frac{7}{10}r_0\int_0^{r_0}\left(1 - \frac{r}{r_0}\right)^{10/7} dr = \frac{7}{10}r_0\left[-\frac{7}{17}r_0\left(1 - \frac{r}{r_0}\right)^{17/7}\right]_0^{r_0} = -\frac{49}{170}r_0^2(0 - 1) = \frac{49}{170}r_0^2$$

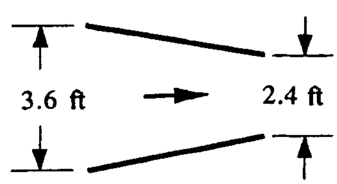
$$\text{and } \int u^3 dA = 2\pi u_m^3(49/170)r_0^2 = (49/85)\pi r_0^2 u_m^3$$

$$\text{Eq. 5.4: } \alpha = \frac{\int u^3 dA}{AV^3} = \frac{(49/85)\pi r_0^2 u_m^3}{(\pi r_0^2)(49u_m/60)^3} = \frac{60^3}{85(49^2)} = 1.058 \quad ◄$$

Sec. 5.2: Equation for Steady Motion of an Ideal Fluid Along a Streamline, and Bernoulli's Theorem -- *Exercises*

5.2.1 *Assume frictionless flow in a long, horizontal, conical pipe, where the diameter is 3.6 ft at entrance and 2.4 ft at exit. The pressure head at the smaller end is 15 ft of water. If water flows through this cone at the rate of 95 cfs, find the velocities at the two ends and the pressure head at the larger end.*

BG



$$\text{Eq. 4.6: } V_1 = \frac{Q}{A_1} = \frac{95}{\pi(1.8)^2} = 9.33 \text{ fps} \quad ◄$$

$$V_2 = \frac{Q}{A_2} = \frac{95}{\pi(1.2)^2} = 21.0 \text{ fps} \quad ◄$$

$$\text{Eq. 5.7 with } z_1 = z_2\text{: } \frac{p_1}{\gamma} + z + \frac{9.33^2}{2(32.2)} = 15 + z + \frac{21.0^2}{2(32.2)}\ ;\quad \frac{p_1}{\gamma} = 20.5 \text{ ft} \quad ◄$$

5.2.2 *Assume the flow to be frictionless in the siphon shown in Fig. X5.2.2, where a = 3ft, b = 12 ft. Find the rate of discharge in cfs and the pressure head at B if the pipe has a uniform diameter of 3 in.*

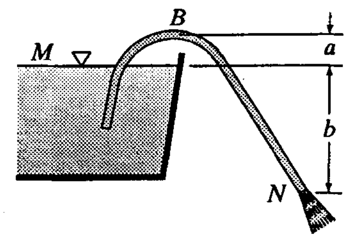


Figure X5.2.2

BG

Eq. 5.7 from M to N (elevation datum at N):

$$0 + 12 + 0 = 0 + 0 + V_N^2/2g \ ; \quad V_N = V_B = 27.8 \text{ fps}$$

$Q = (\pi/4)(3/12)^2\, 27.8 = 1.365$ cfs ◀

Eq. 5.7 from M to B:

$$0 + 12 + 0 = p_B/\gamma + 15 + V_B^2/2g \ ; \quad p_B/\gamma = -15.0 \text{ ft}$$ ◀

5.2.3 *Refer to Fig. X5.2.2. Assume a = 1 m, b = 4 m, and the flow to be frictionless in the siphon. Find the rate of discharge in m³/s and the pressure head at B if the pipe has a uniform diameter of 150 mm.*

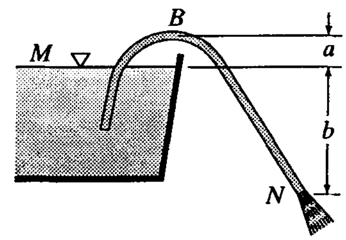


Figure X5.2.2

SI

Eq. 5.7 from M to N (elevation datum at N):

$$0 + 4 + 0 = 0 + 0 + V_N^2/2g \ ; \quad V_N = V_B = 8.86 \text{ m/s}$$

$Q = \pi(0.15/2)^2 8.86 = 0.1565$ m³/s ◀

Eq. 5.7 from M to B:

$$0 + 4 + 0 = p_B/\gamma + 5 + V_B^2/2g \ ; \quad p_B/\gamma = -5.00 \text{ m}$$ ◀

5.2.4 *From point 1, a 25-mm-diameter pipe runs horizontally under the floor and then a 12.5-mm-diameter line runs 1 m up the wall to point 2. To maintain a pressure of 300 kPa at point 2, when 15°C water is flowing at 0.5 L/s, what pressure must be provided at point 1? Neglect friction.*

SI

Table A.1 for water at 15°C: $\gamma = 9798$ N/m³. Given $Q = 0.5$ L/s $= 0.0005$ m³/s.

Continuity Eq. 4.7: $V_1 = \dfrac{4Q}{\pi D_1^2} = \dfrac{4(0.0005)}{\pi(0.025)^2} = 1.019$ m/s; $V_2 = \dfrac{4Q}{\pi D_2^2} = \dfrac{4(0.0005)}{\pi(0.0125)^2} = 4.07$ m/s

From Eq. 5.7: $\dfrac{p_1}{\gamma} = \dfrac{p_2}{\gamma} + z_2 - z_1 + \dfrac{V_2^2 - V_1^2}{2g}$

$$\frac{p_1}{9798} = \frac{300\,000}{9798} + 1 + \frac{4.07^2 - 1.019^2}{2(9.81)} \ ; \quad p_1 = 318\,000 \text{ N/m}^2 \text{ (or Pa)} = 318 \text{ kPa}$$ ◀

5.2.5 *A straight horizontal pipe changes diameter from 6 in at inlet to 3 in at outlet. If the water pressures are 7.5 psi at inlet and 5.0 psi at outlet, find the flow rate of water at 70°F. Neglect friction.*

BG

Table A.1 for water at 70°F: $\gamma = 62.30$ lb/ft³.

Continuity Eq. 4.3: $Q = A_1V_1 = A_2V_2$; $6^2(V_1) = 3^2(V_2)$; so $V_2 = V_1(6/3)^2 = 4V_1$

From Eq. 5.7: $\dfrac{p_1 - p_2}{\gamma} = (z_2 - z_1) + \dfrac{V_2^2 - V_1^2}{2g}$

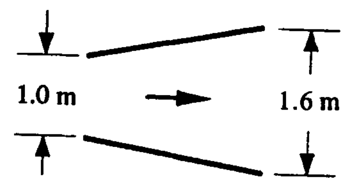
$$\frac{(7.5 - 5.0)144}{62.30} = 0 + \frac{(4V_1)^2 - V_1^2}{2(32.2)} = \frac{15V_1^2}{2(32.2)} \ ; \quad V_1 = 4.98 \text{ fps}$$

$$Q = A_1V_1 = \frac{\pi}{4}\left[\frac{6}{12}\right]^2 4.98 = 0.978 \text{ cfs}$$ ◀

Sec. 5.2: Equation for Steady Motion of an Ideal Fluid Along a Streamline, and Bernoulli's Theorem -- *Problems*

5.3 *Water flows through a long, horizontal, conical diffuser at the rate of 4.2 m^3/s. The diameter of the diffuser changes from 1.0 m to 1.6 m. The pressure at the smaller end is 9.5 kPa. Find the pressure at the downstream end of the diffuser, assuming frictionless flow. Assume also, that the angle of the cone is small enough that the flow does not separate from the walls of the diffuser.*

SI



Eq. 4.6: $V_1 = \dfrac{Q}{A_1} = \dfrac{4.2}{\pi(0.5)^2} = 5.35$ m/s

$V_2 = \dfrac{Q}{A_2} = \dfrac{4.2}{\pi(0.8)^2} = 2.09$ m/s

Eq. 5.7 with $z_1 = z_2$: $p/\gamma + V^2/2g = \text{constant}$

ie: $\dfrac{9.5 \text{ kN/m}^2}{9.81 \text{ kN/m}^3} + \dfrac{5.35^2}{2(9.81)} = \dfrac{p}{\gamma} + \dfrac{2.09^2}{2(9.81)}$; $0.968 \text{ m} + 1.458 \text{ m} = p/\gamma + 0.222$

$p/\gamma = 2.20$ m ; $p = 2.20(9.81) = 21.6 \text{ kN/m}^2$ ◄

5.4 *In Fig. P5.4, ABC is part of a piping system. Water at 50°F flows up AB (15 ft long, 1.5 in diameter), then along BC (10 ft long, 1.0 in diameter). The measured pressure and mean velocity at A are 36.3 psi and 4 ft/sec. (a) Find the pressure at C, neglecting pipe friction and energy losses. (b) Repeat for flow in the opposite direction.*

BG

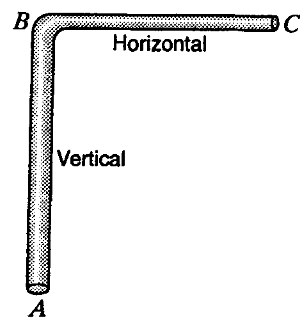


Figure P5.4

Table A.1 for water at 50°F: $\gamma = 62.41 \text{ lb/ft}^3$.

(*a*) Continuity Eq. 4.3: $Q = A_1V_1 = A_2V_2$; $(1.5)^2 4 = (1.0)^2 V_2$; so $V_2 = 9$ fps

From Eq. 5.7: $\dfrac{p_A}{\gamma} + z_A - z_C + \dfrac{V_A^2 - V_C^2}{2g} = \dfrac{p_C}{\gamma}$

$\dfrac{36.3(144)}{62.41} - 15 + \dfrac{4^2 - 9^2}{2(32.2)} = \dfrac{p_C(144)}{62.41}$; $p_C = 29.4$ psi ◄

(*b*) Eqs. are the same regardless of the flow direction. ∴ we get the same answer. $p_C = 29.4$ psi ◄

5.5 *Refer to Fig. P5.4. Water at 10°C flows up AB (5 m long, 40 mm diameter), then along BC (3 m long, 30 mm diameter). The measured pressure at A is 275 kPa. (a) Find the pressure at C if the flow rate is 2.0 L/s. Neglect pipe friction and energy losses. (b) Repeat for the same flow rate in the opposite direction.*

SI

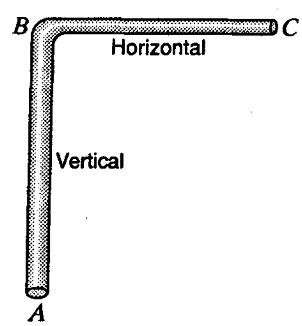


Figure P5.4

Table A.1 for water at 10°C: $\gamma = 9.804 \text{ kN/m}^3$.

Given $Q = 2 \text{ L/s} = 0.002 \text{ m}^3/\text{s}$.

Continuity Eq. 4.7: $V_A = \dfrac{4Q}{\pi D_A^2} = \dfrac{4(0.002)}{\pi(0.040)^2} = 1.592$ m/s;

$V_C = \dfrac{4Q}{\pi D_C^2} = \dfrac{4(0.002)}{\pi(0.030)^2} = 2.83$ m/s

From Eq. 5.7: $\dfrac{p_A}{\gamma} + z_A - z_C + \dfrac{V_A^2 - V_C^2}{2g} = \dfrac{p_C}{\gamma}$

$\dfrac{275\,000}{9804} - 5 + \dfrac{1.592^2 - 2.83^2}{2(9.81)} = \dfrac{p_C}{9804}$; $p_C = 223\,000 \text{ N/m}^2$ (or Pa) $= 223$ kPa ◄

(*b*) Eqs. are the same regardless of the flow direction. ∴ we get the same answer. $p_C = 223$ kPa ◄

5.6 *Part of a vertical piping system consists of 8 ft of 4 in diameter, connected to 8 ft of 2 in diameter above that. Water at 60°F flows up the pipe (no down flow is permitted). (a) Neglecting friction, find the difference in water pressure (psi) between the two ends when the flow rate is 150 gpm. (b) What is the minimum possible value for this pressure difference, and under what circumstances does it occur?*

BG

Table A.1 for water at 60°F: $\gamma = 62.37\ \text{lb/ft}^3$.

(*a*) Inside cover: $Q = 150/448.831 = 0.334$ cfs.

Continuity Eq. 4.7: $V_1 = \dfrac{4Q}{\pi D_1^2} = \dfrac{4(0.334)}{\pi(4/12)^2} = 3.83$ fps; $V_2 = \dfrac{4Q}{\pi D_2^2} = \dfrac{4(0.334)}{\pi(2/12)^2} = 15.32$ fps

From Eq. 5.7: $\dfrac{p_1 - p_2}{\gamma} = z_2 - z_1 + \dfrac{V_2^2 - V_1^2}{2g}$

$$\frac{(p_1 - p_2)144}{62.37} = 3(8) + \frac{15.32^2 - 3.83^2}{2(32.2)} = 24 + 3.42\ ; \quad p_1 - p_2 = 11.87\ \text{psi} \quad ◀$$

(*b*) From Eq. 5.7 above we see $(p_1 - p_2)$ is min when $(V_2^2 - V_1^2)$ is min, i.e., $V_1 = V_2 = 0$

Then Δp is 10.40 psi, when the flow rate is zero. ◀

5.7 *In Fig. 4.12 the velocity of the undisturbed field is 22 fps and the velocities very near the surface at radii from the "source" making angles with the axis of 0, 60, 120, 150° are 0, 17.5, 23.7 and 21.9 fps, respectively. What will be the elevation of the liquid surface relative to that of the free surface of the undisturbed field? (This problem illustrates the way in which the water surface drops alongside a bridge pier or past the side of a moving ship.)*

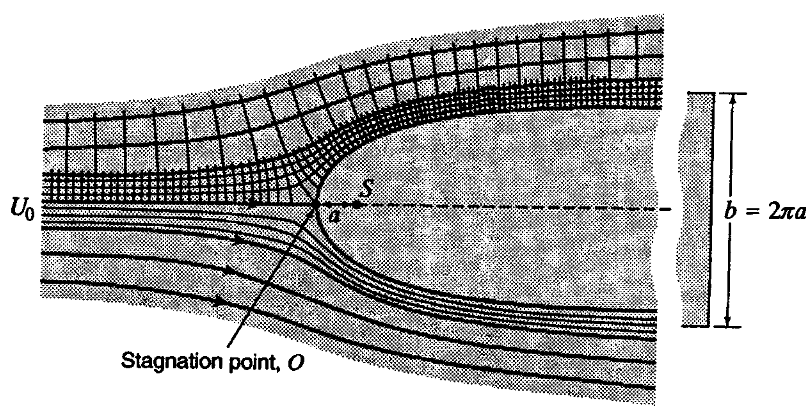


Figure 4.12

BG

Eq. 5.7: $0 + z + V^2/2g = 0 + z_0 + V_0^2/2g$

So $z - z_0 = \Delta z = (V_0^2 - V^2)/2g = (22^2 - V^2)/(2 \times 32.2) = 7.52 - V^2/2g$

At	0°	$V = 0$	$\Delta z = +7.52$ ft
	60°	17.5 fps	+2.76
	120°	23.7 fps	−1.206
	150°	21.9 fps	+0.0682

▲

5.8 *In Fig. 4.12 the velocity of the undisturbed field is 6 m/s and the velocities very near the surface at radii from the "source" making angles with the axis of 0, 60, 120, 150° are 0, 4.8, 6.5, 6.0 m/s respectively. What will be the elevation of the liquid surface relative to that of the free surface of the undisturbed filed? (This problem illustrates that way in which the water surface drops alongside a bridge pier or past the side of a moving ship.)*

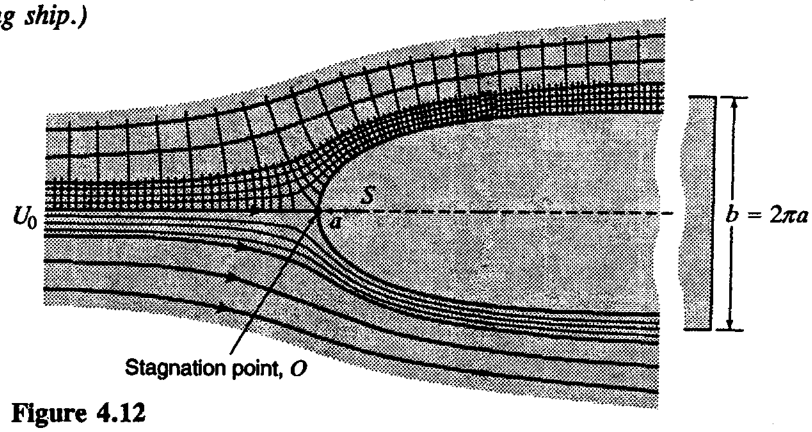


Figure 4.12

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Eq. 5.7: $0 + z + V^2/2g = 0 + z_0 + V_0^2/2g$

So $z - z_0 = \Delta z = (V_0^2 - V^2)/2g = (6^2 - V^2)/(2 \times 9.81) = 1.835 - V^2/2g$

At:	0°	60°	120°	150°
V =	0	4.80	6.50	6.00 m/s
Δz =	+1.835	+0.661	−0.319	0.0 m ◄◄

5.9 *If the body shown in Fig. 4.12 is not two-dimensional but is a solid of revolution about a horizontal axis, the flow will be three-dimensional and the streamlines will be differently spaced. Also, the distance between the stagnation point and the "source" will be d/4, where d is the diameter at a great distance from the stagnation point. At points very near the surface at radii from the source making angles with the axis of 0, 60, 120, and 150°, the velocities are 0, 14.0, 21.3, and 19.8 fps respectively when the velocity of the undisturbed field is 19 fps. If the body is a blimp and the atmospheric pressure in the undisturbed field is 14 psia, what will be the pressure at these points, for an air temperature of 53.9°F?*

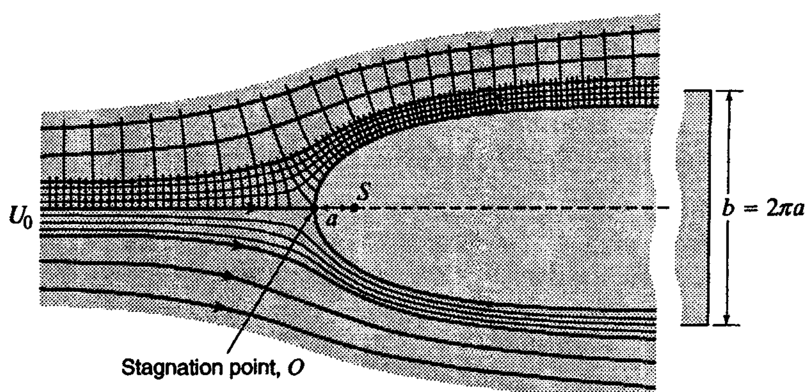

Figure 4.12

BG

Table A.3 for standard atmosphere at 14 psia: T = 54.0°F (by linear interpolation)

Table A.5 for air: $R = 1715$ ft²/(s²·°R) ; Sec. 2.7: $\gamma = \dfrac{gp}{RT} = \dfrac{32.2(14 \times 144)}{1715(460 + 54)} = 0.0736$ lb/ft³

From Eq. 5.7: $\Delta p/\gamma = (V_0^2 - V^2)/2g = (19^2 - V^2)/(2 \times 32.2) = 5.61 - V^2/2g$

At:	0°	60°	120°	150°
V =	0	14.0	21.3	19.8 fps
$\Delta p/\gamma$ =	+5.61	+2.56	−1.439	−0.482 ft
Δp =	+0.413	+0.1887	−0.1060	−0.0355 psf ◄◄

5.10 *In Prob. 5.9 assume the body is a submarine with diameters at the four points of 0, 8.24, 14.28, and 15.90 ft respectively. If the submarine is submerged in the ocean (γ = 64.1 lb/ft^3) with its axis 50 ft below the surface, find the pressures in pounds per square inch at these points along the top and along the bottom.*

Prob. 5.9: The body shown in Fig. 4.12 is a solid of revolution about a horizontal axis, and a = b/4. At points very near the surface at radii from the source making angles with the axis of 0, 60, 120, and 150°, the velocities are 0, 14.0, 21.3, and 19.8 fps respectively when the velocity of the undisturbed field is 19 fps.

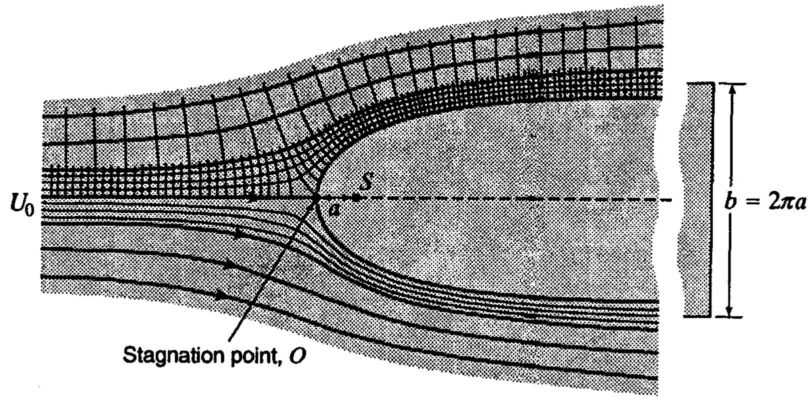

Figure 4.12

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Take datum at submarine centerline 50 feet below water surface and write energy equation 5.7 along the upper and lower boundaries of the submarine.

$p/\gamma + z + V^2/2g = p_0/\gamma + z_0 + V_0^2/2g$; $p/\gamma \pm \Delta z + V^2/2g = 50 + 0 + 19^2/(2\times 32.2)$; Δz = diam/2

p/γ = 55.61 $\mp$ diam/2 − $V^2/2g$; $p = (p/\gamma$ ft)64.1/144 psi

θ	V fps (given)	diam ft (given)	Top p/γ ft	Bot p/γ ft	Top p psi	Bot p psi
0°	0	0	55.61	55.61	24.8	24.8
60°	14.0	8.24	48.44	56.68	21.6	25.2
120°	21.3	14.28	41.42	55.70	18.4	24.8
150°	19.8	15.90	41.57	57.47	18.5	25.6
					▲	▲

Sec. 5.3: Equation for Steady Motion of a Real Fluid Along a Streamline -- *Exercises (7)*

5.3.1 *A vertical pipe of 4 ft diameter and 60 ft long has a pressure head of 22.7 ft of water at its upper end. When the flow of water through it is such that the mean velocity is 16 fps, the pipe friction head loss is h_f = 2.8 ft. Find the pressure head at the lower end of the pipe when the flow is (a) downward; (b) upward.*

BG

(*a*) Eq. 5.14: $22.7 + 60 + \frac{(16)^2}{2(32.2)} - 2.8 = \frac{p_2}{\gamma} + 0 + \frac{(16)^2}{2(32.2)}$ (velocity heads cancel)

p_2/γ = 79.9 ft for downflow ◄

(*b*) Eq. 5.14: $\frac{p_2}{\gamma} + 0 + \frac{(16)^2}{2(32.2)} - 2.2 = 22.7 + 60 + \frac{(16)^2}{2(32.2)}$ (velocity heads cancel)

p_2/γ = 85.5 ft for upflow ◄

5.3.2 *A vertical pipe of 1.5 m diameter and 20 m long has a pressure head at the upper end of 6.3 m of water. When the flow of water through it is such that the mean velocity is 5.6 m/s, the pipe friction head loss is h_f = 1.09 m. Find the pressure head at the lower end of the pipe when the flow is (a) downward (b) upward.*

SI

(*a*) Eq. 5.14: $6.3 + 20 + \frac{5.6^2}{2(9.81)} - 1.09 = \frac{p_2}{\gamma} + 0 + \frac{5.6^2}{2(9.81)}$ (velocity heads cancel)

$\frac{p_2}{\gamma} = 25.2$ m for downflow ◀

(*b*) Eq. 5.14: $\frac{p_2}{\gamma} + 0 + \frac{5.6^2}{2(9.81)} - 1.09 = 6.3 + 20 + \frac{5.6^2}{2(9.81)}$ (velocity heads cancel)

$\frac{p_2}{\gamma} = 27.4$ m for upflow ◀

5.3.3 *A conical pipe has diameters at the two ends of 1.2 and 4.2 ft and is 48 ft long. It is vertical, and the pipe friction head loss is h_f = 7.6 ft for flow of water in either direction when the velocity at the smaller section is 28 fps. If the smaller section is at the top and the pressure head there is 6.4 ft of water, find the pressure head at the lower end when the flow is (a) downward; (b) upward.*

BG

$Q = AV = (\pi/4)D^2V$; so $V_b = V_t\,(D_t/D_b)^2 = 28(1.2/4.2)^2 = 2.29$ fps

(*a*) Eq. 5.14: $6.4 + 48 + \frac{28^2}{2(32.2)} - 7.6 = \frac{p_2}{\gamma} + 0 + \frac{2.29^2}{2(32.2)}$; $p_2/\gamma = 58.9$ ft for downflow ◀

(*b*) Eq. 5.14: $\frac{p_2}{\gamma} + 0 + \frac{2.29^2}{2(32.2)} - 7.6 = 6.4 + 48 + \frac{28^2}{2(32.2)}$; $\frac{p_2}{\gamma} = 74.1$ ft for upflow ◀

5.3.4 *In Figure X5.3.4, the pipe AB is of uniform diameter and h = 28 ft. The pressure at A is 30 psi and at B is 40 psi. In which direction is the flow, and what is the pipe friction head loss in feet of the fluid if the liquid has a specific weight of (a) 35 lb/ft³; (b) 92 lb/ft³?*

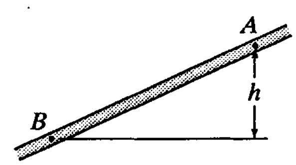

Figure X5.3.4

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Assume flow is from *A* to *B*.

(*a*) For $\gamma = 35$ lb/ft³, and $V_1 = V_2$

Eq. 5.14: $\frac{30(144)}{35} + 28 + \frac{V^2}{2g} - h_f = \frac{40(144)}{35} + 0 + \frac{V^2}{2g}$

$h_f = -13.14$ ft ◀ ; so the flow is from *B* to *A* ◀

(*b*) For $\gamma = 92$ lb/ft³, and $V_1 = V_2$

Eq. 5.14: $\frac{30(144)}{92} + 28 + \frac{V^2}{2g} - h_f = \frac{40(144)}{92} + 0 + \frac{V^2}{2g}$

$h_f = +12.35$ ft ◀ ; so the flow is from *A* to *B* ◀

5.3.5 *If h = 10.5 m in Fig. X5.3.4 and the pressures at A and B are 170 and 275 kPa respectively, find the direction of flow and the pipe friction head loss in meters of liquid. Assume the liquid has a specific gravity of 0.85.*

SI

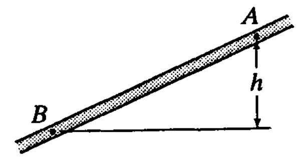

Figure X5.3.4

Assume the flow is from A to B. Eq. 5.14:

$$\frac{170 \text{ kN/m}^2}{(0.85)9.81 \text{ kN/m}^3} + 10.5 + \frac{V^2}{2g} - h_f = \frac{275 \text{ kN/m}^2}{(0.85)9.81 \text{ kN/m}^3} + 0 + \frac{V^2}{2g}$$

$h_f = 20.39 + 10.5 - 32.98 = -2.09$ m ◄ so the flow is from B to A ◄

5.3.6 *Water flows through a pipe at 14 cfs. At a point where the pipe diameter is 18 in, the pressure is 30 psi; at a second point, further along the flow path and 2 ft lower than the first, the diameter is 9 in and the pressure is 18 psi. Find the pipe friction head loss between the two points. Neglect other head losses.*

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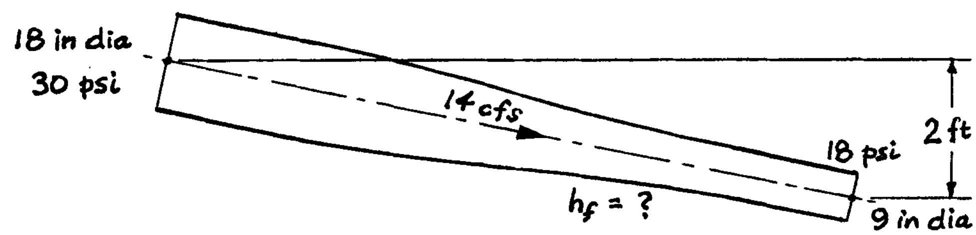

Eq. 4.7: $V_1 = \frac{4Q}{\pi d_1^2} = \frac{4(14)}{\pi(1.5)^2} = 7.92$ fps, $V_2 = \frac{4(14)}{\pi(0.75)^2} = 31.7$ fps. $\gamma = 62.4$ lb/ft^3

Eq. 5.14: $\frac{30 \times 144}{62.4} + 2.0 + \frac{7.92^2}{2(32.2)} - h_f = \frac{18 \times 144}{62.4} + 0 + \frac{31.7^2}{2(32.2)}$

$h_f = 15.07$ ft ◄

5.3.7 *Water at 20°C flows up a straight 180-mm-diameter pipe that slopes at 12° to the horizontal. Find the shear stress at the wall, if the pressure is 100 kPa at point 1, and 25 kPa at higher point 2 that is 30 m further along the pipe.*

SI

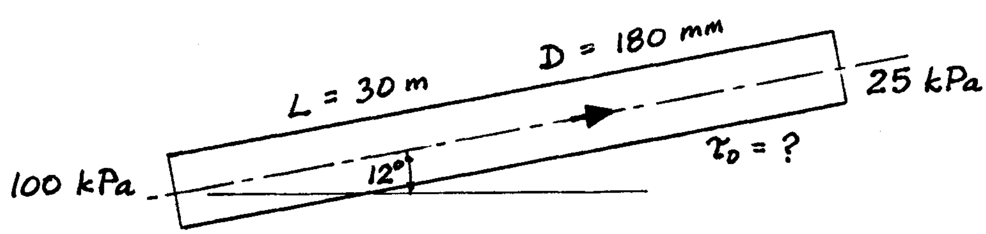

Table A.1 at 20°C: $\gamma = 9.789$ kN/m^3. $z_2 - z_1 = 30 \sin 12° = 6.24$ m

Eq. 5.14: $\frac{100}{9.789} + 0 + \frac{V^2}{2g} - h_f = \frac{25}{9.789} + 6.24 + \frac{V^2}{2g}$; velocity heads cancel

$h_f = 1.424 \text{ m} = \frac{4\tau_0 30}{9789(0.18)}$ per Eq. 5.15. So $\tau_0 = 20.9$ N/m^2 ◄

Sec. 5.3: Equation for Steady Motion of a Real Fluid Along a Streamline – ***Problems*** *5.11–5.15*

5.11 *Refer to Fig. P5.4. ABC is part of a piping system. Water at 50°F flows up AB (15 ft long, 1.5 in diameter), then along BC (10 ft long, 1.0 in diameter). The measured pressure and mean velocity at A are 36.3 psi and 4 ft/sec; at C the pressure is 27.4 psi. (a) Find the pipe friction head loss between A and C. Neglect energy losses caused by the diameter change and bend at B.*

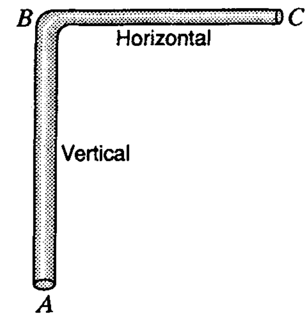

Figure P5.4

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Table A.1 for water at 50°F: $\gamma = 62.41 \text{ lb/ft}^3$.

(*a*) Continuity Eq. 4.3: $Q = A_1V_1 = A_2V_2$; $(1.5)^2 4 = (1.0)^2 V_2$; so $V_2 = 9$ fps

Eq. 5.14: $\dfrac{36.3(144)}{62.41} + 0 + \dfrac{4^2}{2(32.2)} - h_f = \dfrac{27.4(144)}{62.41} + 15 + \dfrac{9^2}{2(32.2)}$

$h_f = 1.424$ m ◄

5.12 *Refer to Fig. P5.4. Water at 10°C flows up pipe AB (5 m long, 40 mm diameter) and along BC (3 m long, 30 mm diameter) at 1.75 L/s. If the measured pressure at A is 250 kPa, and the pipe friction head loss between A and C is 1.45 m, find the pressure at C. Neglect energy losses caused by the diameter change and bend at B.*

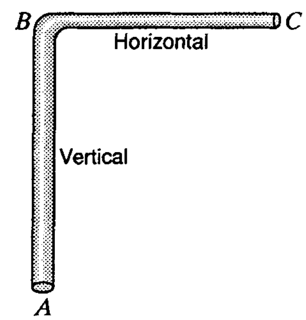


Figure P5.4

SI

Table A.1 for water at 10°C: $\gamma = 9.804 \text{ kN/m}^3$.

Given $Q = 1.75 \text{ L/s} = 0.00175 \text{ m}^3\text{/s}$.

Continuity Eq. 4.7: $V_A = \dfrac{4Q}{\pi D_A^2} = \dfrac{4(0.00175)}{\pi(0.040)^2} = 1.393$ m/s;

$V_C = \dfrac{4Q}{\pi D_C^2} = \dfrac{4(0.00175)}{\pi(0.030)^2} = 2.48$ m/s

Eq. 5.14: $\left(\dfrac{250}{9.804} + 0 + \dfrac{1.393^2}{2(9.81)}\right) - 1.45 = \left(\dfrac{p_C}{9.804} + 5 + \dfrac{2.48^2}{2(9.81)}\right)$; $p_C = 184.7$ kPa ◄

5.13 *Water at 60°F flows at 3 cfs through a 150-ft-long duct of 6 in × 9 in cross section. The pressure at the inlet end is 15 psig, and at the outlet, 20 ft higher than the inlet, it is 4 psig. Find (a) the wall friction head loss, and (b) the friction force on the duct. Neglect energy losses caused by bends.*

BG

(*a*) Table A.1 for water at 60°F: $\gamma = 62.37 \text{ lb/ft}^3$.

Eq. 5.14: $\left(\dfrac{15(144)}{62.37} + 0 + \dfrac{V^2}{2g}\right) - h_f = \left(\dfrac{4(144)}{62.37} + 20 + \dfrac{V^2}{2g}\right)$ since $V_1 = V_2$

$h_f = \dfrac{(15-4)144}{62.37} - 20 = 25.40 - 20 = 5.40$ ft ◄

(*b*) From Eq. 5.13: Force on duct $= \tau_0 PL = h_f\gamma A = 5.40(62.37)6(9)/144 = 126.2$ lb ◄

5.14 *Water at 15°C flows up a 24-m-long conical pipe with its centerline sloping at 3° to the horizontal. At its lower end the diameter is 600 mm, the water pressure is 94.6 kPa, and the velocity is 1.3 m/s; at its upper, outlet end the diameter is 450 mm and the water pressure is 78.4 kPa. Find the shear stress at the wall, assuming it to be nonvarying. (Hint: You may use the mean diameter to find the pipe friction head loss.)*

SI

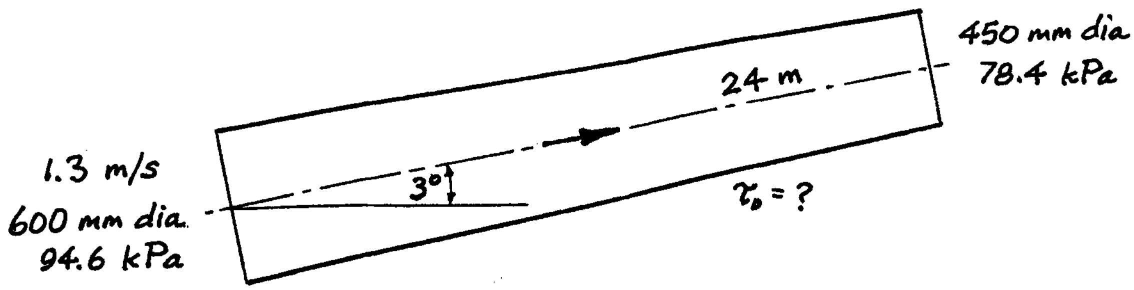


Table A.1 for water at 15°C: $\gamma = 9.798\ \text{kN/m}^3$. $z_2 - z_1 = 25\sin 3° = 1.308$ m

Continuity Eq. 4.3: $Q = A_1V_1 = A_2V_2$; $\frac{1}{4}\pi(600)^2 1.3 = \frac{1}{4}\pi(450)^2 V_2$; so $V_2 = 2.31$ m/s

Eq. 5.14: $\left(\frac{94.6}{9.798} + 0 + \frac{1.3^2}{2(9.81)}\right) - h_f = \left(\frac{78.4}{9.798} + 1.308 + \frac{2.31^2}{2(9.81)}\right)$; $h_f = 0.1589$ m

$\bar{D} = \frac{1}{2}(600 + 450)/1000 = 0.525$ m

From Eq. 5.15: $\tau_0 = \frac{h_f\gamma\bar{D}}{4L} = \frac{0.1589(9798)0.525}{4(25)} = 8.17\ \text{N/m}^2$ ◄

5.15 *Water at 70°F flows up a 50-ft-long conical pipe with its centerline sloping at 5° to the horizontal. At its lower end the diameter is 24 in, the water pressure is 15 psi; at its upper, outlet end the diameter is 18 in and the water pressure is 12.5 psi. By the methods of Sec. 8.5, τ_0 has been calculated to be 0.25 lb/ft². Assuming this value to be nonvarying, calculate the flow rate. (Hint: You may use the mean diameter to find the pipe friction head loss.)*

BG

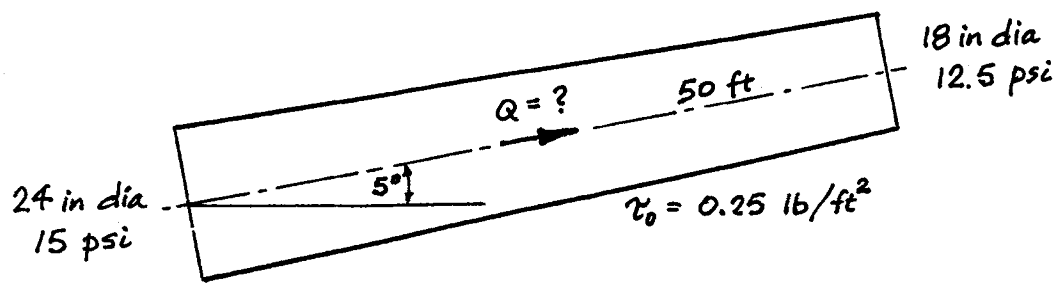


Table A.1 for water at 70°F: $\gamma = 62.30\ \text{lb/ft}^3$. $\bar{D} = \frac{1}{2}(24 + 18) = 21$ in $= 1.75$ ft

$z_2 - z_1 = 50\sin 5° = 4.37$ ft. Eq. 5.15: $h_f = \frac{4\tau_0 L}{\gamma\bar{D}} = \frac{4(0.25)50}{62.30(1.75)} = 0.459$ ft

From Eq. 5.14: $\frac{p_1 - p_2}{\gamma} + z_1 - z_2 + \frac{V_1^2 - V_2^2}{2g} = h_f$

Continuity Eq. 4.3: $Q = A_1V_1 = A_2V_2$; $(D_1)^2V_1 = (D_2)^2V_2$; so $V_2 = V_1(D_1/D_2)^2$

And $\frac{V_1^2 - V_2^2}{2g} = \frac{V_1^2}{2g} - \frac{V_1^2}{2g}\left(\frac{D_1}{D_2}\right)^4 = \frac{V_1^2}{2g}\left[1 - \left(\frac{D_1}{D_2}\right)^4\right]$

Substituting these into Eq. 5.14: $\frac{(15.0 - 12.5)144}{62.30} - 4.37 + \frac{V_1^2}{2(32.2)}\left[1 - \left(\frac{24}{18}\right)^4\right] = 0.459$

$V_1 = 5.33$ fps; $Q = A_1V_1 = \frac{1}{4}\pi(2^2)5.33 = 16.74$ cfs ◄

Sec. 5.4: Pressure in Fluid Flow -- *Exercises (3)*

5.4.1 *Find the stagnation pressure on the nose of a submarine moving at 12 knots in seawater (γ = 64 lb/ft³) when it is 70 ft below the surface.*

BG

From inside cover: 1 knot (1 nautical mile per hour) = 1.688 ft/sec

V = 12 knots = 12(1.688) = 20.3 fps

Energy Eq. 5.7 from the stagnation point to the surface, referring all quantities to the stagnation point:

$p/\gamma + 0 + 0 = 0 + 70 + V^2/2g$

$$p = \gamma\left(z + \frac{V^2}{2g}\right) = 64\left(70 + \frac{20.3^2}{2 \times 32.2}\right) = 4888 \text{ psf} = 33.9 \text{ psi} \quad ◄$$

5.4.2 *Find the stagnation pressure on the nose of a submarine moving at 6 m/s in seawater (γ = 10 050 N/m³) when it is 20 m below the surface.*

SI

Energy Eq. 5.7 from the stagnation point to the surface, referring all quantities to the stagnation point:

$p/\gamma + 0 + 0 = 0 + 20 + V^2/2g$

$$p = \gamma\left(z + \frac{V^2}{2g}\right) = 10\,050\left(20 + \frac{6^2}{2 \times 9.81}\right) = 219\,000 \text{ N/m}^2 = 219 \text{ kPa} \quad ◄$$

5.4.3 *Find the stagnation pressure on the nose of a fish swimming at 22 fps in fresh water (γ = 62.4 lb/ft³) when it is 8 ft below the surface.*

BG

Energy Eq. 5.7 from the stagnation point to the surface, referring all quantities to the stagnation point:

$$\frac{p}{\gamma} + 0 + 0 = 0 + z + \frac{V^2}{2g}; \quad p = \gamma\left(z + \frac{V^2}{2g}\right) = 62.4\left(8 + \frac{22^2}{2(32.2)}\right) = 968 \text{ psf}$$

Dividing by 144, p = 6.72 psi ◄

Sec. 5.4: Pressure in Fluid Flow -- *Problems 5.16–5.19*

5.16 *Find the stagnation pressure on a tree trunk at an elevation of 1000 m if the wind speed is 25 m/s.*

SI

Table A.3 at 1000 m: ρ = 1.112 kg/m³, c = 336 m/s

$$\text{From Eq. 5.18:} \quad (p_0)_{gage} = (p_0)_{abs} - p_{abs} = \frac{1.112(25)^2}{2}\left(1 + \frac{25^2}{4(336)^2}\right) = 348 \text{ Pa} \quad ◄$$

5.17 *Wind blows at a velocity of 20 m/s against the side of a pole at an elevation of 2000 m above sea level. What is the stagnation pressure assuming standard atmospheric conditions? Express your answer as a gage pressure and as an absolute pressure in kN/m², Pa, and mmHg.*

SI

Table A.3 at 2000 m: p = 79.501 kPa abs, ρ = 1.006 kg/m³, c = 333 m/s.

$$\text{From Eq. 5.18:} \quad (p_0)_{gage} = (p_0)_{abs} - p_{abs} = \frac{1.006(20)^2}{2}\left(1 + \frac{20^2}{4(336)^2}\right) = 202 \text{ N/m}^2$$

$(p_0)_{gage}$ = 202 Pa ◄ = 0.202 kN/m² ◄

$$(p_0/\gamma)_{gage} = 0.202\left(\frac{760 \text{ mmHg}}{101.3 \text{ kPa}}\right) = 1.511 \text{ mmHg} \quad ◄$$

$(p_0)_{abs} = (p_0)_{gage} + p_{abs}$ = 202 + 79 501 = 79 700 Pa ◄ = 79.7 kN/m² ◄

$(p_0/\gamma)_{abs}$ = 79.7(760/101.3) = 598 mmHg ◄

5.18 *In Fig. P5.18 water is admitted at the center at a rate of 2 cfs and is discharged into the air around the periphery. The upper circular plate in the figure is horizontal and is fixed in position, while the lower annular plate is free to move vertically and is not supported by the pipe in the center. The annular plate weighs 6 lb, and the weight of the water on it should be considered. (a) If the distance d between the two plates is to be maintained at 1.5 in, what is the total weight W that can be supported? (b) What is the pressure head where the radius is 4 in, and what is it at a radius of 8 in?*

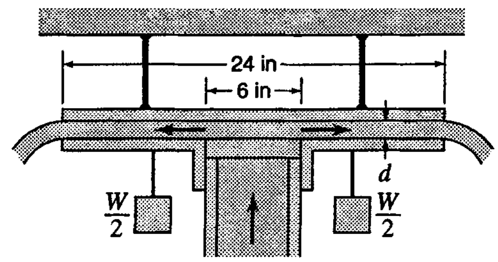
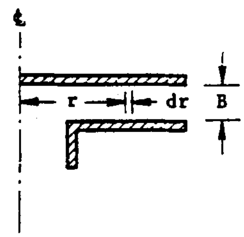


Figure P5.18

BG

(*a*) $V = \dfrac{Q}{2\pi rB} = \dfrac{2}{2\pi r(1.5/12)} = \dfrac{8}{\pi r}$ fps. At low r, V will be high, so p will be low, providing suction to support the lower plate

Energy Equation from any point between the plates to the periphery:



$p_r/\gamma + z + V_r^2/2g = 0 + z + V_R^2/2g$; $R = 1$ ft

$$\therefore \quad \frac{p_r}{\gamma} = \frac{V_R^2 - V_r^2}{2g} = \frac{1}{2g}\left[\left(\frac{8}{\pi R}\right)^2 - \left(\frac{8}{\pi r}\right)^2\right] = \frac{64}{2\pi^2 g}\left(1 - \frac{1}{r^2}\right)$$

To find total pressure force on plate,

Eq. 3.14: $$F = \int_A p\,dA = \int_{0.25}^{1} p(2\pi r\,dr) = \frac{2\pi(64)62.4}{2\pi^2 g}\int_{0.25}^{1}\left(1 - \frac{1}{r^2}\right)r\,dr$$

$$= 39.5\left[\frac{r^2}{2} - \ln r\right]_{0.25}^{1} = -36.2 \text{ lb}$$

Weight of water $= 62.4\dfrac{1.5\pi}{12}\left(1 - \dfrac{1}{4^2}\right) = 23.0$ lb. Then $W = 36.2 - 23.0 - 6 = 7.25$ lb ◄

(*b*) At $r = 4$ in $= 1/3$ ft, $\dfrac{p}{\gamma} = \dfrac{64}{2\pi^2 g}\left[1 - \dfrac{1}{(1/3)^2}\right] = -0.806$ ft ◄

At $r = 8$ in $= 2/3$ ft, $\dfrac{p}{\gamma} = \dfrac{64}{2\pi^2 g}\left[1 - \dfrac{1}{(2/3)^2}\right] = -0.1259$ ft ◄

5.19 *Plot the stagnation pressure (psia) on an object as it passes through air at sea level (standard atmosphere) as a function of velocity. Repeat for movement through air at 10,000 ft elevation. Let V vary from zero to c using 0, 25, 50, 75, and 100% of c.*

BG

At sea level, from Table A.3: $p = 14.70$ psia, $\rho = 0.002\,38$ slug/ft^3, $c = 1116$ fps

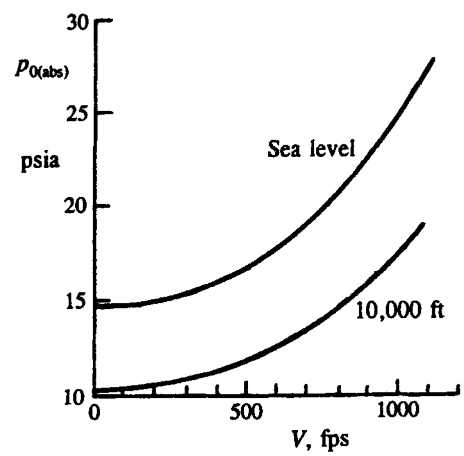
Eq. 5.18: $(p_0)_{abs} = 14.70 + \frac{0.002\,38}{144}\frac{V^2}{2}\left(1 + \frac{V^2}{4c^2}\right) = 14.70 + 8.26 \times 10^{-6} V^2\left(1 + \frac{V^2}{4c^2}\right)$

% of c	V (fps)	V^2	$8.26 \times 10^{-6} V^2$	$\left(1 + \frac{V^2}{4c^2}\right)$	$(p_0)_{abs}$ (psia)
0	0	0	0	1.000	14.70
25	279	7.79×10^4	0.643	1.016	15.35
50	558	3.12×10^5	2.57	1.063	17.43
75	837	7.01×10^5	5.79	1.141	21.3
100	1116	12.45×10^5	10.29	1.250	27.6 ▲

At 10,000 ft, from Table A.3: $p = 10.11$ psia, $\rho = 0.001\,756$ slug/ft^3, $c = 1077$ fps

Eq. 5.18: $(p_0)_{abs} = 10.11 + 6.10 \times 10^{-6} V^2(1 + V^2/4c^2)$

% of c	V fps	$(p_0)_{abs}$ psia
0	0	10.11
25	269	10.56
50	539	11.99
75	808	14.65
100	1077	18.95 ▲



Sec. 5.6: Energy Equation for Steady Flow of Incompressible Fluids, Bernoulli's Theorem -- *Exercises (4)*

5.6.1 *Water is flowing in a pipeline. Due to heat input from the environment and energy dissipation (head loss), the water temperature rises by 3°F between intake and outlet. Find the gain in heat in (a) ft·Lb/lb, and (b) Btu/lb.*

BG

Table A.4 or footnote 3 of Sec. 5.5, for water: $c = 25{,}000$ ft^2/(sec^2·°R)

(*a*) Eq. 5.25: $\Delta I = \frac{c}{g}\Delta T = \frac{25{,}000}{32.2}(3) = 233$ ft·lb/lb ◄

(*b*) From inside cover (and from the definition of a Btu): $\Delta I = 2329/777.649 = 3.00$ Btu/lb ◄

5.6.2 *Head loss and sunshine striking a pipeline cause the temperature of the water flowing inside to rise by 2°C between two measuring points. Find the heat gain in (a) J/N, and (b) cal/N.*

SI

Table A.4 or footnote 3 of Sec. 5.5, for water: $c = 4187\ \mathrm{m^2/(s^2{\cdot}K)}$

(*a*) Eq. 5.25: $\Delta I = \frac{c}{g}\Delta T = \frac{4187}{9.81}(2) = 854\ \mathrm{N{\cdot}m/N} = 854\ \mathrm{J/N}$ ◄

(*b*) From inside cover: $\Delta I = 854(0.239006) = 204\ \mathrm{cal/N}$ ◄

5.6.3 *The pipeline shown in Fig. X5.6.3 supplies water to a hydroelectric power plant, the elevation of which is 2200 ft below the level of the water at intake to the pipe. If 8 per cent of this total, or 176 ft, is the head loss in the line, what will be the value of ΔI in Btu/lb if there is no heat transfer; and what will be the rise in temperature?*

BG

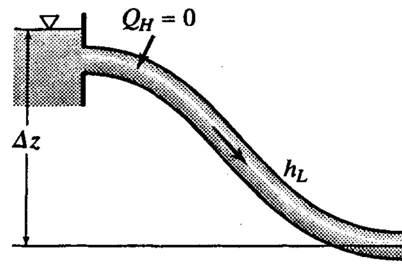


Figure X5.6.3

Eq. 5.25 with $Q_H = 0$:

$$\Delta I = h_L = \frac{176\ \text{ft·lb/lb}}{778\ \text{ft·lb/Btu}} = 0.226\ \text{Btu/lb} \quad ◄$$

Eq. 5.25: $\Delta I = 0.226\ \text{Btu/lb} = (c/g)\Delta T$

where (from Sec. 5.6, footnote 4) $c = 1.0$ Btu/(mass of one lb·°R)

$$\therefore\ \Delta T = \frac{\Delta I g}{c} = \frac{(0.226\ \text{Btu/lb})(32.2\ \text{ft/sec}^2)}{(1.0\ \text{Btu})(32.2\ \text{ft/sec}^2)/(\text{lb·°R})} = 0.226°\text{R} = 0.226°\text{F} \quad ◄$$

5.6.4 *The pipeline shown in Fig. X5.6.3 supplies water to a hydroelectric power plant, the elevation of which is 650 m below the level of the water at intake to the pipe. If 8 per cent of this total, or 52 m, is the head loss in the line, what will be the value of ΔI in J/N if there is no heat transfer, and what will be the rise in temperature?*

SI

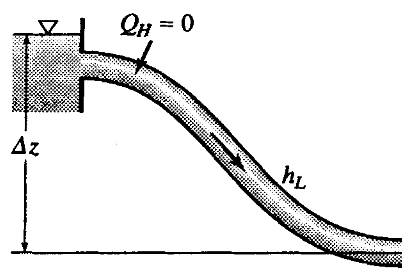


Figure X5.6.3

Eq. 5.25 with $Q_H = 0$:

$$\Delta I = h_L = 52\ \text{m} = 52\ \text{N·m/N} = 52\ \text{J/N} \quad ◄$$

Eq. 5.25: $\Delta I = 52\ \text{N·m/N} = (c/g)\ \Delta T$

where (from Sec. 5.6, footnote 4) $c = 1\ \text{cal/(g·K)} = 4187\ \mathrm{m^2/(s^2{\cdot}K)}$

$$\therefore\ \Delta T = \frac{\Delta I g}{c} = \frac{(52\ \text{N·m/N})(9.81\ \text{m/s}^2)}{4187\ \text{m}^2/(\text{s}^2\text{·K})} = 0.1218\ \text{K} = 0.1218°\text{C} \quad ◄$$

Sec. 5.6: Energy Equation for Steady Flow of Incompressible Fluids, Bernoulli's Theorem – ***Problems** 5.20–5.23*

5.20 *Water is flowing at 12 m³/s through a long pipe. The temperature of the water rises 0.18°C when heat is transferred to the water at the rate of 4500 J/s. Find the head loss in the pipe.*

SI

From Eq. 4.6: $\dot{m}g = G = \gamma Q = (9810\ \mathrm{N/m^3})(12\ \mathrm{m^3/s}) = 117\,720\ \mathrm{N/s}$

Q_H = (Heat energy per second)/G = (5600 kN·m/s)/(117 720 N/s) = 38.2 N·m/N (or m or J/N)

Eq. 5.26: $h_L = (c/g)\Delta T - Q_H = \frac{4187\ \mathrm{m^2/(s^2{\cdot}K)}}{9.81\ \mathrm{m/s^2}}(0.18\ \mathrm{K}) - 38.2$; $h_L = 76.8 - 38.2 = 38.6$ m ◄

5.21 *A pipeline supplies water to a hydroelectric plant from a reservoir in which the water temperature is 61.3°F. (a) Suppose that in the length of the pipe there is a total loss of heat to the surrounding air of 0.26 Btu/lb of water and the temperature of the water at the power house is 61.2°F. What is the head loss per pound of water? (b) With the same flow rate as in (a) what will be the temperature of the water at the power house if the water absorbs heat from hot sunshine at the rate of 2.9 Btu/lb of water?*

BG

From footnote 4 of Sec. 5.6: c = 1.0 Btu/(mass of one lb·°R) = 25,000 ft²/(sec²·°R)

Heat gain (loss), $Q_H = -0.26$ Btu/lb $= -0.26(778) = -202.2$ ft·lb/lb

(*a*) Eq. 5.26: $h_L = (c/g)(T_2 - T_1) - Q_H$, where Q_H is negative in this case

$h_L = (25{,}000/32.2)(61.2 - 61.3) - (-202.2) = 124.5$ ft·lb/lb ◀

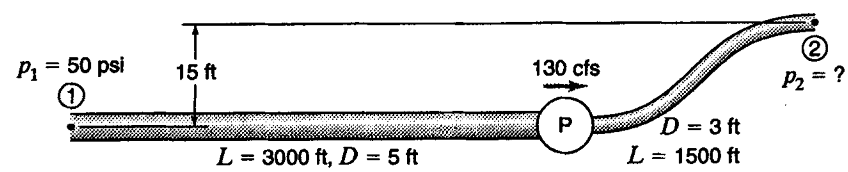
(*b*) Gain in heat = 2.9(778) = 2260 ft·lb/lb

Eq. 5.25 (same h_L): $(c/g)(T_2 - T_1) = Q_H + h_L = 2260 + 125.4 = 2380$ ft·lb/lb

$(T_2 - T_1) = 2380(32.2)/25{,}000 = 3.07$ °R; $T_2 = 61.3 + 3.1 = 64.4$°F ◀

5.22 *A pump lifting water at 3.5 cfs adds 35 ft·lb/lb to the flow. The suction line diameter is 8 in, and at intake (elevation 350 ft) the water pressure is 5.2 psi. The discharge line diameter is 6 in, and at outlet (elevation 370 ft) the water pressure is 3.5 psi. Due to cold weather, 7 ft·lb/lb of thermal energy (heat) are lost to the environment. Find the change (rise or fall?) in water temperature between intake and outlet. Assume the specific weight of the water remains constant at 62.4 lb/ft³.*

BG



Continuity Eq. 4.7: $V_1 = \dfrac{4Q}{\pi D_1^2} = \dfrac{4(3.5)}{\pi(8/12)^2} = 10.03$ fps; $V_2 = \dfrac{4Q}{\pi D_2^2} = \dfrac{4(3.5)}{\pi(6/12)^2} = 17.83$ fps

Eq. 5.27: $$\left(\frac{5.2(144)}{62.4} + 350 + \frac{10.03^2}{2(32.2)}\right) + 35 - h_L = \left(\frac{3.5(144)}{62.4} + 370 + \frac{17.83^2}{2(32.2)}\right)$$

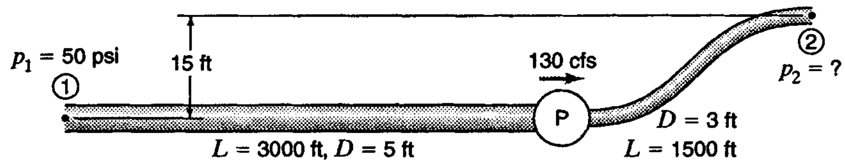
i.e., $363.56 + 35 - h_L = 383.01$; $h_L = 15.55$ ft

Table A.4 or footnote 3 of Sec. 5.5, for water: c = 25,000 ft²/(sec²·°R)

Eq. 5.25: $\dfrac{c}{g}(T_2 - T_1) = Q_H + h_L$; $\dfrac{25{,}000}{32.2}(T_2 - T\ ) = -7 + 15.55$; $T_2 - T_1 = 0.01101$°F rise ◀

5.23 *A pump lifting water at 0.08 m³/s adds 12 N·m/N to the flow. The suction line diameter is 200 mm, and at intake (elevation 50 m) the water pressure is 72 kPa. The discharge line diameter is 150 mm, and at outlet (elevation 56 m) the water pressure is 60 kPa. Sunshine striking the pipe adds 5 J/N of heat to the water. Find the change (rise or fall?) in water temperature between intake and outlet. Assume the specific weight of the water remains constant at 9.81 kN/m³.*

SI



Continuity Eq. 4.7: $V_1 = \dfrac{4Q}{\pi D_1^2} = \dfrac{4(0.08)}{\pi(0.2)^2} = 2.55$ m/s; $V_2 = \dfrac{4Q}{\pi D_2^2} = \dfrac{4(0.08)}{\pi(0.15)^2} = 4.53$ m/s

Eq. 5.27: $\left(\dfrac{72}{9.81} + 50 + \dfrac{2.55^2}{2(9.81)}\right) + 12 - h_L = \left(\dfrac{60}{9.81} + 56 + \dfrac{4.53^2}{2(9.81)}\right)$

i.e, $57.67 + 12 - h_L = 63.16$; $h_L = 6.51$ m

Table A.4 or footnote 3 of Sec. 5.5, for water: $c = 4187$ m²/(s²·K)

Eq. 5.25: $\dfrac{c}{g}(T_2 - T_1) = Q_H + h_L$; $\dfrac{4187}{9.81}(T_2 - T_1) = 5 + 6.51$; $T_2 - T_1 = 0.0270$°C rise ◄

Sec. 5.7: Energy Equation for Steady Flow of Compressible Fluids -- ***Exercises (4)***

5.7.1 Gas flows at a constant temperature though a uniform, horizontal pipe. At section 1 the pressure is 125 psia and the velocity is 65 fps; at section 2 the pressure is 105 psia and the velocity is 72 fps. Between sections 1 and 2, find (a) the change in enthalpy, and (b) the gain or loss of heat per lb. Assume the gas is perfect. Hint: Recall Eq. (2.4).

BG

(*a*) Sec. 5.1 for a perfect gas: When T = const, $I = f(T)$ = const.

Eq. 2.4 with const T: $p/\rho = RT$ = const. So from Eq. 5.32: h = const and $\Delta h = 0$ ◄

(*b*) From Eq. 5.34: $Q_H = \dfrac{h_2 - h_1}{g} + \dfrac{V_2^2 - V_1^2}{2g} = 0 + \dfrac{65^2 - 50^2}{2(32.2)} = +26.8$ ft·lb/lb

$Q_H = 26.8/778 = 0.0344$ Btu/lb gain ◄

5.7.2 *Air flows isothermally (constant temperature) through a horizontal duct of constant cross section. At station 1 the pressure is 860 kPa abs and the velocity is 22 m/s; at station 2 the pressure is 1040 kPa abs and the velocity is 18 m/s. Between stations 1 and 2, find (a) the change in enthalpy, and (b) the gain or loss of heat per newton. Assume the air is a perfect gas. Hint: Recall Eq. (2.4).*

SI

(*a*) Sec. 5.1 for perfect gas: When T = const, $I = f(T)$ = const.

Eq. 2.4 with const T: $p/\rho = RT$ = const. So from Eq. 5.32: h = const and $\Delta h = 0$ ◄

(*b*) From Eq. 5.34: $Q_H = \dfrac{h_2 - h_1}{g} + \dfrac{V_2^2 - V_1^2}{2g} = 0 + \dfrac{18^2 - 22^2}{2(9.81)} = -8.15$ N·m/N

$Q_H = 8.15$ J/N loss ◄

5.7.3 *Oxygen flows without a gain or loss of heat through a horizontal pipe of constant cross section. At section 1 the pressure is 170 psia, the velocity is 75 fps, and the temperature is 50° F; at section 2 the pressure is 125 psia, the velocity is 98 fps, and the temperature is 30° F. Between sections 1 and 2, find (a) the head (mechanical energy) loss in Btu/lb; (b) the change in enthalpy. Assume the oxygen is a perfect gas. Hint: Recall Eq. (2.4).*

BG

(*a*) From Eq. 5.28: $h_L = \left(\dfrac{p}{\gamma} + z + \dfrac{V^2}{2g}\right)_1 - \left(\dfrac{p}{\gamma} + z + \dfrac{V^2}{2g}\right)_2$

$$= \frac{p_1}{\rho_1 g} - \frac{p_2}{\rho_2 g} + \frac{V_1^2 - V_2^2}{2g} \qquad \text{since } z_1 = z_2$$

Eq. 2.4: $p/\rho = RT$; Table A.5 for oxygen: $R = 1554$ ft²/(s²·°R)

So $h_L = \dfrac{R}{g}(T_1 - T_2) + \dfrac{V_1^2 - V_2^2}{2g} = \dfrac{1554}{32.2}(510 - 490) + \dfrac{75^2 - 98^2}{2(32.2)}$

$= 965 - 61.8 = 903$ ft $= 903$ ft·lb/lb; $h_l = 903/778 = 1.161$ Btu/lb ◄

(*b*) From Eq. 5.34 with $Q_H = 0$ (given): $\dfrac{h_1 - h_2}{g} = \dfrac{V_2^2 - V_1^2}{2g} = \dfrac{98^2 - 75^2}{2(32.2)} = 61.8$ ft $= 61.8$ ft·lb/lb

So $\Delta h = h_1 - h_2 = g(61.8) = 32.2(61.8) = 1990$ ft·lb/slug decrease ◄

5.7.4 *Air flows without heat gain or loss through a uniform horizontal pipe. At station 1 the pressure is 1135 kPa abs, the velocity is 25 m/s, and the temperature is 10° C; at station 2 the pressure is 830 kPa abs, the velocity is 33 m/s, and the temperature is 0° C. Between sections 1 and 2, find (a) the head (mechanical energy) loss in J/N; (b) the change in enthalpy. Assume the air is a perfect gas. Hint: Recall Eq. (2.4).*

SI

(*a*) From Eq. 5.28: $h_L = \left(\dfrac{p}{\gamma} + z + \dfrac{V^2}{2g}\right)_1 - \left(\dfrac{p}{\gamma} + z + \dfrac{V^2}{2g}\right)_2$

$$= \frac{p_1}{\rho_1 g} - \frac{p_2}{\rho_2 g} + \frac{V_1^2 - V_2^2}{2g} \qquad \text{since } z_1 = z_2$$

Eq. 2.4: $p/\rho = RT$; Table A.5 for air: $R = 287$ m²/(s²·K)

So $h_L = \dfrac{R}{g}(T_1 - T_2) + \dfrac{V_1^2 - V_2^2}{2g} = \dfrac{287}{9.81}(283 - 273) + \dfrac{25^2 - 33^2}{2(9.81)} = 293 - 23.6 = 269$ m

or $h_L = 269$ N·m/N $= 269$ J/N ◄

(*b*) From Eq. 5.34 with $Q_H = 0$ (given): $\dfrac{h_1 - h_2}{g} = \dfrac{V_2^2 - V_1^2}{2g} = 23.6$ m $= 23.6$ N·m/N

So $\Delta h = h_1 - h_2 = g(23.6) = 9.81(23.6) = 232$ N·m/kg decrease ◄

Sec. 5.9: Power Considerations in Fluid Flow -- *Exercises (6)*

5.9.1 *A turbine, located at an elevation 750 ft below that of the water surface at intake (Fig. X5.9.1), carries a flow of 120 cfs. The head loss in the pipeline leading to it is 25 ft. Find the horsepower delivered by the turbine if its efficiency is 90 percent.*

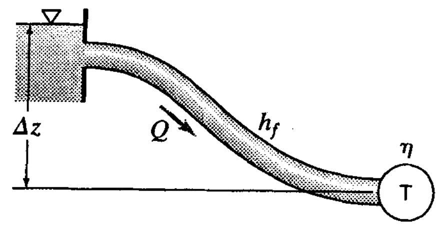


Figure X5.9.1

BG

Eq. 5.40: Water hp $= \dfrac{\gamma Q(\Delta z - h_L)}{550}$

$$= \frac{(62.4)120(750-25)}{550} = 9870 \text{ hp}$$

From Eq. 5.42, Output power = 9870(0.90) = 8880 hp ◄

5.9.2 *A turbine, located 255 m below the water surface at intake (Fig. X5.9.1), carries a flow of 3.5 m^3/s. The head loss in the pipeline leading to it is 10 m. Find the power (kW) delivered by the turbine if its efficiency is 92 percent.*

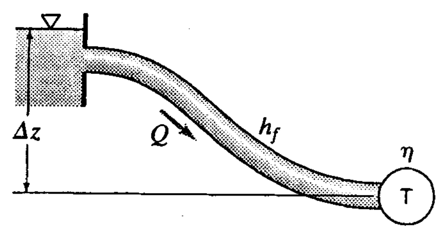


Figure X5.9.1

SI

Eq. 5.41: Water power, $P = \dfrac{\gamma Q(\Delta z - h_L)}{1000}$

$$= \frac{(9810)3.5(255-10)}{1000} = 8410 \text{ kW}$$

From Eq. 5.42, Output power = 8410(0.92) = 7740 kW ◄

5.9.3 *Water entering a pump through an 8-in-diameter pipe at 4 psi has a flow rate of 3.5 cfs. It leaves the pump through a 4-in-diameter pipe at 15 psi. Assuming that the suction and discharge sides of pump are at the same elevation, find the horsepower delivered to the water by the pump.*

BG

$$V_1 = \frac{3.5}{(\pi/4)(8/12)^2} = 10.03 \text{ fps};\quad V_2 = \frac{3.5}{(\pi/4)(4/12)^2} = 40.1 \text{ fps}$$

Eq. 5.35: $H_1 = \dfrac{4(144)}{62.4} + \dfrac{(10.03)^2}{2(32.2)} + z_1 = 10.79 \text{ ft} + z_1$

Eq. 5.35: $H_2 = \dfrac{15(144)}{62.4} + \dfrac{(40.1)^2}{2(32.2)} + z_2 = 59.6 \text{ ft} + z_2$; given $z_1 = z_2$

From Eq. 5.37 with $h_L = 0$: $h_p = +h_m = H_2 - H_1 = 59.6 - 10.79 = 48.8$ ft

Eq. 5.39: $P = \gamma Q h_p = 62.4(3.5)48.8 = 10{,}660$ ft·lb/sec = 19.38 hp ◄

5.9.4 *After entering a pump through an 180-mm-diameter pipe at 35 kN/m^2, oil (s = 0.82) leaves the pump through a 120-mm-diameter pipe at 120 kN/m^2. The suction and discharge sides of the pump are at the same elevation. Find the rate at which energy is delivered to the oil by the pump if the flow rate is 70 L/s.*

SI

$$V_1 = \frac{Q}{A_1} = \frac{0.070}{\pi 0.09^2} = 2.75 \text{ m/s};\quad V_2 = \frac{Q}{A_2} = \frac{0.070}{\pi 0.06^2} = 6.19 \text{ m/s}$$

Eqs. 5.35 and 5.37 with $z_1 = z_2$ and $h_L = 0$: $\dfrac{p_1}{\gamma} + \dfrac{V_1^2}{2g} + h_p = \dfrac{p_2}{\gamma} + \dfrac{V_2^2}{2g}$

$$\frac{35}{0.85(9.81)} + \frac{2.75^2}{2(9.81)} + h_p = \frac{120}{0.82(9.81)} + \frac{6.19^2}{2(9.81)}$$

$4.35 + 0.386 + h_p = 14.92 + 1.953$; $h_p = 12.13$ N· n/N of oil

Eq. 5.39: Rate of energy delivery $= \gamma Q h_p = (0.82 \times 9.81)0.070(12.13) = 6.83$ kW ◄

5.9.5 *Water from a reservoir is being supplied to a powerhouse that is located at an elevation 935 ft below that of the reservoir surface. Discharging through a nozzle, the water has a jet velocity of 240 fps and a jet diameter of 6 in. Find the horsepower lost to friction between the reservoir and the jet, and find the horsepower of the jet.*

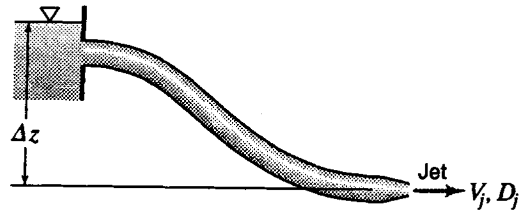


Figure X5.9.5

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Eq. 5.28 from reservoir (1) to jet (2) ,

$$0 + 935 + 0 = 0 + 0 + \frac{(240)^2}{2(32.2)} + h_L \;;\; h_L = 935.0 - 894.4 = 40.6 \text{ ft}$$

$$Q = A_2V_2 = \frac{\pi}{4}\left(\frac{6}{12}\right)^2 240 = 47.1 \text{ cfs} \;;\; h_j = \frac{V_j^2}{2g} = 894 \text{ ft}$$

$$\text{Eq. 5.40: hp in jet} = \frac{\gamma Q h_j}{550} = \frac{62.4(47.1)894}{550} = 4780 \text{ hp} \quad ◄$$

$$\text{Eq. 5.40: hp lost} = \frac{\gamma Q h_L}{550} = \frac{62.4(48.1)40.6}{550} = 217 \text{ hp} \quad ◄$$

5.9.6 *Water from a reservoir is being supplied to a powerhouse that is located at an elevation 325 m below that of the reservoir surface (Fig. X5.9.5). Discharging through a nozzle, the water has a jet velocity of 75 m/s and a jet diameter of 250 mm. Find the kW lost to friction between the reservoir and the jet, and find the power of the jet in kW.*

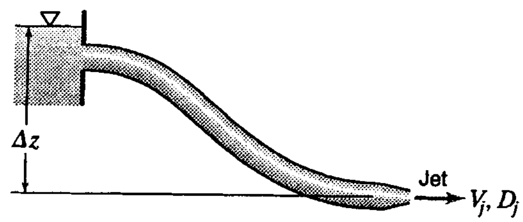

Figure X5.9.5

SI

$$\text{Eq. 5.28: } 0 + 325 + 0 = 0 + \frac{75^2}{2(9.81)} + h_L$$

$$h_L = 325 - 286.7 = 38.3 \text{ m}$$

$$Q = A_2V_2 = (\pi/4)(0.25)^2 75 = 3.68 \text{ m}^3\text{/s} \;;\; h_j = V_j^2/2g = 287 \; m$$

Eq. 5.41: Power in jet = $(3.68 \text{ m}^3\text{/s})(9.81 \text{ kN/m}^3)(287 \text{ m}) = 10\,350 \text{ kN·m/s} = 10\,350 \text{ kW}$ ◄

Eq. 5.41: Power lost = $3.68(9.81)38.3 = 1383$ kW ◄

Sec. 5.9: Power Considerations in Fluid Flow – *Problems* 5.24–25

5.24 *A pump, with an efficiency of 90 per cent, circulates water at the rate of 2500 gpm in a closed circuit that holds 8500 gal. The net head developed by the pump is 360 ft. What is the change in water temperature after one hour, assuming that the bearing friction is negligible and that there is no heat loss from the system?*

BG

Inside cover: 1 cfs = 449 gpm. ∴ $Q = 2500/449 = 5.57$ cfs

Eq. 5.39: Power input = $\gamma Qh/\eta = 62.4(5.57)360/0.90 = 139{,}000$ ft·lb/s

Heat input in 1 hr = $139{,}000(3600) = 5.00 \times 10^8$ ft·lb

Mass heated = $(8500 \text{ gal})(\text{ft}^3/7.48 \text{ gal})(1.940 \text{ slug/ft}^3) = 2200$ slugs

Per Eq. 5.26: $c(\Delta T)(\text{mass}) = 5.00 \times 10^8$ ft·lb

From Sec. 5.6, footnote 4: For water $c = 25{,}000$ ft·lb/(slug·°R)

$$\frac{25{,}000 \text{ ft·lb}}{(\text{slug·°R})}(\Delta T)2200 \text{ slugs} = 5.00 \times 10^8 \text{ ft·lb} \;;\; \Delta T = 9.08\text{°R} = 9.08\text{°F} \quad ◄$$

5.25 *A pump, with an efficiency of 92 per cent, circulates water at the rate of 130 L/s in a closed circuit that holds 45 m³. The net head developed by the pump is 120 m. What is the change in water temperature after one hour, assuming that the bearing friction is negligible and that there is no heat loss from the system?*

SI

Eq. 5.39: Power input $= \gamma Qh/\eta = 9.81(0.130)120/0.92 = 166.3$ kN·m/s

Heat input in 1 h = 16.3(3600) = 599 000 kN·m ; Mass heated = 45 m³(1000 kg/m³) = 45 000 kg

Per Eq. 5.26: $c(\Delta T)(\text{mass}) = 599 \times 10^6$ N·m

From Sec. 5.6, footnote 4: For water $c = 4187$ N·m/(kg·K)

So $\dfrac{4187\ \text{N·m}}{\text{kg·K}}(\Delta T)45\,000\ \text{kg} = 599 \times 10^6\ \text{N·m}$; $\Delta T = 3.18\ \text{K} = 3.18°\text{C}$ ◄

Sec. 5.10: Cavitation -- *Exercises (5)*

5.10.1 *Water at 170°F flows horizontally through the constriction of Fig. S5.9 when the atmospheric pressure is 28.2 inHg. Neglecting head loss, find the flow rate at which cavitation begins.*

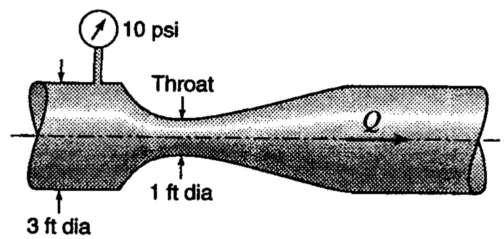

Figure S5.9

BG

Table A.1 at 170°F: $\gamma = 60.80$ lb/ft³, $p_v = 5.99$ psia

From Sec. 3.5: $p_{atm} = \dfrac{28.2\ \text{inHg}}{29.92\ \text{inHg}} 14.70\ \text{psia} = 13.85\ \text{psia}$

Cavitation begins when $p_2 = p_c = p_v$;

Eq. 5.43: $\left(\dfrac{p_{crit}}{\gamma}\right)_{gage} = -\left(\dfrac{13.85 - 5.99}{60.80}\right)144 = -18.63$ ft

Eq. 5.7: $\dfrac{10(144)}{60.80} + 0 + \dfrac{(Q/2.25\pi)^2}{2(32.2)} = -18.63 + 0 + \dfrac{(Q/0.25\pi)^2}{2(32.2)}$; $Q = 41.3$ cfs ◄

5.10.2 *Water at 40°C flows horizontally through a constriction similar to that in Fig. S5.9 when the atmospheric pressure is 715 mmHg. The gage reading is 35 kPa, d_1 =0.5 m, and d_2 =0.15 m. Neglecting head loss, find the flow rate at which cavitation begins.*

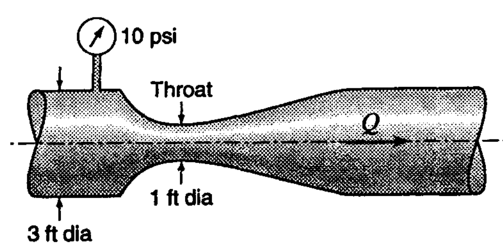

Figure S5.9

SI

Table A.1 at 40°C: $\gamma = 9.731$ kN/m³, $p_v = 7.38$ kN/m² abs

From Sec. 3.5:

$p_{atm} = \left(\dfrac{715\ \text{mmHg}}{760\ \text{mmHg}}\right)101.3\ \text{kN/m}^2\ \text{abs} = 95.3\ \text{kN/m}^2\ \text{abs}$

Cavitation begins when $p_2 = p_c = p_v$. Eq. 5.43: $\left(\dfrac{p_{crit}}{\gamma}\right)_{gage} = -\left(\dfrac{95.3 - 7.38}{9.73}\right) = -9.04$ m

Eq. 5.7: $\dfrac{35}{9.73} + 0 + \dfrac{[Q/(\pi 0.25^2)]^2}{2(9.81)} = -9.04 + 0 + \dfrac{[Q/(\pi 0.075^2)]^2}{2(9.81)}$; $Q = 0.279$ m³/s ◄

5.10.3 *Water at 80°F flows horizontally through the constriction of Fig. S5.9 at a rate of 65 cfs when the atmospheric pressure is 27.9 inHg. Neglecting head loss, find the largest throat constriction diameter d_2 which will cause cavitation.*

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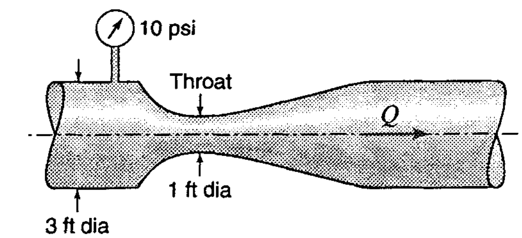

Figure S5.9

Table A.1 at 80°F: $\gamma = 62.22\ \text{lb/ft}^3$, $p_v = 0.51$ psia

From Sec. 3.5:

$$p_{\text{atm}} = \left(\frac{27.9\ \text{inHg}}{29.92\ \text{inHg}}\right)(14.70\ \text{psia}) = 13.71\ \text{psia}$$

Cavitation begins when $p_2 = p_c = p_v$. Eq. 5.43: $\left(\dfrac{p_{\text{crit}}}{\gamma}\right)_{\text{gage}} = -\left(\dfrac{13.71 - 0.51}{62.2}\right)144 = -30.5$ ft

Eq. 5.7: $\dfrac{10(144)}{62.2} + 0 + \dfrac{[65/(\pi 3^2/4)]^2}{2(32.2)} = -30.5 + 0 + \dfrac{[65/(\pi d_2^2/4)]^2}{2(32.2)}$; $d_2 = 1.179$ ft ◄

5.10.4 *Water at 50°C flows horizontally at a rate of 2 m³/s through a constriction similar to that in Fig. S5.9 when the atmospheric pressure is 750 mmHg. The gage reading is 30 kPa and $d_1 = 0.5$ m. Neglecting head loss, find the largest throat constriction diameter d_2 which will cause cavitation.*

SI

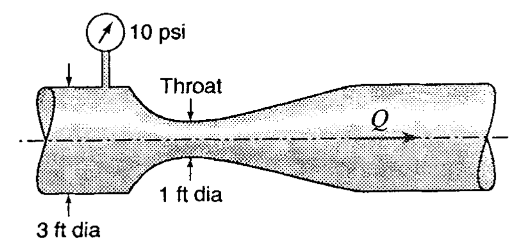

Figure S5.9

Table A.1 at 50°C: $\gamma = 9.689\ \text{kN/m}^3$,

$p_v = 12.33\ \text{kN/m}^2$ abs

From Sec. 3.5:

$$p_{\text{atm}} = \left(\frac{750\ \text{mmHg}}{760\ \text{mmHg}}\right)(101.3\ \text{kN/m}^2\text{abs}) = 100.0\ \text{kN/m}^2\ \text{abs}$$

Cavitation begins when $p_2 = p_c = p_v$. Eq. 5.43: $\left(\dfrac{p_{\text{crit}}}{\gamma}\right)_{\text{gage}} = -\left(\dfrac{100.0 - 12.33}{9.69}\right) = -9.05$ m

Eq. 5.7: $\dfrac{30}{9.69} + 0 + \dfrac{[2/(\pi 0.25^2)]^2}{2(9.81)} = 0 - 9.05 + \dfrac{[2/(\pi d_2^2/4)]^2}{2(9.81)}$; $d_2 = 0.371$ m ◄

5.10.5 *Referring to Fig. S5.9, find the maximum theoretical flow rate for water at 70°C. Neglect head loss. The diameters are 800 mm and 400 mm respectively, the upstream pressure is 20 kPa, and the atmospheric pressure is 750 mmHg*

SI

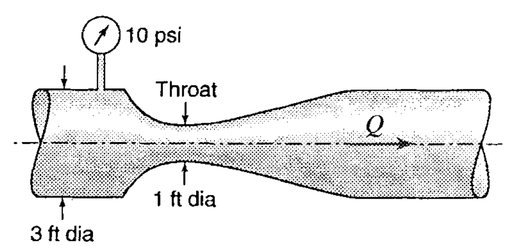

Figure S5.9

From Sec. 3.5: $p_{\text{atm}} = 750(101.3/760) = 100.0\ \text{kN/m}^2$ abs

Table A.1 at 70°C : $\gamma = 9.589\ \text{kN/m}^3$,

$p_v = 31.16\ \text{kN/m}^2$ abs

Q is max when $p_2\ (=p_c) = p_v$ (when cavitation begins).

$V_1 = Q/(\pi 0.40^2) = 1.989Q$; $V_2 = Q/(\pi 0.20^2) = 7.96Q$

Eq. 5.7 from 1 to 2 in absolute pressure heads:

$$\frac{100.0 + 20}{9.59} + z_1 + \frac{(1.989Q)^2}{2(9.81)} = \frac{31.16}{9.59} + z_2 + \frac{(7.96Q)^2}{2(9.81)}\ ;\quad z_1 = z_2$$

$\dfrac{(7.96^2 - 1.989^2)}{2(9.81)}Q^2 = \dfrac{120.0 - 31.16}{9.59}$; Maximum theoretical $Q = 1.749\ \text{m}^3/\text{s}$ ◄

Sec. 5.12: Loss of Head at Submerged Discharge -- *Problems 5.26–5.32*

5.26 *In Fig. P5.26, assume water is flowing and neglect all head losses except at discharge. Find the flow rate if h = 8 ft. Assuming that d = 10 ft, the throat diameter is two-thirds the pipe diameter where it joins the downstream tank, and the atmospheric pressure is equal to the standard atmospheric pressure at 10,000 ft elevation, calculate the gage pressure and the absolute pressure in the constriction. The throat diameter is 14 in.*

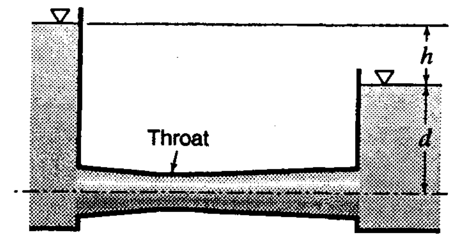

Figure P5.26

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Eq. 5.28 from upper reservoir surface to lower reservoir surface (=elev datum):

$0 + 8 + 0 - V_x^2/2g = 0 + 0 + 0\,;\quad V_x^2/2g = 8$ ft; $\quad V_x = \sqrt{2(32.2)8} = 22.7$ fps

In constriction: $V_c = (3/2)^2 22.7 = 51.1$ fps; $Q = A_c V_c = \pi(7/12)^2 51.1 = 54.6$ cfs ◄

Energy equation using absolute pressure head, from constriction (c, = datum) to lower reservoir surface:

$(p_c/\gamma)_{abs} + 0 + V_c^2/2g - V_x^2/2g = (p_{atm}/\gamma) + 10 + 0$ where $V_x^2/2g$ is again the discharge loss.

Table A.3 at 10,000 feet altitude: $p_{atm} = 10.1083$ psia

$(p_c/\gamma)_{abs} + 0 + 51.1^2/(2\times 32.2) - 8 = 10.1083(144)/62.4 + 10 + 0;\quad (p_c/\gamma)_{abs} = 0.827$ ft

$(p_c)_{abs} = 0.827(62.4) = 51.6$ psfa $= 0.385$ psia ◄

$(p_c)_{gage} = 0.385 - 10.1083 = -9.75$ psi ◄

5.27 *Referring to Fig. P5.26, assume water is flowing and neglect all head losses except at discharge. Find the flow rate if h = 1.6 m. Assuming that d = 4 m, the throat diameter is two-thirds the pipe diameter where it joins the downstream tank, and the atmospheric pressure is equal to the standard atmospheric pressure at 2000 m elevation, calculate the gage pressure and the absolute pressure in the constriction. The throat diameter is 360 mm.*

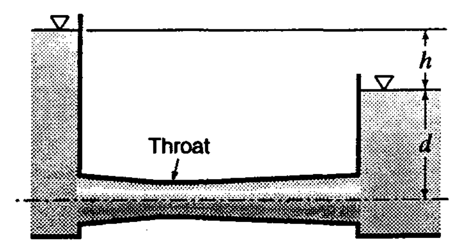

Figure P5.26

SI

Eq. 5.28 from upper reservoir surface to lower reservoir surface (=elev datum):

$0 + 1.6 + 0 - V_x^2/2g = 0 + 0 + 0\,;\quad V_x^2/2g = 1.6$ m; $\quad V_x = \sqrt{2(9.81)1.6} = 5.60$ m/s

In constriction: $V_c = (3/2)^2 5.60 = 12.61$ m/s; $Q = A_c V_c = \pi(0.15)^2 12.61 = 0.891$ m^3/s ◄

Energy equation using absolute pressure head, from constriction (c, datum) to lower reservoir surface:

$(p_c/\gamma)_{abs} + 0 + V_c^2/2g - V_x^2/2g = (p_{atm}/\gamma) + 4 + 0$ where $V_x^2/2g$ is again the discharge loss.

Table A.3 at 2 km altitude: $p_{atm} = 79.501$ kPa abs

$(p_c/\gamma)_{abs} + 0 + 12.61^2/(2\times 9.81) - 1.6 = (79.5/9.81) + 4 + 0;\quad (p_c/\gamma)_{abs} = 5.60$ m

$(p_c)_{abs} = 5.60(9.81) = 55.0$ kPa abs ◄

$(p_c)_{gage} = 55.0 - 79.5 = -24.5$ kPa ◄

5.28 *Repeat Prob. 5.26, assuming head losses are as follows: 6 inches in the converging section and 30 inches in the diverging section.*

Prob. 5.26: Water flows in Fig. P5.26. Given h = 8 ft, d = 10 ft, find Q. The constriction D = 14 in = (2/3)D at exit. The elevation = 10,000 ft, atmosphere = standard. Find the gage and abs pressures in the constriction.

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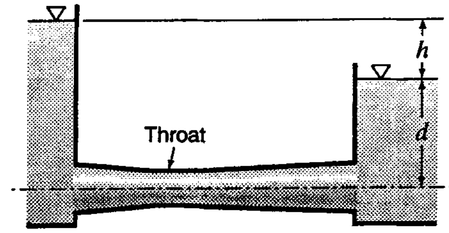


Figure P5.26

Eq. 5.28 from upper reservoir surface to lower reservoir surface (=elev datum):

$0 + 8 + 0 - [(6 + 30)/12 + V_x^2/2g] = 0 + 0 + 0 \qquad V_x^2/2g = 5$ ft; $V_x = \sqrt{2(32.2)5} = 17.94$ fps

In constriction: $V_c = (3/2)^2 17.94 = 40.4$ fps; $V_c^2/2g = 25.3$ ft

$Q = A_c V_c = \pi(7/12)^2 40.4 = 43.2$ cfs ◄

Energy equation using absolute pressure head, from constriction (c, datum) to lower reservoir surface:

$(p_c/\gamma)_{abs} + 0 + V_c^2/2g - [30/12 + V_x^2/2g] = (p_{atm}/\gamma) + 10 + 0$ where $V_x^2/2g$ is again the discharge loss.

Table A.3 at 10,000 feet altitude: $p_{atm} = 10.11$ psia

$(p_c/\gamma)_{abs} + 0 + 25.3 - [2.50 + 5] = 10.11(144)/62.4 + 10 + 0; \quad (p_c/\gamma)_{abs} = 15.51$ ft

$(p_c)_{abs} = 15.51(62.4) = 968$ psfa $= 6.72$ psia ◄

$(p_c)_{gage} = 6.72 - 10.11 = -3.39$ psi ◄

5.29 *Repeat Prob. 5.27, assuming head losses are as follows: 0.12 m in the converging section and 0.65 m in the diverging section.*

Prob. 5.27: Water flows in Fig. P5.26. Given h = 2 m, d = 4 m, find Q. The constriction D = 360 mm = (2/3)D at exit. The elevation = 2000 m, atmosphere = standard. Find the gage and abs pressures in the constriction.

SI

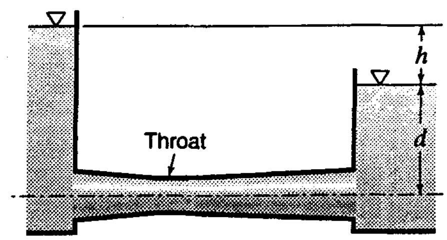


Figure P5.26

Eq. 5.28 from upper reservoir surface to lower reservoir surface (=elev datum):

$0 + 1.6 + 0 - [0.12 + 0.65 + V_x^2/2g] = 0 + 0 + 0$

$V_x^2/2g = 0.830$ m; $V_x = \sqrt{2(9.81)0.830} = 4.04$ m/s

In constriction: $V_c = (3/2)^2 4.04 = 9.08$ m/s; $V_c^2/2g = 4.20$ m

$Q = A_c V_c = \pi(0.15)^2 9.08 = 0.642$ m^3/s ◄

Energy equation using absolute pressure head, from constriction (c, datum) to lower reservoir surface:

$(p_c/\gamma)_{abs} + 0 + V_c^2/2g - [0.65 + V_x^2/2g] = (p_{atm}/\gamma) + 4 + 0$

Table A.3 at 2 km altitude: $p_{atm} = 79.501$ kPa abs

$(p_c/\gamma)_{abs} + 0 + 4.20 - [0.65 + 0.830] = (79.5/9.81) + 4 + 0; \quad (p_c/\gamma)_{abs} = 9.38$ m

$(p_c)_{abs} = 9.38(9.81) = 92.0$ kPa abs ◄

$(p_c)_{gage} = 92.0 - 79.5 = 12.54$ kPa ◄

5.30 *Referring to Fig. P5.26, neglect all head losses except at discharge, and assume water is flowing. If h = 15 ft and d = 12 ft, find the highest permissible water temperature in order that there be no cavitation. The throat diameter is 80 percent of the pipe diameter where it joins the downstream tank. Atmospheric pressure is 13.6 psia.*

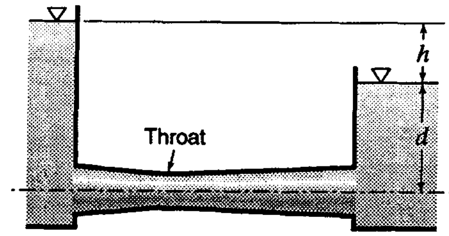


Figure P5.26

BG

$V_x^2/2g = 15$, $V_x = 31.1$ fps; $V_c = 31.1(1/0.8)^2 = 48.6$ fps

Energy equation from upper reservoir to constriction:

$$15 + 12 = \frac{p_c}{\gamma} + \frac{V_c^2}{2g} = \frac{p_c}{\gamma} + \frac{48.6^2}{2(32.2)} = \frac{p_c}{\gamma} + 36.6\ ;\ \frac{p_c}{\gamma} = -9.62\text{ ft}$$

At the maximum temperature, $\left(\frac{p_c}{\gamma}\right)_{abs} = \left(\frac{p_v}{\gamma}\right)_{abs}$; $\left(\frac{p_c}{\gamma}\right)_{abs} = \frac{p_{atm}}{\gamma} + \left(\frac{p_c}{\gamma}\right)_{gage} = \frac{13.6(144)}{\gamma} - 9.62$ ft

Try T	γ (from Table A.1)	$\left(\frac{p_c}{\gamma}\right)_{abs}$	T for this p_v/γ (from Table A.1)
60 °F	62.37	21.78 ft	≈ 191 °F
190 °F	60.36	22.82	* 192.5 °F
192.5 °F	*60.30	22.86	* 192.6 °F
192.6 °F	*60.30		

* By interpolation.

Highest permissible temperature = 192.6 °F ◄

5.31 *In Fig. P5.26 neglect all head losses except at discharge, and assume water is flowing. If h = 4 m and d = 5.5 m, find the highest permissible water temperature in order that there be no cavitation. The throat diameter is 70 percent of the pipe diameter where it joins the downstream tank. Atmospheric pressure is 97 kPa abs.*

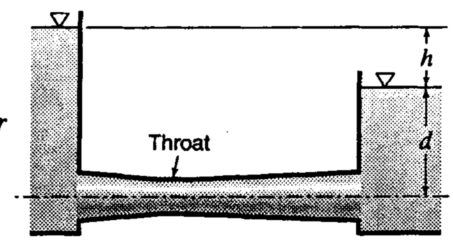


Figure P5.26

SI

$V_x^2/2g = 4$ m, $V_x = \sqrt{2(9.81)4} = 8.86$ m/s

$V_c = 8.86(1/0.7)^2 = 18.08$ m/s

Energy equation from upper reservoir to constriction:

$4.0 + 5.5 = p_c/\gamma + 18.08^2/(2\times 9.81) = p_c/\gamma + 16.66$; $p_c/\gamma = -7.16$ m

At the maximum temperature, $\left(\frac{p_c}{\gamma}\right)_{abs} = \left(\frac{p_v}{\gamma}\right)_{abs}$; $\left(\frac{p_c}{\gamma}\right)_{abs} = \frac{p_{atm}}{\gamma} + \left(\frac{p_c}{\gamma}\right)_{gage} = \frac{97}{\gamma} - 7.16$ m

Try T	γ (from Table A.1)	$\left(\frac{p_c}{\gamma}\right)_{abs}$	T for this p_v/γ (from Table A.1)
15 °C	9.798	2.74 m	≈ 66 °C
70 °C	9.589	2.96	* 67.9 °C
67.9 °C	*9.600	2.94	* 67.8 °C
67.8 °C	*9.601	2.94	

* By interpolation.

Highest permissible temperature = 67.8 °C ◄

5.32 *Redo Prob. 5.30 but this time let the water temperature be 50°F. Find the minimum permissible throat diameter in order to not have cavitation. Express the answer as a fraction of the outlet diameter.*

Prob. 5.30: Water flows in Fig. P5.26. Neglect all losses except discharge. Given: h = 15 ft, d = 12 ft, and p_{atm} = 13.6 psia.

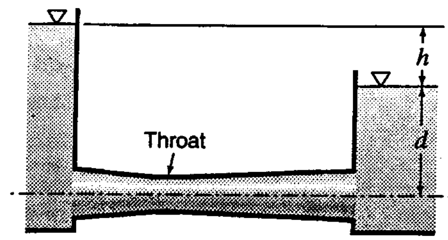


Figure P5.26

BG

Cavitation begins in constriction (subscript *c*) when $p_c = p_v$.

Table A.1 at 60°F: $p_v/\gamma = 0.592$ ft, $\gamma = 62.37$ pcf

Express energy equation from upper reservoir to constriction in terms of absolute pressure head:

$$0 + (15 + 12) + \frac{13.6(144)}{62.37} = \frac{p_c}{\gamma} + 0 + \frac{V_c^2}{2g};\quad 27 + 31.40 = 0.592 + \frac{V_c^2}{2g};\quad \frac{V_c^2}{2g} = 57.8 \text{ ft};\quad V_c = 61.0 \text{ fps}$$

$A_cV_c = A_xV_x$, where subscript *x* indicates outlet (exit), and $V_x = \sqrt{2g(15)} = 31.1$ fps

So $A_c/A_x = 31.1/61.0 = 0.509 = (D_c/D_x)^2$; $D_c/D_x = 0.714$ ◄

Sec. 5.13: Applications of Hydraulic Grade Line and Energy Line -- *Exercises (8)*

5.13.1 *Assume there is friction head loss in the siphon of Fig. X5.13.1, where a = 1 m, b = 4 m. The loss between the intake and B is 0.6 m and between B and N is 0.9 m. What is the rate of discharge and pressure head at B when the diameter is 150 mm?*

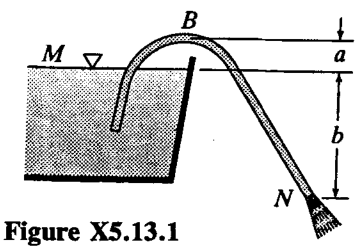


Figure X5.13.1

SI

Energy Eq. 5.14 from *M* to *N*: $0 + 4 + 0 - [0.6 + 0.9]$

$= 0 + 0 + V_N^2/2g = 2.5 \text{ m} = V_B^2/2g$; $V_B = 7.00$ m/s

$Q = \pi(0.15/2)^2 7.00 = 0.1238 \text{ m}^3/\text{s}$ ◄

Energy Eq. 5.14 from *M* to *B*: $0 + 4 + 0 - 0.6 = p_B/\gamma + 5 + V_B^2/2g$; $p_B/\gamma = -4.10$ m ◄

5.13.2 *Assume there is friction head loss in the siphon of Fig X5.13.1, where a = 3ft, b = 12 ft. The loss between the intake and B is 2.5 ft and between B and N is 3 ft. What is the rate of discharge and pressure head at B when the diameter is 6 in?*

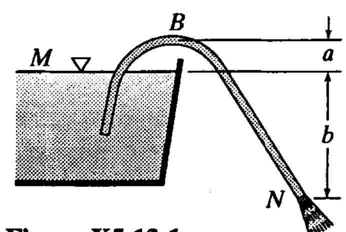


Figure X5.13.1

BG

Energy Eq. 5.14 from *M* to *N*: $0 + 12 + 0 - [2.5 + 3]$

$= 0 + 0 + V_N^2/2g = 6.5 \text{ ft} = V_B^2/2g$

$V_B = \sqrt{2(32.2)6.5} = 20.5$ fps; $Q = \pi(3/12)^2 20.5 = 4.02$ cfs ◄

Eq. 5.14 from *M* to *B*: $0 + 12 + 0 - 2.5 = p_B/\gamma + 15 + V_B^2/2g$; $p_B/\gamma = -12.00$ ft ◄

5.13.3 *Refer to Fig. X5.13.1. Find the maximum value for b if a = 1.1 m. Assume friction is negligible and the minimum pressure allowable in the siphon is a vacuum of −9.8 m of water.*

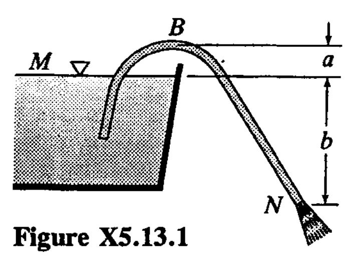


Figure X5.13.1

SI

b = the elevation difference between *M* and *N*.

Eq. 5.7 for *M* to *N*: $0 + b + 0 = 0 + 0 + V_N^2/2g$

$V_N^2/2g = V_B^2/2g = b$

Eq. 5.7 for *M* to *B*: $0 + b + 0 = -9.8 + (b + 1.1) + b$; $b = 8.70$ m ◄

5.13.4 *Refer to Fig. X5.13.1. Find the maximum value for b if a = 3.5 ft. Assume friction is negligible and the minimum pressure allowable in the siphon is a vacuum of −32.8 ft of water.*

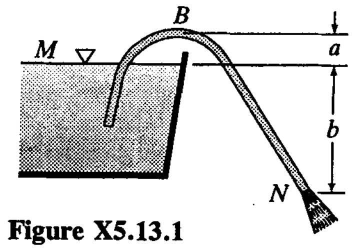


Figure X5.13.1

BG

b = the elevation difference between M and N.

Eq. 5.7 for M to N: $0 + b + 0 = 0 + 0 + V_N^2/2g$

$V_N^2/2g = V_B^2/2g = b$

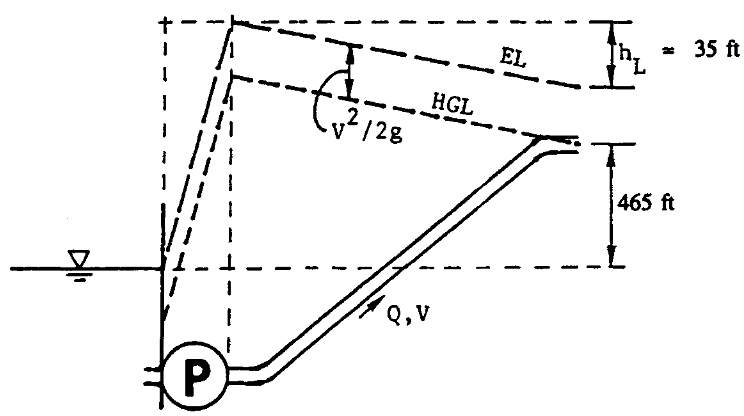
Eq. 5.7 for M to B: $0 + b + 0 = -32.8 + (b + 3.5) + b\,;\ b = 29.3$ ft ◀

5.13.5 *A pump, having an efficiency of 90%, lifts water to a height of 465 ft at the rate of 250 cfs. The friction head loss in the pipe is 35 ft. What is the required horsepower? Also sketch the energy line and the hydraulic grade line of this system.*

BG

Eq. 5.40: Water hp = $\gamma Q(\Delta z + h_L)/550$ = (62.4)250(465 + 35)/550 = 14,180 hp

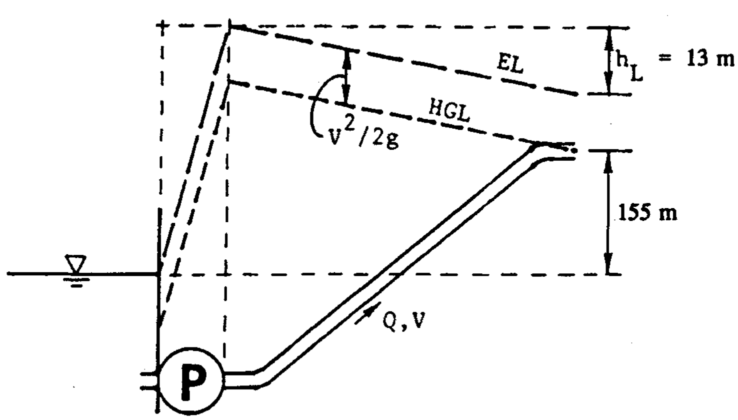
From Eq. 5.42: Input power = 14,180/0.90 = 15,760 hp ◀



5.13.6 *A pump, having an efficiency of 90%, lifts water to a height of 155 m at the rate of 7.5 m³/s. The friction head loss in the pipe is 13 m. What is the required pump power in kW? Also sketch the energy line and the hydraulic grade line of this system.*

SI

From Eqs. 5.41 and 5.42: Input power = $\gamma Q(\Delta z + h_L)/\eta$ = (9.81)7.5(155 + 13)/0.90 = 13 730 kW ◀



5.13.7 *In Fig. X5.13.7 let $a = 25$ ft, $b = 60$ ft, $c = 40$ ft, and $d = 2$ ft. All the losses of energy are to be ignored when the stream discharging into the air at E has a diameter of 4 in. What are pressure heads at B, C, and D if the diameter of the vertical pipe is 5 in?*

BG

Eq. 5.7 from A to E: $0 + 125 + 0 = 0 + 0 + V_E^2/2g$; $V_E^2/2g = 125$ ft

$V_B^2/2g = V_C^2/2g = V_D^2/2g = (4/5)^4 V_E^2/2g = 51.2$ ft

From A to B: $0 + 125 + 0 = p_B/\gamma + 100 + 51.2$ $\quad p_B/\gamma = -26.6$ ft ◄

From A to C: $0 + 125 + 0 = p_C/\gamma + 40 + 51.2$; $p_C/\gamma = 33.8$ ft ◄

From D to E: $p_D/\gamma + 2 + 51.2 = 0 + 0 + V_E^2/2g = 125$

$p_D/\gamma = 71.8$ ft ◄

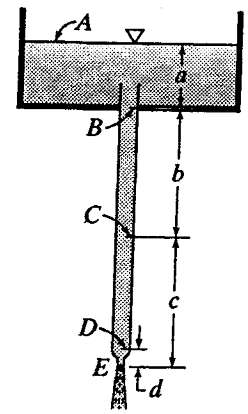


Figure X5.13.7

5.13.8 *In Fig. X5.13.7 let $a = 7.5$ m, $b = c = 15$ m, and $d = 300$ mm. All the losses of energy are to be ignored when the stream discharging into the air at E has a diameter of 80 mm. What are pressure heads at B, C, and D if the diameter of the vertical pipe is 120 mm?*

SI

Eq. 5.7 from A to E: $0 + 37.5 + 0 = 0 + 0 + V_E^2/2g$; $V_E^2/2g = 37.5$ m

$V_B^2/2g = V_C^2/2g = V_D^2/2g = (80/120)^4 V_E^2/2g = 7.41$ m

From A to B: $0 + 37.5 + 0 = \dfrac{p_B}{\gamma} + 30 + 7.41$; $\dfrac{p_B}{\gamma} = 0.0926$ m ◄

From A to C: $0 + 37.5 + 0 = \dfrac{p_C}{\gamma} + 15 + 7.41$; $\dfrac{p_C}{\gamma} = 15.09$ m ◄

From D to E: $\dfrac{p_D}{\gamma} + 0.30 + 7.41 = \dfrac{V_E^2}{2g} = 37.5$; $\dfrac{p_D}{\gamma} = 29.8$ m ◄

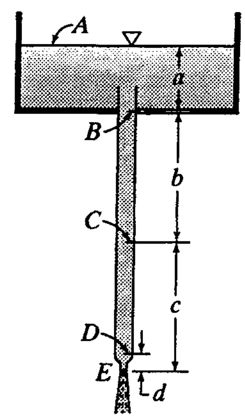


Figure X5.13.7

Sec. 5.13: Applications of Hydraulic Grade Line and Energy Line – ***Problems*** *5.33–5.41*

5.33 *In the constricted pipe of Fig. P5.33 friction loss between A and B is negligible while between B and C it is $0.15(V_B^2/2g)$. Given $h = 750$ mm, $d_A = d_C = 250$ mm, $d_B = 100$ mm. Find the pressure heads at A and C if the liquid is flowing through the circular pipe from A to C at the rate of 280 L/s.*

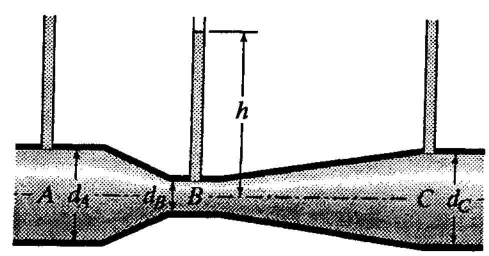


Figure P5.33

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$V_A = Q/A_A = (0.28 \text{ m}^3/\text{s})/(\pi 0.125^2) = 5.70$ m/s $= V_c$

$V_B = Q/A_B = 0.28/(\pi 0.05^2) = 35.7$ m/s;

$p_B/\gamma = 0.750$ m $(= h$, given)

$V_A^2/2g = V_C^2/2g = 5.70^2/(2\times 9.81) = 1.658$ m;

Eq. 5.7 from A to B: $p_A/\gamma + 0 + V_A^2/2g = p_B/\gamma + 0 + V_B^2/2g$

$p_A/\gamma + 0 + 1.658 = 0.750 + 0 + 64.8$; $p_A/\gamma = 63.9$ m ◄

Eq. 5.28 from B to C: $p_B/\gamma + 0 + V_B^2/2g - 0.15V_B^2/2g = p_C/\gamma + 0 + V_C^2/2g$

$0.750 + (1 - 0.15)64.8 = p_C/\gamma + 1.658$; $p_C/\gamma = 54.2$ m ◄

5.34 *In Fig. P5.34 assume the tube flows full. At B, the diameter of the tube is 3 in and the diameter of the water jet discharging into the air is 4.5 in. (a) If all friction losses are negligible, what are the velocity and the pressure head at B if h = 10 ft. (b) What is the rate of discharge in cfs? And what would it be if the tube were cut off at B?*

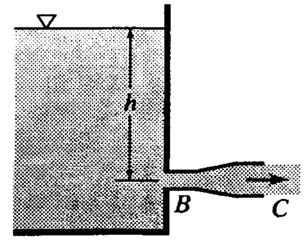


Figure P5.34

BG

(*a*) Eq. 5.7, from water suface to *C*: $0 + 10 + 0 = 0 + 0 + V_C^2/2g$;

$V_C^2/2g = 10$ ft

Eq. 5.7, from *B* to *C*: $p_B/\gamma + 0 + (4.5/3)^4 10 = 0 + 0 + 10$;

$p_B/\gamma = 10(1 - 5.06) = -40.6$ ft ◀

$V_B^2/2g = (4.5/3)^4 10 = 50.6$; $V_B = \sqrt{2(32.2)50.6} = 57.1$ fps ◀

(*b*) $Q = A_B V_B = (\pi/4)(3/12)^2 57.1 = 2.80$ cfs ◀

If tube were cut off at *B*, $V_B^2/2g = 10$ ft; $V_B = 25.4$ fps ; $Q = (\pi/4)(0.25)^2 25.4 = 1.246$ cfs ◀

5.35 *Referring to Fig. P5.34, assume the tube flows full and all friction losses are negligible. The diameter at B is 60 mm and the diameter of the jet discharging into the air is 80 mm. If h = 5 m, what is the flow rate? What is the pressure head at B? What would be the flow rate if the tube were cut off at B?*

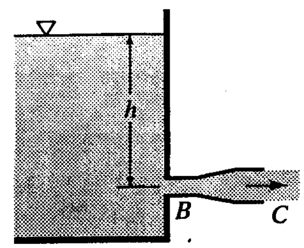


Figure P5.34

SI

Eq. 5.7, from water suface to *C*: $0 + 5 + 0 = 0 + 0 + V_C^2/2g$

$V_C^2/2g = 5$ m ; $V_C = \sqrt{2(9.81)5} = 9.90$ m/s

$Q = A_C V_C = \pi(0.04)^2 9.90 = 0.0498$ m³/s = 49.8 L/s ◀

$V_B = 9.90(8/6)^2 = 17.61$ m/s; $V_B^2/2g = 15.80$ m

Eq. 5.7 from *B* to *C*: $p_B/\gamma + 0 + V_B^2/2g = p_C/\gamma + 0 + V_C^2/2g$

$p_B/\gamma + 0 + 15.80 = 0 + 0 + 5$; $p_B/\gamma = -10.80$ m ◀

If the tube were cut off at *B*: $V_B^2/2g = 5$ m; $V_B = 9.90$ m/s

$Q = A_B V_B = \pi(0.03)^2 9.90 = 0.0280$ m³/s = 28.0 L/s ◀

5.36 *In Fig. P5.36 friction losses in the pipe below pump P are $1.8V^2/2g$ with the barometer pressure at 12.50 psia. The liquid in the suction pipe has a velocity of 7 fps. What would be the maximum allowable value of z if the liquid were (a) water at 70°F; (b) gasoline at a vapor pressure of 9 psia with a specific weight of 47 lb/ft³ ?*

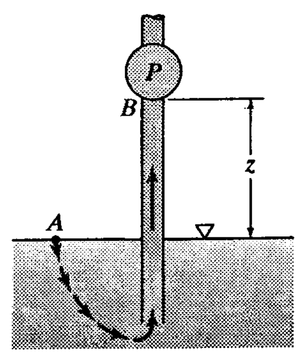


Figure P5.36

BG

From Eq. 5.28 (using absolute pressure heads) from *A* to *B*, where, to prevent cavitation, $p_B = p_{min} = p_v$:

$$\frac{12.50(144)}{\gamma} + 0 + 0 = 144\frac{p_v}{\gamma} + z_{max} + \frac{7^2}{2(32.2)} + \frac{(1.8)7^2}{2(32.2)}$$

$$= 144\frac{p_v}{\gamma} + z_{max} + 2.13$$

from which $z_{max} = (12.50 - p_v)144/\gamma - 2.13$

(*a*) Table A.1 for water at 70°F: $\gamma = 62.30$ pcf, $p_v = 0.363$ psia from which $z_{max} = 25.9$ ft ◀

(*b*) For gasoline ($\gamma = 47$ pcf, $p_v = 9$ psia, given) : $z_{max} = 8.59$ ft ◀

5.37 *In Fig. P5.36 friction losses in the pipe below pump P are $1.6V^2/2g$ with the barometer pressure at 90 kPa. The liquid in the suction pipe has a velocity of 1.8 m/s. What would be the maximum allowable value of z if the liquid were (a) water at 20°C; (b) gasoline at a vapor pressure of 49 kPa abs, with a specific weight of 8 kN/m³?*

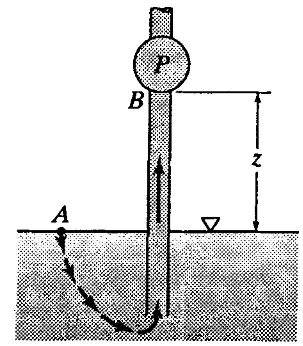


Figure P5.36

SI

From Eq. 5.28 from A to B, where, to prevent cavitation, $p_B = p_{min} = p_v$:

$$\frac{90}{\gamma} + 0 + 0 = \frac{p_v}{\gamma} + z_{max} + \frac{1.8^2}{2 \times 9.81}(1 + 1.6)$$

from which $z_{max} = (90 - p_v)/\gamma - 0.429$

(*a*) Table A.1 for water at 20°C: $\gamma = 9.789$ kN/m³, $p_v = 2.34$ kN/m² abs

from which $z_{max} = 8.53$ m ◄

(*b*) For gasoline ($\gamma = 8$ kN/m³, $p_v = 49$ kN/m² abs, given) : $z_{max} = 4.70$ m ◄

5.38 *A discharge pressure gage reading, taken at a point 6.5 ft above the center line of a pump, is 25 psi. A suction pressure gage reading, taken 2.5 ft below the center line, indicates a vacuum of 12 inHg when gasoline (s = 0.75) is pumped at the rate of 1.5 cfs. The diameter of the suction and discharge pipes of the pump are 8 and 6 in, respectively. What is the power delivered to the fluid? Sketch the energy line and the hydraulic grade line.*

BG

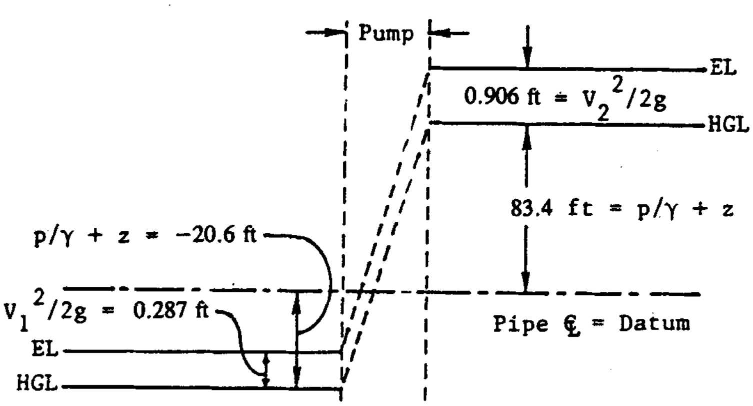
$$V_1 = \frac{Q}{A_1} = \frac{1.5}{(\pi/4)(8/12)^2} = 4.30 \text{ fps} ; \quad V_2 = \frac{Q}{A_2} = \frac{1.5}{(\pi/4)(6/12)^2} = 7.64 \text{ fps}$$

Eq. 5.37 (with $h_M = +h_p$, $h_L = 0$) and Eq. 5.35, from suction to discharge in feet of gasoline:

$$-\left(\frac{12}{12}\right)\frac{13.56}{0.75} - 2.5 + \frac{(4.30)^2}{2(32.2)} + h_p = \frac{25(144)}{0.75(62.4)} + 6.5 + \frac{(7.64)^2}{2(32.2)}$$

$$h_p = 18.08 + 2.5 - 0.287 + 76.9 + 6.5 + 0.906 = 104.6 \text{ ft}$$

Eq. 5.39: Power $= \gamma Q h_p = (0.75 \times 62.4)1.5(104.6) = 7345$ ft·lb/sec $= 13.35$ hp ◄



5.39 *For this problem, use the same data as in Sample Prob. 5.11, except that instead of the pump developing 80 ft of head, it delivers 110 hp to the water. Find the new flow rate. Plot the energy line and the hydraulic grade line. Calculate the pressure on the suction side of the pump.*

Sample Prob. 5.11: In Fig. S5.11, $h_L = V_6^2/2g$ in the 6-in-dia suction line, and $h_L = V_4^2/2g$ in the 4-in-dia discharge line.

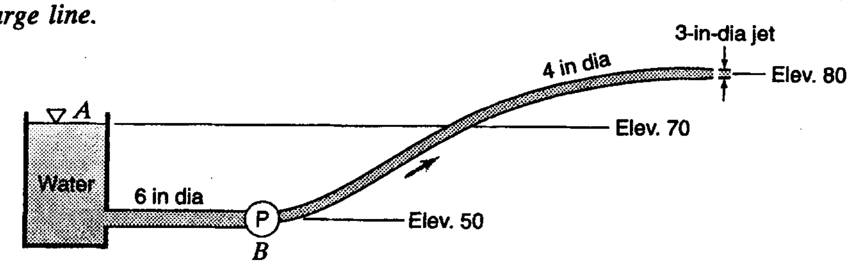


Figure S5.11

BG

$V_6 = Q/A_6 = 5.09Q$; $V_4 = Q/A_4 = 11.46Q$; $V_3 = Q/A_3 = 20.4Q$

Energy equation 5.28 from reservoir surface to jet:

$(0 + 70 + 0) - 5V_6^2/2g + h_p - 12V_4^2/2g = 0 + 80 + V_3^2/2g$

Expressing all velocities in terms of Q:

$70 - 2.01Q^2 + h_p - 24.5Q^2 = 80 + 6.44Q^2$, i.e., $n_p = 10 + 32.9Q^2$

where, from Eq. 5.40, $P = 110 = 62.4Qh_p/550$, i.e , $h_p = 970/Q$

Thus $970/Q = 10 + 32.9Q^2$; by trial $Q = 3.06$ cfs ◄

$(0 + 70 + 0) - 2.01Q_2 = p_s/\gamma + 50 + V_6^2/2g$ where $V_6 = 5.09Q = 15.56$ fps, $V_6^2/2g = 3.76$ ft

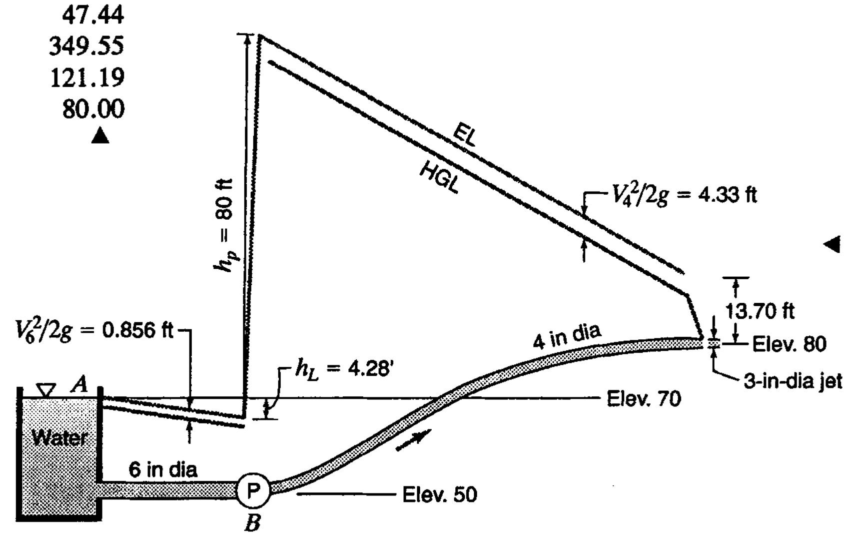
∴ on the suction side of the pump, $p_s/\gamma = 70 - 18.80 - 50 - 3.76 = -2.56$ ft

$p_s = -2.56(62.4)/144 = 1.108$ psi suction ◄

Alternatively (from Sec. 3.5): $p_s/\gamma = -2.56(29.92$ inHg$/33.91$ ft$) = 2.26$ inHg vacuum ◄

The energy line and hydraulic grade line are similar to those in the solution figure of Sample Prob. 5.11, but with the following elevations:

	EL	HGL
Reservoir surface	70.00 ft	70.00 ft
6 inch pipe entrance	70.00	66.24
Pump entrance	51.20	47.44
Pump exit	368.58	349.55
Before nozzle	140.22	121.19
After nozzle	140.22	80.00
	▲	▲



◄

5.40 *Assume ideal fluid. The pressure at section 1 in Fig. P5.40 is 10 psi, V_1 = 15 fps, V_2 = 50 fps, and γ = 60 lb/ft³. (a) Determine the reading on the manometer. (b) If the downstream piezometer were replaced with a pitot tube, what would be the manometer reading? Comment on the practicality of these arrangements.*

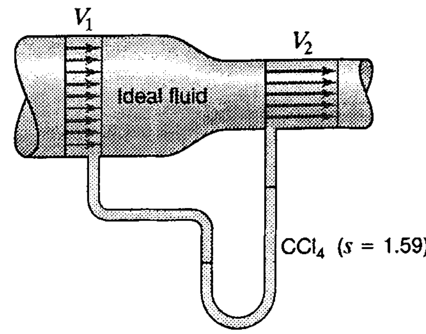


Figure P5.40

BG

(*a*) Flowing fluid: s_F = 60/62.4 = 0.9615

Energy Eq. 5.29 with $z_1 = z_2$: $\frac{15^2}{2g} + \frac{p_1}{\gamma} = \frac{50^2}{2g} + \frac{p_2}{\gamma}$

So $\frac{p_1}{\gamma} - \frac{p_2}{\gamma} = \frac{50^2 - 15^2}{2(32.2)} = 35.3$ ft

Using Eq. 3.12 with $z_A = z_B$:

$$\frac{p_1}{\gamma} - \frac{p_2}{\gamma} = 35.3 \text{ ft} = \left(\frac{s_M}{s_F} - 1\right)R_m = \left(\frac{1.59}{0.9615} - 1\right)R_m$$

from which the manometer reads R_m = 54.1 ft = 649 in ◄

(*b*) $\left[\left(\frac{50^2}{2g} + \frac{p_2}{\gamma}\right) - \frac{p_1}{\gamma}\right] = \frac{15^2}{2(32.2)} = 3.49 = \left(\frac{1.59}{0.9615} - 1\right)R_m$, so R_m = 5.35 ft = 64.1 in ◄

The first reading is too large for practical use. ◄

5.41 *Refer to Fig. P5.40. Assume an ideal fluid with ρ = 900 kg/m³. The pressure at section 1 is 100 kN/m²; V_1 = 10 m/s, V_2 = 20 m/s. (a) Determine the reading on the manometer. (b) If the downstream piezometer were replaced with a pitot tube, what would be the manometer reading? Comment on the practicality of these arrangements?*

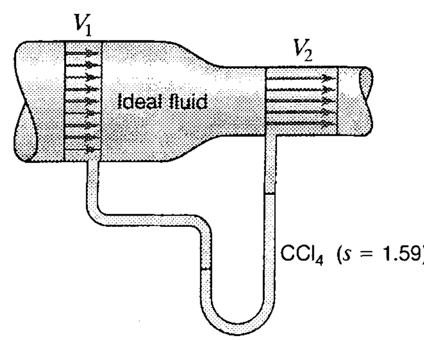


Figure P5.40

SI

(*a*) Flowing fluid: s_F = 900/1000 = 0.90

Energy Eq. 5.29 with $z_1 = z_2$:

$$\frac{10^2}{2g} + \frac{p_1}{\gamma} = \frac{20^2}{2g} + \frac{p_2}{\gamma}; \quad \frac{p_1}{\gamma} - \frac{p_2}{\gamma} = \frac{20^2 - 10^2}{2(9.81)} = 15.29 \text{ m}$$

Using Eq. 3.12 with $z_A = z_B$: $\frac{p_1}{\gamma} - \frac{p_2}{\gamma} = 15.29 \text{ m} = \left(\frac{s_M}{s_F} - 1\right)R_m = \left(\frac{1.59}{0.90} - 1\right)R_m$

from which the manometer reads R_m = 19.94 m ◄

(*b*) $\left[\left(\frac{20^2}{2g} + \frac{p_2}{\gamma}\right) - \frac{p_1}{\gamma}\right] = \frac{10^2}{2(9.81)} = 5.097 = \left(\frac{1.59}{0.90} - 1\right)R_m$, so R_m = 6.65 m ◄

Both readings are too large for practical use. ◄

Sec. 5.14: Method of Solution of Flow Problems – ***Exercises (4)***

5.14.1 *Refer to Fig. S5.12. If the depths upstream and downstream of the gate were 7.5 ft and 3.0 ft respectively, find the flow rate per foot of channel width. Assume no head loss.*

BG

Energy Eq. 5.29: $7.5 + 0 + V_1^2/2g = 3.0 + 0 + V_2^2/2g$

Continuity: $A_1V_1 = A_2V_2$ yields $V_2 = (7.5/3.0)V_1$

Combining these two equations gives $V_1 = 7.43$ fps

Hence $Q = A_1V_1 = (7.5 \times 1)7.43$

$= 55.7$ cfs per ft of channel width ◄

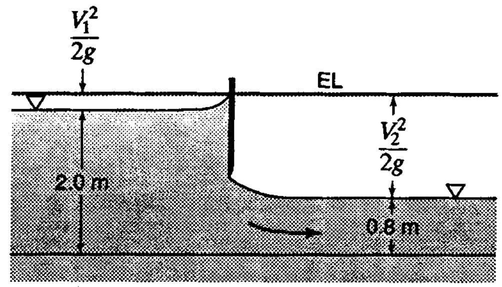


Figure S5.12

5.14.2 *Refer to Fig. S5.12. If the depths upstream and downstream of the gate were 1.5 m and 0.6 m respectively, find the flow rate per meter of channel width. Assume no head loss.*

SI

Energy Eq. 5.29: $1.5 + 0 + V_1^2/2g = 0.6 + 0 + V_2^2/2g$

Continuity: $A_1V_1 = A_2V_2$ yields $V_2 = (1.5/0.6)V_1$

Combining these two equations gives $V_1 = 1.834$ m/s

Hence $Q = A_1V_1 = (1.5 \times 1)1.834$

$= 2.75$ m³/s per m ◄

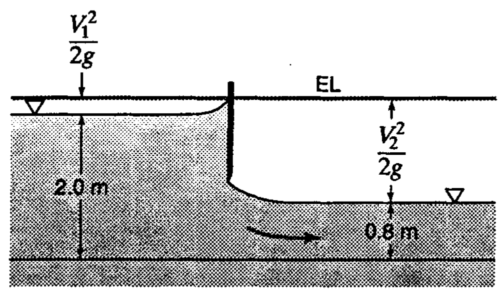


Figure S5.12

5.14.3 *Refer to Fig. S5.12. Suppose the gate opening is set so the depth downstream is 2.0 ft. Find the upstream depth under these conditions if the flow rate is 45 ft³/sec per ft of width. Assume no head loss.*

BG

Energy Eq. 5.29: $y_1 + V_1^2/2g = y_2 + V_2^2/2g$, where z's cancel and water depth $y = p/\gamma$

Continuity: $V_1 = Q/A_1 = 45/(y_1 \times 1) = 45/y_1$

and $V_2 = Q/A_2 = 45/(2.0 \times 1) = 22.5$ ft/sec

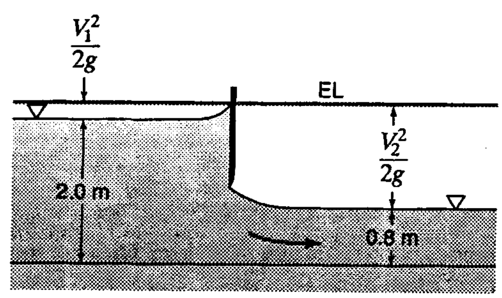


Figure S5.12

so: $y_1 + \dfrac{(45/y_1)^2}{2(32.2)} = 2.0 + \dfrac{22.5^2}{2(32.2)}$ or $y_1 + \dfrac{31.4}{y_1^2} = 9.86$ ft ; By trial $y_1 = 9.51$ or 2.00 ft

2.0 ft is the given downstream depth, so upstream depth = 9.51 ft ◄

5.14.4 *Refer to Fig. 5.12. Suppose the gate opening is set so the depth downstream is 0.7 m. Find the upstream depth under these conditions if the flow rate is 4.24 m³/s per m of width. Assume no head loss.*

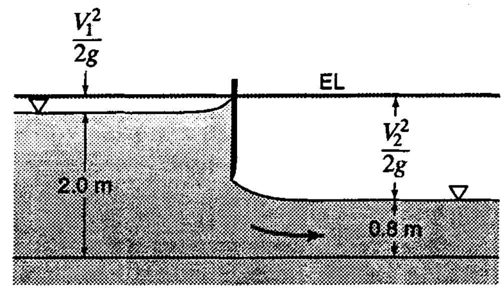


Figure S5.12

SI

Energy Eq. 5.29: $y_1 + V_1^2/2g = y_2 + V_2^2/2g$,

where z's cancel and water depth $y = p/\gamma$

Continuity: $V_1 = Q/A_1 = 4.24/(y_1 \times 1) = 4.24/y_1$

and $V_2 = Q/A_2 = 4.24/(0.7 \times 1) = 6.06$ m/s

So: $y_1 + \dfrac{(4.24/y_1)^2}{2(9.81)} = 0.7 + \dfrac{6.06^2}{2(9.81)}$ or $y_1 + \dfrac{0.916}{y_1^2} = 2.57$ m. By trial, $y_1 = 0.700$ or 2.41 m

0.7 m is the given downstream depth. So upstream depth = 2.41 m ◀

Sec. 5.15: Jet Trajectory – *Exercises (3)*

5.15.1 *A jet issues horizontally from an orifice in the vertical wall of a large tank (Fig. X5.15.1). Neglecting air resistance. determine the velocity of the jet at the orifice for the following variety of trajectories: (a) x = 1.0 m, y = 1.0 m; (b) x = 2.0 m, y = 2.0 m; (c) x = 3.0 m, y = 3.0 m; (d) x = 4.0 m, y = 4.0 m. Express the answers in m/s.*

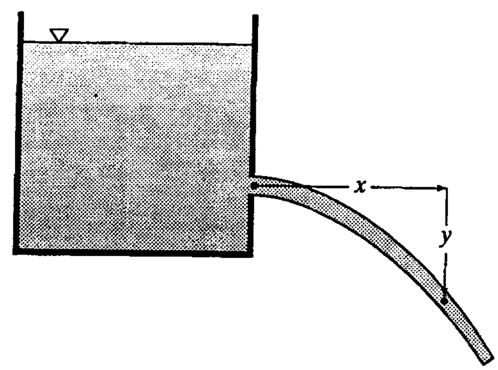


Figure X5.15.1

SI

$x = V_0 t\ ;\ y = (1/2)gt^2 = (1/2)g(x/V_0)^2$

Eq. 5.45: $V_0 = x(g/2y)^{1/2}$

(*a*) $x = 1$ m, $y = 1$ m: $V_0 = 2.21$ m/s ◀
(*b*) $x = 2$ m, $y = 2$ m: $V_0 = 3.13$ m/s ◀
(*c*) $x = 3$ m, $y = 3$ m: $V_0 = 3.84$ m/s ◀
(*d*) $x = 4$ m, $y = 4$ m: $V_0 = 4.43$ m/s ◀

5.15.2 *A water jet must reach the window in the wall shown in Fig. X5.15.2. Assuming a jet velocity of 25 m/s at the nozzle and neglecting air friction, find the angle (or angles) of inclination θ which will achieve this result, given h = 14 m, d = 23 m, and a = 2 m.*

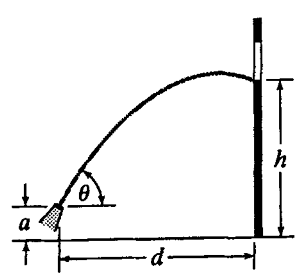


Figure X5.15.2

SI

Eq. 5.44: $(14 - 2) = \dfrac{25 \sin\theta}{25 \cos\theta}(23) - \dfrac{9.81}{2(25)^2 \cos^2\theta}(23)^2$

$12 = 23 \tan\theta - 4.15(1 + \tan^2\theta)$

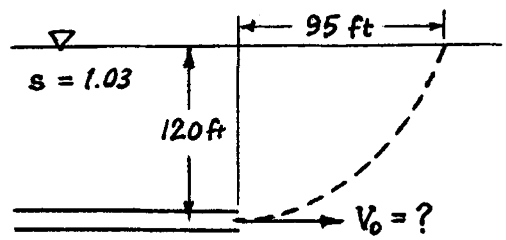
$\tan^2\theta - 5.54 \tan\theta + 3.89 = 0$, quadratic; $\tan\theta = 0.825$ or 4.71

$\theta = 39.5°$ or $78.0°$ (either angle will work) ◀

5.15.3 *Freshwater sewage effluent discharges from a horizontal outfall pipe on the floor of the ocean at a point where the depth is 120 ft. When the ocean is still, the jet is observed to rise to the surface at a point 95 ft horizontally from the end of the pipe. Assuming the ocean water to have a specific gravity of 1.03 and neglecting fluid friction and mixing of the jet with the ocean water, find the velocity at the end of the outfall.*

BG

Replace g in trajectory equations with the force per unit mass ($F/m = a = g'$) of the jet fluid:



Eq. 5.46: $g' = 32.2\left(1 - \dfrac{1.03}{1.00}\right) = -0.966$ lb/slug

$= -0.966$ ft/sec^2

Eq. 5.45, modified: $V_0 = x\sqrt{\dfrac{g'}{2z}} = 95\sqrt{\dfrac{-0.966}{2(-120)}}$

$= 6.03$ fps ◄

Sec. 5.15: Jet Trajectory – *Problems 5.42–5.43*

5.42 *By manipulation of Eq. (5.37), demonstrate that it represents a standard parabola of the form $z - z_0 = a(x - x_0)^2$, where a is a constant and x_0 and z_0 are the coordinates of the vertex.*

N

Eq. 5.37: $z = \dfrac{V_{z0}}{V_{x0}}x - \dfrac{g}{2V_{x0}^2}x^2 = -\dfrac{g}{2V_{x0}^2}\left(x^2 - \dfrac{2V_{x0}V_{z0}}{g}x\right)$

Completing the square, $z = -\dfrac{g}{2V_{x0}^2}\left[x^2 - \dfrac{2V_{x0}V_{z0}}{g}x + \dfrac{V_{x0}^2V_{z0}^2}{g^2}\right] + \dfrac{V_{z0}^2}{2g}$

$\therefore\ z - \dfrac{V_{z0}^2}{2g} = -\dfrac{g}{2V_{x0}^2}\left[x - \left(\dfrac{V_{x0}V_{z0}}{g}\right)\right]^2$ or $(z - z_0) = +a(x - x_0)^2$ ◄

where $z_0 = V_{z0}^2/2g$, $x_0 = V_{x0}V_{z0}/g$ and $a = -g/(2V_{x0}^2)$

5.43 *Find the maximum ideal horizontal range of a jet having an initial velocity of 90 fps. At what angle of inclination is this obtained?*

BG

Eq. 5.44 for horizontal range ($z = 0$): $\dfrac{V_{z0}}{V_{x0}}x = \dfrac{g}{2V_{x0}^2}x^2$ (1)

From Fig. 5.11: $V_{z0}/V_{x0} = \tan\theta$, $V_{x0} = V_0\cos\theta$

Substituting these into (1): $x\tan\theta = \dfrac{gx^2}{2V_0^2\cos^2\theta}$

Thus for $z = 0$, $x = (2V_0^2/g)\cos^2\theta\tan\theta = (V_0^2/g)\sin 2\theta$

For x_{max}, $dx/d\theta = (2V_0^2/g)\cos 2\theta = 0$; $2\theta = 90°$

$\theta = 45°$ ◄ and $x_{max} = V_0^2/g = (90)^2/32.2 = 252$ ft ◄

Sec. 5.16: Flow in a Curved Path -- *Exercises (2)*

5.16.1 *Figure X5.16.1 shows a two-dimensional ideal flow in a vertical plane. Data are as follows: r = 12 ft, b = 5 ft, γ = 62.4 lb/ft³, V = 24 fps. If the pressure at A is 6 psi, find the pressure at B.*

BG

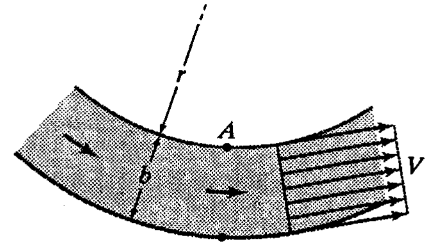


Figure X5.16.1

Pressure increase:

(*a*) Elevation effect: $\Delta p = \gamma(\Delta h) = 5\gamma$

(*b*) Curved path effect (Eq. 5.47): $dp = \rho \frac{V^2}{r} dr$

Integrating: $\int_{12}^{17} dp = \rho V^2 \int_{12}^{17} \frac{dr}{r} = \frac{\gamma}{g} V^2 \int_{12}^{17} \frac{dr}{r}$

So $\Delta p = p_{17} - p_{12} = \gamma\left(\frac{V^2}{g}\right)[\ln r]_{12}^{17} = \gamma\left(\frac{24^2}{32.2}\right)\ln(17/12) = 6.23\gamma$

$\left[\text{Approx. method: } \Delta p \approx \frac{\gamma V^2}{gr}\Delta r = \frac{24^2}{32.2(12 + 2.5)}\gamma(5) = 6.17\gamma\right]$

Thus p_B = 6 psi + (5 + 6.23)62.4/144 = 10.87 psi ◄

5.16.2 *Refer to Fig. X5.16.1. Flow occurs in a vertical plane. Data are as follows: r = 7 m, b = 3 m, γ = 9.81 kN/m³, V = 5 m/s. Find the pressure at A if the pressure at B is 150 kPa.*

SI

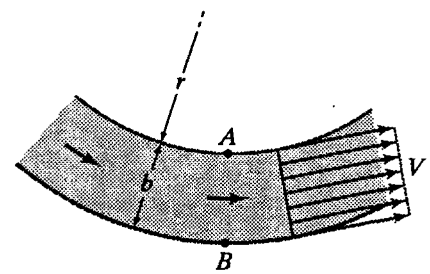


Figure X5.16.1

(*a*) Elevation effect: $\Delta p = \gamma(\Delta h)$ = 9.81(3) = 29.43 kN/m²

(*b*) Curved path effect (Eq. 5.47): $dp = \rho \frac{V^2}{r} dr$

Integrating: $\int_{7}^{10} dp = \rho V^2 \int_{7}^{10} \frac{dr}{r} = \frac{\gamma}{g} V^2 \int_{7}^{10} \frac{dr}{r}$

So $\Delta p = p_{10} - p_7 = \gamma\left(\frac{V^2}{g}\right)[\ln r]_{7}^{10} = 9.81\left(\frac{5^2}{9.81}\right)\ln(10/7) = 8.92$ kPa

$\left[\text{Approx. method: } \Delta p \approx \frac{\gamma V^2}{gr}\Delta r = \frac{9.81(5^2)}{9.81(7 + 1.5)}3 = 8.82 \text{ kPa}\right]$

Thus p_A = 150 - 29.43 - 8.92 = 111.7 kN/m² ◄

Sec. 5.16: Flow in a Curved Path – *Problems 5.44–5.45*

5.44 *Repeat Exer. 5.16.1. Let $V = Q/A = 24$ fps, but assume a parabolic velocity profile.*

Exer. 5.16.1: The two-dimensional ideal flow in Fig. X5.16.1 occurs in a vertical plane. Given $r = 12$ ft, $b = 5$ ft $\gamma = 62.4$ pcf, $p_A = 6$ psi. Find p_B.

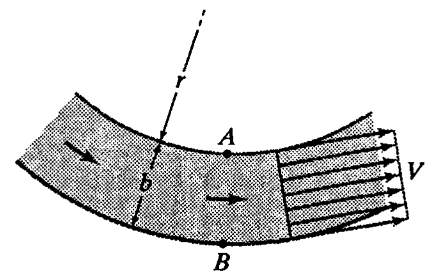


Figure X5.16.1

BG

Table A.7 for parabola: Area = 2/3 of enclosing rectangle.

So $V = (2/3)u_{max}$ and $u_{max} = 1.5V = 1.5(24) = 36$ fps.

Equation of parabola: $(r - 14.5)^2 = k(36 - u)$

Boundary conditions: When $r = 12$ or 17 ft, $u = 0$;

when $r = 14.5$ ft, $u = 36$ fps

Substituting into parabola, $k = (2.5)^2/36 = 1/5.76$ sec·ft. Hence $u = 36 - 5.76(r - 14.5)^2$ fps

Elevation effect: $\Delta p = \gamma(\Delta h) = 62.4(5) = 312$ psf $= 2.17$ psi

Curved path effect:

From Eq. 5.47: $$\Delta p = \int_{12}^{17} \rho\frac{u^2}{r}dr = \frac{\gamma}{g}\int_{12}^{17}[36 - 5.76(r - 14.5)^2]^2\frac{dr}{r}$$

$$= \frac{\gamma}{g}\int_{12}^{17}\left(33.2r^3 - 1924r^2 + 41{,}400r - 393{,}000 + \frac{1{,}381{,}000}{r}\right)dr$$

$$= (\gamma/g)[33.2r^4/4 - 1924r^3/3 + 41{,}400r^2/2 - 393{,}000r + 1{,}381{,}000\ln r]_{12}^{17}$$

$$\Delta p = 239\gamma/g = 239(62.4)/32.2 = 464 \text{ psf} = 3.22 \text{ psi}$$

Thus $p_B = 6 + 2.17 + 3.22 = 11.39$ psi ◄

5.45 *Using Fig. X5.16.1, which depicts a two-dimensional ideal flow in a vertical plane, find the pressure at B if the pressure at A is 32 kPa. Data are as follows: $r = 3$ m, $b = 1.2$ m, $\gamma = 9.81$ kN/m³, $V = Q/A = 5$ m/s. Assume a parabolic velocity profile.*

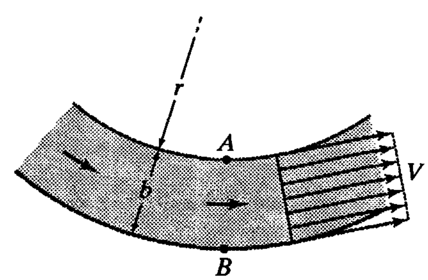


Figure X5.16.1

SI

Table A.7 for parabola: Area = 2/3 of enclosing rectangle.

So $V = (2/3)u_{max}$ and $u_{max} = 1.5V = 1.5(5) = 7.5$ m/s

Equation of parabola: $(r - 3.6)^2 = k(7.5 - u)$

Boundary conditions: When $r = 3$ or 4.2 m, $u = 0$;

when $r = 3.6$ m, $u = 7.5$ m/s.

Substituting into parabola, $k = 0.048$ s·m ; hence $u = 7.5 - \dfrac{(r - 3.6)^2}{0.048}$

Elevation effect: $\Delta p = \gamma(\Delta h) = (9.81)1.2 = 11.77$ kN/m² (or kPa)

Curved path effect: From Eq. 5.47:

$$\Delta p = \int_{3.0}^{4.2}\rho\frac{u^2}{r}dr = \frac{\gamma}{g}\int_{3.0}^{4.2}\left(7.5 - \frac{(r-3.6)^2}{0.048}\right)^2\frac{dr}{r} = \frac{\gamma}{g}\int_{3.0}^{4.2}\left(434r^3 - 6250r^2 + 33440r - 78750 + \frac{68910}{r}\right)dr$$

$$= (\gamma/g)[434r^4/4 - 6250r^3/3 + 33\,440r^2/2 - 78\,750r + 68\,910\ln r]_{3.0}^{4.2}$$

$$\Delta p = 10.08\gamma/g = 10.08(9.81)/9.81 = 10.08 \text{ kN/m}^2 \text{ (or kPa)}$$

Thus, $p_B = 32 + 11.77 + 10.08 = 53.9$ kN/m² $= 53.9$ kPa ◄

Sec. 5.17: Forced or Rotational Vortex – *Exercises (3)*

5.17.1 *A 16-in-diameter closed vessel completely filled with fluid rotates at 1800 rpm. What will be the pressure difference between the circumference and the axis of rotation in feet of the fluid and in pounds per square inch if the fluid is (a) air with a specific weight of 0.076 lb/ft³; (b) water at 70°F; (c) oil with a specific weight of 46 lb/ft³?*

BG

Eq. 5.48: $\dfrac{p_2}{\gamma} - \dfrac{p_1}{\gamma} = \left(\dfrac{1800\times 2\pi}{60}\right)^2 \dfrac{1}{2(32.2)}\left[\left(\dfrac{16}{2\times 12}\right)^2 - 0\right] = 245$ ft of any fluid ◄

(*a*) With air at $\gamma = 0.076\text{ lb/ft}^3$, $\Delta p = (245\times 0.076)/144 = 0.1294$ psi ◄

(*b*) Table A.1 for water at 70°F: $\gamma = 62.30\text{ lb/ft}^3$

∴ With water at 70°F, $\Delta p = (245\times 62.3)/144 = 106.1$ psi ◄

(*c*) With oil at $\gamma = 46\text{ lb/ft}^3$, $\Delta p = (245\times 46)/144 = 78.3$ psi ◄

5.17.2 *A 1.2-m-diameter closed vessel completely filled with oil (γ = 8.3 kN/m³) rotates at 400 rpm. What will be the pressure difference between the circumference and the axis of rotation? Express the answer in Pa.*

SI

$r_1 = 0$; $r_2 = 1.20/2 = 0.60$ m

Eq. 5.48: $\dfrac{p_2}{\gamma} - \dfrac{p_1}{\gamma} = \left(\dfrac{400\times 2\pi}{60}\right)^2 \dfrac{1}{2(9.81)}(0.60^2 - 0) = 32.2$ m of fluid

$\Delta p = \gamma(32.2) = 8.3(32.2) = 267\text{ kN/m}^2 = 267$ kPa ◄

5.17.3 *A 2-ft-diameter open cylindrical vessel partially filled with water rotates about its vertical axis. How many revolutions per minute would cause the water surface at the periphery to be 5 ft higher than the water surface at the axis? What would be the necessary speed for the same conditions if the fluid were mercury?*

BG

$r_1 = 0$; $r_2 = 2/2 = 1.0$ ft

Eq. 5.50: $z_2 - z_1 = 5 = \dfrac{\omega^2}{2g}(1.0^2 - 0)$; $\omega^2 = 5(2g) = 10(32.2) = 322$

So $\omega = 17.94$ radians/sec or $(17.94\times 60)/(2\pi) = 171.4$ rpm ◄

For mercury or any other fluid, the speed would be the same. ◄

Sec. 5.17: Forced or Rotational Vortex -- *Problems 5.46–5.47*

5.46 *In Fig. P5.46 the rotor vanes are all straight and radial, r_1 = 0.3 ft, r_2 = 0.9 ft, and the height perpendicular to the plane of the figure is constant at B = 0.25 ft. Then A = 2πrB. If the rotation speed is 1000 rpm and the flow of liquid is 9.6 cfs, find the difference in the pressure head between the outer and the inner circumferences, neglecting friction losses. Does it make any difference whether the flow is outward or inward?*

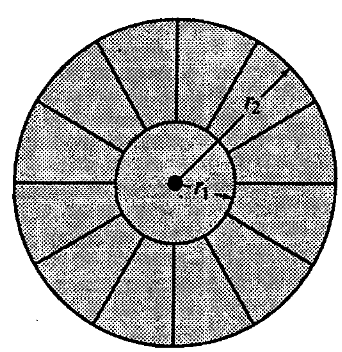


Figure P5.46

BG

$V_1 = \dfrac{Q}{A_1} = \dfrac{9.6}{2\pi(0.30)0.25} = 20.4$ fps; $V_2 = \dfrac{9.6}{2\pi(0.90)0.25} = 6.79$ fps

Eq. 5.53: $\dfrac{p_2}{\gamma} - \dfrac{p_1}{\gamma} =$

$= \left(\dfrac{1000\times 2\pi}{60}\right)^2 \dfrac{1}{2\,(32.2)}(0.90^2 - 0.30^2) + \dfrac{20.4^2 - 6.79^2}{2(32.2)} = 128.3$ ft ◄

Neglecting friction, the answer is the same for flow in either direction. ◄

5.47 *In Fig. 5.46 the vanes are all straight and radial, r_1 = 100 mm, r_2 = 200 mm, and the height perpendicular to the plane of the figure is constant at B = 80 mm. Then A = 2πrB. If the rotation speed is 1000 rpm and the flow of liquid is 0.3 m^3/s, find the difference in the pressure head between the outer and the inner circumferences, neglecting friction losses. Does it make any difference whether the flow is outward or inward?*

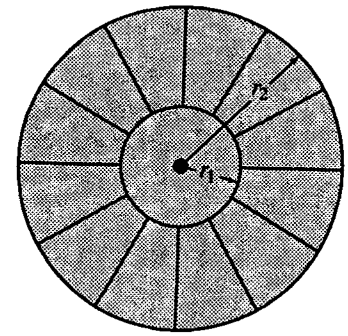


Figure P5.46

SI

$$V_1 = \frac{Q}{A_1} = \frac{Q}{2\pi r_1 B} = \frac{0.3}{2\pi(0.10)0.08} = 5.97 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{Q}{2\pi r_2 B} = \frac{0.3}{2\pi(0.20)0.08} = 2.98 \text{ m/s}$$

Eq. 5.53: $$\frac{p_2}{\gamma} - \frac{p_1}{\gamma} = \left(\frac{1000 \times 2\pi}{60}\right)^2 \frac{1}{2(9.81)}(0.20^2 - 0.10^2) + \frac{5.97^2 - 2.98^2}{2(9.81)} = 18.13 \text{ m} \quad ◄$$

Neglecting friction, the answer is the same for flow in either direction. ◄

Sec. 5.18: Free or Irrotational Vortex – *Exercises (2)*

5.18.1 *Refer to Sample Prob. 5.14. If the impeller diameter is 220 mm, the casing height is 40 mm between a and b, and water leaves the impeller with a velocity of 18 m/s at an angle of 16° with the tangent, find the flow rate, the magnitude and direction of the velocity at b (where r = 160 mm), and the pressure increase from a to b. Neglect friction.*

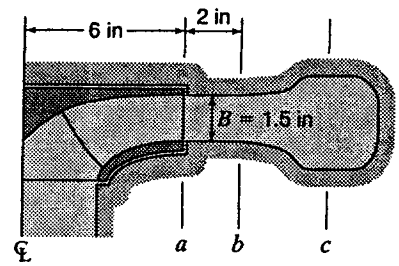


Figure S5.14

SI

Sec. 5.18: $Q = 2\pi r_a B (V_r)_a$ where $(V_r)_a = 18 \sin 16° = 4.96$ m/s

So $Q = 2\pi(0.22/2)0.04(4.96) = 0.1372 \text{ m}^3/\text{s}$ ◄

From continuity: $(V_r)_b/(V_r)_a = r_a/r_b = 110/160$

∴, as in Sample Prob. 5.14, the angle remains at 16° ◄

$V_b = (110/160)18 = 12.38$ m/s ◄

From Eq. 5.29: $$\frac{p_b}{\gamma} - \frac{p_a}{\gamma} = \frac{V_a^2}{2g} - \frac{V_b^2}{2g} = \frac{18^2 - 12.38^2}{2(9.81)} = 8.71 \text{ m}$$

$\Delta p = 8.71(9810 \text{ N/m}^3) = 85\,400 \text{ N/m}^2 = 85.4$ kPa ◄

5.18.2 *Refer to Sample Prob. 5.14. If the impeller diameter is 10 in, the casing height is 1.8 in between a and b, and water leaves the impeller with a velocity of 50 fps at an angle of 16° with the tangent, find the flow rate, the magnitude and direction of the velocity at b (where r = 7 in), and the pressure increase from a to b. Neglect friction.*

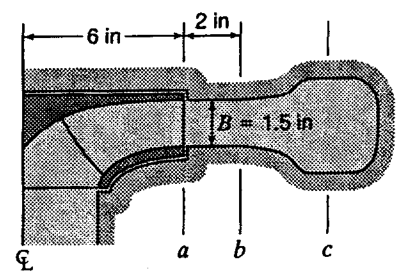


Figure S5.14

BG

Sec 5.18: $Q = 2\pi r_a B (V_r)_a$ where $(V_r)_a = 50 \sin 16° = 13.78$ fps

So $Q = 2\pi(5/12)(1.8/12)13.78 = 5.41$ cfs ◄

From continuity: $(V_r)_b/(V_r)_a = r_a/r_b = 5/7$

∴, as in Sample Prob. 5.14, the angle remains at 16° ◄

$V_b = (5/7)50 = 35.7$ fps ◄

From Eq. 5.29: $$\frac{p_b}{\gamma} - \frac{p_a}{\gamma} = \frac{V_a^2}{2g} - \frac{V_b^2}{2g} = \frac{50^2 - 35.7^2}{2(32.2)} = 19.01 \text{ ft}$$

$\Delta p = 19.01(62.4 \text{ lb/ft}^3) = 1186 \text{ psf} = 8.24$ psi ◄

Sec. 5.18: Free or Irrotational Vortex – *Problems* 5.48–5.49

5.48 *An air duct of 2.5 by 2.5 ft square cross section turns a bend of radius 5 ft as measured to the center line of the duct. If the measured pressure difference between the inside and the outside walls of the bend is 1.5 in of water, estimate the rate of air flow in the duct. Assume standard sea-level conditions in the duct and assume ideal flow around the bend.*

BG

Flow = cylindrical free vortex. Table A.3 for air at sea level: $\gamma = 0.0765$ pcf

Given $\dfrac{p_2}{\gamma} - \dfrac{p_1}{\gamma} = (1.5/12)62.4/0.0765 = 102.0$ ft of air ; Eq. 5.54: $\dfrac{p_2}{\gamma} - \dfrac{p_1}{\gamma} = \dfrac{C^2}{2g}\left(\dfrac{1}{r_1^2} - \dfrac{1}{r_2^2}\right)$

Thus, with $r_1 = 3.75$ ft and $r_2 = 6.25$ ft: $102.0 = \dfrac{C^2}{2(32.2)}\left(\dfrac{1}{3.75^2} - \dfrac{1}{6.25^2}\right)$; $C = 380\ \text{ft}^2/\text{sec}$

Thus, with $Q = \int V\,dA$ and $V = C/r$, while $dA = B\,dr$, where B is the width of the duct

$Q = BC\int_{r_1}^{r_2} dr/r = BC\ln(r_2/r_1) = 2.5(380)\ln(6.25/3\ 75) = 485\ \text{cfs} = 29{,}100\ \text{ft}^3/\text{min}$ ◄

5.49 *An air duct of 1.2 m by 1.2 m square cross section turns a bend of radius 2.4 m as measured to the center line of the duct. If the measured pressure difference between the inside and the outside walls of the bend is 50 mm of water, estimate the rate of air flow in the duct. Assume standard sea-level conditions in the duct and assume ideal flow around the bend.*

SI

Flow = cylindrical free vortex. Table A.3 for air at sea level: $\gamma = 12.01\ \text{N/m}^3$

Given $\dfrac{p_2}{\gamma} - \dfrac{p_1}{\gamma} = 0.05(9810/12.01) = 40.8$ m of air ; Eq. 5.54: $\dfrac{p_2}{\gamma} - \dfrac{p_1}{\gamma} = \dfrac{C^2}{2g}\left(\dfrac{1}{r_1^2} - \dfrac{1}{r_2^2}\right)$

Thus, with $r_1 = 1.8$ m and $r_2 = 3.0$ m: $40.8 = \dfrac{C^2}{2(9.81)}\left(\dfrac{1}{1.8^2} - \dfrac{1}{3.0^2}\right)$; $C = 63.7\ \text{m}^2/\text{s}$

Thus noting $V = C/r$ and $dA = B\,dr$, where B is the width of the duct,

$Q = \int V\,dA = BC\int_{r_1}^{r_2} dr/r = BC\ln(r_2/r_1) = 1.2(63.7)\ln(3.0/1.8) = 39.0\ \text{m}^3/\text{s} = 2340\ \text{m}^3/\text{min}$ ◄

Chapter 6

Momentum and Forces in Fluid Flow

PROBLEM SELECTION GUIDE

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
6.3	***Momentum Correction Factor***						
	X[1]6.3.1	N	Medium	Medium	1		Uses Sec. 4.5; integration
	6.3.2	N	Medium	Medium	1		Uses Sec. 4.5; integration
	P 6.1	N	Medium	Long	1		Uses Sec. 4.5; integration
6.4	***Applications of the Momentum Principle***						
	X 6.4.1	BG	Medium	Short	1		
	6.4.2	SI	Medium	Medium	1	6.4.3	
	6.4.3	BG	Medium	Medium	1	6.4.2	
	6.4.4	BG	Medium	Medium	1	6.4.5	
	6.4.5	SI	Medium	Medium	1	6.4.4	
	P 6.2	BG	Medium	Medium	1	6.3	
	6.3	SI	Medium	Medium	1	6.2	
	6.4	BG	Hard	Long	2		Uses power (Sec. 5.10)
6.5	***Force on Pressure Conduits***						
	X 6.5.1	BG	Easy	Short	1	6.5.2	Uses Sec. 4.5
	6.5.2	SI	Easy	Short	1	6.5.1	Uses Sec. 4.5
	6.5.3	BG	Easy	Short	1		
	6.5.4	BG	Easy	Short	1	6.5.5	
	6.5.5	SI	Easy	Short	1	6.5.4	
	6.5.6	BG	Easy	Medium	1		
	P 6.5	BG	Easy	Medium	3	6.6	
	6.6	SI	Easy	Medium	3	6.5	
	6.7	BG	Easy	Medium	1	6.8	Uses Sec. 4.5
	6.8	SI	Easy	Medium	1	6.7	
	6.9	BG	Medium	Medium	1		
	6.10	BG	Medium	Medium	1	6.11	Includes manometer
	6.11	SI	Medium	Medium	1	6.10	Includes manometer

/cont...

[1] For all Exercises (identified by "X"), answers are given in Appendix F of the textbook.
† Answers are sensitive to values that are measured from figures.
X = Exercise, P = (end-of-chapter) Problem, S = Sample Problem.

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
6.6	***Force of a Free Jet on a Stationary Vane or Blade***						
	X 6.6.1	BG	V Easy	V Short	1	6.6.2	
	6.6.2	SI	V Easy	V Short	1	6.6.1	
	6.6.3	BG	Easy	Short	1	6.6.4	
	6.6.4	SI	Easy	Short	1	6.6.3	
	6.6.5	N	Easy	Short	1		Derivation
	6.6.6	BG	Easy	Short	3	6.6.7	
	6.6.7	SI	Easy	Short	3	6.6.6	
	P 6.12	N	Easy	Short	1		Derivation
	6.13	BG	Easy	Short	3	6.14	
	6.14	SI	Easy	Short	3	6.13	
	6.15	BG	Medium	Medium	1	6.16	
	6.16	SI	Medium	Medium	1	6.15	
	6.17	BG	Medium	Medium	3	6.18	
	6.18	SI	Medium	Medium	3	6.17	
	6.19	BG	Medium	Medium	1	6.20	
	6.20	SI	Medium	Medium	1	6.19	
	6.21	BG	Medium	Medium	1		Integration
	6.22	SI	Hard	Medium	1	6.23	† Scaling from figure: Pappus' theorem
	6.23	BG	Hard	Medium	1	6.22	† Scaling from figure; Pappus' theorem
6.8	***Force of a Jet on One or More Moving Vanes or Blades***						
	X 6.8.1	N	V Easy	V Short	1		Proof
	6.8.2	BG	Easy	Short	2	6.8.3	
	6.8.3	SI	Easy	Short	2	6.8.2	
	6.8.4	BG	Easy	Medium	1	6.8.5	
	6.8.5	SI	Easy	Medium	1	6.8.4	
	6.8.6	BG	Medium	Short	1	S6.4	
	P 6.24	BG	Medium	Short	1	6.25	Scoop
	6.25	SI	Medium	Short	1	6.24	Scoop
	6.26	BG	Easy	Long	1	6.27-29	Single vane
	6.27	BG	Easy	Long	1	6.26-29	Single vane
	6.28	BG	Easy	Long	1	6.26-29	Single vane
	6.29	BG	Easy	Long	1	6.26-28	Single vane
	6.30	BG	Medium	Medium	1	6.31	Single vane
	6.31	SI	Medium	Medium	1	6.30	Single vane
	6.32	BG	Medium	Medium	3	6.33	Series of vanes; uses power (Sec. 5.10)
	6.33	SI	Medium	Medium	3	6.32	Series of vanes; uses power (Sec. 5.10)
	6.34	BG	Hard	Medium	2	6.35	Series of vanes; uses power (Sec. 5.10)
	6.35	SI	Hard	Medium	2	6.34	Series of vanes; uses power (Sec. 5.10)
6.9	***Reaction of a Jet***						
	X 6.9.1	BG	Easy	Short	1	6.9.2	
	6.9.2	SI	Easy	Short	1	6.9.1	
	P 6.36	BG	Easy	Short	1	6.37	
	6.37	SI	Easy	Short	1	6.36	
	6.38	BG	Easy	Medium	1	6.39	
	6.39	SI	Easy	Medium	1	6.38	

/cont...

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
6.10	*Jet Propulsion*						
	X 6.10.1	BG	V Easy	V Short	1	6.10.2	
	6.10.2	SI	Easy	V Short	1	6.10.1	
	P 6.40	BG	Easy	Short	3	6.41	
	6.41	SI	Easy	Short	3	6.40	
6.11	*Rotating Machines: Continuity, Relative Velocities, Torque*						
	X 6.11.1	BG	Easy	Short	1	6.11.2	Uses power (Sec. 5.10)
	6.11.2	SI	Easy	Short	1	6.11.1	Uses power (Sec. 5.10)
6.12	*Head Equivalent of Mechanical Work*						
	P 6.42	SI	Medium	Long	3	6.43	Uses power (Sec. 5.10)
	6.43	BG	Medium	Long	3	6.42	Uses power (Sec. 5.10)
6.13	*Flow Through a Rotating Channel*						
	X 6.13.1	N	Medium	Medium	1		Derivation
	P 6.44	BG	Medium	Long	5	6.45	Uses power (Sec. 5.10)
	6.45	SI	Medium	Long	5	6.44	Uses power (Sec. 5.10)
6.14	*Reaction with Rotation*						
	X 6.14.1	SI	Easy	Short	1		
	6.14.2	BG	Easy	Short	1	6.14.3	Uses power (Sec. 5.10)
	6.14.3	SI	Easy	Short	1	6.14.2	Uses power (Sec. 5.10)
	6.14.4	BG	Medium	Short	1	6.14.5	
	6.14.5	SI	Medium	Short	1	6.14.4	
	P 6.46	BG	Medium	Medium	1	6.47	Uses power (Sec. 5.10)
	6.47	SI	Medium	Medium	1	6.46	Uses power (Sec. 5.10)
	6.48	N	Medium	Medium	1		Derivation
	6.49	BG	Medium	Long	1		Option to use differentiation
6.15	*Momentum Principle Applied to Propellers and Windmills*						
	X 6.15.1	BG	Easy	Medium	3	6.15.2	Uses power (Sec. 5.10)
	6.15.2	SI	Easy	Medium	3	6.15.1	Uses power (Sec. 5.10)
	6.15.3	BG	Medium	Short	2		
	P 6.50	B	Medium	Medium	2	6.51	Uses power (Sec. 5.10)
	6.51	SI	Medium	Medium	2	6.50	Uses power (Sec. 5.10)
	6.52	N	Hard	Long	1		Proof; differentiation

Chapter 6
MOMENTUM AND FORCES IN FLUID FLOW

Sec. 6.3: Momentum Correction Factor -- *Exercises (2)*

6.3.1 *For laminar flow as in Sample Prob 5.1, find β.*
Sample Prob. 5.1: For laminar flow through a circular pipe the velocity profile is parabolic with equation $u = u_m[1 - (r/r_0)^2]$, where u is the velocity at any radius r, u_m is the maximum (centerline) velocity where $r = 0$, and r_0 is the radius of the pipe wall.

N

From Eq. 4.3: $V = \frac{1}{A}\int u\,dA = \frac{1}{\pi r_0^2}\int_0^{r_0} u_m\left[1 - \left(\frac{r}{r_0}\right)^2\right]2\pi r\,dr = \frac{u_m}{2}$

Eq. 6.9: $\beta = \frac{1}{AV^2}\int u^2 dA = \frac{4}{\pi r_0^2 u_m^2}\int_0^{r_0} u_m^2\left[1 - \left(\frac{r}{r_0}\right)^2\right]^2 2\pi r\,dr = \frac{4}{3}$ ◄

6.3.2 *For the turbulent-flow case as approximated in Exer. 5.1.1, find β.*
Exer. 5.1.1: $u = u_m[1 - 0.4(r/r_o)^2]$.

N

Using Eq. 4.3: $V = \frac{1}{A}\int u\,dA = \frac{1}{\pi r_0^2}\int_0^{r_0} u_m\left[1 - 0.4\left(\frac{r}{r_0}\right)^2\right]2\pi r\,dr = 0.8u_m$

By Eq. 6.9: $\beta = \frac{1}{\pi r_0^2(0.8u_m)^2}\int_0^{r_0} u_m^2\left[1 - 0.4\left(\frac{r}{r_0}\right)^2\right]^2 2\pi r\,dr = \frac{49}{48} = 1.021$ ◄

Sec. 6.3: Momentum Correction Factor -- *Problem 6.1*

6.1 *For two-dimensional laminar flow between two stationary parallel plates, find (a) the ratio of mean velocity to maximum velocity; (b) α; (c) β. The velocity profile is parabolic as in Sample Prob. 5.1.*
Sample Prob. 5.1 (replacing r by y): The parabolic velocity profile has equation $u = u_m[1 - (y/y_0)^2]$, where u is the velocity at any distance y, u_m is the maximum (centerline) velocity where $y = 0$, and $2y_0$ is the distance between the plates.

N

$u = u_m[1 - (y/y_0)^2]$; $dA = 2dy$

Eq. 4.3: $V = \frac{1}{A}\int u\,dA = \frac{\int_0^{y_0} u_m[1 - (y/y_0)^2]2\,dy}{\int_0^{y_0} 2\,dy} = \frac{2}{3}u_m$; $\therefore V/u_m = 2/3$ ◄

By Eq. 5.4: $\alpha = \frac{1}{2y_0[(2/3)u_m]^3}\int_0^{y_0} u_m^3[1 - (y/y_0)^2]^3\,2\,dy = 54/35 = 1.543$ ◄

By Eq. 6.9: $\beta = \frac{1}{2y_0[(2/3)u_m]^2}\int_0^{y_0} u_m^2[1 - (y/y_0)^2]^2\,2\,dy = 6/5 = 1.200$ ◄

Sec. 6.4: Applications of the Momentum Principle -- *Exercises* (5)

6.4.1 *A cylindrical drum of radius 2.2 ft is securely held in position in an open channel of rectangular section. The channel is 10 ft wide, and the flow rate is 200 cfs. Water flows beneath the drum as shown in Fig. X6.4.1. Determine the horizontal thrust on the cylinder using the momentum principle. Neglect fluid friction.*

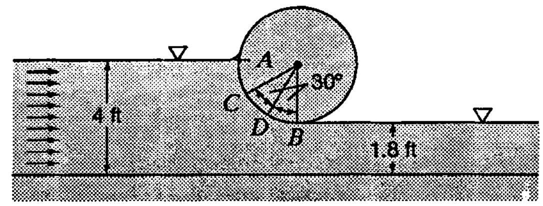


Figure X6.4.1

BG

Eq. 4.6: $V_1 = \dfrac{200}{10(4)} = 5.00$ fps

From continuity $V_2 = 5.00(4/1.6) = 12.50$ fps

Eq. 6.7*a*: $\Sigma F_x = F_1 - F_2 - F_x = \rho Q(V_{2x} - V_{1x})$

$[62.4(4^2)10]/2 - [62.4(1.6^2)10]/2 - F_x = 1.940(200)(12.50 - 5.00)$

$4992 - 799 - F_x = 2910$; $F_x = +1283$ lb to the left ; $(F_{W/C})_x = 1283$ lb to the <u>right</u> ◄

6.4.2 *Find the horizontal thrust of the water on each meter of width of the sluice gate shown in Fig. X6.4.2, given $y_1 = 2.2$ m, $y_2 = 0.4$ m, and $y_3 = 0.5$ m. Neglect friction.*

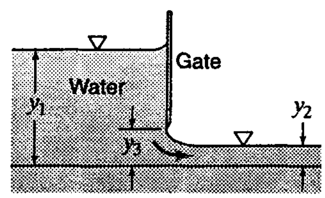


Figure X6.4.2

SI

Let q be the flow rate per meter of width, m^3/s per m.

Continuity, per meter of width: $V_1 = q/2.2$, $V_2 = q/0.4$

Energy, neglecting friction: $2.2 + (q/2.2)^2/2g = 0.4 + (q/0.4)^2/2g$

from which $q = 2.42$ m^3/s per m width ; $V_1 = 1.099$ m/s, $V_2 = 6.04$ m/s

Eq. 6.7*a*: $\Sigma F_x = F_1 - F_2 - F_x = \rho Q(V_2 - V_1)$

$9810(2.2^2)/2 - 9810(0.4^2)/2 - F_x = 1000(2.42)(6.04 - 1.099)$

$23\,700 - 785 - F_x = 11\,950$; $F_x = +11\,000$ N/m to the left ; $(F_{W/G})_x = 11.00$ kN/m to the <u>right</u> ◄

6.4.3 *Refer to Fig. X6.4.2. Find the horizontal thrust of the water on each foot of width of the sluice gate, given $y_1 = 7$ ft, $y_2 = 1.2$ ft, and $y_3 = 1.4$ ft. Neglect friction.*

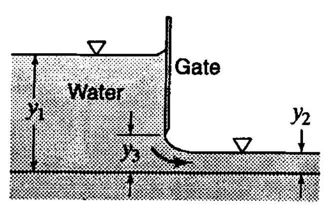


Figure X6.4.2

BG

Let q be the flow rate, per foot of width, cfs per ft.

Continuity, per foot of width: $V_1 = q/7.0$, $V_2 = q/1.2$

Energy, neglecting friction: $7.0 + (q/7.0)^2/2g = 1.2 + (q/1.2)^2/2g$

from which $q = 23.5$ cfs per foot width ; $V_1 = 3.36$ fps, $V_2 = 19.62$ fps

Eq. 6.7*a*: $\Sigma F_x = F_1 - F_2 - F_x = \rho Q(V_{2x} - V_{1x})$

$62.4(7.0^2)/2 - 62.4(1.2^2)/2 - F_x = 1.940(23.5)(19.62 - 3.36)$

$1529 - 44.9 - F_x = 742$; $F_x = +742$ lb/ft to the left ; $(F_{W/G})_x = 742$ lb/ft to the <u>right</u> ◄

6.4.4 *Flow occurs over a spillway of constant section as shown in Fig. X6.4.4. Assuming ideal flow, determine the resultant horizontal force on the spillway per foot of spillway width (perpendicular to the spillway section), given that y_1 = 4.2 ft and y_2 = 0.7 ft.*

BG

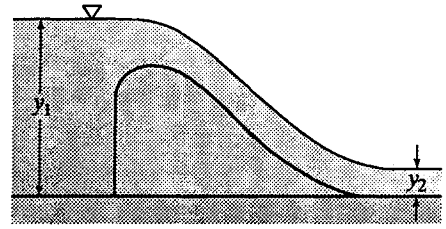


Figure X6.4.4

Energy: $4.2 + V_1^2/(2\times 32.2) = 0.7 + V_2^2/(2\times 32.2)$ (1)

Continuity per foot: $4.2V_1 = 0.7V_2$ (2)

Substituting (2) into (1) yields:

$V_1 = 2.54$ fps, $V_2 = 15.23$ fps, $Q = A_1V_1 = 10.66$ cfs/ft

Eq. 6.7*a*: $\Sigma F_x = F_1 - F_2 - F_x = \rho Q(V_{2x} - V_{1x})$

$62.4(2.1)4.2 - 62.4(0.35)0.7 - F_x = 1.940(10.66)(15.23 - 2.54)$

$F_x = +273$ lb/ft to left. Water $(F_{W/S})_x$ acts on spillway to the <u>right</u> with 273 lb/ft. ◄

6.4.5 *Flow occurs over the spillway of constant section as shown in Fig. X6.4.4. Given that y_1 = 4.2 m and y_2 = 0.7 m determine the horizontal force on the spillway per meter of spillway width (perpendicular to the spillway section). Assume ideal flow.*

SI

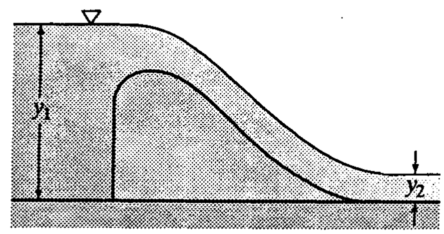


Figure X6.4.4

Energy: $4.2 + V_1^2/(2\times 9.81) = 0.7 + V_2^2/(2\times 9.81)$ (1)

Continuity per m: $4.2V_1 = 0.7V_2$ (2)

Substituting (2) into (1) yields:

$V_1 = 1.401$ m/s, $V_2 = 8.40$ m/s ; $Q = A_1V_1 = 5.88$ m^3/s per meter

Eq. 6.7*a*: $\Sigma F_x = F_1 - F_2 - F_x = \rho Q(V_{2x} - V_{1x})$

$9810(2.1)4.2 - 9810(0.35)0.7 - F_x = 1000(5.88)(8.40 - 1.401)$

$F_x = +42\,900$ N/m to the left. Water $(F_{W/S})_x$ acts on spillway to the <u>right</u> with 42.9 kN/m. ◄

Sec. 6.4: Applications of the Momentum Principle -- *Problems* 6.2–6.4

6.2 *In Sample Prob. 6.1 suppose the passage narrows down to a width of 9 ft at the second section. With the same depths find the flow rate and the horizontal force or the structure.*

Sample Prob. 6.1: The passage at section (1) of Fig. S6.1 is 10 ft wide. Assume ideal flow.

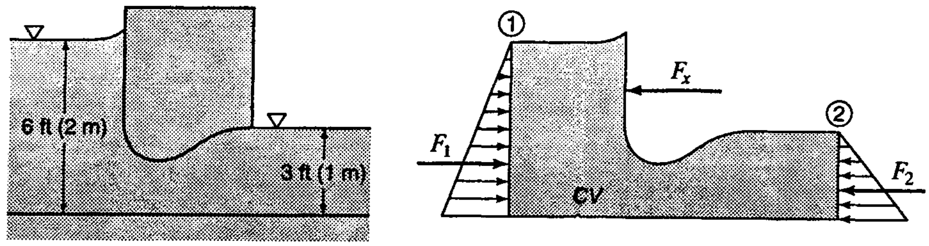


Figure S6.1

BG

Energy: $6 + V_1^2/(2\times 32.2) = 3 + V_2^2/(2\times 32.2)$ (1)

Continuity: $6(10)V_1 = 3(9)V_2$ (2)

Substituting (2) into (1) yields: $V_1 = 7.00$ fps, $V_2 = 15.56$ fps, $Q = A_1V_1 = 60(7.00) = 420$ cfs ◄

Eq. 6.7*a*: $\Sigma F_x = F_1 - F_2 - F_x = \rho Q(V_{2x} - V_{1x})$

$62.4(3)(10 \times 6) - 62.4(1.5)(9 \times 3) - F_x = 1.940(420)(15.56 - 7.00)$

$F_x = +1730$ lb to the left. $(F_{W/S})_x = 1730$ lb to the right ◄

6.3 *In Sample Prob. 6.1 suppose the passage narrows down to a width of 2.5 m at the second section. With the same depths find the flow rate and the horizontal force on the structure.*

Sample Prob 6.1: For Fig. S6.1 refer to Solution 6.2. The passage at section (1) is 3 m wide. Assume ideal flow.

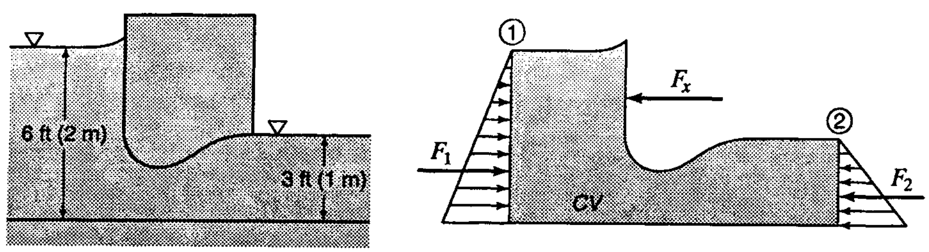


Figure S6.1

BG

Energy: $2 + V_1^2/(2\times 9.81) = 1 + V_2^2/(2\times 9.81)$ (1)

Continuity: $2(3)V_1 = 1(2.5)V_2$ (2)

Substit'g (2) into (1) yields: $V_1 = 2.03$ m/s, $V_2 = 4.87$ m/s ; $Q = A_1V_1 = 6(2.03) = 12.18\ \text{m}^3/\text{s}$ ◄

Eq. 6.7*a*: $\Sigma F_x = F_1 - F_2 - F_x = \rho Q(V_{2x} - V_{1x})$

$9810(1)(2 \times 3) - 9810(0.5)(1 \times 2.5) - F_x = 1000(12.18)(4.87 - 2.03)$

$F_x = +11\,970$ N to the left. $(F_{W/S})_x = 11.97$ kN to the right. ◄

6.4 *A hydraulic jump (Sec. 10.18) occurs in a transparent closed conduit with the diamond-shaped cross section shown in Fig. P6.4. The conduit is horizontal, and the water depth just upstream of the jump is 2.0 ft. The conduit is completely full of water downstream of the jump. Pressure-gage readings are as shown in the figure. (a) Compute the flow rate. Note that, because of turbulence in the jump, there is a substantial energy loss. So we cannot assume ideal flow. However, we may neglect shear forces along the boundary. (b) Determine the horsepower loss in the jump.*

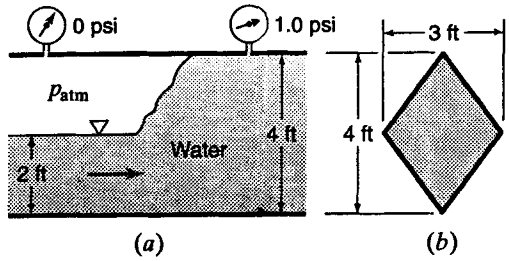


Figure P6.4

BG

(*a*) Since energy is lost, the momentum principle must be used.

Eq. 6.7*a*: $F_1 - F_2 - F_x = \rho Q(V_{2x} - V_{1x})$; $F_1 = 62.4(2/3)3 = 124.8$ lb

$-F_2 = -62.4(2)6 - 144(6) = -1613$ lb ; $F_x = 0$ neglecting shear forces along the boundary

Thus $124.8 - 1613 = 1.940(3V_1)(V_1/2 - V_1)$; $V_1 = 22.6$ fps, $V_2 = 11.31$ fps

$Q = A_1V_1 = 3(22.6) = 67.8$ cfs ◄

(*b*) Taking the channel bed as datum:

Eq. 5.28: $H_1 = 2 + 0 + 22.6^2/(2\times32.2) = 2 + 0 + 7.94 = 9.94$ ft

$H_2 = 4 + 1(144)/62.4 + 0 + 11.3^2/(2\times32.2) = 4 + 2.31 + 0 + 1.985 = 8.29$ ft

From Eq. 5.14: $h_L = H_1 - H_2 = 1.648$ ft

Eq. 5.32: Power loss $= \gamma Q h_L/550 = 62.4(67.8)1.648/550 = 12.69$ hp ◄

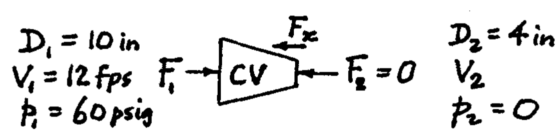
Sec. 6.5: Force Exerted on Pressure Conduits -- ***Exercises*** *(6)*

6.5.1 *A nozzle that discharges a 4-in-diameter water jet into the air is on the right end of a horizontal 10-in-diameter pipe. In the pipe the water has a velocity of 12 fps and a gage pressure of 60 psi. Find the magnitude and direction of the resultant axial force the water exerts on the nozzle, and the head loss in the nozzle.*

BG

Eq. 4.5: $G = \gamma A_1 V_1 = 62.4\pi\left(\frac{5}{12}\right)^2 12 = 408$ lb/sec

From Eq. 4.3: $V_2 = V_1\left(\frac{A_1}{A_2}\right) = 12\left(\frac{10}{4}\right)^2 = 75.0$ fps



By Eq. 6.11: $F_x = 60(144)\pi(5/12)^2 - 0 - (408/32.2)(75.0 - 12) = 3910$ lb to the left on the CV

Equal and opposite force on the nozzle = 3910 lb to the right ◄

From Eq. 5.28: $h_L = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = 138.5 + 2.24 - 87.3 = 53.46$ ft ◄

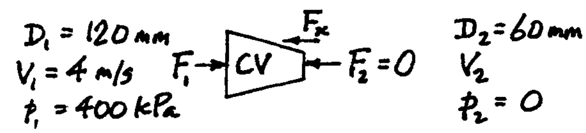
6.5.2 *A nozzle that discharges a 60-mm-diameter water jet into the air is on the right end of a horizontal 120-mm-diameter pipe. In the pipe the water has a velocity of 4 m/s and a gage pressure of 400 kPa. Find the magnitude and direction of the resultant axial force the water exerts on the nozzle, and the head loss in the nozzle.*

SI

Eq. 4.5: $G = 9.81(\pi 0.06^2)4 = 0.444$ kN/s

From Eq. 4.3: $V_2 = 4(120/60)^2 = 16$ m/s

Noting that $\rho Q(\Delta V) = (G/g)(\Delta V)$



In Eq. 6.11: $F_x = 400(\pi 0.06^2) - 0 - 6(0.444/9.81)(16 - 4) = 3.98$ kN to the left on the CV

Equal and opposite force on the nozzle = 3.98 kN to the right ◄

From Eq. 5.28: $h_L = \dfrac{p_1}{\gamma} + \dfrac{V_1^2}{2g} - \dfrac{V_2^2}{2g} = 40.77 + 0.815 - 13.05 = 28.5$ m ◄

6.5.3 *In Fig. X6.5.3, both nozzles discharge water horizontally into the atmosphere at 30 fps. Find θ so that the resultant force on the unit is along the axis of the 8-in-diameter pipe.*

BG

$Q_2 = A_2V_2 = \pi(2/12)^2 30 = 2.62$ cfs

$Q_3 = A_3V_3 = \pi(3/12)^2 30 = 5.89$ cfs

$\Sigma F_z = 0 = \rho Q_2 V_2 \sin 25° - \rho Q_3 V_3 \sin\theta$

i.e., $0 = 1.940(2.62)30\sin 25° - 1.940(5.89)30\sin\theta$

from which $\sin\theta = 0.1878$; $\theta = 10.83°$ ◄

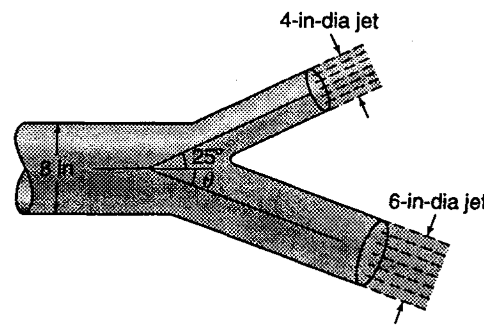


Figure X6.5.3

6.5.4 *Water under a gage pressure of 65 psi flows with a velocity of 12.5 fps through a right-angle bend that has a uniform diameter of 10 inches. The bend lies in a horizontal plane and water enters from the west and leaves towards the north. Assuming no drop in pressure, what is the magnitude and direction of the resultant force acting on the bend?*

BG

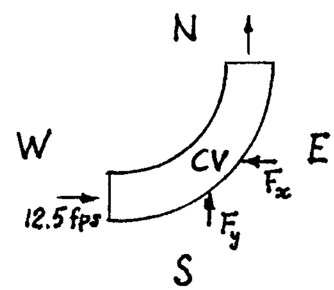
Eq. 4.3: $Q = A_1V_1 = \pi(5/12)^2 12.5 = 6.82$ cfs

By Eq. 6.12: $F_x = 65(\pi 5^2) - 0 - (1.940)6.82(0 - 12) = +5270$ lb to the W

By Eq. 6.15: $F_y = -0 + 65(\pi 5^2) + (1.940)6.82(12.5 - 0) = +5270$ lb to N

The resultant force on the bend is equal and opposite, i.e.,

$F_{W/B} = 5270\sqrt{2} = 7450$ lb at 45° toward the SE ◄



6.5.5 *Water under a gage pressure of 350 kPa flows with a velocity of 5 m/s through a right-angle bend that has a uniform diameter of 250-mm. The bend lies in a horizontal plane and water enters from the west and leaves towards the north. Assuming no drop in pressure, what is the magnitude and direction of the resultant force acting on the bend?*

SI

Eq. 4.3: $Q = A_1V_1 = \pi(0.125)^2 5 = 0.245$ m³/s

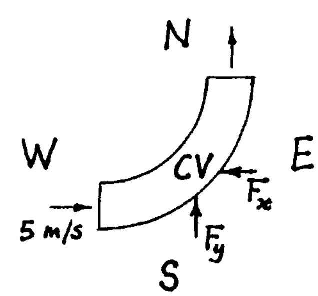
By Eq. 6.12: $F_x = 350(\pi 0.125^2) - 0 - (9.81/9.81)(0.245)(0 - 5)$

$= +18.41$ kN towards the W on the CV

By Eq. 6.15: $F_y = -0 + 350(\pi 0.125^2) + (0.245)(5 - 0) = +18.41$ kN to N

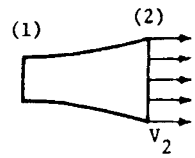
The resultant force on the bend is equal and opposite, i.e.,

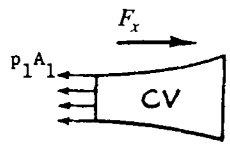
$F_{W/B} = 18.41\sqrt{2} = 26.0$ kN at 45° toward the SE. ◄



6.5.6 *A diverging nozzle that discharges an 8-in-diameter water jet into the air is on the right end of a horizontal 6-in-diameter pipe. If the velocity in the pipe is 12 fps, find the magnitude and direction of the resultant axial force the water exerts on the nozzle. Neglect fluid friction.*

BG





Eq. 5.15: $p_1/\gamma + V_1^2/2g = V_2^2/2g$

$V_1 = 12$ fps, $V_2 = 12(6/8)^2 = 6.75$ fps

$p_1/\gamma = 6.75^2/(2\times32.2) - 12^2/(2\times32.2) = -1.53$ ft

$p_1 = -(1.53\times62.4)/144 = -0.662$ psi ; $p_2 = 0$ (given)

From Eq. 6.11: $F_x = -[-0.662(\pi 3^2)] + 0 + 1.940(\pi 3^2/144)12(6.75-12)$

$F_x = 18.73 - 24.00 = -5.27$ lb to the right $= +5.27$ lb to the left

$(F_{W/N})_x = 5.27$ lb to the right ◄

Sec. 6.5: Force Exerted on Pressure Conduits -- ***Problems*** *6.5−6.11*

6.5 *The diameters in Fig. 6.3 are 42 in and 30 in. At the larger end the pressure is 90 psi and the velocity is 12 fps. Neglecting friction, find the resultant force on the horizontal conical reducer if water flows (a) to the right; and (b) to the left. (c) What would happen to the two previous answers if we did not neglect friction?*

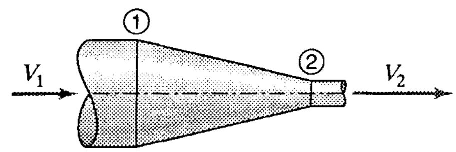


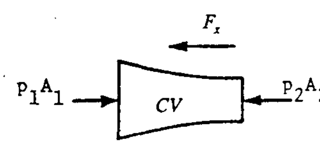
Figure 6.3(*a*)

BG

(*a*) $V_2 = 12(42/30)^2 = 23.5$ fps

By Eq. 5.28 with $z_1 = z_2$: $$\frac{90(144)}{62.4} + \frac{12^2}{2(32.2)} = \frac{p_2}{\gamma} + \frac{23.5^2}{2(32.2)}$$

$$\frac{p_2}{\gamma} = 201 \text{ ft}, \quad p_2 = 87.2 \text{ psi}$$



$A_1 = (\pi/4)42^2 = 1385$ in^2 ; $A_2 = (\pi/4)30^2 = 707$ in^2

$Q = A_1V_1 = (1385/144)12 = 115.5$ cfs

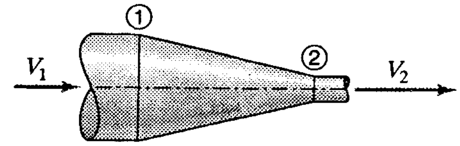
Eq. 6.11: $F_x = 90(1385) - 87.2(707) - (1.940)115.5(23.5-12) = +60{,}400$ lb to the left on the CV.

The equal and opposite force on the reducer = 60,400 lb to the right ◄

(*b*) Reversing the flow direction gives the identical result, in the same direction as before if no friction is considered. ◄

(*c*) Assuming the same pressure at the larger section in both cases, as given, friction will reduce F_2 and therefore increase F_x in case (*a*) and do the opposite in case (*b*). ◄

6.6 *The diameters in Fig. 6.3 are 750 mm and 500 mm. At the larger end the pressure is 650 kPa and the velocity is 2.8 m/s. Neglecting friction, find the resultant force on the horizontal conical reducer if water flows (a) to the right; and (b) to the left. (c) What would happen to the two previous answers if we did not neglect friction?*



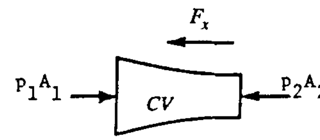
(*a*)

Figure 6.3(*a*)

SI

(*a*) $V_2 = 2.8(750/500)^2 = 6.30$ m/s

Eq. 5.29 with $z_1 = z_2$: $\dfrac{650}{9.81} + \dfrac{2.8^2}{2(9.81)} = \dfrac{p_2}{9.81} + \dfrac{6.30^2}{2(9.81)}$



$p_2 = 634$ kPa

Eq. 6.11:

$F_x = 650(\pi\, 0.375^2) - 634(\pi\, 0.25^2) - (9.81/9.81)\pi 0.375^2/2.8)(6.30 - 2.8)$

$F_x = 287.2 - 124.5 - 4.33 = +158.3$ kN to the left on the CV

Equal and opposite force $(F_{W/R})_x$ of water on the reducer acts to the <u>right</u> with 158.3 kN. ◀

(*b*) Reversing the flow direction gives the identical result, in the same direction as before if no friction is considered. ◀

(*c*) Assuming the same pressure at the larger section in both cases, as given, friction will reduce F_2 and therefore increase F_x in case (*a*) and do the opposite in case (*b*). ◀

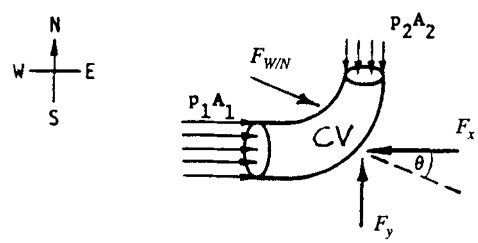
6.7 *A reducing right-angle bend lies in a horizontal plane. Water enters from the west with a velocity of 6 fps and a pressure of 4 psi, and it leaves toward the north. The diameter at entrance is 22 in and at exit is 20 in. Neglecting any friction loss, find the magnitude and direction of the resultant force on the bend.*

BG

$V_2 = (22/20)^2 6 = 7.26$ fps.

If $p_1 = 4$ psi, by Eq. 5.15 with $z_1 = z_2$:

$4(144)/62.4 + 6^2/(2\times 32.2) = p_2/\gamma + 7.26^2/(2\times 32.2)$



$p_2/\gamma = 9.23 + 0.559 - 0.818 = 8.97$ ft ;

$p_2 = 560$ psf $= 3.89$ psi

Eq. 4.5: $G = 62.4\pi(11/12)^2 6 = 998$ lb/sec ;

$\rho Q = \dot{m} = G/g = 988/32.2 = 30.7$ slugs/sec

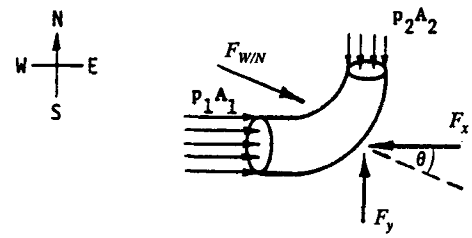
By Eq. 6.12: $4(\pi 11^2) - 0 + F_x = 30.7(0 - 6)$; $F_x = -1705$ lb

By Eq. 6.15: $0 - 3.89(\pi 10^2) + F_y = 30.7(7.26 - 0)$; $F_y = +1444$ lb

$N = \sqrt{1705^2 + 1444^2} = 2230$ lb at 40.3° S of E ◀

6.8 *A reducing right-angled bend lies in a horizontal plane. Water enters from the west with a velocity of 3 m/s and a pressure of 30 kPa, and it leaves toward the north. The diameter at the entrance is 500 mm and at the exit it is 400 mm. Neglecting any friction loss, find the magnitude and direction of the resultant force on the bend.*

SI



$V_2 = 3(50/40)^2 = 4.69$ m/s

Eq. 5.15: $\frac{30}{9.81} + \frac{3^2}{2(9.81)} = \frac{p_2}{\gamma} + \frac{4.69^2}{2(9.81)}$

$\frac{p_2}{\gamma} = 3.06 + 0.459 - 1.12 = 2.40$ m ;

$p_2 = 2.40(9.81) = 23.5$ kN/m^2

Eq. 6.12: $30(\pi 0.25^2) - 0 - F_x = \rho Q(V_2 - V_1)$

$5.89 - F_x = 10^3(\pi 0.25^2)3(0 - 3)/1000 = -1.767$; $F_x = 7.66$ kN to the west on the CV.

Eq. 6.15: $0 - 23.5(\pi 0.20^2) + F_y = 10^3(\pi 0.25^2)3(4.69 - 0)/1000$

$-2.95 + F_y = -2.76$; $F_y = 5.72$ kN to the north on the CV.

Equal and opposite $F_{W/N} = \sqrt{7.66^2 + 5.72^2} = 9.56$ kN at 36.8° S of E ◄

6.9 *Both nozzle jets in Fig. P6.9 discharge horizontally into the atmosphere with a velocity of 40 fps. The liquid has a specific weight of 62.4 lb/ft³. The axes of the pipe and both nozzles all lie in a horizontal plane. Find the magnitude and direction of the resultant force on this double nozzle while neglecting friction.*

BG

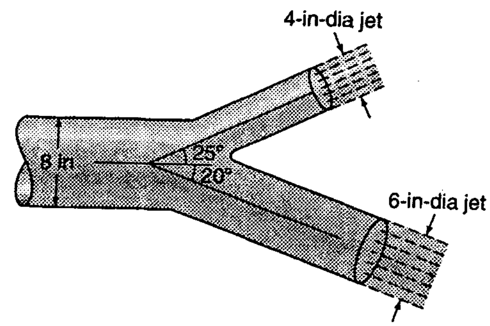


Figure P6.9

Continuity:

$A_1V_1 = A_2V_2 + A_3V_3$

$64V_1 = 16(40) + 36(40)$; $V_1 = 32.5$ fps

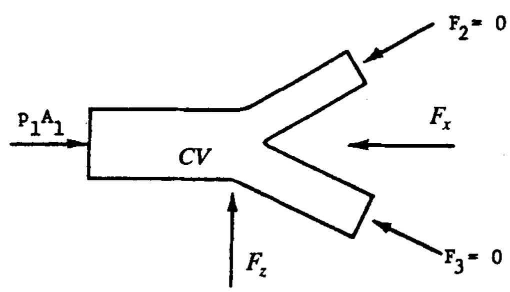
Eq. 5.29 with $z_1 = z_2$: $p_1/\gamma + 32.5^2/2g = 0 + 40^2/2g$

$p_1/\gamma = 24.8 - 16.40 = 8.44$ ft

$p_1 = 8.44(62.4/144) = 3.66$ psi, $p_1A_1 = 183.9$ lb

$\rho Q_1 = 1.940\pi(4/12)^2 32.5 = 22.0$ slugs/sec,

$\rho Q_2 = 6.77$ slugs/sec, $\rho Q_3 = 15.24$ slugs/sec



From Eq. 6.12: $p_1A_1 - 0 - F_x = \rho Q_2V_{2x} + \rho Q_3V_{3x} - \rho Q_1V_{1x}$

$183.9 - F_x = 6.77(40\cos 25°) + 15.24(40\cos 20°) - 22.0(32.5)$

$F_x = 183.9 - 245.5 - 572.7 + 715.3 = 81.0$ lb to the left on the CV

$F_z = \rho Q_2V_{2z} + \rho Q_3V_{3z} - \rho Q_1V_{1z} = 6.77(40\sin 25°) + 15.24(-40\sin 20°) - 0 = 114.5 - 208.5$

$= -94.0$ lb upward on the CV $= +94.0$ lb downward on the CV

Resultant force $\sqrt{81.0^2 + 94.0^2} = 124.1$ lb at $\tan^{-1}(94.0/81.0) = 49.2°$ from the 8-in pipe axis.

Equal and opposite force $F_{W/N}$ of water on nozzle: 121.4 lb at 49.2° ↗ ◄

6.10 *Assuming ideal flow, determine the total pull on the bolts in Fig. P6.10, where y = 6 ft, d_1 = 2 in, d_2 = 4 in, and d_3 = 1 in, and γ_M of the manometer fluid (oil) is 52 pcf.*

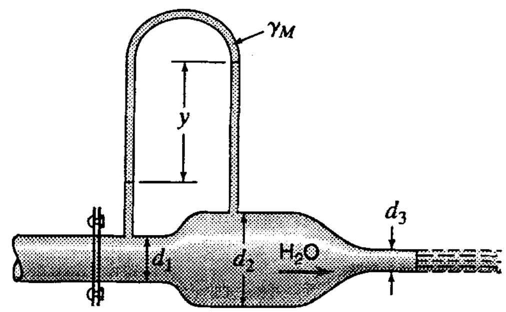


Figure P6.10

BG

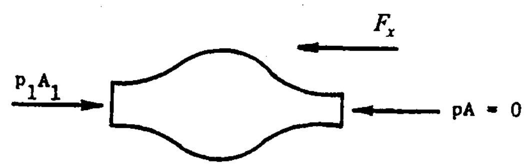
Manometer: $6\left(\frac{52}{62.4}-1\right) = \frac{V_{4''}^2}{2g}-\frac{V_{2''}^2}{2g}$

(Note: $V_{1''} = 16V_{4''}$, $V_{2''} = 4V_{4''}$)

So: $6(-0.1667) = \frac{V_{4''}^2}{2g}-\frac{(4V_{4''})^2}{2g} = -\frac{15V_{4''}^2}{2g}$

$V_{4''}$ = 2.07 fps ; $V_{2''}$ = 8.29 fps, $V_{1''}$ = 33.2 fps

$\frac{p_1}{62.4}+\frac{8.29^2}{2(32.2)} = 0+\frac{33.2^2}{2(32.2)}$;



p_1 = 998 psf = 6.93 psi

Eq. 6.10: $p_1A_1 - F_x = \rho Q(V_{2x}-V_{1x})$

$6.93\pi(1)^2 - F_x = 1.940\pi(1/12)^2 8.29(33.2-8.29)$; F_x = 13.06 lb

Equal and opposite is the pull on the bolts = 13.06 lb. ◄

6.11 *Repeat Prob. 6.10 for the case where y = 1.80 m, d_1 = 50 mm, d_2 = 100 mm, d_3 = 25 mm, and the manometer liquid has a specific gravity of 0.80.*

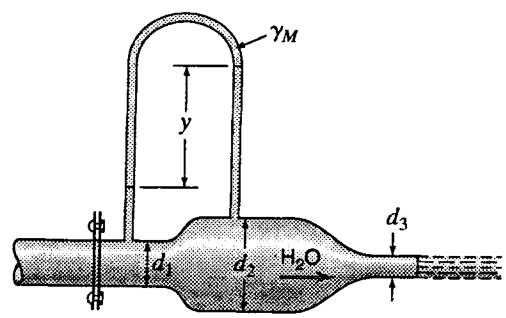


Figure P6.10

SI

Manometer: $1.80(0.8-1.0) = \frac{V_{10}^2}{2g}-\frac{V_5^2}{2g}$

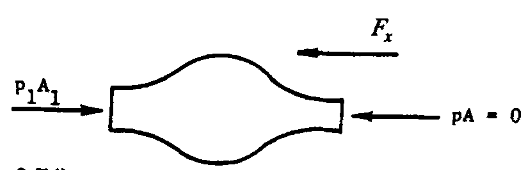
Continuity: $V_5 = 4V_{10} = (1/4)V_{2.5}$

So: $-0.36 = \frac{V_{10}^2}{2g}-\frac{(4V_{10})^2}{2g} = -15\frac{V_{10}^2}{2g}$

$V_{10}^2 = 2(9.81)0.36/15 = 0.471$

V_{10} = 0.686 m/s, V_5 = 2.74 m/s, $V_{2.5}$ = 10.98 m/s

$\frac{p_5}{9.81}+\frac{2.74^2}{2(9.81)} = \frac{10.98^2}{2(9.81)}$; p_5 = 56.5 kN/m^2



Eq. 6.10: $p_1A_1 - F_x = \rho Q(V_{2x}-V_{1x})$

$A_1 = \pi 0.025^2 = 0.001\,963$ m^2

$56.5(0.001\,963) - F_x = (9.81/9.81)(0.001963\times 2.74)(10.98-2.74)$

$F_x = 0.1109 - 0.0444 = 0.0666$ kN = 66.6 N to the left

Equal and opposite is the pull on the bolts = 66.6 N. ◄

Sec. 6.6: Force of a Free Jet on a Stationary Vane or Blade – *Exercises (7)*

6.6.1 *A 1-in-diameter jet has a velocity of 95 fps. Calculate the resultant force on a large flat plate if this jet were to strike it normally.*

BG

Eq. 6.7*a*: $F = F_x = \dot{m}\Delta V = \rho A V_1(V_1 - V_2\cos\theta)$; $A = \pi(1/12)^2/4 = 0.00545 \text{ ft}^2$; $\theta = 90°$

$F = 1.940(0.00545)95(95 - 0) = 1.005(95) = 95.5 \text{ lb}$ ◄

6.6.2 *A 40-mm-diameter jet has a velocity of 25 m/s. If this jet were to strike a large flat plate normally, what would be the resultant force on the plate?*

SI

Eq. 6.7*a*: $F = F_x = \dot{m}\Delta V = \rho A V_1(V_1 - V_2\cos\theta)$; $\theta = 90°$

$F = 1000(\pi 0.04^2/4)25(25 - 0) = 31.4(25) = 785 \text{ N}$ ◄

6.6.3 *In Exer. 6.6.1 assume the center of the jet is coincident with the center of the circular plate. Find (a) the stagnation pressure, and (b) the average pressure on the plate if the area of the plate is 22 times the area of the jet.*

Exer. 6.6.1: The jet, with V_1 = 95 fps and D_1 = 1 in, strikes the plate normally.

BG

(*a*) From Eq. 5.15: $p_O = \gamma V^2/2g = 62.4\times 95^2/(2\times 32.2) = 8744 \text{ psf} = 60.7 \text{ psi}$ ◄

(*b*) Eq. 6.7*a*: $F = F_x = \dot{m}\Delta V = \rho A V_1(V_1 - V_2\cos\theta)$; $A = \pi(1/12)^2/4 = 0.00545 \text{ ft}^2$; $\theta = 90°$

$F = 1.940(0.00545)95(95 - 0) = 1.005(95) = 95.5 \text{ lb}$

Average pressure $= F/A = \dfrac{95.5}{22(0.00545)} = 796 \text{ psf} = 5.53 \text{ psi}$ ◄

6.6.4 *In Exer. 6.6.2 assume the center of the jet is coincident with the center of the circular plate. Find (a) the stagnation pressure, and (b) the average pressure on the plate if the area of the plate is 25 times the area of the jet.*

Exer. 6.6.2: The jet, with V_1 = 25 m/s and D_1 = 40 mm, strikes the plate normally.

SI

(*a*) From Eq. 5.15: $p_O = \gamma V^2/2g = 9.81(25^2)/(2\times 9.81) = 313 \text{ kN/m}^2$ ◄

(*b*) Eq. 6.7*a*: $F = F_x = \dot{m}\Delta V = \rho A V_1(V_1 - V_2\cos\theta)$; $\theta = 90°$

$F = 1000(\pi 0.04^2/4)25(25 - 0) = 31.4(25) = 785 \text{ N}$

Average pressure $= F/A = \dfrac{785}{25(\pi/4)(4/100)^2} = 25\,000 \text{ N/m}^2 = 25.0 \text{ kN/m}^2$ ◄

6.6.5 *A jet containing any type of fluid with a velocity V and an area A is deflected through an angle θ without changing the velocity magnitude. Derive an equation for the dynamic force exerted.*

N

Choose axes so that the y axis is the axis of symmetry. Then

$$\Delta V_x = 0\ ; \quad \Delta V_y = V\sin\frac{\theta}{2} - \left(-V\sin\frac{\theta}{2}\right) = 2V\sin\frac{\theta}{2}$$

From Eq. 6.6: $F = \dot{m}\Delta V = \left(\dfrac{\gamma A V}{g}\right)2V\sin\dfrac{\theta}{2} = \dfrac{2\gamma A}{g}V^2\sin\dfrac{\theta}{2}$ ◄

6.6.6 *In Fig. X6.6.6 assume that friction is negligible, that $\theta = 115°$, and that the water jet has a velocity of 95 fps and a diameter of 1 in. Find (a) the component of the force acting on the blade in the direction of the jet; (b) the force component normal to the jet; and (c) the magnitude and direction of the resultant force exerted on the blade.*

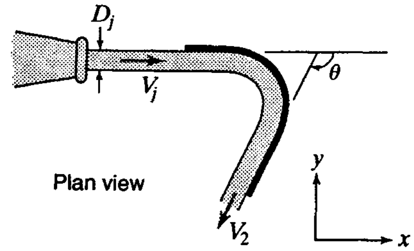


Figure X6.6.6

BG

(*a*) From Eq. 6.7*a*: $F_x = \dot{m}\Delta V = \rho A V_1(V_1 - V_2\cos\theta)$

$A = \pi(1/12)^2/4 = 0.00545\ \text{ft}^2$

$F_x = 1.940(0.00545)95(95 - 95\cos 115°) = 1.005(135.1)$

$= +135.9\ \text{lb} = 135.9\ \text{lb} \leftarrow$ on the CV

$\therefore$ equal and opposite force on the blade, in the direction of the jet, $= 135.9\ \text{lb} \rightarrow$ ◀

(*b*) From Eq. 6.7*b*: $F_y = \dot{m}V_2\sin\theta = 1.005(95\sin 115° - 0) = +86.5\ \text{lb} = 86.5\ \text{lb} \downarrow$ on the CV

$\therefore$ equal and opposite force on the blade, $= 86.5\ \text{lb} \uparrow$ ◀

(*c*) Resultant $F = \sqrt{135.9^2 + 86.5^2} = 161.1\ \text{lb}$ ◀ ; α from $V_1 = \tan^{-1}(86.5/135.9) = 32.5° \nearrow$ ◀

Note: The direction of the resultant can also be determined by inspection to be the extension of the line bisecting the angle between entering and leaving jets.

6.6.7 *Refer to Fig. X6.6.6. Assume that friction is negligible, that $\theta = 115°$, and that the water jet has a velocity of 25 m/s and a diameter of 40 mm. Find (a) the component of the force acting on the blade in the direction of the jet; (b) the force component normal to the jet; and (c) the magnitude and direction of the resultant force exerted on the blade.*

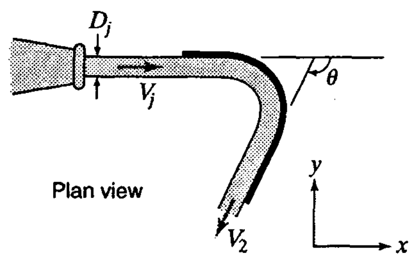


Figure X6.6.6

SI

(*a*) From Eq. 6.7*a*: $F_x = \dot{m}\Delta V = \rho A V_1(V_1 - V_2\cos\theta)$

$= 1000(\pi 0.04^2/4)25(25 - 25\cos 115°)$

$= 31.4(35.6) = +1117\ \text{N} = 1117\ \text{N} \leftarrow$ on the CV

Equal and opposite is the force on the blade $= 1117\ \text{N} \rightarrow$ ◀

(*b*) From Eq. 6.7*b*: $F_y = \dot{m}V_2\sin\theta = 31.4(25\sin 115° - 0) = +712\ \text{N} = 712\ \text{N} \downarrow$ on the CV.

Equal and opposite is the force on the blade $= 712\ \text{N} \uparrow$ ◀

(*c*) Resultant $F = \sqrt{1117^2 + 712^2} = 1325\ \text{N}$ ◀ α from $V_1 = \tan^{-1}(712/1117) = 32.5° \nearrow$ ◀

Note: The direction of the resultant can also be determined by inspection to be the extension of the line bisecting the angle between entering and leaving jets

Sec. 6.6: Force of a Free Jet on a Stationary Vane or Blade -- *Problems* 6.12–6.23

6.12 *If friction reduces the velocity V_2 of a jet of any fluid to $0.8V_1$ while deflecting it through an angle θ, derive an equation for the dynamic force exerted in terms of $\dot{m}$, V_1, and θ.*

N

From Eq. 6.7: $F_x = \dot{m}(V_1 - V_2\cos\theta) = \dot{m}V_1(1 - 0.8\cos\theta)$; $F_y = \dot{m}V_2\sin\theta = \dot{m}\,0.8\,V_1\sin\theta$

$F = \sqrt{F_x^2 + F_y^2} = \dot{m}\sqrt{V_1^2(1 - 0.8\cos\theta)^2 + 0.64\,V_1^2\sin^2\theta} = mV_1\sqrt{1.64 - 1.6\cos\theta}$ ◀

6.13 *Solve Exer. 6.6.6 assuming that friction reduces V_2 to 80 fps.*

Exer. 6.6.6: Given $\theta = 115°$, $V_1 = 95$ fps, and $D_1 = 1$ in. Find the force components (a) parallel to and (b) perpendicular to the jet, and (c) find the magnitude and direction of the resultant force acting on the blade.

BG

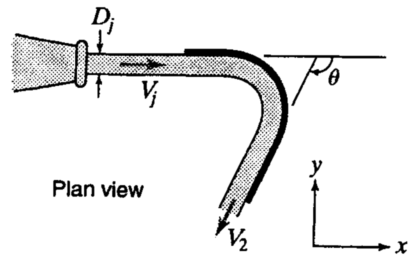


Figure X6.6.6

(*a*) From Eq. 6.7*a*: $F_x = \dot{m}\Delta V = \rho A V_1(V_1 - V_2\cos\theta)$

$A = \pi(1/12)^2/4 = 0.00545\ \text{ft}^2$

$F_x = 1.940(0.00545)95(95 - 80\cos 115°)$

$= 1.005(128.8) = +129.5$ lb $= 129.5$ lb ← on the CV

Equal and opposite is the force on the blade = 129.5 lb → ◀

(*b*) From Eq. 6.7*b*: $F_y = \dot{m}V_2\sin\theta = 1.005(80\sin 115° - 0) = +72.9$ lb $= 72.9$ lb ↓ on the CV.

Equal and opposite is the force on the blade = 72.9 lb ↑ ◀

(*c*) Resultant $F = \sqrt{129.5^2 + 72.9^2} = 148.6$ lb ◀ α from $V_1 = \tan^{-1}(72.9/148.6) = 29.4°$ ↗ ◀

6.14 *Solve Exer. 6.6.7 assuming that friction reduces V_2 to 22 m/s.*

Exer. 6.6.7: Given $\theta = 115°$, $V_1 = 25$ m/s, and $D_1 = 40$ mm. Find the force components (a) parallel to and (b) perpendicular to the jet, and (c) find the magnitude and direction of the resultant force acting on the blade.

SI

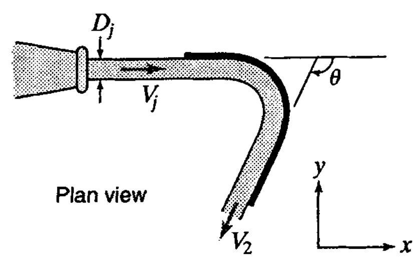


Figure X6.6.6

(*a*) From Eq. 6.7*a*: $F_x = \dot{m}\Delta V = \rho A V_1(V_1 - V_2\cos\theta)$

$A = \pi(0.04)^2/4 = 0.001257\ \text{m}^2$

$F_x = 1000(0.001257)25(25 - 22\cos 115°)$

$= 31.4(34.3) = +1077$ N $= 1077$ N ← on the CV

Equal and opposite is the force on the blade = 1077 N → ◀

(*b*) From Eq. 6.7*b*: $F_y = \dot{m}V_2\sin\theta = 31.4(22\sin 115° - 0) = +626$ N $= 626$ N ↓ on the CV.

Equal and opposite is the force on the blade = 626 N ↑ ◀

(*c*) Resultant $F = \sqrt{1077^2 + 626^2} = 1246$ N ◀ α from $V_1 = \tan^{-1}(626/1077) = 30.2°$ ↗ ◀

6.15 *Assuming ideal flow in a horizontal plane, calculate the magnitude and direction of the resultant force on the stationary blade in Fig. P6.15, knowing that V_j = 50 fps and D_j = 6 in. Note that the jet is divided by the splitter so that one-third of the water is diverted toward A.*

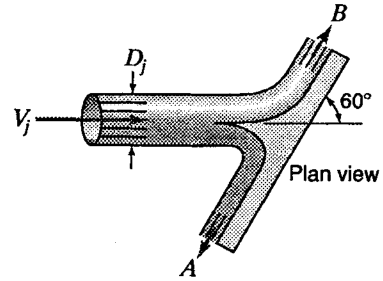


Figure P6.15

BG

$Q = \frac{\pi 0.5^2}{4} 50 = 9.82$ cfs ; $Q_A = \frac{Q}{3} = 3.27$ cfs, $Q_B = 6.54$ cfs

Eq. 6.7*a* for CV: $F_x = \rho Q_A(V_{2x} - V_{1x}) + \rho Q_B(V_{2x} - V_{1x})$

$= 1.940(3.27)(-50\cos 60° - 50) + 1.940(6.54)(50\cos 60° - 50)$

$= -476 - 317 = -794$ lb $= 794$ lb ← on the CV

Eq. 6.7*b* for CV:

$F_y = \rho Q_A(V_{2y} - V_{1y}) + \rho Q_B(V_{2y} - V_{1y}) = 1.940(3.27)(-50\cos 30° - 0) + 1.940(6.54)(50\cos 30° - 0)$

$= -275 + 550 = +275$ lb $= 275$ lb ↑ on the CV.

The water acts on the blade with an opposite force $(F_{W/B})_x$ of 794 lb along the centerline to the right; and perpendicularly with an opposite force $(F_{W/B})_y$ of 275 lb (towards *A*).

Resultant force on the blade = 840 lb at 19.11° ↘ from the centerline (towards *A*) ◀

6.16 *Refer to Fig. P6.15. Assuming ideal flow in a horizonta plane, calculate the magnitude and direction of the resultant force on the stationary blade. Note that the jet (V_j = 12 m/s, D_j = 150 mm) is divided by the splitter so that one-third of the water is diverted toward A.*

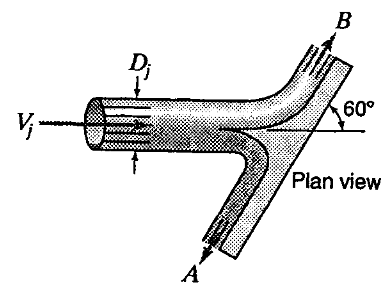


Figure P6.15

SI

$Q = \pi(0.075)^2 12 = 0.212$ m³/s

$Q_A = Q/3 = 0.0707$ m³/s, $Q_B = 0.1414$ m³/s

Eq. 6.7*a* for CV: $F_x = (9.81/9.81)0.0707(12\cos 60° - 12)$

$+ (9.81/9.81)0.1414(12\cos 60° - 12$

$= -1.272 - 0.848 = -2.12$ kN $= 2.12$ kn ← on CV.

Eq 6.7*b* for CV: $F_y = 1(0.0707)(-12\cos 30° - 0) + 1(0.1414)(12\cos 30° - 0) = 0.735$ kN (↑)

Water acts on the blade with an opposite force $(F_{W/B})_x$ of 2.12 kN along the centerline to the right, and perpendicularly with an opposite force $(F_{W/B})_y$ of 0.735 kN (towards *A*).

Resultant force on blade = 2.24 kN at 19.11° ↘ from the centerline (toward *A*). ◀

6.17 *A horizontal jet of water issues from an orifice in the side of a tank under a head h_1 and strikes a large plate a short distance away which covers the end of a horizontal tube in the side of a second tank (Fig. P6.17). The second tank contains oil of specific weight 50 lb/ft^3 at rest. The height of the oil above the tube is h_2. The jet diameter is three-fourths of the inside diameter of the tube. The jet and the tube are at the same elevation. (a) If the impact of the water is just sufficient to hold the plate in place, find the relation between h_1 and h_2. Neglect the weight of the plate and assume ideal flow. (b) Consider the effect of the weight of the plate. Find h_2 if h_1 = 9 ft, the weight of the plate = 8 lb, the jet diameter = 1.4 in, and the coefficient of friction between the plate and tube = 0.4. (c) Repeat part (b) with the plate weighing only 4 lb.*

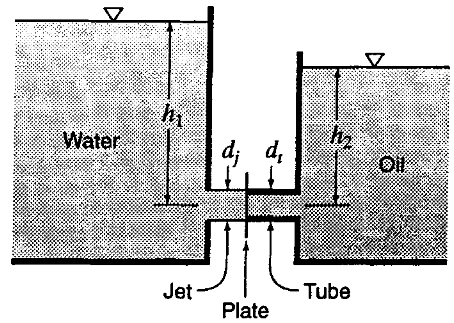


Figure P6.17

BG

(a) $F_j = \rho_1 Q_1 \Delta V_1 = \rho_1 (A_1 V_j) V_j = \rho_1 (\pi/4) D_j^2 V_j^2$ where $V_j = \sqrt{2gh_1}$

$\therefore \; F_j = (\gamma_1/g)(\pi/4)D_j^2(2gh_1) = (\pi/2)\gamma_1 h_1 D_j^2$

$F_{oil} = A_2 p_2 = (\pi/4)D_t^2(\gamma_2 h_2)$

To hold (weightless) plate in place (with $D_j = 0.75D_t$), $F_j = F_{oil}$

$\therefore \; (\pi/2)\gamma_1 h_1 (0.75D_t)^2 = (\pi/4)D_t^2 \gamma_2 h_2$

$h_2 = 2(\gamma_1/\gamma_2)0.75^2 h_1 = 2(62.4/50)0.75^2 h_1 = 1.404 h_1$ ◄

(b) $F_j = (\pi/2)\gamma_1 h_1 D_j^2 = (\pi/2)62.4(9)(1.4/12)^2 = 12.01$ lb

For equilibrium we need friction force $F = \mu N = \mu(F_j - F_{oil}) = W$

i.e., $0.4(12.01 - F_{oil}) = 8$ or $12.01 - F_{oil} = 20$

F_{oil} cannot be negative, so this is impossible, the 8-lb plate cannot be held in place. ◄

(c) If W = 4 lb we need: $0.4(12.01 - F_{oil}) = 4$

$\therefore \; F_{oil} = 12.01 - 10 = 2.01 \text{ lb} = \frac{\pi}{4}\left(\frac{D_j}{0.75}\right)^2 \gamma_2 h_2 = \frac{\pi}{4}\left(\frac{1.4}{12 \times 0.75}\right)^2 50 h_2$

from which $h_2 = 2.11$ ft ◄

6.18 *Repeat Prob. 6.17 with the following data: (a) oil specific weight = 7500 N/m³ ; (b) h_1 = 2.8 m, W = 25 N, jet diameter = 35 mm, μ = 0.4; (c) W = 18 N.*

SI

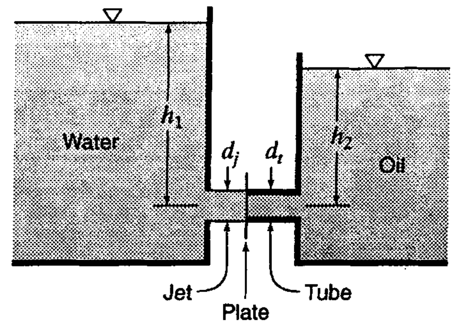


Figure P6.17

(a) $F_j = \rho_1 Q_1 \Delta V_1 = \rho_1(A_1 V_j)V_j = \rho_1(\pi/4)D_j^2 V_j^2$

where $V_j = \sqrt{2gh_1}$

$\therefore \quad F_j = (\gamma_1/g)(\pi/4)D_j^2(2gh_1) = (\pi/2)\gamma_1 h_1 D_j^2$

$F_{oil} = A_2 p_2 = (\pi/4)D_t^2(\gamma_2 h_2)$

To hold (weightless) plate in place (with $D_j = 0.75D_t$)

we need $F_j = F_{oil}$

$\therefore \quad (\pi/2)\gamma_1 h_1 (0.75D_t)^2 = (\pi/4)D_t^2 \gamma_2 h_2$

$h_2 = 2(\gamma_1/\gamma_2)0.75^2 h_1 = 2(9810/7500)0.75^2 h_1 = 1.472 h_1$ ◄

(b) $F_j = (\pi/2)\gamma_1 h_1 D_j^2 = (\pi/2)9810(2.8)(0.035)^2 = 52.9$ N

For equilibrium we need friction force $F = \mu N = \mu(F_j - F_{oil}) = W$

i.e., $0.4(52.9 - F_{oil}) = 25$ or $52.9 - F_{oil} = 62.5$

F_{oil} cannot be negative, so this is impossible: the 25-N plate cannot be held in place ◄

(c) If W = 18 N we need: $0.4(52.9 - F_{oil}) = 18$

$$\therefore \quad F_{oil} = 52.9 - 45 = 7.85 \text{ N} = \frac{\pi}{4}\left(\frac{D_j}{0.75}\right)^2 \gamma_2 h_2 = \frac{\pi}{4}\left(\frac{0.035}{0.75}\right)^2 7500 h_2$$

from which h_2 = 0.612 m = 612 mm ◄

6.19 *At section P a 5-in-diameter water jet with a velocity of 28 fps is directed vertically upward against the cone shown in Fig. P6.19. Neglecting friction and assuming the streamlines at Q are parallel, find the weight of the cone if a = 1.5 ft, b = 0.6 ft, and c = 4 ft.*

BG

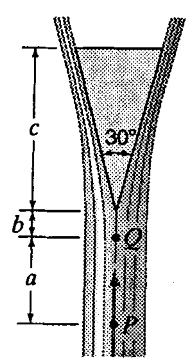


Figure P6.19

Find V at section Q: Eq. 5.15: $V_P^2/2g = 1.5 + V_Q^2/2g$

$V_Q^2/2g = 28^2/2g - 1.5 = 10.67$ ft ; $V_Q = 26.2$ ft/sec

Find V_d at discharge: $V_Q^2/2g = 4.6 + V_d^2/2g$

$V_d^2/2g = 26.2^2/2g - 4.6 = 6.07$; $V_d = 19.78$ fps

From Eq. 6.10: $-(F_{C/W})_z - W_w = \rho Q(V_{2z} - V_{1z})$

where W_w = weight of water = γQ × time to flow about 4.6 ft

$$\therefore \quad \text{flow time} = \frac{4.6}{(26.2 + 19.78)/2} = 0.200 \text{ sec} ;$$

$W_w = \gamma Q t = 62.4(\pi/4)(5/12)^2 28(0.200) = 47.7$ lb

$W_{cone} = (F_{C/W})_z = -W_w - \rho Q(V_{2z} - V_{1z})$

$W_{cone} = -47.7 - 1.940(\pi/4)(5/12)^2 28(19.78\cos 15° - 26.2) = -47.7 + 52.7 = 5.04$ lb ◄

6.20 *Repeat Prob. 6.19 if the jet diameter is 0.6 m, the jet velocity is 22 m/s at P, and a = 1.6 m, b = 0.6 m, and c = 3.5 m.*

Prob. 6.19: A water jet is directed vertically upward against the cone shown in Fig. P6.19. Neglecting friction and assuming the streamlines at Q are parallel, find the weight of the cone.

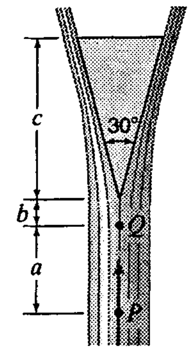


Figure P6.19

SI

Find V at section Q: Eq. 5.15: $22^2/(2\times9.81) = 1.6 + V_Q^2/2g$; $V_Q = 21.27$ m/s

Find V_d at discharge: $21.27^2/(2\times9.81) = 4.1 + V_d^2/2g$; $V_d = 19.29$ m/s

From Eq. 6.10: $-F_z - W_w = \rho Q(V_{2z} - V_{1z})$

where W_w = weight of water = $\gamma Q \times$ time to flow about 4.1 m

$$\therefore \text{ flow time } = \frac{4.1}{(19.29+21.27)/2} = 0.202 \text{ s}$$

$W_w = \gamma Q t = 9.81(\pi 0.30^2)22(0.202) = 12.33$ kN

$W_{cone} = F_z = -W_w - \rho Q(V_{2z} - V_{1z})$

$W_{cone} = -12.33 - (9.81/9.81)(\pi 0.30^2)22(19.29\cos 15° - 21.27) = -12.33 + 16.42 = 4.09$ kN ◄

6.21 *Repeat Prob. 6.19 with a jet velocity of 19 fps and a = 0 , b = 2 ft, and c = 4 ft.*

Prob. 6.19: At section P a 5-in-diameter water jet is directed vertically upward against the cone shown in Fig. P6.19. Neglecting friction and assuming the streamlines at Q are parallel, find the weight of the cone.

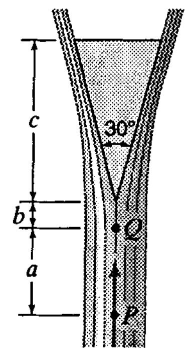


Figure P6.19

BG

$$\frac{19^2}{2g} = 6 + \frac{V_d^2}{2g} \; ; \; \frac{V_d^2}{2g} = 5.61 - 6 = -0.394 \text{ ft}$$

Thus the jet does not have enough energy to rise to the top of the cone.

$$\frac{19^2}{2g} = z + 0 \; ; \; z = 5.61 \text{ ft} = \text{maximum rise above point Q}$$

Thus the jet rises up 5.61 − 2 = 3.61 ft in contact with cone,

$$\text{when } \frac{19^2}{2g} = z + \frac{V^2}{2g} \text{ or}$$

$$V = \sqrt{2g(5.61 - z)} \; ; \; V = \frac{dz}{dt}, \quad \therefore t = \int_0^t dt = \int_0^{5.61} \frac{dz}{V} = \int_0^{5.61} \frac{dz}{\sqrt{2g(5.61 - z)}}$$

$$= -\frac{2\sqrt{5.61 - z}}{\sqrt{2g}} \Bigg|_0^{5.61} = 0.590 \text{ sec}$$

$W_w = \gamma Q t = \gamma A V t = 62.4(\pi/4)(5/12)^2 19(0.590) = 95.4$ lb

$\rho Q(V_{2z} - V_{1z}) = 1.940(\pi/4)(5/12)^2 19(0 - 19) = -95.5$ b

From Eq. 6.10: $-F_z - W_w = \rho Q \Delta V$

$\therefore W_{cone} = F_z = -W_w - \rho Q \Delta V = -95.4 - (-95.5) = 0.1040$ lb ◄

6.22 *In Fig. P6.22 streamlines are plotted to scale in the plane of the center of a free jet impinging vertically on a horizontal circular plate. The jet diameter is 280 mm and stagnation pressure at point O is 5.5 kPa. By scaling off the pertinent dimensions determine as accurately as possible the velocity of the water as it leaves the plate and the total resultant force exerted by the water on the plate.*

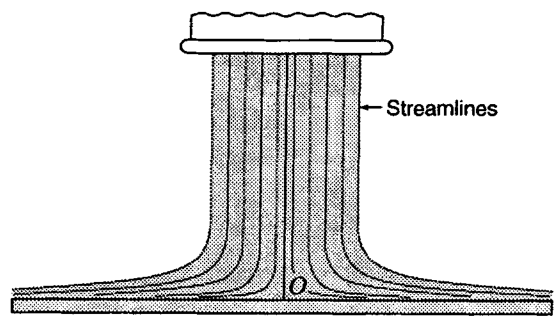


Figure P6.22

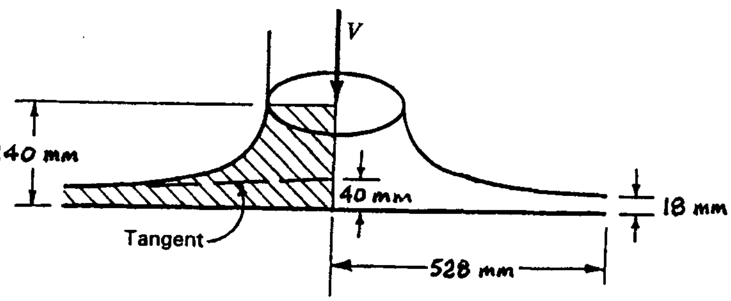
SI

Energy equation from where streamlines diverge (z = 140 mm) to stagnation point:

$$0 + 0.14 + \frac{V^2}{2g} = \frac{p_O}{\gamma} + 0 + 0 = \frac{5.5}{9.81} = 0.561 \text{ m};$$

$$\frac{V^2}{2g} = 0.421 \text{ m} \; ; \quad V = \sqrt{2(9.81)0.421} = 2.87 \text{ m/s}$$

$$Q = AV = (\pi 0.14^2)2.87 = 0.1769 \text{ m}^3\text{/s}$$



By scaling from Fig. P6.22: Water leaves the plate (r = 528 mm) at an average angle of about 1.2° and at a depth of about 18 mm, for example. (Scaling at different points will give rise to somewhat different answers.)

$$V_2 = Q/A_2 \approx 0.1769/[(2\pi(0.528)0.018)] \approx 3.0 \text{ m/s} \quad ◄$$

$$\rho Q(\Delta V) \approx (9810/9.81)0.1769(2.87 - 2.96\sin 1.2°) = 497 \text{ N}$$

By scaling from Fig. P6.22, and by calculation, for water to one side of the vertical centerline and below the point where the jet first begins to widen: Area $\approx A$ = 33 300 mm^2 = 0.0333 m^2, distance from centerline to centroid = $r_c \approx$ 165.7 mm = 0.1657 m.

Using Pappus' theorem: $W = \gamma \forall = 9810(2\pi r_c A) \approx 19\,620\pi(0.1657)0.0333 = 340$ N

Total Force = $W + \rho Q(\Delta V) \approx 340 + 497 = 837 \text{ N} \approx 840$ N ◄

6.23 *Repeat Prob. 6.22 with a jet diameter of 12 in and a jet velocity of 10 fps.*

Prob. 6.22: In Fig. P6.22 streamlines are plotted to scale in the plane of the center of a free jet impinging vertically on a horizontal circular plate. By scaling off the pertinent dimensions determine as accurately as possible the velocity of the water as it leaves the plate and the total resultant force exerted by the water on the plate.

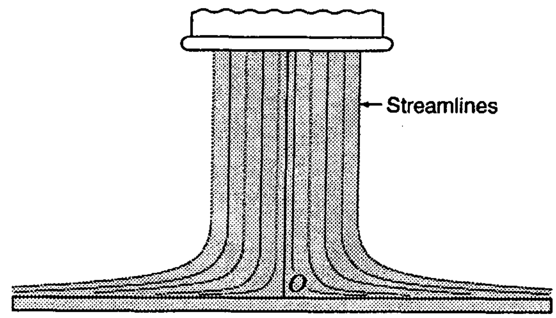
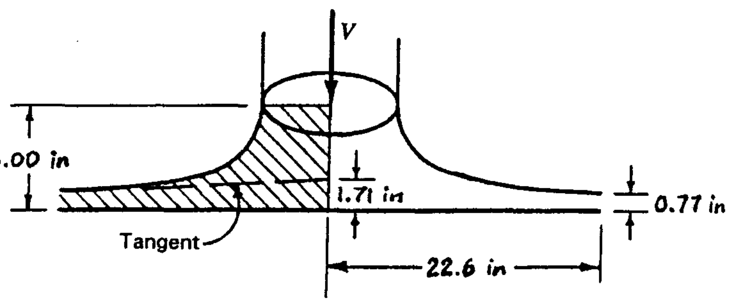


Figure P6.22

BG

$Q = \pi(0.5)^2 10 = 7.85$ cfs



By scaling from Fig. P6.22: Water leaves the plate (r = 22.6 in) at an average angle of 1.2° and at a depth of about 0.77 in, for example. (Scaling at different points will give rise to somewhat different answers.)

$$V_2 = Q/A_2 \approx 7.85 \Big/ \left(2\pi \frac{22.6}{12} \frac{0.77}{12}\right) = 10.3 \approx 10 \text{ fps} \quad \blacktriangleleft$$

$$\rho Q(\Delta V) \approx (1.940)7.85(10.3 - 10.3\sin 1.2°) = 154.3 \text{ lb}$$

By scaling from Fig. P6.22, and by calculation, for water to one side of the vertical centerline and below the point where the jet first begins to widen: Area = $A \approx 61.1$ in^2 = 0.425 ft^2, distance from centerline to centroid = $r_c \approx 7.11$ in = 0.593 ft.

Using Pappus' theorem: $W = \gamma \mathcal{V} = 62.4(2\pi r_c A) \approx 124.8\pi(0.593)0.425 = 98.8$ lb

Total Force = $W + \rho Q(\Delta V) \approx 98.8 + 154.3 = 253 \approx 250$ lb ◀

Sec. 6.8: Force of a Jet on One or More Moving Vanes or Blades -- *Exercises (6)*

6.8.1 *If a jet of fluid strikes a single body moving in the same direction with a velocity u, flows over it without friction loss, and leaves with a relative velocity in the same direction of β_2, prove that $F_u = (\gamma A_1/g)(1 - \cos\beta_2)(V_1 - u)^2$.*

N

By Eq 6.23: $F_u = \dfrac{\gamma A_1}{g} v_1 \Delta v_u = \dfrac{\gamma A_1}{g} v_1(v_1 - v_2 \cos\beta_2)$

But, $v_1 = v_2 = V_1 - u$, and $\therefore$ $F_u = (\gamma A_1/g)(1 - \cos\beta_2)(V_1 - u)^2$ QED ◀

6.8.2 *A jet of water strikes a single vane which reverses it through 180° without friction loss. If the jet has an area of 3.5 in² and a velocity of 175 fps, find the force exerted if the vane moves (a) in the same direction as the jet with a velocity of 75 fps, or (b) in a direction opposite to that of the jet with a velocity of 75 fps.*

BG

(*a*) $v_1 = 175 - 75 = 100$ fps ; Eq 6.20: $\rho Q' = (62.4/32.2)(3.5/144)100 = 4.71$ slugs/sec

By Eq. 6.22: $F = 4.71[100 - (-100)] = 942$ lb ◀

(*b*) $v_1 = 175 - (-75) = 250$ fps ; $\rho Q' = (62.4/32.2)(3.5/144)250 = 11.78$ slugs/sec

By Eq. 6.22: $F = 11.78[250 - (-250)] = 5890$ lb ◀

6.8.3 *A jet of water strikes a single vane which reverses it through 180° without friction loss. If the jet has an area of 2500 mm² and a velocity of 55 m/s, find the force exerted if the vane moves (a) in the same direction as the jet with a velocity of 20 m/s; (b) in a direction opposite to that of the jet with a velocity of 20 m/s.*

SI

(*a*) $v_1 = 55 - 20 = 35$ m/s ; Eq. 6.20: $\rho Q' = (9810/9.81)(25/10\,000)35 = 87.5$ N·s/m

By Eq. 6.22: $F = 87.5[35 - (-35)] = 6125$ N $= 6.13$ kN ◀

(*b*) $v_1 = 55 - (-20) = 75$ m/s ; $\rho Q' = (9810/9.81)(25/10\,000)75 = 187.5$ N·s/m

By Eq. 6.22: $F = 187.5[75 - (-75)] = 28\,100$ N $= 28.1$ kN ◀

6.8.4 *A 4-in-diameter water jet with a velocity of 105 fps acts on a series of vanes with $\alpha_1 = \beta_1 = 0$. Neglect friction and find the required blade angle β_2 in order that the resultant force acting on the vane in the direction of the jet is 200 lb. Solve using vane velocities of 0, 15, 45, and 75 fps. Also find the maximum possible vane velocity.*

BG

$A = \pi(4/12)^2/4 = 0.0873$ ft² ; $Q = AV = (0.0873)105 = 9.16$ cfs ; $v_2 = v_1 = v = 105 - u$

From Eqs. 6.24 and 6.17: $F = -200 = \rho Q(V_{2x} - V_{1x}) = 1.940(9.16)(u + v\cos\beta_2 - 105)$

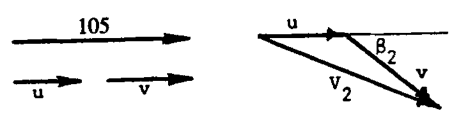
i.e. $93.7 = u + (105 - u)\cos\beta_2$; $\cos\beta_2 = (93.7 - u)/(105 - u)$

For $u = 0$ fps ; $\beta_2 = 26.8°$ ◀

$u = 15$ fps ; $\beta_2 = 29.0°$ ◀

$u = 45$ fps ; $\beta_2 = 35.7°$ ◀

$u = 75$ fps ; $\beta_2 = 51.3°$ ◀



For $u = u_{max}$, $\beta_2 = 180°$,

yielding $u_{max} = 99.4$ fps ◀

6.8.5 *A 100-mm-diameter water jet with a velocity of 35 m/s acts on a series of vanes with $\alpha_1 = \beta_1 = 0$. Neglect friction and find the required blade angle β_2 in order that the resultant force acting on the vane in the direction of the jet is 950 N. Solve using vane velocities of 0, 5, 15, and 25 m/s. Also find the maximum possible vane velocity.*

BG

$A = \pi(0.01)^2/4 = 0.00785\ \text{m}^2$; $Q = AV = 0.00785(35) = 0.275\ \text{m}^3/s$; $v_2 = v_1 = v = 35 - u$

From Eqs. 6.24 and 6.17: $F = -950 = \rho Q(V_{2x} - V_{1x}) = 1000(0.275)(u + v\cos\beta_2 - 35)$

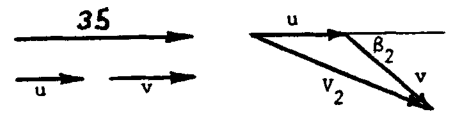
i.e. $31.5 = u + (35 - u)\cos\beta_2$; $\cos\beta_2 = (31.5 - u)/(35 - u)$

For $u = 0\ \text{m}/s$; $\beta_2 = 25.7°$ ◄

$u = 5\ \text{m}/s$; $\beta_2 = 27.8°$ ◄

$u = 15\ \text{m}/s$; $\beta_2 = 34.2°$ ◄

$u = 25\ \text{fps}$; $\beta_2 = 49.1°$ ◄



For $u = u_{max}$, $\beta_2 = 180°$,

$$\text{yielding } u_{max} = \frac{35 - 31.5\cos 180°}{1 - \cos 180°} = 33.3\ \text{m}/s$$ ◄

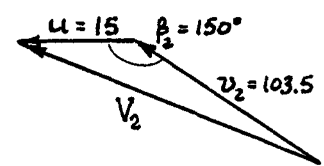
6.8.6 *What would be the resultant force components on the single vane of Sample Prob. 6.4 if it were traveling to the left toward the nozzle at 15 fps?*

Sample Prob. 6.4: A 2-in-diameter water jet, $V = 100$ fps, strikes a vane with $\beta_2 = 150°$. Friction causes $v_2 = 0.9 v_1$.

BG

At entrance $V_1 = 100$, $u = -15$, $\alpha_1 = \beta_1 = 180°$, $\therefore\ v_1 = V_1 - u = 115$;

$\therefore\ v_2 = 0.9(115) = 103.5$ fps. $A_j = \pi(2/12)^2/4 = 0.0218\ \text{ft}^2$



At exit, Eq. 6.17: $V_2\cos\alpha_2 = v_2\cos\beta_2 + u_2 = 103.5(\cos 30°) + 15$

$= 104.6$ fps to the left

$\Delta V_x = 100 - (-104.6) = 204.6$ fps

Eqs. 6.19 and 6.21: $F_x = \rho Q'(\Delta V_x) = 1.940(0.0218 \times 115)204.6 = 996$ lb ◄

Eq. 6.18: $V_{2y} = V_2\sin\alpha_2 = v_2\sin\beta_2 = 103.5\sin 30° = 51.8$ fps ;

$\Delta V_y = V_{2y} - V_{1y} = 51.8 - 0 = 51.8$ fps

From Eqs 6.19 and 6.21: $F_y = \rho Q'(\Delta V_y) = 1.940(0.0218 \times 115)51.8 = 252$ lb ◄

Sec. 6.8: Force of a Jet on One or More Moving Vanes or Blades – *Problems 6.24–6.35*

6.24 *A locomotive tender running at 20 mph scoops up water from a trough between the rails, as shown in Fig. P6.24. The scoop delivers water at a height h = 8 ft above its origional level and in the direction of motion. The area of the stream of water at entrance is 40 in². The water is everywhere under atmospheric pressure. Neglecting all losses, what is the absolute velocity of the water as it leaves the scoop? What force on the tender does the water cause? At what minimum speed will water rise to the height h above the original level?*

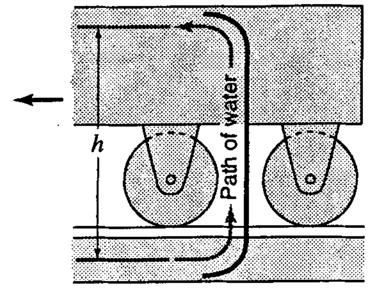


Figure P6.24

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$u_1 = u_2 = -20(44/30) = -29.3$ fps ; $v_1 = 29.3$ fps

Relative to moving car, Eq. 5.29: $0 + 0 + \dfrac{29.3^2}{2(32.2)} = 0 + 8 + \dfrac{v_2^2}{2(32.2)}$

$v_2 = -18.58$ fps, $V_2 = u_2 + v_2 = -29.3 - 18.58 = -47.9$ fps ◄

Eq. 6.23: $F = -\dfrac{62.4}{32.2}\dfrac{40}{144}29.3(-47.9 - 0) = 757$ lb ◄

Minimum tender speed $= \sqrt{2gh} = \sqrt{2(32.2)8} = 22.7$ fps $= 15.48$ mph ◄

6.25 *Solve Prob. 6.24 for the following data: locomotive speed = 12 m/s, h = 2.3 m, stream area = 0.03 m².*

Prob. 6.24: A locomotive tender scoops up water from a trough between the rails, as shown in Fig. P6.24. The scoop delivers water at a height h above its origional level and in the direction of motion. The given area of the stream of water is at entrance. The water is everywhere under atmospheric pressure. Neglecting all losses, what is the absolute velocity of the water as it leaves the scoop? What force on the tender does the water cause? At what minimum speed will water rise to the height h above the original level?

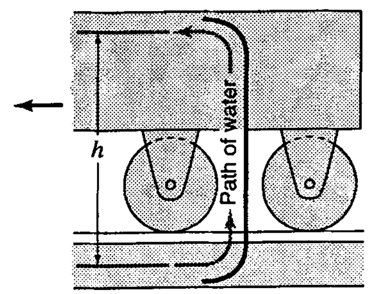

Figure P6.24

SI

$u_1 = u_2 = -12$ m/s ; $v_1 = 12$ m/s

Relative to the moving car, Eq. 5.15: $0 + 0 + \dfrac{12^2}{2(9.81)} = 0 + 2.3 + \dfrac{v_2^2}{2(9.81)}$; $v_2 = -9.94$ m/s

$V_2 = u_2 + v_2 = -12 - 9.94 = -21.94$ m/s ◄

Eq. 6.23: $F = -(9.81/9.81)(0.03)12(-21.94 - 0) = 7.90$ kN ◄

Minimum tender speed $= \sqrt{2gh} = \sqrt{2(9.81)2.3} = 6.72$ m/s ◄

6.26 *A 4-in-dia water jet has a velocity of 130 fps. It strikes a single vane, which has an angle β_2 = 90° and which is moving in the same direction as the jet with a velocity u. When u has values of 0, 45, 65, 85, 105, and 130 fps, find the values of (a) $\dot{m}'g$; (b) $V_2\cos\alpha_2$; (c) ΔV_u; (d) Δv_u; (e) F_u. Assume $v_2 = 0.9v_1$. Present answers in a neat tabular form or on a spreadsheet.*

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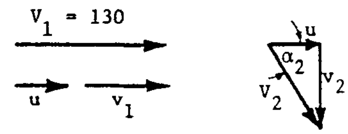


Eq. 6.17: $V_u = u + v_u \quad \therefore \quad V_1 = u + v_1$

$\therefore \quad v_1 = V_1 - u = 130 - u = v_{1u}$ (since $\beta_1 = 0$) (for Col. 2)

From Eq. 6.20: $\dot{m}'g = (\rho A_1 v_1)g = \gamma A_1 v_1$

$$= 62.4\frac{\pi}{4}\left(\frac{4}{12}\right)^2 v_1 = 5.45\,v_1 \quad \text{(for Col. } a\text{)}$$

Eq. 6.17: $V_2\cos\alpha_2 = u + v_2\cos\beta_2 = u + v_2\cos 90° = u$ (for Col. *b*)

Given $v_2 = 0.9\,v_1$ at $\beta_2 = 90°$; $v_{2u} = v_2\cos\ _2 = v_2\cos 90° = 0$

$\therefore \quad \Delta v_u = v_{2u} - v_{1u} = 0 - v_1 = -v_1$ (for Col. *d*)

Sec. 6.8: $\Delta V = \Delta v, \quad \therefore \quad \Delta V_u = \Delta v_u$ (for Col. *c*)

From Eqs. 6.23 and 6.20: $F_u = \pm(\gamma/g)A_1 v_1 \Delta v_u = (\rho A_1 v_1)\Delta v_u = \dot{m}'\Delta v_u$

$= \text{Col. } (a) \times \text{Col. } (d)/g$ (for Col. *e*)

u fps	v_1 fps	(a) $\dot{m}'g$ lb/sec	(b) $V_2\cos\alpha_2$ fps	(c) ΔV_u fps	(d) Δv_u fps	(e) $F_u = \pm\dot{m}'\Delta v_u$ lb
0	130	708	0	−130	−130	2860
45	85	463	45	−85	−85	1222
65	65	354	65	−65	−65	715
85	45	245	85	−45	−45	342
105	25	136.1	105	−25	−25	105.7
130	0	0	130	0	0	0
		▲	▲	▲	▲	▲

6.27 *Assume all the data in Prob. 6.26 are the same except that β_2 = 180°. Find the values of (a) $\dot{m}'g$; (b) v_2; (c) V_2; (d) ΔV; (e) Δv; (f) F_u. Assume $v_2 = 0.8 v_1$. Present answers in a neat tabular form or on a spreadsheet.*

Prob. 6.26: A jet with V = 130 fps, diameter = 4 in, strikes a single vane moving in the same direction at velocity u. Solve for u = 0, 45, 65, 85, 105, and 130 fps.

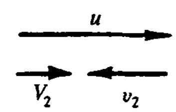
BG

Eq. 6.17: $V_u = u + v_u \quad \therefore \quad V_1 = u + v_1$

$\therefore \quad v_1 = V_1 - u = 130 - u = v_{1u}$ (since $\beta_1 = 0$) (for Col. 2)

From Eq. 6.20: $\dot{m}'g = (\rho A_1 v_1)g = \gamma A_1 v_1 = 62.4[\pi(4/12)^2/4]v_1 = 5.45\, v_1$ (for Col. *a*)

Given: $\beta_2 = 180°$ and $v_2 = 0.8\, v_1$ (for Col. *b*)



Eq. 6.17: $V_2 = V_{2u} = u + v_{2u} = u + v_2\cos 180° = u - v_2$ (for Col. *c*)

$\Delta V = V_2 - V_1 = V_2 - 130$ (for Col. *d*). Sec. 6.8: $\Delta V = \Delta v$ (for Col. *e*)

From Eqs. 6.23 and 6.20: $F_u = \pm(\gamma/g)A_1 v_1 \Delta v_u = (\rho A_1 v_1)\Delta v_u = \dot{m}'\Delta v_u$

$= \text{Col. } (a) \times \text{Col. } (e)/g$ (for Col. *f*)

u fps	v_1 fps	(*a*) $\dot{m}'g$ lb/sec	(*b*) v_2 fps	(*c*) V_2 fps	(*d, e*) $\Delta V = \Delta v$ fps	(*f*) $F_u = \pm\dot{m}'\Delta v_u$ lb
0	130	708	104	−104	−234	5140
45	85	463	68	−23	−153	2200
65	65	354	52	13	−117	1286
85	45	245	36	49	−81	616
105	25	136.1	20	85	−45	190.3
130	0	0	0	130	0	0
		▲	▲	▲	▲	▲

6.28 *Solve Prob. 6.27 for the case when $v_2 = 0.7v_1$.*

Prob. 6.27: A jet with V = 130 fps, diameter = 4 in, strikes a single vane (β_2 = 180°) moving in the same direction at velocity u. Find the values of (a) $\dot{m}'g$; (b) v_2; (c) V_2; (d) ΔV; (e) Δv; (f) F_u. Solve for u = 0, 45, 65, 85, 105, and 130 fps. Present answers in a neat tabular form or on a spreadsheet.

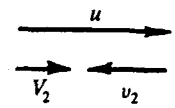
BG

Eq. 6.17: $V_u = u + v_u \quad \therefore \quad V_1 = u + v_1$

$\therefore \quad v_1 = V_1 - u = 130 - u = v_{1u}$ (since $\beta_1 = 0$) (for Col. 2)

From Eq. 6.20: $\dot{m}'g = (\rho A_1 v_1)g = \gamma A_1 v_1 = 62.4\frac{\pi}{4}\left(\frac{4}{12}\right)^2 v_1 = 5.45\,v_1$ (for Col. *a*)

Given: $\beta_2 = 180°$ and $v_2 = 0.7\,v_1$ (for Col. *b*)



Eq. 6.17: $V_2 = V_{2u} = u + v_{2u} = u + v_2\cos 180° = u - v_2$ (for Col. *c*)

$\Delta V = V_2 - V_1 = V_2 - 130$ (for Col. *d*). Sec. 6.8: $\Delta V = \Delta v$ (for Col. *e*)

From Eqs. 6.23 and 6.20: $F_u = \pm(\gamma/g)A_1 v_1 \Delta v_u = (\rho A_1 v_1)\Delta v_u = \dot{m}'\Delta v_u$

$=$ Col. (*a*) × Col. (*e*)/*g* (for Col. *f*)

		(*a*)	(*b*)	(*c*)	(*d, e*)	(*f*)
u	v_1	$\dot{m}'g$	v_2	V_2	$\Delta V = \Delta v$	$F_u = \pm\dot{m}'\Delta v_u$
fps	fps	lb/sec	fps	fps	fps	lb
0	130	708	91.0	−91.0	−221	4870
45	85	463	59.5	−14.5	−144.5	2080
65	65	354	45.5	19.5	−110.5	1215
85	45	245	31.5	53.5	−76.5	582
105	25	136.1	17.5	87.5	−42.5	179.7
130	0	0	0	130	0	0
		▲	▲	▲	▲	▲

6.29 *Assume that all the data are the same as in Prob. 6.26, except that $\beta_2 = 145°$ and $v_2 = 0.7v_1$. Find the values of (a) $v_2\cos\beta_2$; (b) $V_2\cos\alpha_2$; (c) ΔV_u; (d) Δv_u; (e) F_u. Present answers in a neat tabular form or on a spreadsheet.*

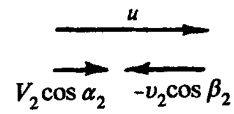
Prob 6.26: A jet with V = 130 fps, diameter = 4 in, strikes a single vane moving in the same direction at velocity u. Solve for u = 0, 45, 65, 85, 105, and 130 fps.

BG

Eq. 6.17: $V_u = u + v_u \quad \therefore \quad V_1 = u + v_1$

$\therefore \quad v_1 = V_1 - u = 130 - u = v_{1u}$ (since $\beta_1 = 0$) (for Col. 2)

From Eq. 6.20: $\dot{m}'g = (\rho A_1 v_1)g = \gamma A_1 v_1 = 62.4\frac{\pi}{4}\left(\frac{4}{12}\right)^2 v_1 = 5.45\, v_1$



$v_2\cos\beta_2 = 0.7\, v_1\cos 145° = -0.573\, v_1$ (for Col. *a*)

Eq. 6.17: $V_2\cos\alpha_2 = u + v_2\cos\beta_2$ (for Col. *b*)

Using Eq. 6.17: $\Delta V_u = V_{2u} - V_{1u} = V_2\cos\alpha_2 - V_1 = V_2\cos\alpha_2 - 130$ (for Col. *c*)

Sec. 6.8: $\Delta V = \Delta v, \quad \therefore \quad \Delta V_u = \Delta v_u$ (for Col. *d*)

From Eqs. 6.23 and 6.20: $F_u = \pm(\gamma/g)A_1 v_1 \Delta v_u = (\rho A_1 v_1)\Delta v_u = \dot{m}'\Delta v_u$

$= \text{Col. (3)} \times \text{Col. } (d)/g$ (for Col. *e*)

			(*a*)	(*b*)	(*c, d*)	(*e*)
u fps	v_1 fps	$\dot{m}'g$ lb/sec	$v_2\cos\beta_2$ fps	$V_2\cos\alpha_2$ fps	$\Delta V_u = \Delta v_u$ fps	$F_u = \pm\dot{m}'\Delta v_u$ lb
0	130	708	−74.5	−74.5	−205	4500
45	85	463	−48.7	−3.74	−133.7	1922
65	65	354	−37.3	27.7	−102.3	1124
85	45	245	−25.8	59.2	−70.8	539
105	25	136.1	−14.34	90.7	−39.3	166.3
130	0	0	0	130	0	0
			▲	▲	▲	▲

6.30 *For the conditions of Prob. 6.15, compute the magnitude and direction of the resultant force on the single blade if it is moving to the right at a velocity of 20 fps.*

Prob. 6.15: The water jet shown in Fig. P6.15 (V_j = 50 fps, D_j = 6 in) strikes the blade and is divided so that one third of the water is diverted toward A. Assume ideal flow in a horizontal plane.

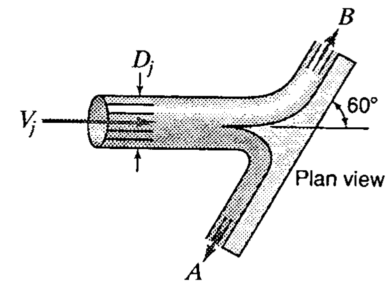


Figure P6.15

BG

$v_1 = V_1 - u = 50 - 20 = 30$ fps

Eq. 6.19: $Q' = A_1 v_1 = \pi\left(\frac{0.5}{2}\right)^2 30 = 5.89$ cfs

$Q'_A = 1.963$ cfs, $Q'_B = 3.93$ cfs

Eq. 6.23: $-F_x = 1.940(1.963)(-30\cos 60° - 30) + 1.940(3.93)(30\cos 60° - 30)$

$-F_x = -171.4 - 114.3 = -286$ lb ; $F_x = 286$ lb

Similarly: $F_y = 1.940(1.963)(-30\sin 60° - 0) + 1.940(3.93)(30\sin 60° - 0)$

$F_y = -99.0 + 197.9 = 99.0$ lb

Water acts on the blade with a force of 286 lb along the centerline to the right, and perpendicularly with a force of 99.0 lb (towards *A*).

Resultant force = 302 lb at 19.11° from the centerline (toward *A*). ◄

6.31 *For the conditions of Prob. 6.16, compute the magnitude and direction of the resultant force on the single blade if it is moving to the right at a velocity of 5 m/s.*

Prob. 6.16: Fig. P6.15 represents ideal flow in a horizontal plane. The jet (V = 12 m/s, D = 150 mm) is divided by the splitter so that one third of the water is diverted toward A.

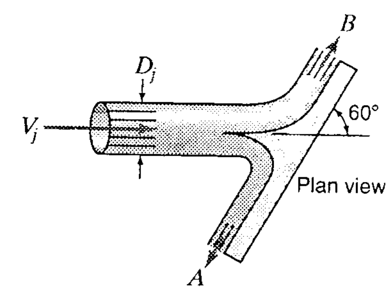


Figure P6.15

SI

$A_1 = \pi(0.15)^2/4 = 0.01767$ m²; $v_1 = V_1 - u = 12 - 5 = 7$ m/s

Eq. 6.19: $Q' = A_1 v_1 = 0.01767(7) = 0.1237$ m³/s

$Q'_A = 0.0412$ m³/s, $Q'_B = 0.0825$ m³/s

Eq. 6.23: $-F_x = 1000(0.0412)(-7\cos 60° - 7) + 1000(0.0825)(7\cos 60° - 7)$

$-F_x = -433 - 289 = -722$ N

Similarly: $F_y = 1000(0.0412)(-7\sin 60° - 0) + 1000(0.0825)(7\sin 60° - 0) = -250 + 500 = 250$ N

Water acts on the blade with a force of 722 N along the centerline to the right, and perpendicularly with a force of 250 N (toward *A*).

Resultant force = 764 N at 19.11° from the centerline (toward *A*). ◄

6.32 *Suppose the blade of Prob. 6.15 is one of a series of blades which are moving to the right at 15 fps. (a) Determine the resultant horizontal force on the blade system, and (b) compute the power transferred to the blades. (c) Compute the power of the jet and of the water leaving the blade system to verify an energy balance (see Sample Prob. 6.4).*

Prob. 6.15: Assume ideal flow in a horizontal plane. Given $V_{1x} = 50$ fps, $D_j = 6$ in, and that one-third of the water is directed by the splitter towards A.

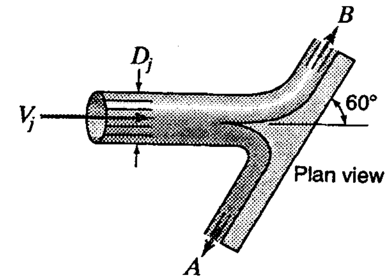


Figure P6.15

BG

$Q = AV = (\pi 0.5^2/4)50 = 9.82$ cfs

$Q_A = Q/3 = 3.27$ cfs, $Q_B = 6.54$ cfs

(*a*) From Eq. 6.17: $v_1 = v_{1x} = V_{1x} - u = 50 - 15 = 35$ fps. Ideal flow, $\therefore\ v_2 = v_1 = 35$ fps

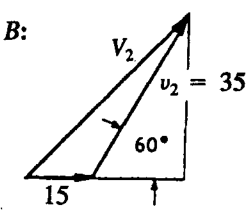
Eq. 6.24: $F_u = 1.940(3.27)(-35\cos 60° - 35) + 1.940(6.54)(35\cos 60° - 35)$

$F_u = -333 - 222 = -556$ lb. Horizontal force on blades is 556 lb to the right ◄

(*b*) Power $= Fu/550 = 556(15)/550 = 15.15$ hp transferred to blades ◄

(*c*) Energy balance check: Velocities at exit:

B:

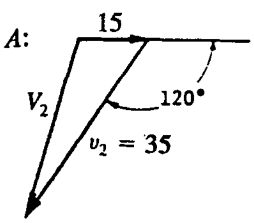


$V_{2x} = 15 + 35\cos 60° = 32.5$ fps

$V_{2y} = 35\sin 60° = 30.8$ fps

$V_2 = \sqrt{32.5^2 + 30.3^2} = 44.4$ fps

A:



$V_{2x} = 15 - 35\cos 60° = -2.5$ fps

$V_{2y} = -35\sin 60° = -30.3$ fps

$V_2 = \sqrt{2.5^2 + 30.3^2} = 30.4$ fps

Energy entering: $H_1 = 50^2/2g = 38.8$ ft ;

Energy leaving: $H_2 = (2/3)44.4^2/2g + (1/3)30.4^2/2g = 25.2$ ft

Power transferred to blades: $P = \gamma QH/550 = 62.4(9.82)(38.8 - 25.2)/550 = 15.13$ hp ◄

6.33 *Suppose the blade of Prob. 6.15 is one of a series of blades which are moving to the right at 4 m/s. (a) Determine the resultant horizontal force on the blade system, and (b) compute the power transferred to the blades. (c) Compute the power of the jet and of the water leaving the blade system to verify an energy balance (Sample Prob. 6.4).*

Prob. 6.15: Assume ideal flow in a horizontal plane. Given V_{1x} = 12 m/s, D_j = 150 mm, and that one-third of the water is directed by the splitter towards A.

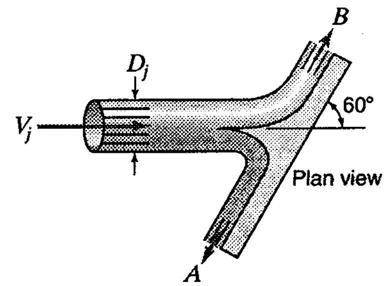


Figure P6.15

SI

$Q = AV = \pi(0.075)^2 12 = 0.212 \text{ m}^3/\text{s}$

$Q_A = Q/3 = 0.0707 \text{ m}^3/\text{s}; \quad Q_B = 0.1414 \text{ m}^3/\text{s}$

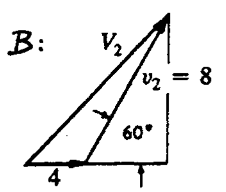
(*a*) From Eq. 6.17: $v_1 = v_{1x} = V_{1x} - u = 12 - 4 = 8$ m/s. Ideal flow, $\therefore\ v_2 = v_1 = 8$ m/s

Eq.6.24: $F_u = 1000(0.0707)(-8\cos 60° - 8) + 1000(0.1414)(8\cos 60° - 8)$

$F_u = -848 - 565 = -1414$ N. Horizontal force on blades is 1414 N to the right ◄

(*b*) Power transferred to blades $= F_u/1000 = 1414(4)\ 1000 = 5.65$ kW ◄

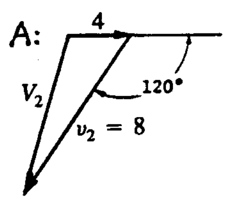
(*c*) Energy balance check: Velocities at exit:



$V_{2x} = 4 + 8\cos 60° = 8$ m/s

$V_{2y} = 8\sin 60° = 6.93$ m/s

$V_2 = \sqrt{8^2 + 6.93^2} = 10.58$ m/s



$V_{2x} = 4 - 8\cos 60° = 0$

$V_{2y} = -8\sin 60° = -6.93$ m/s

$V = 6.93$ m/s

Energy entering: $H_1 = 12^2/2g = 7.34$ m

Energy leaving: $H_2 = (2/3)10.58^2/2g + (1/3)6.93^2/2g = 4.62$ m

Power transferred to blades: $P = \gamma QH/1000 = 9810(0.212)(7.34 - 4.62)/1000 = 5.65$ kW ◄

6.34 *A 3-in-diameter air jet impinges on a series of blades, entering smoothly, and having absolute velocities V_1 = 200 fps and V_2 = 150 fps as shown in Fig. P6.34. Assume γ = 0.075 lb/ft³, that the pressure is the same on both sides, and neglect friction. (a) What is the velocity of the blades and the power being transmitted to them? (b) Determine the blade angles necessary at entrance and exit.*

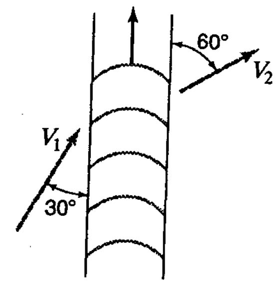


Figure P6.34

BG

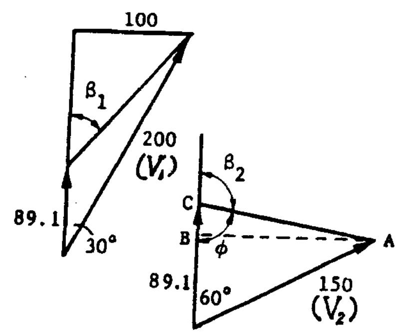
(*a*) $H = \dfrac{200^2}{2g} - \dfrac{150^2}{2g} = 272$ ft·lb/lb

$\gamma Q = \gamma A V = 0.075 \dfrac{\pi}{4}\left(\dfrac{3}{12}\right)^2 200 = 0.736$ lb/sec

Eq. 5.32: $\text{hp} = \dfrac{\gamma Q H}{550} = \dfrac{0.736(272)}{550} = 0.364$ ◄

Eq. 6.23: $F_x = \rho Q(V_{2x} - V_{1x}) = \dfrac{0.736}{32.2}(75 - 200\sin 60°) = -2.25$ lb

$\text{hp} = 0.364 = \dfrac{F_x v_x}{550}$; $v_x = \dfrac{0.364(550)}{2.25} = 89.1$ fps ◄



(*b*) $\tan\beta_1 = \dfrac{100}{200\cos 30° - 89.1} = 1.189$; $\beta_1 = 49.9$

$\tan\phi = \dfrac{AB}{BC} = \dfrac{150(\sin 60°)}{89.1 - 150\cos 60°} = 9.21$; $\phi = 83.8°$

$\beta_2 = 180 - 83.8 = 96.2°$ ◄

6.35 *A 60-mm-diameter air jet impinges on a series of blades, entering smoothly, and having absolute velocities V_1 = 60 m/s and V_2 = 45 m/s as shown in Fig. P6.34. Assume γ = 11 N/m³, that the pressure is the same on both sides, and neglect friction. (a) What is the velocity of the blades and the power being transmitted to them? (b) Determine the blade angles necessary at entrance and exit.*

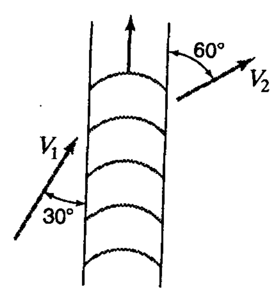


Figure P6.34

SI

(*a*) $H = \dfrac{60^2}{2(9.81)} - \dfrac{45^2}{2(9.81)} = 80.3$ m

$\gamma Q = \gamma A V = 11(\pi/4)(6/100)^2 60 = 1.866$ N/s

Eq. 5.31: Power $= \gamma Q H = 1.866(80.3) = 149.8$ N·m/s $= 149.8$ W ◄

Eq. 6.23: $F_x = \rho Q(V_{2x} - V_{1x}) = (1.866/9.81)(45\cos 60° - 60\cos 30°) = -5.60$ N

Power $= 149.8$ N·m/s $= F_x v_x$; $\therefore v_x = 149.8/5.60 = 26.7$ m/s ◄

(*b*) $\tan\beta_1 = 30/(60\cos 30° - 26.7) = 1.189$; $\beta_1 = 49.9°$

$\tan\phi = AB/BC = 45\sin 60°/(26.7 - 45\cos 60°) = 9.21$; $\phi = 83.8°$

$\beta_2 = 180 - 83.8 = 96.2°$ ◄

Sec. 6.9: Reaction of a Jet -- *Exercises (2)*

6.9.1 *Find the thrust developed when water is pumped through a 9-in-diameter pipe in the bow of a boat at v = 6 fps and emitted through a 5-in-diameter pipe in the stern of the boat.*

BG

Eq. 6.7*a*: $F = \rho Q(V_{2x} - V_{1x})$, where $\rho Q = 1.940\frac{\pi}{4}\left(\frac{9}{12}\right)^2 6 = 5.14$ slugs/sec

$V_{2x} = 6(9/5)^2 = 19.44$ fps ; $\therefore\ F = 5.14(19.44 - 6) = 69.1\ \text{slug·ft/sec}^2 = 69.1$ lb ◄

6.9.2 *Find the thrust developed when water is pumped through a 225-mm-diameter pipe in the bow of a boat at v = 2.5 m/s and emitted through a 125-mm-diameter pipe in the stern of the boat.*

SI

$\rho Q = 1000(\pi\, 0.225^2/4)2.5 = 99.4$ kg/s

$V_{2x} = 2.5(225/125)^2 = 8.10$ m/s ; Eq. 6.7*a*: $F = 99.4(8.1 - 2.5) = 557\ \text{kg·m/s}^2 = 557$ N ◄

Sec. 6.9: Reaction of a Jet -- *Problems 6.36–6.39*

6.36 *An ideal liquid (γ = 52 lb/ft^3) flows from a 2-ft-diameter tank as shown in Fig. P6.36. The jet diameter is 3 in and a = 1 ft. If the static coefficient of friction between the tank and floor is 0.52, determine the minimum value of h at which the tank will start to move to the left. The tank itself weighs 80 lb.*

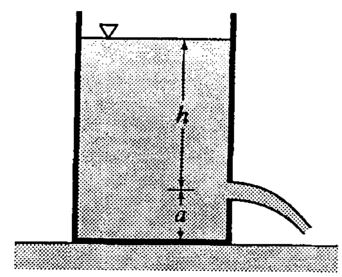


Figure P6.36

BG

$W = 80 + \pi(2^2/4)(1 + h)52$; $\mu W = 0.52W = 126.5 + 84.9h$

Eq. 6.25: $F_x = \rho QV = \rho AV^2 = (52/32.2)\pi(1.5/12)^2(2gh) = 5.11h$

$F_x = \mu W$; Thus $126.5 + 84.9h = 5.11h$; $h = -1.585$ ft

h is negative, so the tank will not move regardless of the value of *h*. ◄

6.37 *An ideal liquid (γ = 9810 N/m^3) flows from a 400-mm-diameter tank as shown in Fig. P6.36. The jet diameter is 80 mm and a = 250 mm. If the static coefficient of friction between the tank and floor is 0.6, determine the minimum value of h at which the tank will start to move to the left. The tank itself weighs 500 N.*

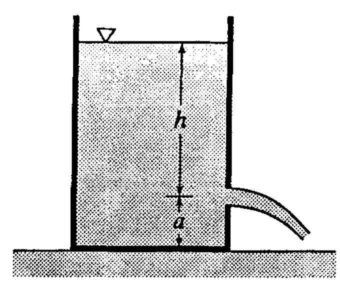


Figure P6.36

SI

$W = 500 + \pi(0.4^2/4)(0.25 + h)9810$; $\mu W = 0.6W = 808 + 1233h$

Eq. 6.25: $F_x = \rho QV = \rho AV^2 = 1000\pi 0.04^2(2gh) = 98.6h$

$F_x = \mu W$; Thus $98.6h = 808 + 1233h$; $h = -0.713$ m

h is negative, so the tank will not move regardless of the value of *h*. ◄

6.38 *Find the magnitude and direction of the resultant force of the fluid on the compressor shown in Fig. P6.38. Air (γ = 0.075 lb/ft^3) enters at A through a 4-ft^2 area at a velocity of 15 fps. Air discharges at B through a 3-ft^2 area with a velocity of 17 fps.*

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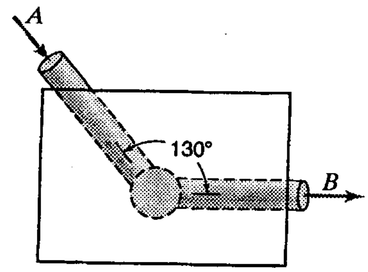
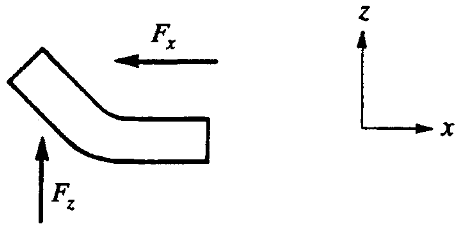


Figure P6.38



$$\rho Q = \frac{\gamma}{g} Q = \frac{0.075}{32.2}(4 \times 15) = 0.1398 \text{ slugs/sec}$$

Eq. 6.23: $-F_x = \rho Q(V_{2x} - V_{1x}) = 0.1398(17 - 15\cos 50°) = 1.028$ slugs/sec

$\therefore$ $F_x = -1.028$ lb. The minus sign indicates that the compressor pushes on the fluid to the right.

Eq. 6.23: $F_z = \rho Q(V_{2z} - V_{1z}) = 0.1398(0 + 15\cos 40°) = +1.606$ lb

The positive sign indicates that the assumed direction of the compressor force on the fluid is correct.

Equal and opposite force of fluid on compressor: 1.028 lb to the left, 1.606 lb downward

Resultant force = 1.907 lb at 57.4° to F_x ↙. ◄

6.39 *Solve Prob. 6.38 for the case where a gas (γ = 12.1 N/m^3) enters at A through a 600-mm-diameter pipe at 5 m/s and leaves at B through a 500-mm-diameter pipe at 7 m/s.*

Prob. 6.35: Refer to figures with Solution 6.35.

SI

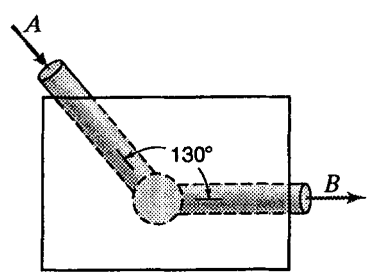
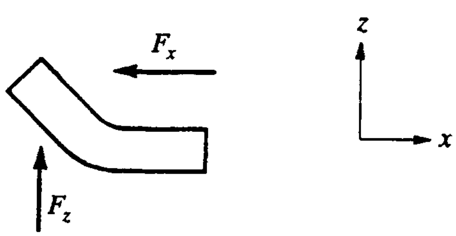


Figure P6.38



$\rho = \gamma/g = (12.1 \text{ N/m}^3)/(9.81 \text{ m/s}^2) = 1.233 \text{ kg/m}^3$; $Q = \pi(0.30^2)5 = 1.414 \text{ m}^3\text{/s}$

$\dot{m} = \rho Q = 1.233(1.414) = 1.744$ kg/s

$\Delta V_x = V_{2x} - V_{1x} = 7 - 5\cos 50° = 3.79$ m/s ; $\Delta V_z = V_{2z} - V_{1z} = 0 - 5\cos 40° = -3.83$ m/s

$-F_x = \rho Q(\Delta V_x) = 1.744(3.79) = 6.60$ N ; $F_z = \rho Q(\Delta V_z) = 1.744(-3.83) = -6.68$ N

Equal and opposite force on fluid on compressor: 6.60 N to the left, 6.68 N downward.

Resultant force = 9.39 N at 45.3° ↙ to F_x. ◄

Sec. 6.10: Jet Propulsion -- ***Exercises*** *(2)*

6.10.1 *Find the thrust of a turbojet whose speed is 750 fps and whose air intake rate is 50 lb/sec. The air/fuel ratio is 30:1 and the exhaust velocity is 1800 fps.*

BG

$(\dot{m}g)_{\text{fuel}} = 50/30 = 1.667$ lb/sec ; Eq. 6.27: $F = [(50 + 1.667)1800 - 50(750)]/32.2 = 1724$ lb ◄

6.10.2 *Find the thrust of a turbojet whose speed is 280 m/s and whose air intake rate is 12 kg/s. The air/fuel ratio is 25:1 and the exhaust velocity is 550 m/s.*

SI

$\dot{m}_{\text{fuel}} = 12/25 = 0.480$ kg/s

Eq. 6.27 can be expressed as: $F = (\dot{m}_{\text{air}} + \dot{m}_{\text{fuel}})v_2 - \dot{m}_{\text{air}}u = 12.48(550) - 12(280) = 3500$ N ◄

Sec. 6.10: Jet Propulsion -- ***Problems*** *6.40–6.41*

6.40 *A rocket has a propellant flow rate of 21.6 lb/sec through a nozzle with a throat area of 9.3 in². The nozzle expands the gases down to 14.7 psia at exit. The exit area of the nozzle is 48.5 in² and the exhaust velocity is 6370 fps. Find the rocket's thrust (a) at sea level and (b) at an elevation of 20,000 ft where the barometer pressure is 6.75 psia; (c) find the specific weight of the exhaust gas.*

BG

(*a*) Eq. 6.26 with $p_n = p_a = 14.7$ psia at sea level:

$F = \rho A v_2^2 = \dot{m} v_2 = (21.6/32.2)6370 = 4270$ lb at sea level ◄

(*b*) $p_n = 14.7$ psia, $p_a = 6.75$ psia (given) at 20,000 ft elev.

From Eq. 6.26: $F = \dot{m} v_2 + (p_n - p_a)A_2$

$F = 4270 + (14.7 - 6.75)\,48.5 = 4660$ lb at 20,000 ft ◄

(*c*) $\dot{m}g = 21.6$ lb/sec $= \gamma A v = \gamma(48.5/144)6370$; $\gamma = 0.0101$ pcf ◄

6.41 *A rocket has a propellant flow rate of 10 kg/s througl ı nozzle with a 90-mm-diameter throat. The nozzle is designed to expand the gases down to 101.3 kPa at ex . The exit diameter of the nozzle is 200 mm, and the exhaust velocity is 2100 m/s. Find the rocket's thrust a) at sea level and (b) at an elevation of 6 km where the barometer pressure is 47.2 kPa abs; (c) find the d ısity of the exhaust gas.*

SI

(*a*) Eq. 6.26 with $p_n = p_a = 101.3$ kPa abs at sea level:

$F = \rho A v_2^2 = \dot{m} v_2 = 10(2100) = 21\,000$ lb at sea level ◄

(*b*) $p_n = 101.3$ kPa abs, $p_a = 47.2$ kPa abs (given) at 6 km elev.

From Eq. 6.26: $F = \dot{m} v_2 + (p_n - p_a)A_2$

$F = 21\,000 + (101.3 - 47.2)\,1000(\pi 0.2^2/4) = 22\,700$ N at 6 km ◄

(*c*) $\dot{m}g = 10$ kg/s $= \rho A v = \rho(\pi 0.2^2/4)2100$; $\rho = 0.1516$ kg/m³ ◄

Sec. 6.11: Rotating Machines: Continuity, Relative Velocities, Torque -- ***Exercises (2)***

6.11.1 *The absolute velocity of a jet of steam impinging on the blades of a steam turbine is 3800 fps, and that leaving is 2600 fps.* $\alpha_1 = 20°$, $\alpha_2 = 150°$, $u_1 = u_2 = 500$ *fps, and* $r_1 = r_2 = 0.5$ *ft. Find the torque exerted on the rotor and the power delivered to it if the steam flows at 0.4 lb/sec.*

BG

Noting that $\rho Q = \dot{m} = G/g = 0.4/32.2$ slug/sec

By Eq. 6.31: $T = (0.4/32.2)0.5(3800\cos 20° - 2600\cos 150°) = 36.2$ ft·lb ◄

Eqs. 5.40 and 5.38: Power $= T\omega/550 = T(u/r)/550 = 36.2(500/0.5)/550 = 65.8$ hp ◄

6.11.2 *The absolute velocity of a jet of steam impinging on the blades of a steam turbine is 1200 m/s, and that leaving is 950 m/s.* $\alpha_1 = 20°$, $\alpha_2 = 150°$, $u_1 = u_2 = 180$ *m/s, and* $r_1 = r_2 = 120$ *mm. Find the torque exerted on the rotor and the power delivered to it if the steam flows at 2 N/s.*

SI

Noting that $\rho Q = \dot{m} = G/g = 2/9.81$ kg/s

By Eq. 6.31: $T = (2/9.81)0.12(1200\cos 20° - 950\cos 150°) = 47.7$ N·m ◄

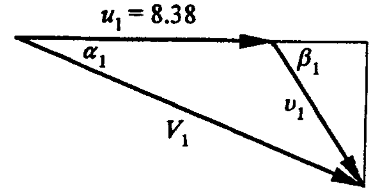
Eq. 5.38: Power $= T\omega = Tu/r = 47.7(180/0.12) = 71\,600$ N·m/s $= 71.6$ kW ◄

Sec. 6.12: Head Equivalent of Mechanical Work -- ***Problems 6.42–6.43***

6.42 *A radial-flow turbine has the following dimensions:* $r_1 = 0.5$ *m,* $r_2 = 0.3$ *m,* $\beta_1 = 74°$*, and* $\beta_2 = 126°$*. The width B of the flow passage between the two sides of the turbine is 0.2 m. When operating at 160 rpm the flow rate through the turbine is 1.5 m³/s. Find (a) the torque exerted by the water, (b) the power delivered to the shaft, and (c) the head converted into mechanical work.*

SI

At the outer periphery ($r_1 = 0.5$ m, $\beta_1 = 74°$):

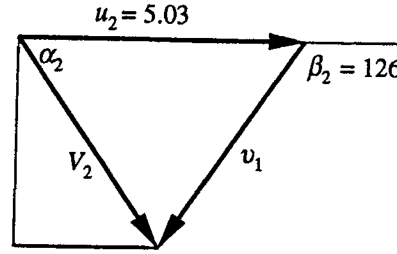


$u_1 = \omega r_1 = (2\pi/60)160(0.5) = 16.76(0.5) = 8.38$ m/s

From continuity: $Q = 1.5 = 2\pi(0.5)0.2V_{r1}$; $V_{r1} = 2.39$ m/s

$V_1\cos\alpha_1 = u_1 + V_{r1}/\tan\beta_1 = 8.38 + 2.39/\tan 74° = 9.06$ m/s

At the inner periphery ($r_2 = 0.3$ m, $\beta_2 = 126°$):



$u_2 = \omega r_2 = u_1(r_2/r_1) = 8.38(0.3/0.5) = 5.03$ m/s

$V_{r2} = V_{r1}(r_1/r_2) = 2.39(0.5/0.3) = 3.98$ m/s

$v_2 \sin 126° = v_{r2} = 3.98$, $v_2 = 4.92$ m/s

$v_2\cos\alpha_2 = u_2 + v_2\cos\beta_2 = 5.03 + 4.92\cos 126° = 2.14$ m/s

(*a*) Eq. 6.31: $T = 1000(1.5)(0.5\times 9.06 - 0.3\times 2.14)$

$= 5840$ N·m ◄

(*b*) Eq. 5.38: Power $= T\omega/1000 = 5840(2\pi/60)160/1000 = 97.8$ kW ◄

(*c*) Eq. 6.32: $h_m = (8.38\times 9.06 - 5.03\times 2.14)/9.81 = 6.64$ m ◄

Check: Power $= \gamma Q h_m = 9810(1.5)6.64/1000 = 97.8$ kW. Good!

6.43 *A radial-flow turbine has the following dimensions: $r_1 = 3.2$ ft, $r_2 = 1.6$ ft, $\beta_1 = 76°$, and $\beta_2 = 135°$. The width B of the flow passage between the two sides of the turbine is 0.65 ft. When operating at 150 rpm the flow rate through the turbine is 60 cfs. Find (a) the torque exerted by the water, (b) the horsepower delivered to the shaft, and (c) the head converted into mechanical work.*

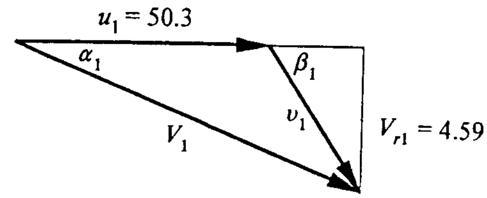
BG

At the outer periphery ($r_1 = 3.2$ ft, $\beta_1 = 76°$):

$u_1 = \omega r_1 = (2\pi/60)150(3.2) = 15.71(3.2) = 50.3$ fps

From continuity: $Q = 60 = 2\pi(3.2)0.65V_{r1}$, $V_{r1} = 4.59$ fps

$V_1 \cos\alpha_1 = u_1 + V_{r1}/\tan\beta_1 = 50.3 + 4.59/\tan 76° = 51.4$ fps



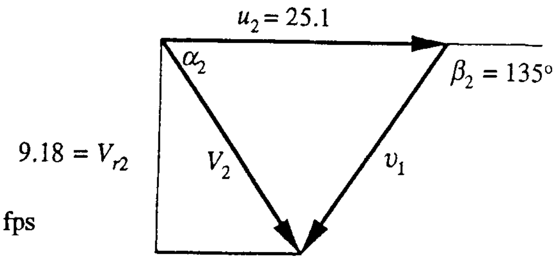
At the inner periphery ($r_2 = 1.6$ ft, $\beta_2 = 135°$):

$u_2 = \omega r_2 = u_1(r_2/r_1) = 50.3(1.6/3.2) = 25.1$ fps

$V_{r2} = V_{r1}(r_1/r_2) = 4.59(2.2/1.6) = 9.18$ fps

$v_2 \sin 135° = v_{r2} = 9.18, \quad v_2 = 12.98$ fps

$v_2 \cos\alpha_2 = u_2 + v_2 \cos\beta_2 = 25.1 + 12.98 \cos 135° = 15.95$ fps



(*a*) Eq. 6.31: $T = 1.940(60)(3.2 \times 51.4 - 1.6 \times 15.95)$

$= 16{,}180$ ft·lb ◄

(*b*) Eq. 5.38: Power $= T\omega/550 = 16{,}180(2\pi/60)150/550 = 462$ hp ◄

(*c*) Eq. 6.32: $h_m = (50.3 \times 51.4 - 25.1 \times 15.95)/32.2 = 67.8$ ft ◄

Check: Power $= \gamma Q h_m/550 = 62.4(60)67.8/550 = 462$ hp. Good!

Sec. 6.13: Flow Through a Rotating Channel -- ***Exercise*** *(1)*

6.13.1 *Develop Eq. (6.34) by making the substitutions indicated in the text.*

N

Given Eq. 6.33: $$\left(\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g}\right) - \left(\frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}\right) = h_L + \frac{u_1 V_1 \cos\alpha_1 - u_2 V_2 \cos\alpha_2}{g}$$

But $V^2 = v^2 + u^2 + 2vu\cos\beta$ and $V\cos\alpha = u + v\cos\beta$

Substituting: $p_1/\gamma + z_1 + (v_1^2 + u_1^2 + 2v_1u_1\cos\beta)/2g - p_2/\gamma - z_2 - (v_2^2 + u_2^2 + 2v_2u_2\cos\beta)/2g$

$= h_L + (2u_1^2 + 2u_1v_1\cos\beta - 2u_2^2 - 2u_2v_2\cos\beta)/2g$

i.e., $$\frac{p_1}{\gamma} + z_1 - \left(\frac{p_2}{\gamma} + z_2\right) + \frac{v_1^2 + u_1^2 - v_2^2 - u_2^2 - 2u_1^2 + 2u_2^2}{2g} = h_L$$

i.e., $$\left(\frac{p_1}{\gamma} + z_1 + \frac{v_1^2 - u_1^2}{2g}\right) - \left(\frac{p_2}{\gamma} + z_2 + \frac{v_2^2 - u_2^2}{2g}\right) = h_L$$ which is Eq. 6.34. QED ◄

Sec. 6.13: Flow Through a Rotating Channel -- *Problems 6.44–6.45*

6.44 *A paddle wheel with vanes that are all straight and radial is to be used as a crude centrifugal pump for water (Fig. P6.44): r_1 = 3 in, r_2 = 9 in, and the dimension B perpendicular to the plane of the figure is 0.2 ft. If the speed is 1200 rpm and the flow is 3380 gpm, (a) at the centerline elevation find the difference in pressure (psi) between the inner and outer circumferences, neglecting friction losses, (b) find which of these two points has the higher pressure, (c) compute the torque required to drive the pump, (d) calculate the horsepower requirement, and (e) verify that the horsepower requirement is equal to the difference between the horsepower of the outflow minus the horsepower of the inflow.*

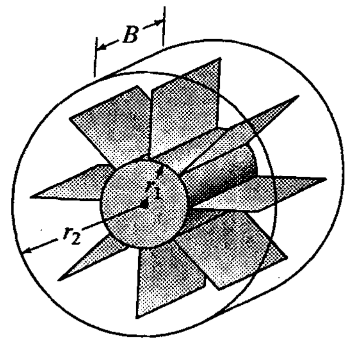


Figure P6.44

BG

$\omega = 1200(2\pi/60) = 40\pi$ rad/sec

$u_1 = \omega r_1 = 40\pi(3/12) = 10\pi$ fps ; $u_2 = \omega r_2 = 30\pi$ fps

$a_1 = 2\pi r_1 B = 2\pi(3/12)(0.2) = \pi/10$ ft^2

$a_2 = 2\pi(9/12)(0.2) = 0.3\pi$ ft^2

Q = 3380 gpm = 7.53 cfs

$v_1 = 7.53/(0.10\pi) = 24.0$ fps ; $v_2 = 7.53/(0.30\pi) = 7.99$ fps

(*a*) Eq. 6.34: $$\left[\frac{p_1}{\gamma} + \frac{24.0^2 - (10\pi)^2}{2g}\right] - \left[\frac{p_2}{\gamma} + \frac{7.99^2 - (30\pi)^2}{2g}\right] = 0$$

$$p_1/\gamma - p_2/\gamma = (63.8 - 8880 - 575 + 987)/(2g) = -130.5 \text{ ft}$$

$$p_1 - p_2 = 62.4(-130.5)/144 = -56.6 \text{ psi} \quad ◀$$

(*b*) p_2 is 56.6 psi greater than p_1. ◀

(*c*) Eq. 6.31: $T = \rho Q(r_1 u_1 - r_2 u_2) = 1.940(7.53)[(3/12)10\pi - (9/12)30\pi] = -918$ ft·lb (on water)

Thus torque required is 918 ft·lb

(*d*) Eq. 5.38: Power = $T\omega/550$ = 918(40π)/550 = 210 hp ◀

(*e*) $V_1 = \sqrt{u_1^2 + v_1^2} = 39.5$ fps ; $V_2 = \sqrt{u_2^2 + v_2^2} = 94.6$ fps

Head = $(p_2/\gamma + V_2^2/2g) - (p_1/\gamma + V_1^2/2g) = 130.5 + 138.9 - 24.3 = 245$ ft

Eq. 5.40: Power = $\gamma QH/550$ = 62.4(7.53)245/550 = 210 hp, checks out ◀

6.45 *Repeat Prob. 6.44 where the data in SI units are as follows: r_1 = 65 mm, r_2 = 215 mm, and dimension B perpendicular to the plane of the figure = 48 mm. The speed is 1200 rpm and the flow is 150 L/s. Express pressure difference in kPa and power in kW.*

A paddle wheel with vanes that are all straight and radial is to be used as a crude centrifugal pump for water (Fig. P6.44). (a) At the centerline elevation find the difference in pressure (psi) between the inner and outer circumferences, neglecting friction losses, (b) find which of these two points has the higher pressure, (c) compute the torque required to drive the pump, (d) calculate the horsepower requirement, and (e) verify that the horsepower requirement is equal to the difference between the horsepower of the outflow minus the horsepower of the inflow.

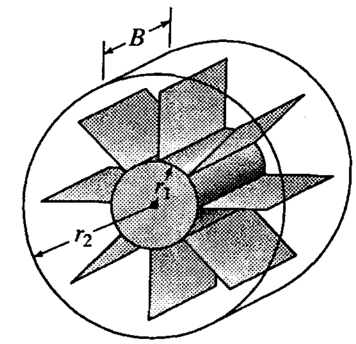


Figure P6.44

SI

$\omega = 1200(2\pi/60) = 40\pi$ rad/s

$u_1 = \omega r_1 = 2.6\pi$ m/s ; $u_2 = \omega r_2 = 8.6\pi$ m/s

$a_1 = 2\pi r_1 B = 2\pi(0.065)0.048 = 0.01960\text{ m}^2,\ a_2 = 0.0648\text{ m}^2$

$Q = 0.15\text{ m}^3/\text{s}$; $v_1 = Q/a_1 = 7.65$ m/s ; $v_2 = 2.31$ m/s

(*a*) Eq. 6.34: $$\left[\frac{p_1}{\gamma}+\frac{7.65^2-(2.6\pi)^2}{2(9.81)}\right]-\left[\frac{p_2}{\gamma}+\frac{2.31^2-(8.6\pi)^2}{2(9.81)}\right] = 0$$

$$\frac{p_1}{\gamma}-\frac{p_2}{\gamma} = \frac{-58.5 + 66.7 + 5.35 - 730}{2(9.81)} = -36.5\text{ m}$$

$\Delta p = p_2 - p_1 = -36.5(9.81) = -358\text{ kN/m}^2 = -358$ kPa ◄

(*b*) p_2 is 358 kPa greater than p_1 ◄

(*c*) Eq. 6.31: $T = \rho Q(r_1u_1 - r_2u_2) = (1)0.15(0.065\times 2.6\pi - 0.215\times 8.6\pi) = -0.792$ kN·m

Thus torque required is 0.792 kN·m ◄

(*d*) Eq. 5.38: Power $= T\omega = 0.792(40\pi) = = 99.5$ kW ◄

(*e*) $V_1 = \sqrt{u_1^2+v_1^2} = 11.19$ m/s ; $V_2 = \sqrt{u_2^2 + v_2^2} = 27.1$ m/s

Head $= (p_2/\gamma + V_2^2/2g) - (p_1/\gamma + V_1^2/2g) = 36.5 + 37.5 - 6.38 = 67.6$ m

Power $= \gamma QH = 9.81(0.15)67.6 = 99.5$ kW, checks out ◄

Sec. 6.14: Reaction with Rotation -- *Exercises (5)*

6.14.1 *The flow from a lawn sprinkler such as Fig. 6.14 is 140 L/min, $\beta_2 = 180°$, and the total area of the jets is 120 mm². The jets are located 200-mm from the center of rotation. Determine the speed of rotation if there is no friction.*

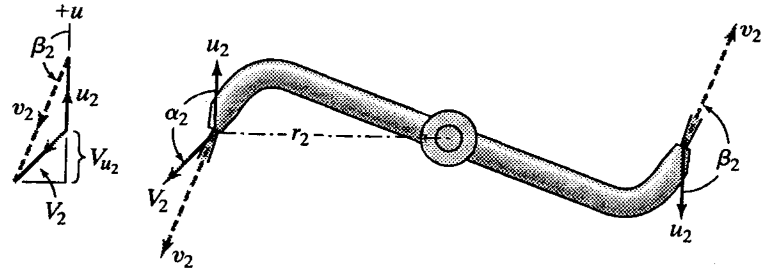


Figure 6.14

SI

$Q = 0.14/60 = 0.00233\ \text{m}^3/\text{s} = a_2 v_2 = (120\times10^{-6})v_2\ ;\quad v_2 = 19.44\ \text{m/s}$

Eq. 6.36: $T = F_u r_2 = -\rho a_2 v_2 r_2(u_2 + v_2\cos\beta_2) = 0$ for no friction

Thus $u_2 + v_2\cos 180° = 0\ ;\quad u_2 - 19.44 = 0,\quad u_2 = 19.44$ m/s

$u_2 = 19.44 = \omega r = \omega(0.20)$; thus $\omega = 97.2$ rad/s; $n = 97.2(60/2\pi) = 928$ rpm ◄

6.14.2 *A lawn sprinkler like that of Fig. 6.14 with $\beta_2 = 156°$ has a total jet area of 0.000 86 ft² and a radius of 16 inches. Compute the discharge rate, the torque exerted by the water, and the power developed, when h = 120 ft and the sprinkler is prevented from rotating.*

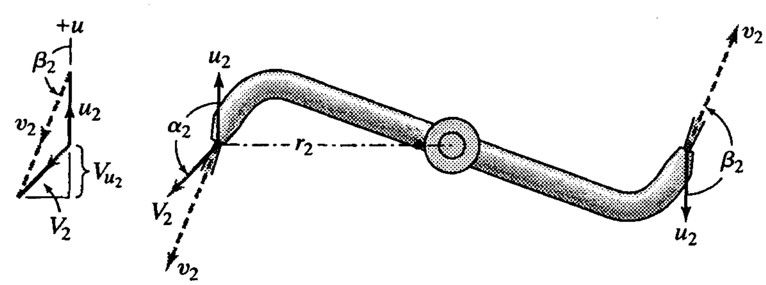


Figure 6.14

BG

Eq. 6.35 with $u_2 = 0$: $v_2 = \sqrt{2(32.2)120} = 87.9$ fps; $Q = a_2 v_2 = 0.0756$ cfs ◄

Eq. 6.36: $T = -1.940(0.0756)(16/12)(0 + 87.9\cos 156°) = 15.70$ ft·lb ◄

Since no motion occurs, Power = zero. ◄

6.14.3 *Repeat Exer. 6.14.2 with the following changes: total jet area = 75 mm², radius = 360 mm, h = 45 m.*
Exer. 6.14.2: The sprinkler (Fig. 6.14) has β_2 = 156° and is prevented from rotating. Compute Q, T, and the power developed.

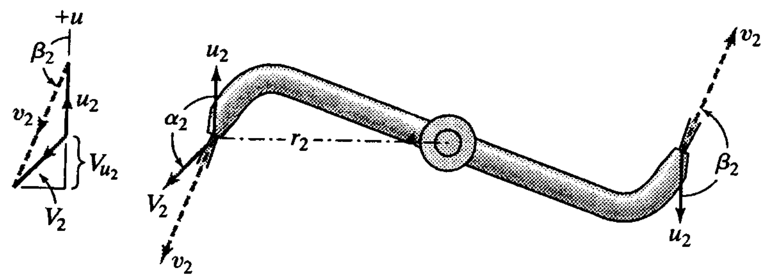


Figure. 6.14

SI

Eq. 6.35 with $u_2 = 0$: $v_2 = \sqrt{2(9.81)45} = 29.71$ m/s

$Q = a_2 v_2 = 0.000\,075(29.71) = 0.002\,23$ m³/s ◄

Eq. 6.36: $T = -1000(0.00223)0.36(0 + 29.71 \cos 156°) = 21.8$ N·m ◄

As $\omega = 0$, Power = 0 ◄

6.14.4 *How fast would the sprinkler of Exer. 6.14.2 rotate if there were no mechanical friction or air resistance (i.e., consider the case of runaway speed, where T = 0)?*
Exer. 6.14.2: The sprinkler (Fig. 6.14) has β_2 = 156° , a = 0.000 86 ft², r = 16 in, h = 120 ft.

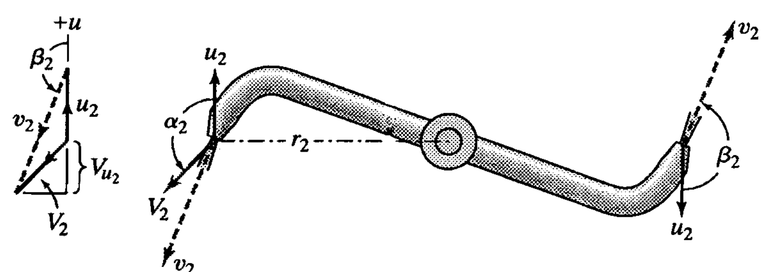


Figure. 6.14

BG

$u_2 = \omega r_2 = (16/12)\omega = 1.333\omega$; Eq. 6.35: $v_2 = \sqrt{2(32.2)120 + (16\omega/12)^2}$; $Q = a_2 v_2$

Eq. 6.36: $T = -\rho Q r_2(u_2 + v_2 \cos \beta_2) = 0$ for runaway speed

$\therefore \quad u_2 = -v_2 \cos\beta_2 = -v_2 \cos 156° = 0.914\, v_2$

$(16/12)\omega = 0.914\sqrt{2(32.2)120 + (16\omega/12)^2}$; $\omega = 148$ rad/s = 1414 rpm ◄

6.14.5 *How fast would the sprinkler of Ex. 6.14.3 rotate if there were no mechanical friction or air resistance (i.e. consider the case of runaway speed, where $T = 0$)?*

Exer. 6.14.3: The sprinkler (Fig. 6.14) has $\beta_2 = 156°$, $a = 75\ mm^2$, $r = 360\ mm$, $h = 45\ m$.

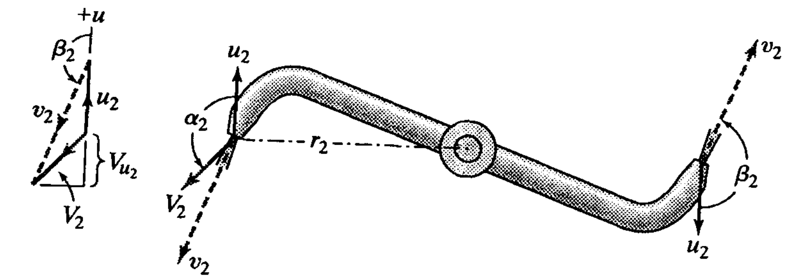


Figure. 6.14

SI

$u_2 = \omega r_2 = 0.36\omega$; Eq. 6.35: $v_2 = \sqrt{2g(45) + (0.36\omega)^2}$; $Q = a_2 v_2$

Eq. 6.36: $T = -(\gamma/g)Qr_2(u_2 + v_2 \cos\beta_2) = 0$ for runaway speed

$\therefore\ u_2 = -v_2 \cos\beta_2 = -v_2 \cos 156° = 0.914 v_2$

$\therefore\ 0.36\omega = 0.914\sqrt{2g(45) + (0.36\omega)^2}$; $\omega = (185.4)60/(2\pi) = 1770$ rpm ◄

Sec. 6.14: Reaction with Rotation -- *Problems 6.46–6.49*

6.46 *Given a lawn sprinkler such as that in Fig. 6.14 with $\beta_2 = 160°$, in which the total area of the jets at a radius of 15 in is 0.0008 ft^2. When $h = 144$ ft, compute the rate of discharge, the torque exerted by the water, and the power developed if the rotative speed of the sprinkler is 400 rpm. Neglect fluid friction, but note that the calculated torque must overcome mechanical friction and air resistance.*

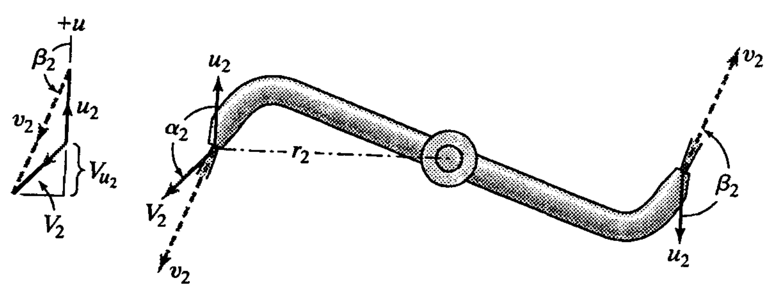


Figure. 6.14

BG

$\omega = 400(2\pi/60) = 41.9$ rad/sec ; $u_2 = \omega r_2 = 41.9(15/12) = 52.4$ fps

Eq. 6.35: $v_2 = \sqrt{2(32.2)144 + (52.4)^2} = 109.6$ fps

$Q = a_2 v_2 = (0.0008)109.6 = 0.0877$ cfs ◄

Eq. 6.36: $T = -1.940(0.0877)(15/12)(52.4 + 109.6 \cos 160°) = 10.77$ ft·lb ◄

Eq. 5.38: Power $= T\omega/550 = 10.76(41.9)/550 = 0.820$ hp ◄

Check on power: Eq. 5.40: $P = \gamma QH/550$ hp where $H = 144 - V_2^2/2g$

From Fig. 6.14: $V^2 = (v_2 \cos 20° - u_2)^2 + (v_2 \sin 20°)^2 = (109.6 \cos 20° - 52.4)^2 + (109.6 \sin 20°)^2$

$= 50.6^2 + 37.5^2 = 3970$; $V = 63.0$ fps

$H = 144 - 63.0^2/(2 \times 32.2) = 82.4$ ft

Power $= 62.4(0.088)82.4/550 = 0.819$ hp, checks closely

6.47 *Repeat Prob 6.46 with the following changes: radius = 400 mm, total jet area = 70 mm², h = 42 m.*
Prob. 6.46: The sprinkler (Fig. 6.14) has $\beta_2 = 160°$ and a rotative speed of 400 rpm. Compute Q, T, and the paver developed. Neglect fluid friction, but note that the calculated torque is that required to overcome mechanical friction and air resistance.

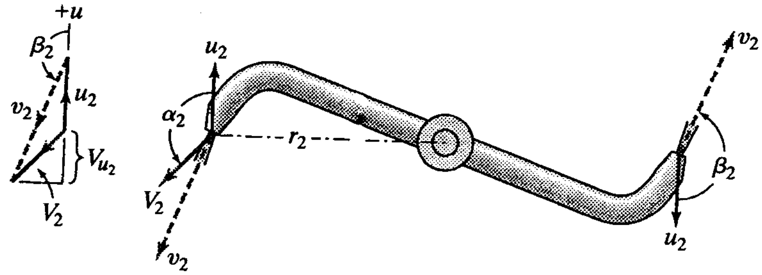


Figure. 6.14

SI

$\omega = 400(2\pi/60) = 41.9$ rad/s ; $u_2 = \omega r_2 = 41.9(0.4) = 16.76$ m/s

Eq. 6.35: $v_2 = \sqrt{2(9.81)42 + 16.76^2} = 33.2$ m/s ; $Q = a_2 v_2 = 0.000\,07(33.2) = 0.002\,33\ \text{m}^3/\text{s}$ ◄

Eq. 6.36: $T = -(1000)0.00233(0.4)(16.76 + 33.2\cos 160°) = 13.47$ N·m ◄

Eq. 5.38: Power $= T\omega = 13.47(41.9) = 564$ N·m/s $= 564$ W ◄

6.48 *For a lawn sprinkler like that of Fig. 6.14, develop an expression for the runaway speed ω in terms of h, r, and β_2. This would occur if there were no mechanical friction or air resistance, i.e., zero torque.*

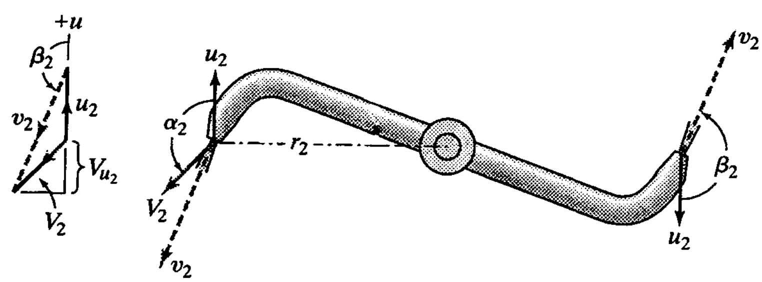


Figure. 6.14

N

From Eq. 6.36 with $T = 0$: $u_2 = -v_2\cos\beta_2$. ∴ using Eq. 6.35: $u_2 = -\cos\beta_2\sqrt{2gh + u_2^2}$

i.e. $\dfrac{u_2^2}{\cos^2\beta_2} = 2gh + u_2^2$ or $u_2^2\left(\dfrac{1}{\cos^2\beta_2} - 1\right) = 2gh$ or $u_2^2\left(\dfrac{1 - \cos^2\beta_2}{\cos^2\beta_2}\right) = u_2^2\tan^2\beta_2 = 2gh$

$\therefore\ \omega r_2|\tan\beta| = \sqrt{2gh}$; $\omega = \dfrac{\sqrt{2gh}}{r_2|\tan\beta|}$ ◄

6.49 *At what approximate speed will the sprinkler of Prob. 6.46 develop maximum horsepower?*

Prob. 6.46: The sprinkler (Fig. 6.14) has β_2 = 160°, a = 0.0008 ft², r = 15 in, h = 144 ft. Neglect fluid friction, but note that the calculated torque is that required to overcome mechanical friction and air resistance.

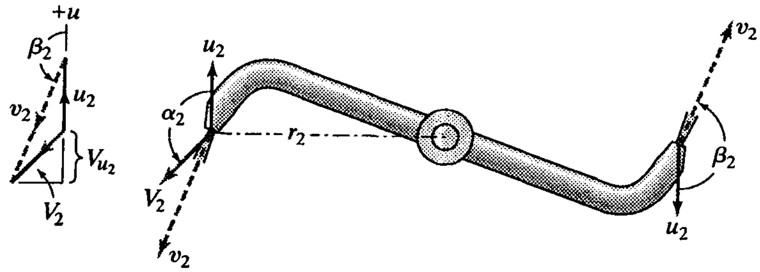


Figure 6.14

BG

Two methods of solution may be used:

(*a*) Express power as a function of ω. Take ∂ Power/$\partial\omega$ = 0 and solve for ω.

(*b*) Assume various ω's and compute the corresponding Powers.

Here we will use method (*b*).

$\omega = n(2\pi/60)$ rad/sec; $u_2 = \omega r_2 = \omega(15/12)$ fps

Eq. 6.35: $v_2 = \sqrt{2gh + u_2^2} = \sqrt{2(32.2)144 + u_2^2}$ fps ; $Q = a_2 v_2 = 0.0008 v_2$ cfs

$\rho Q r_2 = 1.940Q(15/2) = 2.425Q$ lb·sec

Sec. 6.14: $\Delta V_u = u_2 + v_2\cos\beta_2 = u_2 + v_2\cos 160° = u_2 - 0.940v_2$ fps

From Eq. 6.36: $T = -\rho Q r_2(\Delta V_u) = -2.425Q(u_2 - 0.940v_2)$ ft·lb

Eqs. 5.38 and 5.40: Power = $T\omega/550$ hp /cont...

n rpm	ω rad/sec	u_2 fps	v_2 fps	Q cfs	$\rho Q r_2$ lb·sec	ΔV_u fps	T ft·lb	Power hp	
200	20.9	26.2	99.8	0.0798	0.1936	-67.6	13.09	0.498	
600	62.8	78.5	124.3	0.0994	0.241	-38.2	9.22	1.053	
1000	104.7	130.9	162.5	0.1300	0.315	-21.8	6.87	1.309	
1200	125.7	157.1	184.2	0.1474	0.357	-16.06	5.74	1.311	(max)
1400	146.6	183.3	207	0.1656	0.402	-11.28	4.53	1.207	
1500	157.1	196.3	219	0.1750	0.424	-9.15	3.88	1.109	
1980	207	259	276	0.221	0.536	-0.637	0.342	0.1289	

Maximum power occurs at a rotative speed of approximately 1200 rpm ◄

Sec. 6.15: Momentum Principle Applied to Propellers and Windmills -- *Exercises (3)*

6.15.1 *A 18-in-diameter household fan drives air (γ = 0.076 lb/ft³) at a rate of 1.80 lb/sec. (a) Find the thrust exerted by the fan. (b) What is the pressure difference on the two sides of the fan? (c) Find the required horsepower to drive the fan. Neglect losses.*

BG

Sec. 6.15 for fan: $V_1 = 0$

Eq. 6.40: $\gamma Q = \gamma A(0 + \Delta V/2) = 1.8$ lb/sec ; $\Delta V = \dfrac{1.8(2)}{(0.076)\dfrac{\pi \times 18^2}{4 \times 144}} = 26.8$ fps

(*a*) Eq. 6.38: $F_T = \rho Q(\Delta V) = (1.8/32.2)26.8 = 1.498$ lb ◀

(*b*) Eq. 6.39 with $V_1 = 0$: $\dfrac{\Delta p}{\gamma} = \dfrac{(\Delta V)^2}{2g} = \dfrac{26.8^2}{2(32.2)} = 11.16$ ft ; $\Delta p = 0.076(11.16) = 0.848$ psf ◀

(*c*) Eqs. 5.35 and 5.38: Power required = $\dfrac{F_T V}{550} = \dfrac{F_T(\Delta V/2)}{550} = 0.0365$ hp ◀

or (Eq. 5.35) Power required = $\dfrac{\gamma QH}{550} = \dfrac{1.8(11.16)}{550} = 0.0365$ hp ◀

6.15.2 *A 1.8-m-diameter fan drives air (γ = 12 N/m³) at a rate of 50 N/s. (a) Find the thrust exerted by the fan. (b) What is the pressure difference on the two sides of the fan? (c) Find the required power to drive the fan. Neglect losses.*

SI

Sec. 6.15 for fan: $V_1 = 0$.

Eq. 6.40: $\gamma Q = \gamma A(0 + \Delta V/2) = 50$ N/s $= 12(\pi/4)1.8^2(\Delta V/2)$; $\Delta V = 3.27$ m/s

(*a*) Eq. 6.38: $F_T = \rho Q(\Delta V) = (50/9.81)3.27 = 16.69$ N ◀

(*b*) Eq. 6.39 with $V_1 = 0$: $\dfrac{\Delta p}{\gamma} = \dfrac{(\Delta V)^2}{2g} = \dfrac{3.27^2}{2(9.81)} = 0.547$m ; ◀

$\Delta p = 12(0.547) = 6.56$ N/m²

(*c*) Eq. 5.38: Power $= F_T V = F_T(\Delta V/2) = 16.69(3.27/2) = 27.3$ watts ◀

or (Eq. 5.39) Power $= \gamma QH = 50(0.547) = 27.3$ watts ◀

6.15.3 *A fan sucks air from outside to inside a building through a 20-in-diameter duct. The density of air is 0.0022 slug/ft³ and the pressure difference across the two sides of the fan is 4.0 in of water. (a) What thrust must the fan support be designed to withstand? (b) Determine the flow rate of the air in cubic feet per second.*

BG

(*a*) Δp = 4.0 inches of water = 0.333 ft of water ; $\Delta p = 0.333(62.4/144) = 0.1444$ psi

Eq. 6.37: $F_T = A\Delta p = (\pi 20^2/4)0.1444 = 45.4$ lb ◀

(*b*) Sec. 6.15 for fan: $V_1 = 0$. Eq. 6.37 with $V_1 = 0$: $Q = A(\Delta V/2)$

Eq. 6.38: $F_T = \rho Q(\Delta V) = \rho[A(\Delta V/2)](\Delta V)$; $45.4 = 0.0022(\pi/4)(20/12)^2(\Delta V^2/2)$;

$\Delta V = 137.5$ fps ; $Q = A(\Delta V/2) = 2.18(137.5/2) = 150.0$ cfs ◀

Sec. 6.15: Momentum Principle Applied to Propellers and Windmills -- *Problems 6.50–6.52*

6.50 By placing a 12-in electric fan on a frictionless mount it is observed to exert a thrust of 0.8 lb. (a) Find the approximate velocity of the slipstream of standard air (sea level) which it produces. (b) If 45 percent of the power supplied to the blades is lost in eddies and friction and if the driving motor has an efficiency of 60 percent, find the required electrial input in watts.

B

(*a*) Eq. 6.37: $F_T = 0.8\text{ lb} = (\Delta p)\pi 1^2/4 = 0.785\Delta p$; $\Delta p = 1.019$ psf

Table A.3 for standard air at sea level: $\gamma = 0.0765\ \text{lb/ft}^3$

Sec 6.15 for fan: $V_1 = 0$. ∴ Eq. 6.36 becomes: $\Delta p/\gamma = (\Delta V)^2/2g$

from which $\Delta V = \sqrt{2g(\Delta p/\gamma)} = \sqrt{2(32.2)1.019/0.0765} = 29.3$ fps

Slipstream velocity $= V_1 + \Delta V = 0 + \Delta V = 29.3$ fps ◄

(*b*) Eq. 6.40 with $V_1 = 0$: $Q = A(\Delta V/2) = 0.785(29\ 3/2) = 11.50$ cfs

Eq. 5.39: $\text{Power}_{\text{blades}} = Q\Delta p = 11.50(1.019) = 11.71$ ft·lb/sec

Eqs. 5.39 and 5.42: Required power

$$= \frac{11.71}{(1 - 0.45)0.60}\ \text{ft·lb/sec} = \frac{11.71}{(1 - 0.45)0.60} \times \frac{745.7}{550} = 48.1\ \text{watts}$$ ◄

6.51 *By placing a 300-mm electric fan on a frictionless mount it is observed to exert a thrust of 2.5 N. (a) Find the approximate velocity of the slipstream of standard air (sea level) which it produces. (b) If 45 percent of the power supplied to the blades is lost in eddies and friction and if the driving motor has an efficiency of 60 percent, find the required electrial input in watts.*

SI

(*a*) Eq. 6.37: $F_T = 2.5\text{ N} = \Delta p(\pi 0.15^2) = 0.0707\Delta p$ $\Delta p = 35.4\ \text{N/m}^2$

Table A.3 for standard air at sea level: $\gamma = 12.01\ \text{N/m}^3$

Sec 6.15 for fan: $V_1 = 0$. ∴ Eq. 6.36 becomes: $\Delta p/\gamma = (\Delta V)^2/2g$

from which $\Delta V = \sqrt{2g(\Delta p/\gamma)} = \sqrt{2(9.81)35.4/12.01} = 7.60$ m/s

Slipstream velocity $= V_1 + \Delta V = 0 + \Delta V = 7.60$ m/s ◄

(*b*) Eq. 6.40 with $V_1 = 0$: $Q = A(\Delta V/2) = 0.707(7.60/2) = 0.269$ m/s

Eq. 5.39: $\text{Power}_{\text{blades}} = Q\Delta p = 0.269(35.4) = 9.50$ watts

From Eq. 5.42: Required power $= \dfrac{9.50}{(1 - 0.45)0.60} = 28.8$ watts ◄

6.52 *Apply the momentum and energy principles to the case of a windmill (essentially the opposite of a propeller), to determine the maximum theoretical effciency based on an input energy available from the wind velocity in a stream tube having a cross section equal to that of the windmill blade circle.*

N

Wind velocity V_1 ; Velocity through windmill = $V_1 - \Delta V/2$ where $\Delta V = V_1 - V_4$

Eq. 5.39: $P_{out} = \gamma Q\left[\frac{V_1^2}{2g} - \frac{(V_1 - \Delta V)^2}{2g}\right]$ where $Q = A\left(V_1 - \frac{\Delta V_1}{2}\right)$

Eq. 5.39: $P_{in} = \gamma Q V_1^2/2g$ where $Q = AV_1$

$$\frac{P_{out}}{P_{in}} = \text{efficiency} = \frac{[V_1 - (\Delta V/2)][V_1^2 - (V_1 - \Delta V)^2]}{V_1(V_1^2)} = \frac{[V_1 + (V_1 - \Delta V)][V_1^2 - (V_1 - \Delta V)^2]}{2V_1^3}$$

$$= \text{efficiency} = \frac{[1 + (V_1 - \Delta V)/V_1]^2[1 - (V_1 - \Delta V)/V_1]}{2}$$

$$\frac{\partial(\text{eff})}{\partial\left(\frac{V_1 - \Delta V}{V_1}\right)} = \frac{\left[1 + \left(\frac{V_1 - \Delta V}{V_1}\right)\right]^2(-1) + \left[1 - \left(\frac{V_1 - \Delta V}{V_1}\right)\right]2\left[1 + \left(\frac{V_1 - \Delta V}{V_1}\right)\right]}{2} = 0$$

This simplifies to: $3\left(\frac{1 - \Delta V}{V_1}\right)^2 + 2\left(\frac{1 - \Delta V}{V_1}\right) - 1 = 0$; $\frac{V_1 - \Delta V}{V_1} = \frac{-2 \pm \sqrt{4 + 12}}{6} = 1/3$ or -1

Hence efficiency is maximum when $\frac{V_1 - \Delta V}{V_1} = 1/3$ or when $\Delta V = (2/3)V_1$

$$(\text{eff})_{max} = \frac{(1 + 1/3)^2(1 - 1/3)}{2} = \frac{16}{27} = 0.593 \quad ◀$$

Chapter 7

Similitude and Dimensional Analysis

PROBLEM SELECTION GUIDE

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
7.4	*Dynamic Similarity*						
	X[1]7.4.1	BG	Easy	V Short	1		Interpolation
	7.4.2	SI	V Easy	V Short	1		
	7.4.3	BG	Easy	Short	1	P7.12	
	7.4.4	BG	V Easy	Medium	1		Uses $\nu = \mu/\rho$ (Sec. 2.11)
	7.4.5	BG	Easy	Short	1		
	7.4.6	N	Easy	Medium	1		Uses Secs. 2.7 & 2.11
	7.4.7	SI	Medium	Medium	1		
	7.4.8	SI	Medium	Medium	1		
	P 7.1	BG	Easy	Short	1	7.2	Interpolation; uses Secs. 2.7 & 2.11
	7.2	SI	Easy	Short	1	7.1	Interpolation; uses Secs. 2.7 & 2.11
	7.3	BG	Easy	Short	1		
	7.4	SI	Easy	Short	1		Uses $pv = RT$ (Sec. 2.7)
	7.5	N	Medium	Short	8		
	7.6	BG	Medium	Medium	1	7.8	Uses Secs. 2.7 and 4.5
	7.7	SI	Medium	Medium	1		Uses $pv = RT$ (Sec. 2.7)
	7.8	SI	Medium	Medium	1	7.6	Uses Secs. 2.7 and 4.5
	7.9	B	Hard	Medium	1		Unit convers'ns; uses $\nu = \mu/\rho$, Fig. A.2
	7.10	BG	Medium	Medium	1		
	7.11	N	Medium	Medium	2		
	7.12	BG	Medium	Short	1	X7.4.3	
	7.13	BG	Medium	Short	1		
	7.14	SI	Medium	Short	1		
	7.15	BG	Medium	Short	2		
	7.16	BG	Medium	Medium	2	7.17	Uses Secs. 2.8 and 13.3
	7.17	SI	Medium	Medium	1	7.16	
	7.18	BG	Medium	Medium	2	7.19	Interpolation: uses Secs. 2.7-8
	7.19	SI	Medium	Medium	2	7.18	Interpolation: uses Secs. 2.7-8
7.5	*Scale Ratios*						
	P 7.20	N	Medium	Short	1		
	7.21	N	Medium	Short	1		
	7.22	N	Medium	Medium	1		
	7.23	N	Medium	Medium	1		
	7.24	N	Medium	Medium	1		
	7.25	BG	Hard	Medium	2	7.26	Uses Sec. 4.5
	7.26	SI	Hard	Medium	2	7.25	Uses Sec. 4.5

/cont...

[1] For all Exercises (identified by "X"), answers are given in Appendix F of the textbook.
X = Exercise, P = (end-of-chapter) Problem, S = Sample Problem.

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
7.7	***Dimensional Analysis***						
	X 7.7.1	N	Medium	Medium	2		
	7.7.2	N	Medium	Medium	2		
	7.7.3	N	Medium	Long	1		
	7.7.4	N	Medium	Long	1		
	7.7.5	N	Medium	Long	1		
	P 7.27	N	Medium	Medium	2		Needs literature search for part (*b*)
	7.28	N	Medium	Medium	2		Needs literature search for part (*b*)
	7.29	N	Medium	Medium	2		Needs literature search for part (*b*)
	7.30	N	Medium	Long	1		Uses Sec. 2.12
	7.31	N	Medium	Long	1		
	7.32	N	Medium	Long	1		
	7.33	N	Hard	Long	1		

Chapter 7
SIMILTUDE AND DIMENSIONAL ANALYSIS

Sec. 7.4: Dynamic Similarity -- *Exercises (8)*

7.4.1 *Water at 65°F in an 8-in-diameter pipe flows with a velocity of 5 fps. What is the Reynolds number? Note that in the Reynolds number the significant length L = D, and that Appendix A gives physical properties of water.*

BG

Table A.1 for water at 65°F: $\nu = 11.38 \times 10^{-6}$ ft²/sec (by interpolation)

Eq 7.6: $\mathbf{R} = \dfrac{LV}{\nu} = \dfrac{DV}{\nu} = \dfrac{(8/12)5}{11.38 \times 10^{-6}} = 293{,}000$ ◄

7.4.2 *Oil (s = 0.85 and μ = 0.24 N·s/m²) in a 100-mm-diameter pipe flows with a velocity of 3.5 m/s. What is the Reynolds number? Note that in the Reynolds number the significant length L = D.*

SI

Eq 7.6: $\mathbf{R} = \dfrac{LV}{\nu} = \dfrac{DV\rho}{\mu} = \dfrac{(0.10\text{ m})(3.5\text{ m/s})(0.85 \times 10^3\text{ kg/m}^3)}{0.24\text{ N·s/m}^2}\left(\dfrac{\text{N}}{\text{kg·m/s}^2}\right) = 1240$ ◄

7.4.3 *A vertical jet of water issuing upward from a nozzle at a velocity of 76 fps will rise to a height of approximately 90 ft on the earth. To get a water jet to rise to a height of 120 ft on the moon, where gravity is one-sixth of that on earth, what must be the jet velocity? Neglect atmospheric resistance.*

BG

Gravity and inertia dominate, so **F** governs, and $\mathbf{F}_e = \mathbf{F}_m$.

Eq. 7.1: $L_r = L_e/L_m = 90/120 = 0.75$; $g_r = g_e/g_m = 6$

Eq. 7.9: $[V/(gL)^{1/2}]_e = [V/(gL)^{1/2}]_m$; $\therefore V_e/V_m = \sqrt{(g_e/g_m)(L_e/L_m)} = \sqrt{6(0.75)} = 2.12 = V_r$ (Eq. 7.2)

$V_m = V_e/V_r = 76/2.12 = 35.8$ fps ◄

7.4.4 *A wind tunnel test on a 1:40 scale model of a submarine is planned to find the drag on the submarine when it is moving at 10 knots through 50°F ocean water. What should the test velocity be in the wind tunnel if it contains 60°F air at atmospheric pressure? If the drag on the model is 75 lb, what will the drag on the prototype be?*

BG

R governs because the significant forces are inertia and fluid friction due to viscosity.

$\therefore$ (Eq. 7.6) $\mathbf{R} = DV/\nu$ = constant. Table A.1 for fresh water at 50°F: $\nu = 14.10 \times 10^{-6}$ ft²/sec.

Using Table A.4 and Eq. 2.11: $\dfrac{\nu_{\text{ocean}}}{\nu_{\text{fresh}}} = \dfrac{\mu_o\,\rho_f}{\rho_o\,\mu_f} = \dfrac{(2.25 \times 10^{-6})1.936}{1.985(2.10 \times 10^{-6})} = 1.045$

$\therefore$ at 50°F, $\nu_{\text{ocean}} = \nu_p = 1.045(14.10 \times 10^{-6}) = 14.73 \times 10^{-6}$ ft²/sec

Table A.2 for air at 60°F: $\nu_m = 0.158 \times 10^{-3}$ ft²/sec ; From inside cover: 10 knots = 16.88 fps = V_p

$\mathbf{R}_m = \mathbf{R}_p$: $\dfrac{(1/40)V_m}{0.158 \times 10^{-3}} = \dfrac{(1)16.88}{14.73 \times 10^{-6}}$; $V_m = 7240$ fps = 4940 mph ◄

Since V_m greatly exceeds sonic velocity (Table A.3), the model will not properly indicate prototype behavior, including drag, due to compressibility effects (Sec. 2.4). ◄

Subscript *m* denotes model, subscript *p* denotes prototype, and subscript *r* denotes relative or ratio in this chapter.

7.4.5 *A 500-ft-long ship is to operate at a speed of 20 mph If a model of this ship is 10 ft long, what should its speed be in fps to give the same Froude number? What is the value of this Froude number?*

BG

Given $\mathbf{F}_p = \mathbf{F}_m$; Eq 7.8: $\mathbf{F} = V/(gL)^{1/2}$

$[V/(gL)^{1/2}]_p = [V/(gL)^{1/2}]_m$; $V_m = V_p(g_m/g_p)^{1/2}(L_m/L_p)^{1/2} = V_p(10/500)^{1/2}$

$V_m = 20$ mph/7.07 = 2.83 mph = 4.15 fps ◄ $\mathbf{F} = 4.15/(32.2 \times 10)^{1/2} = 0.231$ ◄

7.4.6 *The dimensions of a model airplane are 1/30 those oj its prototype. In planned tests in a pressure wind tunnel the model will operate at the same speed and air temperature as the prototype. What pressure in the wind tunnel, relative to the atmospheric pressure, will make the Reynolds number the same?*

N

Given $L_r = L_m/L_p = 1/30$ and $V_m = V_p$. Substituting these into

$L_m V_m/\nu_m = L_p V_p/\nu_p$ (since $\mathbf{R}_m = \mathbf{R}_p$, given) we obtain $\nu_m/\nu_p = (L_m/L_p)(V_m/V_p) = (1/30)1 = 1/30$

Eliminating ρ from Eqs. 2.4 and 2.11: $p\nu/\mu = RT$ or $p\nu = \mu RT$.
Since μ changes little with pressure (Sec. 2.11), for constant temperature μRT = constant.

So $p\nu$ = constant, and $p_m = (\nu_p/\nu_m)p_p = 30p_p = 30$ atmospheres ◄

7.4.7 *Water flows over the crest of a 1:35 model spillway, and the velocity measured at a particular point is 0.46 m/s. What velocity does this represent in the prototype? The force exerted on a certain area of the model is measured to be 0.12 N. What would be the force on the corresponding area in the prototype? Develop your own dimensionless ratios.*

SI

Gravity and inertia govern this flow, so the Froude criterion must be satisfied.

Equating Froude numbers, Eq. 7.9: $[V/(gL)^{1/2}]_m = [V/(gL)^{1/2}]_p$

$\therefore\ V_r = V_p/V_m = (L_p/L_m)^{1/2} = 35^{1/2} = 5.92$; $V_p = V_r V_m = 5.92(0.46) = 2.72$ m/s ◄

$$F_r = \frac{(F_I)_p}{(F_I)_m} = \frac{(\rho V^2 L^2)_p}{(\rho V^2 L^2)_m} = \rho_r V_r^2 L_r^2 = (1)L_r(L_r^2) = L_r^3$$

$$\text{or:}\quad F_r = \frac{(F_G)_p}{(F_G)_m} = \frac{(\rho L^3 g)_p}{(\rho L^3 g)_m} = \rho_r L_r^3 g_r = (1)L_r^3(1) = L_r^3. \quad \therefore\ F_p = L_r^3 F_m = 35^3(0.12) = 5150\ \text{N}$$ ◄

7.4.8 *A certain missile travels at 2500 km/h through the atmosphere at an elevation of 5 km. Air at 20°C and standard atmospheric conditions will flow around a model of the missile in a wind tunnel. What air speed in the wind tunnel will achieve dynamic similarity?*

SI

Compressibility is important at supersonic speeds, so $\mathbf{M}$ governs, and $\mathbf{M}_p = \mathbf{M}_m$.

Table A.3 at 5 km: $p = 54.0$ kPa abs, $\rho = 0.736$ kg/m³

Table A.2 at 20°C: $\rho = 1.205$ kg/m³. Eq. 7.10: $\mathbf{M} = V/(E_v/\rho)^{1/2}$ where (Eq. 2.8) $E_v = np$

Thus $[V/(p/\rho)^{1/2}]_p = [V/(p/\rho)^{1/2}]_m$ as n is the same for both.

$$\frac{2500\ \text{km/h}}{(54.0/0.736)^{1/2}} = \frac{V_m}{(101.3/1.205)^{1/2}}\ ;\quad V_m = 2680\ \text{km/h} = 743\ \text{m/s}$$ ◄

Sec. 7.4: Dynamic Similarity -- *Problems 7.1−7.19*

7.1 *What is the Reynolds number for 110°F air at a pressure of 90 psia when it flows at a velocity of 120 fps through a 4-in-diameter pipe? Note that Appendix A gives physical properties of air.*

BG

Table A.2 for air at 110°F and 14.7 psia: $\nu = 0.1845 \times 10^{-3}$ ft²/sec (by interpolation)

Eliminating ρ from Eqs. 2.4 and 2.11: $p\nu/\mu = RT$ or $p\nu = \mu RT$.
Since μ changes little with pressure (Sec. 2.11), for constant temperature μRT = constant.

so $p\nu$ = constant and $\nu_2 = (p_1/p_2)\nu_1 = (14.7/90)0.1845 \times 10^{-3} = 30.1 \times 10^{-6}$ ft²/sec

Eq 7.6: $\mathbf{R} = LV/\nu = DV/\nu = (4/12)120/(30.1 \times 10^{-6}) = 1{,}327{,}000$ ◄

7.2 *What is the Reynolds number for 50°C air at a pressure of 650 kPa abs when it flows at a velocity of 35 m/s through a 100-mm-diameter pipe? Note that Appendix A gives physical properties of air.*

SI

Table A.2 for air at 50°C and 101.3 kN/m²: $\nu = 17.75 \times 10^{-6}$ m²/s (by interpolation)

Eliminating ρ from Eqs. 2.4 and 2.11: $p\nu/\mu = RT$ or $p\nu = \mu RT$.
Since μ changes little with pressure (Sec. 2.11), for constant temperature μRT = constant.

so $p\nu$ = constant and $\nu_2 = (p_1/p_2)\nu_1 = (101.3/650)17.75 \times 10^{-6} = 2.77 \times 10^{-6}$ m²/s

Eq 7.6: $\mathbf{R} = LV/\nu = DV/\nu = (0.10)35/(2.77 \times 10^{-6}) = 1\ 265\ 000$ ◄

7.3 *The linear dimensions of a model airplane are 1/15 those of its prototype. If the protoype is to fly at 450 mph, what must be the air velocity in a wind tunnel for the same Reynolds number if the air temperature and pressure are the same?*

BG

$L_r = L_m/L_p = 1/15$; $\nu_m = \nu_p$ due to same air temperature and pressure.

$\mathbf{R}_m = \mathbf{R}_p$, so $L_mV_m/\nu_m = L_pV_p/\nu_p$; $V_m = V_p(L_p/L_m) = 450(15/1) = 6750$ mph ◄

This answer, which indicates a velocity about 9 times the speed of sound, illustrates why it is not generally possible to test model aircraft by the Reynolds model law in atmospheric wind tunnels.

7.4 *Air at 150°C and a pressure of 240 kPa abs flows at a velocity of 16 m/s through an 180-mm-diameter pipe. What is the Reynolds number?*

SI

Table A.5 for air: R = 287 N·m/(kg·K). From Eq. 2.4: $\rho = \dfrac{p}{RT} = \dfrac{240 \times 10^3}{287(150+273)} = 1.977$ kg/m³

From Table A.2 by interpolation (or Fig. 2.3): $\mu = 23.8 \times 10^{-6}$ N·s/m²

Eq 7.6: $\mathbf{R} = LV\rho/\mu = DV\rho/\mu = 0.18(16)1.977/(23.8 \times 10^{-6}) = 239\ 000$ ◄

7.5 *Models are to be built of the following prototypes: (a) tides; (b) oil flowing through a full pipeline; (c) a water jet; (d) flow over the spillway of a dam; (e) a deep submersible vehicle; (f) an airplane flying at low speed; (g) a supersonic aircraft; (h) a supersonic missile. For dynamic similarity, indicate which single dimensionless ratio will govern, and give reasons why.*

N

Governing dimensionless ratios are:

R for parts (*b*), (*e*), and (*f*), because for these the significant forces are inertia and fluid friction due to viscosity (air compressibility is not appreciable at low airplane speeds). ◄

F for parts (*a*), (*c*), and (*d*), because for these the significant forces are inertia and gravity. ◄

M for parts (*g*) and (*h*), because for these compressibility is important. ◄

7.6 *Air at 68°F and 60 psia flows in a 1.5-in-diameter pipe. What weight flow rate of this air will give dynamic similarity to 60°F water flowing at 200 gpm through a 3-in-diameter pipe?*

BG

Table A.1 for water at 60°F: $\nu = 12.17 \times 10^{-6}$ ft²/sec

Velocity of water, $V = Q/A = (200/449)/[\pi(3/12)^2/4] = 9.08$ fps

Table A.5 for air: $R = 1715$ ft²/(sec²·°R)

Eq. 2.5 for air at 68°F and 60 psia: $\gamma = \dfrac{gp}{RT} = \dfrac{32.2(60)144}{1715(68+460)} = 0.307$ lb/ft³

Table A.2 for air at 68°F and any pressure (Sec 2.11), by interpolation: $\mu = 0.380 \times 10^{-6}$ lb·sec/ft²

$\therefore\ \nu_{air} = \mu/\rho = 0.380 \times 10^{-6}/(0.307/32.2) = 39.9 \times 10^{-6}$ ft²/sec

For dynamic similarity: $\mathbf{R}_{water} = \mathbf{R}_{air}$.

Using Eq. 7.6: $\dfrac{(3/12)9.08}{12.17 \times 10^{-6}} = \dfrac{(1.5/12)Q/[(\pi/4)(1.5/12)^2]}{39.9 \times 10^{-6}}$; $Q = 0.730$ cfs ◄

Eq 4.5: Weight flowrate of air, $G = \gamma Q = 0.307(0.730) = 0.224$ lb/sec ◄

7.7 *When a submerged sphere moves through 15°C water at 2.0 m/s (Fig. P7.7), the drag force exerted on it is 16 N. In wind tunnel tests with another sphere of three times the diameter the air pressure and temperature are 1.8 MPa abs and 290 K respectively. What must the air velocity be for dynamic similarity, and what will the drag force then be on the larger sphere?*

SI

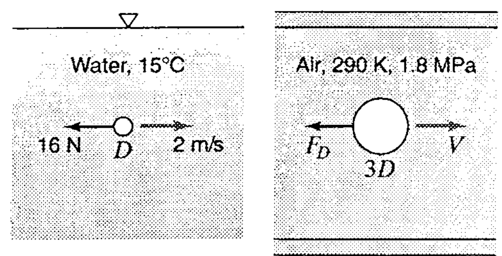


Figure P7.7

Table A.1 for water at 15°C:

$\rho = 999$ kg/m³, $\mu = 0.001\,139$ N·s/m²

Table A.2 for air at 17°C (= 290K) and any pressure (Sec 2.11), by interpol'n: $\mu = 17.95 \times 10^{-6}$ N·s/m²

Table A.5 for air: $R = 287$ N·m/(kg·K)

For air, from Eq. 2.4: $\rho = \dfrac{p}{RT} = \dfrac{1.8 \times 10^6\ \text{N/m}^2}{(287\ \text{N·m/kg·K})290\ \text{K}} = 21.6$ kg/m³

For dynamic similarity: $\mathbf{R}_{water} = \mathbf{R}_{air}$.

Using Eq. 7.6: $\therefore\ \dfrac{D(2.0)999}{0.001\,139} = \dfrac{3D(V_a)21.6}{17.95 \times 10^{-6}}$; $V_a = 0.485$ m/s ◄

$$F_r = \frac{(F_l)_a}{(F_l)_w} = \frac{(\rho V^2 L^2)_a}{(\rho V^2 L^2)_w} = \left(\frac{21.6}{999}\right)\left(\frac{0.485}{2.0}\right)^2\left(\frac{3D}{D}\right)^2 = 0.01147$$

$\therefore\ F_a = F_r F_w = 0.01147(16) = 0.1836$ N ◄

7.8 *Air at 80°C and 475 kPa abs pressure flows in a 50-mm-diameter pipe. What air flow rate (kg/s) will give dynamic similarity to 70 L/s of 60°C water flowing in a 540-mm-diameter pipe?*

SI

Table A.5 for air: $\mathbf{R} = 287\ \text{N·m/(kg·K)}$

Air, Eq. 2.4: $\rho_a = \left(\frac{p}{RT}\right)_a = \frac{475 \times 10^3\ \text{N/m}^2}{(287\ \text{N·m/kg·K})(80 + 273)\ \text{K}} = 4.69\ \text{kg/m}^3$

Table A.2 for air at 80°C and any pressure (Sec. 2.11): $\mu_a = 20.9 \times 10^{-6}\ \text{N·s/m}^2$

Water, Eq. 4.7: $V_w = Q_w/A_w = 0.070/(\pi 0.27^2) = 0.306\ \text{m/s}$

Table A.1 for water at 60°C: $\nu_w = 0.474 \times 10^{-6}\ \text{m}^2\text{/s}$

For dynamic similarity: $\mathbf{R}_{\text{air}} = \mathbf{R}_{\text{water}}$.

Using Eq. 7.6: $\therefore\ \frac{0.05(V_a)4.69}{20.9 \times 10^{-6}} = \frac{0.54(0.306)}{0.474 \times 10^{-6}}$; $V_a = 31.0\ \text{m/s}$

Eq. 4.4: $\dot{m}_a = (\rho A V)_a = 4.69(\pi 0.025^2)31.0 = 0.286\ \text{kg/s}$ ◄

7.9 *A 500-ft-long ship will operate at a speed of 20 mph in ocean water whose viscosity is 1.2 cP and specific weight is 64 lb/ft³. What should be the kinematic viscosity of the liquid used with a 10-ft-long model of the ship so that both the Reynolds number and the Froude number would be the same? Does such a liquid exist?*

B

$\mathbf{R}_p = \mathbf{R}_m$; $\mathbf{F}_p = \mathbf{F}_m$; $L_r V_r/\nu_r = 1$; $V_r/(g_r L_r)^{1/2} = 1$

To satisfy both **R** and **F**, $[LV/\nu]_r = [V/(gL)^{1/2}]_r$, i.e. $\nu_r = [(gL)^{1/2}L]_r = L_r^{3/2}$ assuming $g_r = 1.0$

$L_r = L_p/L_m = 500/10 = 50$; $\nu_r = \nu_p/\nu_m = (50)^{3/2} = 354$

$\mu_p = 1.2\ \text{cP} = 1.2 \times 10^{-3}\ \text{N·s/m}^2 = 0.0209(1.2 \times 10^{-3})\ \text{lb·sec/ft}^2 = 2.51 \times 10^{-5}\ \text{lb·sec/ft}^2$

Eq 2.11: $\nu_p = \mu_p/\rho = 2.51 \times 10^{-5}/(64/32.2) = 1.261 \times 10^{-5}\ \text{ft}^2\text{/sec}$

$\nu_m = \nu_p/\nu_r = 1.261 \times 10^{-5}/354 = 3.57 \times 10^{-8}\ \text{ft}^2\text{/sec}$ ◄

Fig A.2: There is no such liquid available. ◄

7.10 *Water flows over a spillway at 5000 cfs. For dynamic similarity, what should the model scale be if the flow rate over the model is to be 45 cfs? The force exerted on a certain area of the model is 1.0 lb. What would the force be on the corresponding area of the prototype?*

BG

Gravity and inertia govern, so satisfies Froude's number. Equating these, we get

Eq 7.2: $V_r = V_p/V_m = (L_p/L_m)^{1/2} = L_r^{1/2}$ assuming $g_m = g_p$

$Q_r = (A \times V)_r = (L^2 \times L^{1/2})_r = L_r^{5/2}$; $(5000/45) = L_r^{5/2}$;

$L_r = L_p/L_m = 6.58$; $\lambda = L_m/L_p = 1:6.58$ ◄

$F_r = (\rho V^2 L^2)_r = \rho_r V_r^2 L_r^2 = 1(L_r)L_r^2 = L_r^3$ assuming $\rho_p = \rho_m$. [Alternatively, $F_r = (\rho Q V)_r$.]

$(F_p/1.0) = (6.58)^3 = 285$; $F_p = 1.0(285) = 285\ \text{lb}$ ◄

7.11 *A 1:600 scale model is built to study tides. (a) What length of time in the model corresponds to one day in the prototype? (b) Suppose this model could be tested on the moon where g is one-sixth of that on earth. What then would be the time relationship between the model and prototype?*

N

Gravity and inertia dominate, so **F** governs, and $\mathbf{F}_p = \mathbf{F}_m$ (Eq. 7.9).

(*a*) $[V/(gL)^{1/2}]_p = [V/(gL)^{1/2}]_m$; $\therefore\ V_r = (gL)_r^{1/2}$. Eq. 7.3: $T_r = L_r/V_r = L_r/(gL)_r^{1/2} = \sqrt{L_r/g_r}$

$T_m = T_p/T_r = T_p/\sqrt{L_r/g_r} = T_p\sqrt{g_r/L_r} = T_p\sqrt{(g_p/g_m)/(L_p/L_m)}$

On earth: $T_m = (24\text{ hr})\sqrt{(1/1)/(600/1)} = 24\text{ hr}/24.5 = 0.980\text{ hr}$ ◄

(*b*) On the moon: $T_m = (24\text{ hr})\sqrt{(6/1)/(600/1)} = 24\text{ hr}/10 = 2.40\text{ hr}$ ◄

7.12 *A vertical jet of water issuing upward from a nozzle at a velocity of 44 fps will rise to a height of approximately 30 ft on the earth. To get a water jet to rise to a height of 120 ft on the moon, where the gravity in one-sixth of that on earth, what must the jet velocity be? Neglect atmospheric resistance.*

BG

Gravity and inertia dominate, so **F** governs, and $\mathbf{F}_e = \mathbf{F}_m$.

Eq. 7.1: $L_r = L_e/L_m = 30/120 = 0.25$; $g_r = g_e/g_m = 6$

Eq. 7.9: $[V/(gL)^{1/2}]_e = [V/(gL)^{1/2}]_m$; $\therefore\ V_e/V_m = \sqrt{(g_e/g_m)(L_e/L_m)} = \sqrt{6(0.25)} = 1.225 = V_r$ (Eq. 7.2)

$V_m = V_e/V_r = 44/1.225 = 35.9\text{ fps}$ ◄

7.13 *A 3-ft-high sectional model of a spillway is built in a 1-ft-wide laboratory flume. The flow is 0.86 cfs under a head of 0.380 ft. If the model scale is 1:20 and the prototype spillway is 600 ft long, what flow does this represent in the prototype?*

BG

$L_r = L_p/L_m = 20$; $g_r = 1.0$. Gravity and inertia dominate, so **F** governs, and $\mathbf{F}_m = \mathbf{F}_p$ (Eq. 7.9).

$[V/(gL)^{1/2}]_p = [V/(gL)^{1/2}]_m$ so $V_r = (gL)_r^{1/2} = \sqrt{g_rL_r} = \sqrt{1(20)} = 4.47$; $A_r = L_r^2 = 20^2 = 400$

$Q_r = A_rV_r = 400(4.47) = 1789$; $Q_m = (0.86\text{ cfs/ft})(600/20\text{ft}) = 25.8\text{ cfs}$

$Q_p = Q_mQ_r = (25.8\text{ cfs})1789 = 46{,}200\text{ cfs}$ ◄

7.14 *A model spillway has a flow of 100 L/s per m of width. What is the actual flow for the prototype spillway if the model scale is 1:20?*

SI

$L_r = L_p/L_m = 20$; $g_r = 1.0$. Gravity and inertia dominate, so F governs, and $\mathbf{F}_m = \mathbf{F}_p$ (Eq. 7.9).

$[V/(gL)^{1/2}]_p = [V/(gL)^{1/2}]_m$ so $V_r = (gL)_r^{1/2} = \sqrt{g_rL_r} = \sqrt{1\times 20} = 4.47$

$q = Q/b = AV/b = byV/b = yV$; $q_r = y_rV_r = L_rV_r = 20(4.47) = 89.4$

$q_p = q_rq_m = 89.4(0.1\text{ m}^3\text{/s per m}) = 8.94\text{ m}^3\text{/s per m}$ ◄

7.15 *When traveling at a velocity of 3 fps a 1:50 scale model of a ship has a wave resistance of 0.07 lb. This is kinematically similar to the design velocity of the prototype ship. (a) What is the design velocity of the prototype in the same water, and (b) what is its wave resistance at that velocity?*

BG

Gravity and inertia dominate, so **F** governs, and $\mathbf{F}_p = \mathbf{F}_m$. Eq. 7.9: $[V/(gL)^{1/2}]_p = [V/(gL)^{1/2}]_m$

(*a*) Eq. 7.2: $V_r = V_p/V_m = \sqrt{\dfrac{g_p}{g_m}\dfrac{L_p}{L_m}} = \sqrt{\dfrac{1}{1}\dfrac{50}{1}} = 7.07$; $V_p = V_rV_m = 7.07(3\text{ fps}) = 21.2\text{ fps}$ ◄

(*b*) $F_r = (\rho V^2L^2)_r = \rho_rV_r^2L_r^2 = 1(7.07)^2(50/1)^2 = 125{,}000$ since $\rho_p = \rho_m$.

$\therefore\ F_p = F_rF_m = 125{,}000(0.07\text{ lb}) = 8750\text{ lb}$ ◄

7.16 *An aircraft company is investigating the flow about a model of a supersonic plane in a variable-density wind tunnel at 1500 fps; the air, at 80°F, has a pressure of 20 psia. What should the velocity be to maintain dynamic similarity if the air temperature is raised to 90°F and the pressure is increased to 30 psia? Solve this two ways: (a) using specific weights and densities, and (b) not using them.*

BG

Compressibility is important at supersonic speeds, so **M** governs, and $\mathbf{M}_1 = \mathbf{M}_2$.

(*a*) Table A.5 for air: $R = 1715\ \text{ft}^2/(\text{sec}^2\cdot°\text{R})$. Eq. 2.5: $\gamma = pg/RT$

$\gamma_1 = 20(144)32.2/[1715(460+80)] = 0.1001\ \text{lb/ft}^3$

$\gamma_2 = 30(144)32.2/[1715(460+90)] = 0.1475\ \text{lb/ft}^3$

Eq. 7.10: $\mathbf{M} = V/(E_v/\rho)^{1/2} = V\sqrt{\rho/E_v}$ where $\rho = \gamma/g$ and $E_v = np$ (Eq. 2.8)

Thus $[V\sqrt{\gamma/p}]_1 = [V\sqrt{\gamma/p}]_2$ as g and n are the same for both.

$1500\sqrt{0.1001/20} = V_2\sqrt{0.1475/30}$; $V_2 = 1514$ fps ◀

(*b*) Eq. 7.10: $\mathbf{M} = V/c$ where (Eq. 13.15) $c = \sqrt{kRT}$

Thus $[V/\sqrt{T}]_1 = [V/\sqrt{T}]_2$ as k and R are the same for both.

$1500/\sqrt{460+80} = V_2/\sqrt{460+90}$; $V_2 = 1514$ fps ◀

7.17 *An aircraft company is investigating the flow about a model of a supersonic plane in a variable density wind tunnel at 400 m/s; the air, at 40°C, has a pressure of 150 kPa abs. What should the velocity be to maintain dynamic similarity if the air temperature is raised to 75°C and the pressure is increased to 200 kPa abs?*

SI

Compressibility is important at supersonic speeds, so **M** governs, and $\mathbf{M}_1 = \mathbf{M}_2$.

Table A.5 for air: $R = 287\ \text{m}^2/(\text{s}^2\cdot\text{K})$. Eq. 2.5: $\gamma = pg/RT$

$\gamma_1 = 150\,000(9.81)/[287(273+40)] = 16.38\ \text{N/m}^3$; $\gamma_2 = 200\,000(9.81)/[287(273+75)] = 19.64\ \text{N/m}^3$

Eq. 7.10: $\mathbf{M} = V/(E_v/\rho)^{1/2} = V\sqrt{\rho/E_v}$ where $\rho = \gamma/g$ and $E_v = np$ (Eq. 2.8)

Thus $[V\sqrt{\gamma/p}]_1 = [V\sqrt{\gamma/p}]_2$ as g and n are the same for both.

$400\sqrt{16.38/150\,000} = V_2\sqrt{19.64/200\,000}$; $V_2 = 422$ m/s ◀

7.18 *Researchers plan to test a 1:13 model of a ballistic missile in a high-speed wind tunnel. The prototype missile will travel at 1250 fps through air at 68°F and 14.5 psia. (a) If the air in the wind tunnel test section has a temperature of 32°F and a pressure of 12.1 psia, what must its velocity be? (b) Estimate the drag force on the prototype if the drag force on the model is 95 lb.*

BG

Compressibility is important at supersonic speeds, so **M** governs, and $\mathbf{M}_p = \mathbf{M}_m$.

Table A.5 for air: $R = 1715\ \text{ft}^2/(\text{sec}^2\cdot°\text{R})$. Eq. 2.5: $\gamma = pg/RT$

$\gamma_p = 14.5(144)32.2/[1715(460+68)] = 0.0742\ \text{lb/ft}^3$

$\gamma_m = 12.1(144)32.2/[1715(460+32)] = 0.0665\ \text{lb/ft}^3$

Eq. 7.10: $\mathbf{M} = V/(E_v/\rho)^{1/2} = V\sqrt{\rho/E_v}$ where $\rho = \gamma/g$ and $E_v = np$ (Eq. 2.8)

Thus $[V\sqrt{\gamma/p}]_p = [V\sqrt{\gamma/p}]_m$ as g and n are the same for both.

$1250\sqrt{0.0742/14.5} = V_m\sqrt{0.0665/12.1}$; $V_m = 1207$ fps ◀

$F_r = (\rho V^2L^2)_r = \rho_r V_r^2 L_r^2 = (\gamma/g)_r V_r^2 L_r^2 = (0.0742/0.0665)(1250/1207)^2(13/1)^2 = 203$

$\therefore\ F_p = F_r F_m = 203(95\ \text{lb}) = 19{,}240\ \text{lb}$ ◀

7.19 *Researchers plan to test a 1:13 model of a ballistic missile in a high-speed wind tunnel. The prototype missile will travel at 380 m/s through air at 23°C and 95.0 kPa abs. (a) If the air in the wind tunnel test section has a temperature of −20°C at a pressure of 89 kPa abs, what must its velocity be? (b) Estimate the drag force on the prototype if the drag force on the model is 400 N.*

SI

Compressibility is important at supersonic speeds, so **M** governs, and $\mathbf{M}_p = \mathbf{M}_m$.

Table A.5 for air: $R = 287\ \text{m}^2/(\text{s}^2{\cdot}\text{K})$. Eq. 2.5: $\gamma = pg/RT$

$\gamma_p = 95\,000(9.81)/[287(273+23)] = 10.97\ \text{N/m}^3$; $\gamma_m = 89\,000(9.81)/[287(273-20)] = 12.02\ \text{N/m}^3$

Eq. 7.10: $\mathbf{M} = V/(E_v/\rho)^{1/2} = V\sqrt{\rho/E_v}$ where $\rho = \gamma/g$ and $E_v = np$ (Eq. 2.8)

Thus $[V\sqrt{\gamma/p}]_p = [V\sqrt{\gamma/p}]_m$ as g and n are the same for both.

$380\sqrt{10.97/95} = V_m\sqrt{12.02/89}$; $V_m = 351\ \text{m/s}$ ◄

$F_r = (\rho V^2L^2)_r = \rho_r V_r^2 L_r^2 = (\gamma/g)_r V_r^2 L_r^2 = (10.97/12.02)(380/351)^2(13/1)^2 = 180.4$

$\therefore\ F_p = F_r F_m = 180.4(400\ \text{N}) = 72\,200\ \text{N}$ ◄

Sec. 7.5: Scale Ratios -- ***Problems*** *7.20−7.26*

7.20 *Find the dimensions of pressure, energy, and momentum in the FLT system. Repeat for the MLT system.*

N

	FLT	MLT	Eq. No.	Dimens. Equiv.
Pressure	FL^{-2}	$ML^{-1}T^{-2}$	3.4	γh
Energy	FL	ML^2T^{-2}	5.21	$\frac{1}{2}mV^2$
Momentum	FT	MLT^{-1}	6.5	ρQV
	▲	▲		

7.21 *Find the dimensions of force, torque, and power in the FLT system. Repeat for the MLT system.*

N

	FLT	MLT	Eq. No.	Dimens. Equiv.
Force	F	MLT^{-2}	3.14	pA
Torque	FL	ML^2T^{-2}	6.31	$\rho Q_r V$
Power	FLT^{-1}	ML^2T^{-3}	5.39	γQh
	▲	▲		

7.22 *Develop the scale ratios given in Table 7.1 for the case where prototype and model Reynolds numbers are the same.*

N

Using Eq 7.6: $V_r = V_p/V_m = [\mathbf{R}\nu/D]_p \times [D/(\mathbf{R}\nu)]_m = (\nu/L)_r = (\mu/L\rho)_r$ ◄

$T_r = (L/V)_r = (L^2/\nu)_r = (L^2\rho/\mu)_r$ ◄ $Q_r = A_r V_r = L_r^2(\nu/L)_r = (L\nu)_r = (L\mu/\rho)_r$ ◄

$F_r = (\rho V^2L^2)_r = (\rho L^2)_r(\mu/L\rho)_r^2 = (\mu^2/\rho)_r$ ◄ $P = F_r V_r = (\mu^2/\rho)_r(\mu/L\rho)_r = (\mu^3/L\rho^2)_r$ ◄

The other relationships (for acceleration, mass, pressure, impulse and momentum, energy and work) can be determined using their dimensions, the above, and algebra.

7.23 *Develop the scale ratios given in Table 7.1 for the case where prototype and model Froude numbers are identical.*

N

Using Eq 7.8: $V_r = V_p/V_m = [\mathbf{F}(gL)^{1/2}]_p/[\mathbf{F}(gL)^{1/2}]_m = (L^{1/2}g^{1/2})_r$ ◄

$T_r = (L/V)_r = [L/(L^{1/2}g^{1/2})]_r = (L^{1/2}g^{-1/2})_r$ ◄ $Q_r = A_rV_r = L_r^2(L^{1/2}g^{1/2})_r = (L^{5/2}g^{1/2})_r$ ◄

$F_r = (\rho V^2L^2)_r = (\rho L^2)_r(Lg)_r = (L^3\rho g)_r$ ◄ $P_r = F_rV_r = (L^3\rho g)_r(L^{1/2}g^{1/2})_r = (L^{7/2}\rho g^{3/2})_r$ ◄

The other relationships (for acceleration, mass, pressure, impulse and momentum, energy and work) can be determined using their dimensions, the above, and algebra.

7.24 *Develop the scale ratios given in Table 7.1 for the case where prototype and model Mach numbers are the same.*

N

Using Eq. 7.10: $V_r = V_p/V_m = (\mathbf{M}E_v^{1/2}/\rho^{1/2})_p(\rho^{1/2}/\mathbf{M}E_v^{1/2})_m = (E_v/\rho)_r^{1/2}$ ◄

$T_r = (L/V)_r = L_r(E_v/\rho)_r^{-1/2} = (L\rho^{1/2}/E_v^{1/2})_r$ ◄ $Q_r = A_rV_r = L_r^2(E_v/\rho)_r^{1/2} = (L^2E_v^{1/2}/\rho^{1/2})_r$ ◄

$F_r = (\rho V^2L^2)_r = (\rho L^2)_r(E_v/\rho)_r = (L^2E_v)_r$ ◄ $P_r = F_rV_r = (L^2E_v)_r(E_v/\rho)_r^{1/2} = (L^2E_v^{3/2}/\rho^{1/2})_r$ ◄

The other relationships (for acceleration, mass, pressure, impulse and momentum, energy and work) can be determined using their dimensions, the above, and algebra.

7.25 *Gas (γ = 0.32 lb/ft^3, μ = 2.0 × 10^{-6} lb·sec/ft^2) is flowing in an 0.75-in-diameter pipe. When a gas flowmeter measures the flow as being 0.13 lb/sec, it registers a pressure drop of 1.17 psi. Investigators plan to test an enlarged model that is geometrically similar in a 6-in-diameter pipe. (a) What flow rate of 80°F water will achieve dynamic similarity? (b) What would the pressure drop across the water meter be?*

BG

Inertia and fluid friction due to viscosity dominate for pipe flow, so it must satisfy Reynolds number.

(*a*) From Table 7.1 for Reynolds number: $Q_r = (L\mu/\rho)_r = L_r\nu_r$; $L_r = L_p/L_m = 0.75/6 = 0.125$

$\nu_p = \mu_p/\rho_p = \mu_p/(\gamma/g)_p = 2.0\times10^{-6}/(0.32/32.2) = 2.01\times10^{-4}$ ft^2/sec

Table A.1 for water at 80°F: $\nu_m = 9.30\times10^{-6}$ ft^2/sec, $\rho_m = 1.934$ slugs/ft^3

$\nu_r = \nu_p/\nu_m = (2.01\times10^{-4})/(9.30\times10^{-6}) = 21.6$; $Q_r = Q_p/Q_m = L_r\nu_r = 0.125(21.6) = 2.70$

From Eq. 4.5: $Q_p = G/\gamma = 0.13/0.32 = 0.406$ cfs ; $Q_m = Q_p/Q_r = 0.406/2.70 = 0.1502$ cfs ◄

(*b*) From Table 7.1 , $p_r = (\mu^2/L^2\rho)_r = (\rho\nu^2/L^2)_r$; $p_r = [(0.32/32.2)/1.934](21.6/0.125)^2 = 154.0$

$\Delta p_r = \Delta p_p/\Delta p_m = 154.0$; $\Delta p_m = \Delta p_p/\Delta p_r = 1.17/154.0 = 0.007\,60$ psi ◄

7.26 *Gas (ρ = 5.25 kg/m^3, ν = 2.0 × 10^{-5} m^2/s) is flowing in a 20-mm-diameter pipe. When a gas flowmeter measures the flow as being 0.064 kg/s, it registers a pressure drop of 8.5 kPa. Investigators plan to test an enlarged model that is geometrically similar in a 180-mm-diameter pipe. (a) What flow rate of 25°C water will achieve dynamic similarity? (b) What would the pressure drop across the water meter be?*

SI

Inertia and fluid friction due to viscosity dominate for pipe flow, so it must satisfy Reynolds number.

(*a*) From Table 7.1 for Reynolds number: $Q_r = (L\mu/\rho)_r = L_r \nu_r$; $L_r = L_p/L_m = 2/18 = 0.1111$

Table A.1 for water at 25°C: $\nu_m = 0.893 \times 10^{-6}$ m^2/s, $\rho_m = 997.0$ kg/m^3

$\nu_r = \nu_p/\nu_m = (2.0 \times 10^{-5})/(0.893 \times 10^{-6}) = 22.4$ $\quad Q_r = Q_p/Q_m = L_r \nu_r = 0.1111(22.4) = 2.49$

From Eq. 4.4: $Q_p = \dot{m}/\rho = 0.064/5.25 = 0.012\,19$ m^3/s ;

$Q_m = Q_p/Q_r = 0.001\,219/2.49 = 0.004\,90$ m^3/s ◄

(*b*) From Table 7.1: $p_r = (\mu^2/L^2\rho)_r = (\rho\nu^2/L^2)_r$; $p_r = (5.25/997.0)(22.4/0.1111)^2 = 214$

$\Delta p_r = \Delta p_p/\Delta p_m = 214$; $\Delta p_m = \Delta p_p/\Delta p_r = 8.5/214 = 0.0397$ kPa = 39.7 Pa ◄

Sec. 7.7: Dimensional Analysis -- ***Exercises*** *(5)*

7.7.1 *Use dimensional analysis to arrange the following groups into dimensionless parameters: (a) τ, V, ρ; (b) V, L, ρ, σ. Use the MLT system.*

N

(*a*) $\Pi = \tau^a V^b \rho^c = (ML^{-1}T^{-2})^a (LT^{-1})^b (ML^{-3})^1$

M: $0 = a + 1$; L: $0 = -a + b - 3$; T: $0 = -2a - b$. Solving: $a = -1$, $b = 2$

$\Pi = \tau^{-1}V^2\rho$; $(1/\Pi) = \tau/\rho V^2$ ◄

(*b*) $\Pi = V^a L^b \rho^c \sigma = (LT^{-1})^a L^b (ML^{-3})^c (MT^{-2})^1$

M: $0 = c + 1$; L: $0 = a + b - 3c$; T: $0 = -a - 2$. Solving: $a = -½$, $b = ½$, $c = 1$

$\Pi = V^{-2}L^{-1}\rho^{-1}\sigma = \sigma/(\rho L V^2)$ ◄

7.7.2 *Use dimensional analysis to arrange the following groups into dimensionless parameters: (a) Δp, V, γ, g; (b) F, ρ, L, V. Use the MLT system.*

N

(*a*) $\Pi = \Delta p^a V^b \gamma^c g = (ML^{-1}T^{-2})^a (LT^{-1})^b (ML^{-2}T^{-2})^c (LT^{-2})^1$

M: $0 = a + c$; L: $0 = -a + b - 2c + 1$; T: $0 = -2a - b - 2c - 2$

Solving: $a = 1$, $b = ½$, $c = 1$. $\Pi = \Delta p V^{-2} \gamma^{-1} g = \Delta p g/(\gamma V^2)$ ◄

(*b*) $\Pi = F^a \rho^b L^c V = (MLT^{-2})^a (ML^{-3})^b L^c (LT^{-1})^1$

M: $0 = a + b$; L: $0 = a - 3b + c + 1$; T: $0 = -2a - 1$

Solving: $a = -½$, $b = ½$, $c = 1$

$\Pi = F^{-1/2}\rho^{1/2}LV$; $(1/\Pi^2) = F/(\rho L^2 V^2)$ ◄

7.7.3 *Use dimensional analysis to derive an expression for the power developed by an engine in terms of the torque T and rotative speed ω.*

N

Step 1: No. of variables (P, T, ω), $n = 3$

Step 2: Their dimensions are: $P = [FL/T]$, $T = [FL]$, $\omega = [1/T]$

So no. of fundamental dimensions (M,L,T) involved, $m = 3$.

Step 3: Our 3 variables can be formed into a dimensionless group,

$\therefore\ k \neq m = 3$. Try $k = 2$. Any 2 variables can not be so formed, so $k = 2$.

Step 4: No. of dimensionless Π groups needed $= n - k = 3 - 2 = 1$; name it Π. $\therefore\ \phi(\Pi) = 0$

Steps 5 and 6: $\Pi = P^a T^b \omega\ ;\ \ M^0L^0T^0 = \left(\frac{FL}{T}\right)^a (FL)^b \left(\frac{1}{T}\right)^1$

F: $0 = a + b$; L: $0 = a + b$; T: $0 = -a - 1$

Solving: $a = -1$, $b = 1$. So $\Pi = P^{-1}T\omega = T\omega/P$

Step 7: $\phi(\Pi) = 0$, i.e., $1/\Pi = \text{const}$ or $P/T\omega = C$, $P = CT\omega$ ◄

7.7.4 *Derive an expression for the shear stress at the pipe wall when an incompressible fluid flows through a pipe under pressure. Use dimensional analysis with the following significant parameters: pipe diameter D, flow velocity V, and viscosity μ and density ρ of the fluid.*

N

Step 1: No. of variables (D, V, μ, ρ, τ), $n = 5$

Step 2: Their dimensions are: $D = [L]$, $V = \left[\frac{L}{T}\right]$, $\mu = \left[\frac{M}{LT}\right]$, $\rho = \left[\frac{M}{L^3}\right]$, $\tau = \left[\frac{F}{L^2} = \frac{M}{LT^2}\right]$

So no. of fundamental dimensions (M,L,T) involved, $m = 3$.

Step 3: The 3 variables $[L]$, $[L/T]$, and $[M/L^3]$ can <u>not</u> be formed into a dimensionless group,

so the reduction number $k = 3$.

Step 4: No. of dimensionless Π groups needed $= n - k = 5 - 3 = 2$; name them Π_1 and Π_2.

$\therefore\ f(\Pi_1, \Pi_2) = 0$

Steps 5 and 6: Select the following 3 (= k) variables to be primary (repeating): V, D, ρ.

For $\Pi_1 = V^{a_1}D^{b_1}\rho^{c_1}\mu$: $M^0L^0T^0 = \left(\frac{L}{T}\right)^{a_1}(L)^{b_1}\left(\frac{M}{L^3}\right)^{c_1}\left(\frac{M}{LT}\right)^1$

L: $0 = a_1 + b_1 - 3c_1 - 1$; M: $0 = c_1 + 1$; T: $0 = -a_1 - 1$

Solving: $a_1 = -1$, $b_1 = -1$, $c_1 = -1$. So $\Pi_1 = V^{-1}D^{-1}\rho^{-1}\mu = \mu/VD\rho = 1/\mathbf{R}$

For $\Pi_2 = V^{a_2}D^{b_2}\rho^{c_2}\tau$: $M^0L^0T^0 = \left(\frac{L}{T}\right)^{a_2}(L)^{b_2}\left(\frac{M}{L^3}\right)^{c_2}\left(\frac{M}{LT^2}\right)^1$

L: $0 = a_2 + b_2 - 3c_2 - 1$; M: $0 = c_2 + 1$; T: $0 = -a_2 - 2$

Solving: $a_2 = -2$, $b_2 = 0$, $c_2 = -1$. So $\Pi_2 = V^{-2}D^0\rho^{-1}\tau = \tau/\rho V^2$

Step 7: $f(\Pi_1, \Pi_2) = 0$, i.e. $f(1/\mathbf{R}, \tau/\rho V^2) = 0$

so also $\Pi_2 = \phi(\Pi_1^{-1})$ or $\tau/\rho V^2 = \phi(\mathbf{R})$, $\tau = \rho V^2\phi(\mathbf{R})$ ◄

7.7.5 *Refer to the example of the drag on a submerged sphere, which illustrates the seven-step procedure of Sec. 7.7. Using the same parameters, but also including the acceleration due to gravity g to account for the effect of wave action, derive an expression for the drag on a surface vessel.*

N

Step 1: $f'(F_D,D,V,\rho,\mu,g) = 0$, so no. of variables, $n = 6$.

Step 2: $F_D = \left[\frac{ML}{T^2}\right]$, $D = [L]$, $V = \left[\frac{L}{T}\right]$, $\rho = \left[\frac{M}{L^3}\right]$, $\mu = \left[\frac{M}{LT}\right]$, $g = \left[\frac{L}{T^2}\right]$

No. of fundamental dimensions (M,L,T) involved, $m = 3$.

Step 3: The 3 variables D, V, and ρ can not be formed into a dimensionless group, so the reduction number $k = 3$.

Step 4: No. of dimensionless Π groups needed $= n - k = 6 - 3 = 3$.. $\therefore f(\Pi_1, \Pi_2, \Pi_3) = 0$

Steps 5 and 6: Select the following 3 (= k) variables to be primary (repeating): ρ, D, V

For $\Pi_1 = \rho^{a_1}D^{b_1}V^{c_1}\mu$: $M^0L^0T^0 = \left(\frac{M}{L^3}\right)^{a_1}(L)^{b_1}\left(\frac{L}{T}\right)^{c_1}\left(\frac{M}{LT}\right)^1$

M: $0 = a_1 + 1$; L: $0 = -3a_1 + b_1 + c_1 - 1$; T: $0 = -c_1 - 1$

Solving: $a_1 = -1$, $b_1 = -1$, $c_1 = -1$. So $\Pi_1 = \rho^{-1}D^{-1}V^{-1}\mu = \mu/(DV\rho) = \mathbf{R}^{-1}$

For $\Pi_2 = \rho^{a_2}D^{b_2}V^{c_2}F_D$: $M^0L^0T^0 = \left(\frac{M}{L^3}\right)^{a_2}(L)^{b_2}\left(\frac{L}{T}\right)^{c_2}\left(\frac{ML}{T^2}\right)^1$

M: $0 = a_2 + 1$; L: $0 = -3a_2 + b_2 + c_2 + 1$; T: $0 = -c_2 - 2$

Solving: $a_2 = -1$, $b_2 = -2$, $c_2 = -2$. So $\Pi_2 = \rho^{-1}D^{-2}V^{-2}F_D = F_D/(\rho D^2V^2)$

For $\Pi_3 = \rho^{a_3}D^{b_3}V^{c_3}g$: $M^0L^0T^0 = \left(\frac{M}{L^3}\right)^{a_3}(L)^{b_3}\left(\frac{L}{T}\right)^{c_3}\left(\frac{L}{T^2}\right)^1$

M: $0 = a_3$; L: $0 = -3a_3 + b_3 + c_3 + 1$; T: $0 = -c_3 - 2$

Solving: $a_3 = 0$, $b_3 = 1$, $c_3 = -2$. So $\Pi_3 = \rho^0 DV^{-2}g = gD/V^2 = \mathbf{F}^{-2}$

Step 7: $f(\Pi_1, \Pi_2, \Pi_3) = 0$, i.e. $f(\mathbf{R}^{-1}, F_D/(\rho D^2V^2), \mathbf{F}^{-2}) = 0$

so also $\Pi_2 = \phi(\Pi_1^{-1}, \Pi_3^{-1/2})$, $F_D/(\rho D^2V^2) = \phi(\mathbf{R}, \mathbf{F})$, $F_D = \rho D^2V^2\phi(\mathbf{R}, \mathbf{F})$ ◄

Sec. 7.7: Dimensional Analysis -- *Problems 7.27–7.33*

7.27 *(a) Use dimensional analysis and the MLT system to arrange the following into a dimensionless number: g, L, ρ, and μ. (b) Name the dimensionless number.*

N

(*a*) $\Pi = g^aL^b\rho^c\mu^d = (LT^{-2})^aL^b(ML^{-3})^c(ML^{-1}T^{-1})^d = M^0L^0T^0$

M: $0 = c + d$; L: $0 = a + b - 3c - d$; T: $0 = -2a - d$

Solving: $b = 3a$, $c = 2a$, $d = -2a$. So $\Pi = g^aL^{3a}\rho^{2a}\mu^{-2a} = \left[\frac{gL^3\rho^2}{\mu^2}\right]^a$

$\Pi^{1/a} = \mathbf{N} = \frac{gL^3\rho^2}{\mu^2}$ ◄

(*b*) Ref. 49, p. B–8: This dimensionless number is known as the Gallileo number ◄

7.28 *(a) Use dimensional analysis and the MLT system to arrange the following into a dimensionless number: g, ρ, μ, and σ. (b) Name the dimensionless number.*

N

(*a*) $\Pi = g^a \rho^b \mu^c \sigma^d = (LT^{-2})^a (ML^{-3})^b (ML^{-1}T^{-1})^c (MT^{-2})^d = M^0 L^0 T^0$

M: $0 = b + c + d$; L: $0 = a - 3b - c$; T: $0 = -2a - c - 2d$

Solving: $a = -b$, $c = -4b$, $d = 3b$. So $\Pi = g^{-b}\rho^b\mu^{-4b}\sigma^{3b} = \left[\frac{\rho\sigma^3}{g\mu^4}\right]^b$

$\Pi^{-1/b} = N = \frac{g\mu^4}{\rho\sigma^3}$ ◄

(*b*) Ref. 49, p. B−10: This dimensionless number is known as the Morton number ◄

7.29 *(a) Use dimensional analysis and the MLT system to arrange the following into a dimensionless number: L, ρ, μ, and σ. (b) Name the dimensionless number.*

N

(*a*) $\Pi = L^a \rho^b \mu^c \sigma^d = L^a (ML^{-3})^b (ML^{-1}T^{-1})^c (MT^{-2})^d = M^0 L^0 T^0$

M: $0 = b + c + d$; L: $0 = a - 3b - c$; T: $0 = -c - 2d$

Solving: $a = d$, $b = d$, $c = -2d$. So $\Pi = L^d\rho^d\mu^{-2d}\sigma^d = \left[\frac{L\rho\sigma}{\mu^2}\right]^d$

$\Pi^{-1/2d} = N = \frac{\mu}{\sqrt{L\rho\sigma}}$ ◄

(*b*) Ref. 49, p. B−11: This dimensionless number is known as the Ohnesorge number ◄

7.30 *Use dimensional analysis to derive an expression for the height of capillary rise in a glass tube.*

N

Step 1: $f'(h, r, \sigma, \gamma) = 0$, so no. of variables, $n = 4$. Using FLT:

Step 2: $h = [L]$, $r = [L]$, $\sigma = \left[\frac{F}{L}\right]$, $\gamma = \left[\frac{F}{L^3}\right]$; $m = 2$

Step 3: The 3 variables h, r, and σ can not be formed into a dimensionless group, so the reduction number $k = 3$.

Step 4: No. of Π groups $= n - k = 1$, $\therefore f(\Pi) = 0$.

Steps 5 and 6: For $\Pi = r^a\sigma^b\gamma^c h$: $F^0L^0T^0 = (L)^a\left(\frac{F}{L}\right)^b\left(\frac{F}{L^3}\right)^c(L)^1$

F: $0 = b + c$; L: $0 = a - b - 3c + 1$; T: $0 = 0$

Solving: $a = 2c - 1$, $b = -c$. So $\Pi = r^{2c-1}\sigma^{-c}\gamma^c h = (h/r)(\gamma r^2/\sigma)^c$

Step 7: We can write $f(\Pi) = 0$, or $\Pi = C = \text{const.}$ Then $h = Cr\left(\frac{\sigma}{\gamma r^2}\right)^c$ ◄

Experimental observation suggests that $c = 1$ (see Sec 2.12), in which case $h = \frac{C\sigma}{\gamma r}$

7.31 N *Derive an expression for the drag on an aircraft flying at supersonic speed.*

At supersonic speed the volume modulus of elasticity E_v is also a factor (see Eq 7.10). $\therefore$ we expect both **M** and **R** to govern.

Step 1: $f'(F_D, L, V, \rho, \mu, E_v) = 0$, so no. of variables, $n = 6$. Using FLT:

Step 2: $F_D = [F]$, $L = [L]$, $V = \left[\dfrac{L}{T}\right]$, $\rho = \left[\dfrac{FT^2}{L^4}\right]$, $\mu = \left[\dfrac{FT}{L^2}\right]$, $E_v = \left[\dfrac{F}{L^2}\right]$; $m = 3$

Step 3: The 3 variables V, L, and ρ can <u>not</u> be formed into a dimensionless group,

so the reduction number $k = 3$.

Step 4: No. of Π groups $= n - k = 3$, $\therefore f(\Pi_1, \Pi_2, \Pi_3) = 0$

Steps 5 and 6: Select for the 3 $(=k)$ primary (repeating) variables: ρ, L, V

For $\Pi_1 = \rho^a L^b V^c \mu$: $F^0L^0T^0 = \left(\dfrac{FT^2}{L^4}\right)^a (L)^b \left(\dfrac{L}{T}\right)^c \left(\dfrac{FT}{L^2}\right)^1$

F: $0 = a + 1$; L: $0 = -4a + b + c - 2$; T: $0 = 2a - c + 1$

Solving: $a = -1$, $b = -1$, $c = -1$. So $\Pi_1 = \rho^{-1}L^{-1}V^{-1}\mu = \mu/(LV\rho) = \mathbf{R}^{-1}$

For $\Pi_2 = \rho^a L^b V^c F_D$: $F^0L^0T^0 = \left(\dfrac{FT^2}{L^4}\right)^a (L)^b \left(\dfrac{L}{T}\right)^c (F)^1$

F: $0 = a + 1$; L: $0 = -4a + b + c$; T: $0 = 2a - c$

Solving: $a = -1$, $b = -2$, $c = -2$. So $\Pi_2 = \rho^{-1}L^{-2}V^{-2}F_D = F_D/(\rho L^2V^2)$

For $\Pi_3 = \rho^a L^b V^c E_v$: $F^0L^0T^0 = \left(\dfrac{FT^2}{L^4}\right)^a (L)^b \left(\dfrac{L}{T}\right)^c \left(\dfrac{F}{L^2}\right)^1$

F: $0 = a + 1$; L: $0 = -4a + b + c - 2$; T: $0 = 2a - c$

Solving: $a = -1$, $b = 0$, $c = -2$. So $\Pi_3 = \rho^{-1}L^0V^{-2}E_v = E_v/(\rho V^2) = \mathbf{M}^{-2}$

Step 7: We can write $\Pi_2 = \phi(\Pi_1^{-1}, \Pi_3^{-1/2})$, i.e. $F_D/(\rho L^2V^2) = \phi(\mathbf{R},\mathbf{M})$, or $F_D = \rho L^2V^2\phi(\mathbf{R},\mathbf{M})$ ◄

7.32 *Derive an expression for small flow rates over a spillway, in the form of a function including dimensionless quantities. Use dimensional analysis with the following parameters: height of spillway P, head on the spillway H, viscosity of liquid μ, density of liquid ρ, surface tension σ, and acceleration due to gravity g.*

N

Step 1: $f'(q, P, H, g, \mu, \rho, \sigma) = 0$, so no. of variables, $n = 7$. Using MLT:

Step 2: $q = \left[\frac{L^2}{T}\right]$, $\rho = [L]$, $H = [L]$, $g = \left[\frac{L}{T^2}\right]$, $\mu = \left[\frac{M}{LT}\right]$, $\rho = \left[\frac{M}{L^3}\right]$, $\sigma = \left[\frac{M}{T^2}\right]$; $m = 3$

Step 3: The 3 variables q, H, and ρ can not be formed into a dimensionless group, so the reduction number $k = 3$.

Step 4: No. of Π groups needed $= n - k = 4$, $\therefore f(\Pi_1, \Pi_2, \Pi_3, \Pi_4) = 0$.

Steps 5 and 6: Select for the 3 (=k) primary (repeating) variables: q, H, ρ.

For $\Pi_1 = q^a H^b \rho^c P$: $M^0 L^0 T^0 = \left(\frac{L^2}{T}\right)^a (L)^b \left(\frac{M}{L^3}\right) (L)^1$

M: $0 = c$; L: $0 = 2a + b - 3c + 1$; T: $0 = -a$

Solving: $a = 0, b = -1, c = 0$. So $\Pi_1 = q^0 H^{-1} \rho^0 P = P/H$

For $\Pi_2 = q^a H^b \rho^c g$: $M^0 L^0 T^0 = \left(\frac{L^2}{T}\right)^a (L)^b \left(\frac{M}{L^3}\right) \left(\frac{L}{T^2}\right)^1$

M: $0 = c$; L: $0 = 2a + b - 3c + 1$; T: $0 = -a - 2$

Solving: $a = -2$, $b = 3$, $c = 0$. So $\Pi_2 = q^{-2} H^3 g = gH^3/q^2$

For $\Pi_3 = q^a H^b \rho^c \mu$: $M^0 L^0 T^0 = \left(\frac{L^2}{T}\right)^a (L)^b \left(\frac{M}{L^3}\right)^c \left(\frac{M}{LT}\right)^1$

M: $0 = c + 1$; L: $0 = 2a + b - 3c - 1$; T: $0 = -a - 1$

Solving: $a = -1, b = 0, c = -1$. So $\Pi_3 = q^{-1} H^0 \rho^{-1} \mu = \mu/q\rho = \mathbf{R}^{-1}$ (as $q = HV$)

For $\Pi_4 = q^a H^b \rho^c \sigma$: $M^0 L^0 T^0 = \left(\frac{L^2}{T}\right)^a (L)^b \left(\frac{M}{L^3}\right)^c \left(\frac{M}{T^2}\right)^1$

M: $0 = c + 1$; L: $0 = 2a + b - 3c$; T: $0 = -a - 2$

Solving: $a = -2$, $b = 1$, $c = -1$. So $\Pi_4 = q^{-2} H \rho^{-1} \sigma = \sigma H/(\rho q^2) = \mathbf{W}^{-2}$ (*as* $q = HV$)

Step 7: We can write $\Pi_2^{-1/2} = \phi(\Pi_1, \Pi_3^{-1}, \Pi_4^{-1/2})$

i.e., $q/(g^{1/2} H^{3/2}) = \phi(H/P, \mathbf{R}, \mathbf{W})$ or $q = g^{1/2} H^{3/2} \phi(H/P, \mathbf{R}, \mathbf{W})$ ◄

7.33 *Derive an expression for the velocity of rise of an air bubble in a stationary liquid. Consider the effect of surface tension as well as other variables.*

N

Bubble velocity will be governed by its depth h (pressure and volume) and the properties including surface tension (given).

Step 1: $f'(V,h,\mu,\rho,\sigma) = 0$, so no. of variables, $n = 5$

Step 2: $V = \left[\frac{L}{T}\right],\ h = [L],\ \mu = \left[\frac{M}{LT}\right],\ \rho = \left[\frac{M}{L^3}\right],\ \sigma = \left[\frac{M}{T^2}\right];\ m = 3$

Step 3: The 3 variables V, h, and ρ can not be formed into a dimensionless group,

so the reduction number $k = 3$.

Step 4: No. of Π groups needed $= n - k = 2$, $\therefore f(\Pi_1, \Pi_2) = 0$.

Steps 5 and 6: Select for the 3 (= k) primary (repeating) variables: V, h, ρ.

For $\Pi_1 = V^a h^b \rho^c \mu$: $M^0L^0T^0 = \left(\frac{L}{T}\right)^a (L)^b \left(\frac{M}{L^3}\right)^c \left(\frac{M}{LT}\right)^1$

M: $0 = c + 1$; L: $0 = a + b - 3c - 1$; T: $0 = -a - 1$

Solving: $a = -1,\ b = -1,\ c = -1$. So $\Pi_1 = V^{-1}h^{-1}\rho^{-1}\mu = \mu/(hV\rho) = \mathbf{R}^{-1}$

For $\Pi_2 = V^a h^b \rho^c \sigma$: $M^0L^0T^0 = \left(\frac{L}{T}\right)^a (L)^b \left(\frac{M}{L^3}\right)^c \left(\frac{M}{T^2}\right)^1$

M: $0 = c + 1$; L: $0 = a + b - 3c$; T: $0 = -a - 2$

Solving: $a = -2,\ b = -1,\ c = -1$. So $\Pi_2 = V^{-2}h^{-1}\rho^{-1}\sigma = \sigma/(V^2h\rho) = \mathbf{W}^{-2}$

Step 7: Thus we can write $\Pi_2^{-1} = \phi(\Pi_1^{-1})$, i.e. $V^2h\rho/\sigma = \phi(\mathbf{R})$ or $V = \sqrt{\sigma/h\rho}\,\phi(\mathbf{R})$ ◄

Alternatively $\Pi_1^{-1} = \phi(\Pi_2^{-1/2})$, i.e. $hV\rho/\mu = \phi(\mathbf{W})$ or $V = (\mu/h\rho)\phi(\mathbf{W})$ ◄

Chapter 8

Steady Incompressible Flow in Pressure Conduits

PROBLEM SELECTION GUIDE

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
8.2	*Critical Reynolds Number*						
	X[1]8.2.1	BG	V Easy	V Short	1	8.2.2	
	8.2.2	SI	V Easy	V Short	1	8.2.1	
	8.2.3	BG	V Easy	V Short	1		
	P 8.1	BG	Medium	Short	1		Uses $pv = RT$ (Sec 2.7)
	8.2	SI	Medium	Medium	1		Uses $pv = RT$ (Sec 2.7)
8.3	*Hydraulic Radius, Hydraulic Diameter*						
	X 8.3.1	BG	V Easy	V Short	1		
	8.3.2	SI	V Easy	V Short	1		
	P 8.3	N	Easy	Short	1		
8.5	*Friction in Circular Conduits*						
	X 8.5.1	BG	V Easy	V Short	1	8.5.2	
	8.5.2	SI	V Easy	V Short	1	8.5.1	
	8.5.3	BG	Easy	Short	1	8.5.4	Laminar flow
	8.5.4	SI	Easy	Short	1	8.5.3	Laminar flow
	P 8.4	N	Easy	Short	1		
	8.5	N	Easy	Short	1		
	8.6	BG	Easy	Short	1		
8.6	*Friction in Noncircular Conduits*						
	X 8.6.1	BG	Easy	Short	1	8.6.2	Power (Sec 5.10)
	8.6.2	SI	Easy	Short	1	8.6.1	Power (Sec 5.10)
	P 8.7	BG	Medium	Medium	1		† f varies. Lam'r; uses $pv = RT$ (§ 2.7).
	8.8	BG	Hard	Medium	1	8.9	† f varies. T & E
	8.9	SI	Hard	Medium	1	8.8	† f varies. T & E
8.7	*Laminar Flow in Circular Pipes*						
	X 8.7.1	BG	Easy	Short	1		f varies
	8.7.2	N	Easy	Short	1		
	8.7.3	N	Medium	Medium	1		Integration
	P 8.10	N	Easy	Medium	1		Plot
	8.11	N	Medium	Medium	1		Integration
	8.12	N	Medium	Medium	1		Integration
	8.13	SI	Easy	Medium	1		f varies. Power (Sec 5.10)

/cont...

[1] For all Exercises (identified by "X"), answers are given in Appendix F of the textbook.
† Answers are sensitive to values (f, μ, ν, k') that are or may be read from a graph.
X = Exercise, P = (end-of-chapter) Problem, S = Sample Problem. ☐ = could use computing aids.

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
8.8	***Entrance Conditions in Laminar Flow***						
	X 8.8.1	BG	Easy	Short	1	8.8.2	
	8.8.2	SI	Easy	Short	1	8.8.1	
8.9	***Turbulent Flow***						
	X 8.9.1	BG	Medium	Medium	3		
	8.9.2	SI	Medium	Medium	4		
8.10	***Viscous Sublayer in Turbulent Flow***						
	X 8.10.1	BG	V Easy	V Short	1		
	8.10.2	BG	Easy	Short	2	8.10.3	Given const *f*
	8.10.3	SI	Easy	Short	2	8.10.2	Given const *f*
	8.10.4	BG	Easy	Short	1	8.10.5	Given const *f*
	8.10.5	SI	Easy	Short	1	8.10.4	Given const *f*
	8.10.6	SI	Easy	Short	2		Given const *f*
	P 8.14	BG	Easy	Short	2	8.15	
	8.15	SI	Easy	Short	2	8.14	
8.11	***Velocity Profile in Turbulent Flow***						
	X 8.11.1	SI	Easy	Short	1		
	8.11.2	N	Easy	Short	1		*f* cancels out
	8.11.3	BG	Medium	Medium	1		† *f* varies
	P 8.16	BG	Medium	Short	1		*f* varies
	8.17	BG	Medium	Medium	1	8.18	† *f* varies
	8.18	SI	Medium	Medium	1	8.17	† *f* varies
	8.19	BG	Medium	Medium	1	8.20	† *f* varies. Laminar, turbulent
	8.20	SI	Medium	Medium	1	8.19	† *f* varies. Laminar, turbulent
	8.21	SI	Medium	Medium	1		† *f* varies
	8.22	BG	Medium	Long	1	8.23	*f* varies. Plot
	8.23	SI	Medium	Long	1	8.22	*f* varies. Plot
8.12	***Pipe Roughness***						
	X 8.12.1	N	Medium	Medium	4		□ *f* varies
	8.12.2	SI	Medium	Medium	1	8.12.3	Given const *f*
	8.12.3	SI	Medium	Medium	1	8.12.2	Given const *f*
	P 8.24	N	Medium	Medium	2		*f* varies. Laminar, turbulent
	8.25	BG	Medium	Medium	2	8.26	† *f* varies, Blasius' Eq.
	8.26	SI	Medium	Medium	2	8.25	† *f* varies, Blasius' Eq.
	8.27	SI	Medium	Medium	1		Given const *f*
	8.28	SI	Hard	Medium	2		□ *f* varies. T & E (Trial & Error)

/cont...

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
8.13	***Chart for Friction Factor***						
	X 8.13.1	BG	Easy	Short	1	8.13.2	† *f* varies
	8.13.2	SI	Easy	Short	1	8.13.1	† *f* varies
	8.13.3	SI	Medium	Medium	2		† *f* varies
	8.13.4	BG	Medium	Medium	2		† *f* varies. Interpolation
	P 8.29	BG	Easy	Short	1	8.30	† *f* varies
	8.30	SI	Easy	Short	1	8.29	† *f* varies
	8.31	SI	Medium	Medium	1		† *f* varies. Power (Sec 5.10)
	8.32	BG	Medium	Medium	1		† *f* varies. Laminar
	8.33	BG	Medium	Medium	1		† *f* varies. Uses Secs 2.7, 2.11, 3.2, 4.5
	8.34	SI	Medium	Medium	1	8.35	† *f* varies
	8.35	SI	Medium	Medium	1	8.34	† *f* varies
	8.36	N	Easy	Long	1		† ☐ *f* varies. L, T; uses Secs 5.1, 6.3
	8.37	BG	Medium	Long	1		† ☐ *f* varies. Laminar, turbulent
8.15	***Single-Pipe Flow: Solution by Trials***						
	X 8.15.1	BG	Medium	Medium	6		† *f* varies. Find h_L
	8.15.2	BG	Medium	Medium	3		† *f* varies. Find h_L
	8.15.3	BG	Hard	Long	1	8.40	† *f* varies, find *Q*. T&E (Trial & Error)
	8.15.4	SI	Hard	Long	1	8.41	† *f* varies, find *Q*. T & E.
	P 8.38	BG	Medium	Long	1	8.39	† *f* varies, find *D*. T & E.
	8.39	SI	Medium	Long	1	8.38	† *f* varies, find *D*. T & E.
	8.40	SI	Hard	Long	1	8.15.3	† *f* varies, find *Q*. T & E.
	8.41	BG	Hard	Long	1	8.15.4	† *f* varies, find *Q*. T & E.
	8.42	BG	Hard	Long	1	8.43	† *f* varies, assume regime, find *D*. T&E
	8.43	SI	Hard	Long	1	8.42	† *f* varies, assume regime, find *D*. T&E
	8.44	BG	Hard	Long	1	8.45	† *f* varies, find *Q*. T & E.
	8.45	SI	Hard	Long	1	8.44	† *f* varies, find *Q*. T & E.
8.16	***Single-Pipe Flow: Direct Solutions***						
	X 8.16.1	BG	Medium	Medium	1	8.16.2	*f* varies. Find *Q*, check **R**.
	8.16.2	SI	Medium	Medium	1	8.16.1	*f* varies. Find *Q*, check **R**.
	P 8.46	SI	Medium	Medium	1	8.40	*f* varies. Find *Q*, check **R**.
	8.47	SI	Medium	Medium	1	8.45	*f* varies. Find *Q*, check **R**.
	8.48	SI	Medium	Long	1	8.43	*f* varies. Find *D*, check 2 flow regimes.
	8.49	SI	Medium	Long	1	8.39	*f* varies. Find *D*, check 2 flow regimes.
8.17	***Single-Pipe Flow: Automated Solutions***						
	X 8.17.1	SI	Medium	Long	2		☐ *f* varies, find *Q*
	8.17.2	BG	Medium	Long	2	8.17.3	☐ *f* varies, find *Q*
	8.17.3	SI	Medium	Long	2	8.17.2	☐ *f* varies, find *Q*
	P 8.50	B	Medium	Long	4	8.51	☐ *f* varies. Find *Q*, check **R**.
	8.51	B	Medium	Long	4	8.50	☐ *f* varies. Find *D*, check **R**.

/cont...

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
8.18	***Empirical Equations for Single-Pipe Flow***						
	X 8.18.1	BG	Easy	V Short	1	8.18.2	C_{HW}
	8.18.2	SI	Easy	V Short	1	8.18.1	C_{HW}
	8.18.3	BG	Easy	V Short	1		Manning's n.
	P 8.52	BG	Medium	Medium	1	8.53	† Manning's n.
	8.53	SI	Medium	Medium	1	8.52	† Manning's n.
8.22	***Loss of Head at Submerged Discharge***						
	X 8.22.1	BG	Easy	Short	3	8.22.2	Given const f. h' (minor losses).
	8.22.2	SI	Easy	Short	3	8.22.1	Given const f. h'
	8.22.3	BG	Easy	Medium	2		
	8.22.4	SI	Medium	Medium	1	P8.54	† f varies. h'
	8.22.5	BG	Medium	Medium	1	P8.55	† f varies. h'
	P 8.54	BG	Medium	Medium	1	8.22.4	† f varies. h'
	8.55	SI	Medium	Medium	1	8.22.5	† f varies. h'
8.24	***Loss Due to Expansion***						
	X 8.24.1	SI	Easy	Medium	3		† h' (minor losses)
	8.24.2	BG	Medium	Medium	2		† Given const f. h'
	P 8.56	BG	Medium	Medium	1	8.57	† ☐ f varies. h'
	8.57	SI	Medium	Medium	1	8.56	† f varies. h'
8.26	***Loss in Bends and Elbows***						
	X 8.26.1	BG	Medium	Medium	1	8.26.2	† f varies. h'. Interpolation
	8.26.2	SI	Medium	Medium	1	8.26.1	† f varies. h'. Interpolation
8.27	***Single-Pipe Flow with Minor Losses***						
	X 8.27.1	BG	Medium	Medium	1	8.27.2	Given const f. h'. Jet, find Q.
	8.27.2	SI	Medium	Medium	1	8.27.1	Given const f. h'. Jet, find Q.
	8.27.3	BG	Medium	Medium	2	8.27.4	Given const f. h'. Find p.
	8.27.4	SI	Medium	Medium	2	8.27.3	Given const f. h'. Find p.
	8.27.5	BG	Medium	Medium	2		Given const f. h'. Find Q.
	8.27.6	BG	Medium	Long	1		† ☐ f varies. h'. Find Q. T & E.
	P 8.58	BG	Medium	Medium	2	8.59	Given const f. h'. Find p.
	8.59	SI	Medium	Medium	2	8.58	Given const f. h'. Find p.
	8.60	BG	Medium	Medium	2	8.61	Given const f. h'. Find Q, p.
	8.61	SI	Medium	Medium	2	8.60	Given const f. h'. Find Q, p.
	8.62	BG	Medium	Medium	1	8.63	Given const f. Jet, find p.
	8.63	SI	Medium	Medium	1	8.62	Given const f. Jet, find p.
	8.64	BG	Medium	Medium	2		Given const f. h'. Jet, find V, p.
	8.65	BG	Medium	Long	1	8.66	☐ Given const f. h', jet, find D, power
	8.66	SI	Medium	Long	1	8.65	☐ Given const f. h', jet, find D, power
	8.67	BG	Hard	Long	2	8.68	† f varies. h'. Find Q. T & E.
	8.68	BG	Hard	Long	2	8.67	☐ f varies. h'. Find Q.
	8.69	SI	Hard	Long	1		† f varies. h'. Find D. T & E for D!
	8.70	SI	Hard	Long	1	8.69	☐ f varies. h'. Find D.

/cont…

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
8.28	***Pipeline with Pump or Turbine***						
	X 8.28.1	BG	Easy	Short	1		Given const f. Power (Sec 5.10).
	8.28.2	BG	Easy	Short	1	8.28.3	Given const f. Power (Sec 5.10), pump.
	8.28.3	SI	Easy	Short	1	8.28.2	Given const f. Power (Sec 5.10), pump.
	8.28.4	BG	Medium	Short	1		Const f. h', power, cavit (§5.11), pump
	8.28.5	BG	Medium	Short	2	8.28.6	Const f. h', power (§ 5.10), turbine.
	8.28.6	SI	Medium	Short	2	8.28.5	Const f. h', power (§ 5.10), turbine.
	8.28.7	BG	Medium	Medium	1	8.28.8	Given const f. Power (§5.10), pump, jet
	8.28.8	SI	Medium	Medium	1	8.28.7	Given const f. Power (§5.10), pump, jet
	8.28.9	BG	Medium	Medium	1	8.28.10	Power (Sec 5.10), pump.
	8.28.10	SI	Medium	Medium	1	8.28.9	Power (Sec 5.10), pump.
	P 8.71	SI	Medium	Medium	1		Given const f. h', cavit (§5.11), pump.
	8.72	BG	Medium	Medium	2		Given const f. h', power (§5.10), pump
	8.73	BG	Hard	Medium	1	8.74	Given const f. h', power, turbine.
	8.74	SI	Hard	Medium	1	8.73	Given const f. h', power, turbine.
	8.75	BG	Hard	Medium	1		Given const f. h', turbine.
	8.76	BG	Hard	Medium	1		Given const f. Power (Sec 5.10), pump.
	8.77	BG	Hard	Long	1		☐ Given const f. h', power, cavitation,
	〃						pump, jet. Plot graph.
	8.78	BG	Hard	Long	3		Const f. h', power, pump; sketch.
8.29	***Branching Pipes***						
	P 8.79	BG	Hard	Long	1	8.80	† f varies. Case 1
	8.80	SI	Hard	Long	1	8.79	† f varies. Case 1
	8.81	SI	Hard	Long	1	8.80	☐ f varies. Case 1
	8.82	BG	Hard	Long	1	8.83, 84	f varies. T&E, interp, check **R**. Case 2
	8.83	BG	Hard	Long	1	8.82, 84	f varies. T&E, interp, check **R**. Case 2
	8.84	BG	Hard	Long	1	8.82, 83	☐ f varies Case 2
	8.85	SI	Hard	Long	1	8.86	f varies. T&E, interp, check **R**. Case 2
	8.86	SI	Hard	Long	1	8.85	f varies. T&E, interp, check **R**. Case 2
	8.87	SI	Hard	Long	1		f varies. T&E, interp, check **R**. Case 3
	8.88	SI	Hard	Long	1	8.87	☐ f varies. Case 3
	8.89	BG	Hard	Long	1		f varies. T&E, interp, check **R**. Case 3
	8.90	BG	Hard	Long	1	8.89	☐ f varies. Case 3
	8.91	BG	Hard	Long	1		☐ f varies. Check **R**. Special case
8.30	***Pipes in Series***						
	P 8.92	BG	Medium	Medium	1	8.93, 94	Given const f. h', jet.
	8.93	SI	Medium	Medium	1	8.92, 95	Given const f. h', jet, interpolation.
	8.94	BG	Medium	Medium	1	8.92, 95	Given const f. Jet.
	8.95	SI	Medium	Medium	1	8.93, 94	Given const f. Jet.
	8.96	BG	Hard	Long	1	8.97	† ☐ f varies. T & E
	8.97	SI	Hard	Long	1	8.96	† ☐ f varies. T & E
	8.98	BG	Hard	Long	1		† ☐ f varies. T & E
	8.99	BG	Hard	Long	1	8.100	f varies. Laminar flow, check **R**.
	8.100	SI	Hard	Long	1	8.99	f varies. Laminar flow, check **R**.
	8.101	BG	Hard	Long	1		† ☐ f varies. h', jet. T & E.

/cont...

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
8.31	***Pipes in Parallel***						
	P 8.102	BG	Hard	Medium	1	8.105	† f varies. Assume/verify Blasius' Eq
	8.103	BG	Medium	Medium	1	8.106	† f varies. Laminar flow, check **R**.
	8.104	BG	Hard	Long	1	8.107	† ☐ f varies. T & E.
	8.105	SI	Hard	Medium	1	8.102	† f varies. Assume/verify Blasius' Eq.
	8.106	SI	Medium	Medium	1	8.103	† f varies. Make/verify assump; Lam.
	8.107	SI	Hard	Long	1	8.104	† ☐ f varies. T & E.
8.32	***Pipe Networks***						
	P 8.108	BG	Medium	Medium	2	8.109,110	☐ Given const f. With pump, jet.
	8.109	BG	Medium	Medium	2	8.108,110	☐ Given const f. With pump, jet.
	8.110	BG	Medium	Medium	2	8.108,109	☐ Given const f. With pump, jet.
	8.111	BG	Medium	Long	2	8.112	☐ Given const f
	8.112	SI	Medium	Long	2	8.111	☐ Given const f. See ending comment.
	8.113	BG	Medium	Long	1	8.114	† f varies.
	8.114	SI	Medium	Long	1	8.113	† f varies.
	8.115	BG	Hard	Long	2		† ☐ f varies. T & E.
	8.116	BG	Medium	Long	2		☐ Given const f
	8.117	BG	Hard	Long	1	8.118	☐ Given K values. Repet Cross apprxs.
	8.118	SI	Hard	Long	1	8.117	☐ Given K values. Repet Cross apprxs.
	8.119	SI	Hard	V Long	1		† ☐ Const f. Repet'v H. Cross apprxns.
	8.120	BG	Hard	V Long	1		† ☐ Const f. Repet'v H. Cross apprxns.

Chapter 8
STEADY INCOMPRESSIBLE FLOW IN PRESSURE CONDUITS

Sec. 8.2: Critical Reynolds Number – *Exercises (3)*

8.2.1 *Oil with a kinematic viscosity of 0.00015 ft²/sec is flowing through a 3-in-diameter pipe. Below what velocity will the flow be laminar?*

BG

Eq. 8.1: $\mathbf{R} = (3/12)V/0.000\,15 < 2000 = \mathbf{R}_{crit}$; so $V < 1.200$ fps ◄

8.2.2 *Oil with a kinematic viscosity of 0.185 St is flowing through a 150-mm-diameter pipe. Below what velocity will the flow be laminar?*

SI

Inside cover: $\nu = 0.142$ St $= 0.142 \times 10^{-4}$ m²/s

Eq. 8.1: $\mathbf{R} = 0.15V/(0.185 \times 10^{-4}) < 2000 = \mathbf{R}_{crit}$; so $V < 0.247$ m/s ◄

8.2.3 *Oil with a kinematic viscosity of 0.0035 ft²/sec flows through a 4-in-diameter pipe with a velocity of 15 fps. Is the flow laminar or turbulent?*

BG

Eq. 8.1: $\mathbf{R} = (4/12)15/0.0035 = 1429 < \mathbf{R}_{crit} = 2000$, so flow is laminar ◄

Sec. 8.2: Critical Reynolds Number – *Problems 8.1–8.2*

8.1 *Hydrogen at 30°F and atmospheric pressure has a kinematic viscosity of 0.0011 ft²/sec. Determine the minimum turbulent flow rate in pounds per second through a 2-in-diameter pipe. At this flow rate what is the average velocity?*

BG

Eqs. 8.1–8.2: $\mathbf{R} = (2/12)V/0.0011 < 2000 = \mathbf{R}_{crit}$, so max. avg. $V = 13.20$ fps ◄

Table A.5 for hydrogen: ρ at 68°F = 0.000 162 slug/ft³ ; $\gamma = 0.000\,162(32.2) = 0.005\,22$ lb/ft³

At constant (atmos) p, from Eq. 2.5: $\gamma = \text{const}/T$

$\therefore$ at 30°F: $\gamma = 0.005\,22(460 + 68)/(460 + 30) = 0.005\,62$ lb/ft³

Eq. 4.5: $G = \gamma A V = (0.005\,62)(\pi/4)(2/12)^2 13.80 = 0.001\,692$ lb/sec ◄

8.2 *Air at 80°C and a pressure of approximately 1350 kPa abs flows in a 20-mm-diameter tube. What is the minimum turbulent flow rate? Express the answer in liters per second, newtons per second, and kilograms per second. At this flow rate what is the average velocity?*

SI

From Sec. 2.7: $p/\rho = RT$, $\rho = p/RT$; Eq. 8.1: $\mathbf{R} = DV\rho/\mu = DVp/\mu RT$

Table A.2: $\mu = 2.09 \times 10^{-5}$ N·s/m²; Table A.5: $R = 287$ m²/(s²·K)

Eq. 8.2: For laminar flow, max $\mathbf{R} = \dfrac{(0.02 \text{ m})(V)(1350 \times 10^3 \text{ N/m}^2)}{(2.09 \times 10^{-5} \text{ N·s/m}^2)[287 \text{ m}^2/(\text{s}^2\text{·K})](273 + 80 \text{ K})} = 2000$

i.e. $12\,750\,V$ m/s = 2000; max $V = 0.1569$ m/s ◄

Eq. 4.3: $Q = AV = \pi(0.01)^2 0.1569 = 0.000\,0493$ m³/s = 0.0493 L/s ◄

From Sec. 2.7: $\rho = p/RT = 1350 \times 10^3/[287(273 + 80)] = 13.32$ kg/m³

Eq. 4.4: $\dot{m} = \rho Q = (13.32)0.0493 \times 10^{-3} = 0.657 \times 10^{-3}$ kg/s ◄

Eq. 4.5: $G = \gamma Q = \rho g Q = 13.32(9.81)0.0493 \times 10^{-3} = 6.44 \times 10^{-3}$ N/s ◄

Sec. 8.3: Hydraulic Radius, Hydraulic Diameter -- *Exercises (2)*

8.3.1 BG *What is the hydraulic radius of a 12 in by 16 in rectangular air duct?*

Eq. 8.3: $R_h = A/P = (12 \times 16)/[2(12 + 16)] = 3.43$ inch $= 0.286$ feet ◄

8.3.2 SI *What is the percentage difference between the hydraulic radii of a 300-mm-diameter duct and a 300-mm-square duct?*

Eq. 8.3: Circular duct: $R_h = A/P = (\pi/4)(300^2)/(300\pi) = 75$ mm

Square duct: $R_h = A/P = 300^2/(4 \times 300) = 75$ mm; Therefore % difference = zero ◄

Sec. 8.3: Hydraulic Radius, Hydraulic Diameter -- *Problem 8.3*

8.3 N *Two pipes, one circular and one square, have the same cross-sectional area (Fig. P8.3). Which has the larger hydraulic radius, and by what percentage?*

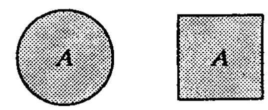


Figure P8.3

Circular pipe, diameter D, and square pipe, side a, have same cross-sectional area so $a^2 = (\pi/4)D^2$, i.e., $a = \pi^{1/2}D/2$

Eq. 8.3, circular: $R_h = A/P = (\pi/4)D^2/\pi D = D/4 = 0.25D$

Square: $R_h = A/P = a^2/4a = a/4 = \pi^{1/2}D/8 = 0.222D$

R_h ratio $= 0.25D/0.222D = 1.128$; so circular pipe has larger R_h, by 12.8% ◄

Sec. 8.5: Friction in Circular Conduits -- *Exercises (4)*

8.5.1 BG *Steam with a specific weight of 0.32 lb/ft³ is flowing with a velocity of 94 fps through a circular pipe with f = 0.0171. What is the shear stress at the pipe wall?*

Eq. 8.19: $\tau_0 = (0.0171/4)0.32(94)^2/[2(32.2)] = 0.1877$ psf ◄

8.5.2 SI *Steam with a specific weight of 38 N/m³ is flowing with a velocity of 35 m/s through a circular pipe with f = 0.0154. What is the shear stress at the pipe wall?*

Eq. 8.19: $\tau_0 = (0.0154/4)38(35)^2/[2(9.81)] = 9.13$ N/m² ◄

8.5.3 BG *Oil (s = 0.93) of viscosity of 0.004 ft²/s flows in a 4-in-diameter pipe at a rate of 6.5 gpm. Find the head loss per unit length.*

$Q = 6.5$ gpm $= 0.014\,48$ cfs; Eq. 4.7: $V = 4Q/(\pi D^2) = 4(0.014\,48)/[\pi(4/12)^2] = 0.1660$ fps

Eq. 8.1: $\mathbf{R} = [(4/12)(0.1660)]/(0.004) = 13.83$, flow is laminar, $f = 64/13.83 = 4.63$

Eq. 8.14: $h_f/L = 4.63(12/4)(0.1660)^2/[2(32.2)] = 0.005\,94$ feet per feet ◄

8.5.4 SI *Oil (s = 0.92) of viscosity of 0.000 38 m²/s flows in a 100-mm-diameter pipe at a rate of 0.64 L/s. Find the head loss per unit length.*

Eq. 4.7: $V = Q/A = 4(0.000\,64)/(\pi 0.10^2) = 0.0815$ m/s

Eq. 8.1: $\mathbf{R} = (0.1)0.0815/0.000\,38 = 21.44$, flow is laminar; Eq. 8.29: $f = 64/21.44 = 2.98$

Eq. 8.14: $h_f/L = 2.98(1/0.10)(0.0815)^2/[2(9.81)] = 0.010\,10$ m per meter ◄

Sec. 8.5: Friction in Circular Conduits -- ***Problems*** *8.4–8.6*

8.4 *Prove that for a constant rate of discharge and a constant value of f the friction head loss in a pipe varies inversely as the fifth power of the diameter.*

N

Eq. 8.13: $h_f = f(L/D)V^2/(2g)$ where $V = Q/A = Q/(\pi D^2/4) = 4Q/(\pi D^2)$

$h_f = f(L/D)[4Q/(\pi D^2)]^2/(2g) = f(L/D^5)16Q^2/(2g\pi^2)$

Thus for f = constant and Q = constant, $h_L = 1/D^5$ Q.E.D. ◄

8.5 *Two long pipes convey water between two reservoirs whose water surfaces are at different elevations. One pipe has a diameter twice that of the other; both pipes have the same length and the same value of f. If minor losses are neglected, what is the ratio of the flow rates through the two pipes?*

N

Eq. 8.13: $\Delta\text{elev} = h_f = f(L/D)V^2/(2g)$ where $V = Q/A = Q/(\pi D^2/4)$

$\therefore\ h_f = f(L/D)[Q/(\pi D^2/4)]^2/2g = fL4^2Q^2/(2gD\pi^2D^4)$

Thus $h_f \propto Q^2/D^5$; $(h_L)_1 = (h_L)_2$; $\therefore\ Q_1^2/D_1^5 = Q_2^2/D_2^5$ and $Q_2/Q_1 = (D_2/D_1)^{5/2} = 2^{5/2} = 5.66$

The flow in the larger pipe will be 5.66 times that in the smaller pipe. ◄

8.6 *Tests were made with 60°F water flowing in a 9-in-diameter pipe. They showed that, when V = 12 fps, f = 0.0165. Find the unit shear at the pipe wall and at radii of 0, 0.25, 0.4, 0.6, 0.85 times the pipe radius.*

BG

Table A.1 for water at 60°F: $\rho = 1.938$ slugs/ft³

(*a*) Eq. 8.19: $\tau_0 = (0.0165/4)1.938(12^2/2) = 0.576$ psf, at wall ◄

(*b*) Stress distribution is linear (Eq. 8.18):

r/r_0	τ (psf)
0	0
0.25	0.1439
0.4	0.230
0.6	0.345
0.85	0.489

▲

Sec. 8.6: Friction in Noncircular Conduits -- ***Exercises*** *(2)*

8.6.1 *When fluid of specific weight 46 lb/ft³ flows in a 8-in-diameter pipe, the frictional stress between the fluid and the pipe wall is 0.65 lb/ft². Calculate the friction head loss per foot of pipe. If the flow rate is 1.8 cfs, how much power is lost per foot of pipe?*

BG

Eq. 8.20: $R_h = \frac{D}{4} = \frac{8/12}{4} = 0.1667$ ft. Eq. 8.8: $h_f = \frac{\bar{\tau}_0 L}{R_h \gamma} = \frac{(0.65)1}{0.1667(46)} = 0.0848$ ft per ft ◄

Eq. 5.40: Power loss/ft = 46(1.8)0.0848/550 = 0.012 76 hp per ft ◄

8.6.2 *When fluid of specific weight 8.2 kN/m³ flows in a 150-mm-diameter pipe, the frictional stress between the fluid and the pipe wall is 25 N/m². Calculate the friction head loss per meter of pipe. If the flow rate is 42 L/s, how much power is lost per meter of pipe?*

SI

Eq. 8.20: $R_h = D/4 = 0.15/4 = 0.0375$ m

Eq. 8.8: $h_f = \bar{\tau}_0 L/(R_h\gamma)$ = (25 N/m²)(1 m)/[(0.0375 m)8200 N/m³] = 0.0813 m/m ◄

From Eq. 5.41: Power loss/m = $\gamma Q h_L$ = (8200 N/m³)(0.042 m³/s)(0.0813 m) = 28.0 W/m ◄

Sec. 8.6: Friction in Noncircular Conduits -- ***Problems*** *8.7–8.9*

8.7 *Air at 160°F and standard sea-level atmospheric pressure flows in a 15-in by 21-in rectangular air duct (e = 0.0007 in) at the rate of 1.2 lb/min. Find the friction head loss per 100 ft of duct. Express the answer in feet of air flowing and in pounds per square inch.*

BG

Eq. 8.20: $R_h = A/P = 1.25(1.75)/6 = 0.365$ ft $= 4.38$ inches

Using Eq. 8.21: $e/D = e/(4R_h) = 0.0007/(4 \times 4.38) = 0.000\,0400$

Table A.5 for air: $R = 1715$ ft²/(sec²·°R). Eq. 2.5: $\gamma = \dfrac{pg}{RT} = \dfrac{60(144)32.2}{1715(460 + 160)} = 0.262$ lb/ft³

$Q = (1.2$ lb/min)(ft³/0.262 lb)(min/60 sec) $= 0.0764$ ft³/s; $V = Q/A = 0.0764/2.1875 = 0.0350$ fps

Table A.2 for air at 160°F: $\nu = 0.212 \times 10^{-3}$ ft²/sec

From Eq. 8.23: $\mathbf{R} = 4R_hV/\nu = 4(0.365)0.0350/(0.212 \times 10^{-3}) = 240$

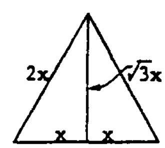
Flow is laminar, so *e/D* is insignificant and from Eq. 8.29: $f = 64/\mathbf{R} = 64/240 = 0.267$

Eq. 8.22: $h_f = 0.267[100/(4 \times 0.365)]0.0350^2/[(2(32.2)] = 0.000\,347$ ft ◄

$\Delta p = h_L\gamma = 0.000\,347(0.262)/144 = 6.32 \times 10^{-7}$ psi ◄

8.8 *Water at 80°F flows in a conduit with a cross section shaped in the form of an equilateral triangle. The cross-sectional area of the duct is 160 in² and e = 0.0018 in. If the frictionhead loss is 3 ft in 150 ft, find the approximate flow rate.*

BG



$A = 3^{1/2}x^2 = 160$ in², $x = 9.61$ in $= 0.801$ ft

Eq. 8.20: $R_h = A/P = 160/(3 \times 2 \times 9.61) = 2.77$ in $= 0.231$ ft

Eq. 8.21: $D = 4R_h = 0.925$ ft; $e/4R_h = 0.0018/(4 \times 2.77) = 0.000\,1622$

Table A.1 for water at 80°F: $\nu = 0.930 \times 10^{-5}$ ft²/sec

From Eq. 8.23: $\mathbf{R} = 0.925V/(0.930 \times 10^{-5}) = 9.945 \times 10^4 V$ (1)

Eq. 8.22: $h_f = 3 = f\dfrac{150}{0.925}\dfrac{V^2}{2(32.2)}$; $1.191 = fV^2$ (2)

Solve by T and E to satisfy (1), (2), and Fig. 8.11

If $f = 0.015$, $V = 8.91$ fps, $\mathbf{R} = 8.86 \times 10^5$; Fig. 8.11 or Eq. 8.52: $f = 0.0143$ Try again.

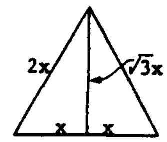
If $f = 0.0143$, $V = 9.13$ fps, $\mathbf{R} = 9.08 \times 10^5$; Fig. 8.11 or Eq. 8.52: $f = 0.0143$ Close enough.

Thus $V = 9.13$ fps and $Q = 9.13(160/144) = 10.14$ cfs ◄

8.9 *Water at 20°C flows in a conduit with a cross section shaped in the form of an equilateral triangle. The cross-sectional area of the duct is 0.100 m² and e = 0.045 mm. If the friction head loss is 1 m in 50 m, find the approximate flow rate.*

SI

$A = 3^{1/2}x^2 = 0.100 \text{ m}^2, \; x = 0.2403 \text{ m}$



Eq. 8.20: $R_h = A/P = 0.100/(6 \times 0.2403) = 0.0694$ m

Eq. 8.21: $D = 4R_h = 0.278$ m; $e/(4R_h) = 0.045/(4 \times 69.4) = 0.000\,1622$

Table A.1 for water at 20°C: $\nu = 1.003 \times 10^{-6}$ m²/s

From Eq. 8.23: $\mathbf{R} = 0.278V/(1.003 \times 10^{-6}) = 2.77 \times 10^5 V$ (1)

Eq. 8.22: $h_f = 1 = f\dfrac{50}{0.278}\dfrac{V^2}{2(9.81)}$; $0.1091 = fV^2$ (2)

Solve by T and E to satisfy (1), (2), and Fig. 8.11 or Eq. 8.52

If $f = 0.015$, $V = 2.70$ m/s, $\mathbf{R} = 7.47 \times 10^5$; Fig. 8.11 or Eq. 8.52: $f = 0.0143$ Try again.

If $f = 0.0143$, $V = 2.76$ m/s, $\mathbf{R} = 7.65 \times 10^5$; Fig. 8.11 or Eq. 8.52: $f = 0.0145$ Close enough!

Thus $V = 2.76$ m/s and $Q = AV = (0.100)2.76 = 0.276$ m³/s ◄

Sec. 8.7: Laminar Flow in Circular Pipes -- *Exercises (3)*

8.7.1 *An oil with kinematic viscosity 0.004 ft²/sec weighs 62 lb/ft³. What will be its flow rate and head loss in a 2750-ft length of a 3-in-diameter pipe when the Reynolds number is 950?*

BG

Eq. 8.1: $\mathbf{R} = DV/\nu = (3/12)V/0.004 = 950$, $V = 15.20$ fps; $\mathbf{R} < 2000$, so flow is laminar

Eq. 4.3: $Q = AV = (\pi/4)(3/12)^2 15.20 = 0.746$ cfs ◄

Eq. 8.29: $f = 64/\mathbf{R} = 64/950 = 0.0673$; Eq. 8.10: $h_L = 0.0673(2750 \times 12/3)15.20^2/(2g) = 2660$ ft ◄

8.7.2 *With laminar flow in a circular pipe, at what distance from the centerline (in terms of the pipe radius) does the average velocity occur?*

N

Sec. 8.6: Velocity profile is a paraboloid with $V_{mean} = 0.5V_c$, so

Eq. 8.25: $u = V_c(1 - r^2/r_0^2)$; $(1/2)V_c = V_c(1 - r^2/r_0^2)$; $(r/r_0)^2 = 0.5$; $r/r_0 = 0.707$; $r = 0.707r_0$ ◄

8.7.3 *For laminar flow in a two-dimensional passage, find the relation between the average and maximum velocities.*

N

Let passage width be $2y_0$, so that $u = u_{max} = V_c$ at $y = y_o$.

Eq. 4.3: $Q = AV = (2y_0)V$. But from Eq. 8.25: $Q = 2\int_0^{y_0} u\,dy = 2\int_0^{y_0} V_c(1 - y^2/y_0^2)dy$

By equating the two: $V = 2V_c\left[y - (y^3/y_0^2)\right]_0^{y_0}/(2y_0) = (2/3)V_c$ ◄

Sec. 8.7: Laminar Flow in Circular Pipes – ***Problems*** *8.10–8.13*

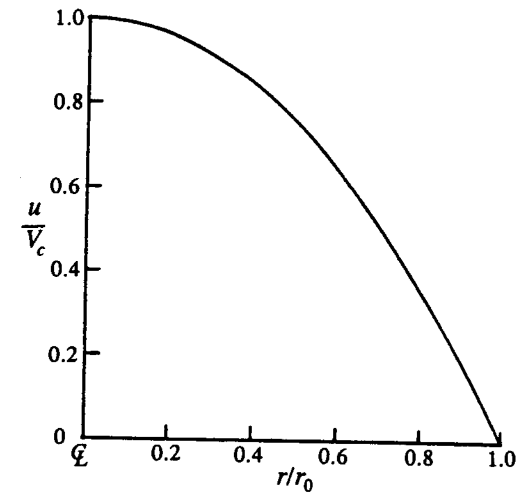
8.10 *For laminar flow in a circular pipe, find the velocities at 0.1r, 0.3r, 0.5r, 0.7r, and 0.9r. Plot the velocity profile.*

N

From Eq. 8.25:

r/r_0	u/V_c
0.1	0.99
0.3	0.91
0.5	0.75
0.7	0.51
0.9	0.19

▲



8.11 *Prove that the centerline velocity is twice the average velocity when laminar flow occurs in a circular pipe.*

N

Eq. 8.25: $u = V_c(1 - r^2/r_0^2)$

Eq. 4.3: $Q = AV = \int u\,dA = \int_0^{r_0} V_c(1 - r^2/r_0^2)2\pi r\,dr = 2\pi V_c \int_0^{r_0}(r - r^3/r_0^2)dr = 2\pi V_c(r_0^2/4)$

Eq. 4.6: $V = Q/A = [2\pi V_c(r_0^2/4)]/(\pi r_0^2) = (1/2)V_c$ Q.E.D. ◄

8.12 *For laminar flow between two parallel, flat plates a small distance d apart, at what distance from the centerline (in terms of d) will the velocity be equal to the mean velocity?*

N

Let y_m = distance from centerline where $u = V_{mean}$. From solution to Exer. 8.7.3, $V_{mean} = (2/3)V_c$.

So when $y = y_m$, $u = (2/3)V_c$, i.e. $(2/3)V_c = V_c(1 - y_m^2/y_0^2)$; $y_m^2 = (1/3)y_0^2 = (1/3)(d/2)^2$;

$y_m = 0.289d$ ◄

8.13 *Oil with an absolute viscosity of 0.16 N·s/m² and a density of 925 kg/m³ is flowing in a 200-mm-diameter pipe at 0.50 L/s. How much power is lost per meter of pipe length?*

SI

Eq. 4.7: $V = \$(0.000\,50)/(\pi 0.200^2) = 0.015\,92$ m/s

Eq. 8.1: $\mathbf{R} = [0.20(0.015\,92)925]/0.16 = 18.40$, flow is laminar. Eq. 8.29: $f = 64/18.40$

Eq. 8.14: $h_f/L = (64/18.40)(1/0.20)0.015\,92^2/[2(9.81)] = 0.000\,1796$ meter per meter

Eq. 5.41: Power loss = $(925 \times 9.81)(\pi 0.100^2)0.015\,92(0.000\,1796) = 0.000\,815$ watts per meter ◄

Sec. 8.8: Entrance Conditions in Laminar Flow -- *Exercises (2)*

8.8.1 *In Exer. 8.2.3 what will be the approximate distance from the pipe entrance to the first point at which the flow is established?*

Exer. 8.2.3: Oil with a kinematic viscosity of 0.0035 ft²/sec flows through a 4-in-diameter pipe with a velocity of 15 fps.

BG

Eq. 8.1: $\mathbf{R} = (4/12)15/0.0035 = 1429$

Eq. 8.30: $L_e = 0.058\mathbf{R}D = 0.058(1429)(4/12) = 27.6$ feet ◄

8.8.2 *In Exer. 8.5.4 what will be the approximate distance from the pipe entrance to the first point at which the flow is established?*

Exer. 8.5.4: Oil (s = 0.92) of viscosity of 0.00038 m²/s flows in a 100-mm-diameter pipe at a rate of 0.64 L/s.

SI

Eq. 4.7: $V = Q/A = 4Q/(\pi D^2) = 4(0.000\,64)/(\pi 0.10^2) = 0.0815$ m/s

Eq. 8.1: $\mathbf{R} = (0.10)0.0815/0.000\,38 = 21.44$

Eq. 8.30: $L_e = 0.058\mathbf{R}D = 0.058(21.44)0.10 = 0.1244$ m ◄

Sec. 8.9: Turbulent Flow -- *Exercises (2)*

8.9.1 *Tests on 70°F water flowing through a 9-in-diameter pipe showed that when V = 13 fps, f = 0.0162. (a) If, at a distance of 3 in from the center of the pipe, τ = 0.388 psf and the velocity profile gives a value for du/dy of 6.97 per second, find at that radius (a) the viscous shear, (b) the turbulent shear, and (c) the mixing length ℓ.*

BG

Table A.1 for water at 70°F: $\rho = 1.936$ slugs/ft³, $\mu = 0.000\,0205$ lb·sec/ft²

(*a*) Eq. 8.32 at $r = 3$ in: $\tau = \mu du/dy + \eta du/dy = 0.388$ psf (given)

$\mu du/dy = 0.000\,0205(6.97) = 0.000\,1429$ psf, viscous shear ◄

(*b*) $\eta du/dy = \tau - \mu dy/dy = 0.388 - 0.000\,1429 = 0.388$ psf, turbulent shear ◄

(*c*) Eq. 8.34: $\tau_{turb} = \rho\ell^2(du/dy)^2 = 0.388$ psf

$\ell = (\tau/\rho)^{1/2}/(du/dy) = [0.388(1.936)]^{1/2}/6.97 = 0.0642$ ft or 0.771 inch ◄

8.9.2 *Water at 20°C flows through a 240-mm-diameter pipe. Tests have determined that at a distance of 60 mm from the pipe centerline the mixing length ℓ is 19.8 mm and from the velocity profile du/dy = 5.33 s⁻¹. Find at that radius (a) the total shear stress, (b) the eddy viscosity, (c) the viscous shear, and (d) the turbulent shear.*

BG

Table A.1 for water at 20°C: $\rho = 998.2$ kg/m³, $\mu = 0.001002$ N·s/m²

(*a*) Eq. 8.24: $\tau = 998.2\left[\frac{19.8}{1000}\right]^2 5.33^2 = 11.12\ \text{kg·m}^{-1}\text{s}^{-2} = 11.12\ \text{M/m}^2$ ◄

(*b*) Eq. 8.22: $\tau = (\mu + \eta)\frac{du}{dy}$; $11.12 = (0.001\,002 + \eta)5.33$; $\eta = 2.08$ N·s/m² ◄

(*c*) Viscous shear $= \mu\, du/dy = 0.001\,002(5.33) = 0.005\,34$ N/m² ◄

(*d*) Turbulent shear $= \eta\, du/dy = 2.08(5.33) = 11.11$ N/m² ◄

Sec. 8.10: Viscous Sublayer in Turbulent Flow -- *Exercises (6)*

8.10.1 *Compute δ_v for the data of Sample Prob. 8.3.*

Sample Prob. 8.3: The pipe friction head loss in 200 ft of 6-in-diameter pipe is 25 ft·lb/lb when oil (s = 0.90) of viscosity 0.0008 lb·sec/ft² flows at 2.0 cfs. $V = Q/A = 10.19$ fps; $f = h_f D(2g)/(LV^2) = 0.0388$.

BG

Eq. 2.11: $\nu = \mu/\rho = 0.0008/[0.90(1.940)] = 0.000\,458$ ft²/sec

Eq. 8.38: $\delta_v = \dfrac{14.14\nu}{Vf^{1/2}} = \dfrac{14.14(0.000\,458)}{10.19(0.0388)^{1/2}} = 0.00323$ feet $= 0.387$ in ◄

8.10.2 *Water in a pipe (f = 0.018) is at a temperature of 70°F. (a) If the mean velocity is 14 fps, what is the nominal thickness δ_v of the viscous sublayer? (b) What will δ_v be if we increase the velocity to 24 fps and f does not change?*

BG

Table A.1 for water at 70°F: $\nu = 1.059 \times 10^{-5}$ ft²/sec

(*a*) Eq. 8.38: $\delta_v = \dfrac{14.14\nu}{Vf^{1/2}} = \dfrac{14.14(1.059 \times 10^{-5})}{14(0.018)^{1/2}} = 0.000\,0797$ ft $= 0.000\,957$ inch ◄

(*b*) For $V = 24$ fps and the same f, $\delta_v' = (14/24)\delta_v = 0.000\,558$ inch ◄

8.10.3 *Water in a pipe (f = 0.012) is at a temperature of 15°C. (a) If the mean velocity is 3.2 m/s, what is the nominal thickness δ_v of the viscous sublayer? (b) What will δ_v be if we increase the velocity to 5.5 m/s and f does not change?*

SI

Table A.1 for water at 15°C: $\nu = 1.139 \times 10^{-6}$ m²/s

(*a*) Eq. 8.38: $\delta_v = \dfrac{14.14\nu}{Vf^{1/2}} = \dfrac{14.14(1.139 \times 10^{-6})}{3.2(0.012)^{1/2}} = 4.59 \times 10^{-5}$ m $= 0.0459$ mm ◄

(*b*) For $V = 5.5$ m/s and the same f, $\delta_v' = (3.2/5.5)\delta_v = 0.0267$ mm ◄

8.10.4 *For the data in Exer. 8.10.2(a), what is the distance from the wall to the assumed limit of the transition region where true turbulent flow begins?*

Exer. 8.10.2(a): Water at 70°F flows in a pipe (f = 0.018) with a mean velocity of 14 fps.

BG

Table A.1 for water at 70°F: $\nu = 1.059 \times 10^{-5}$ ft²/sec

Eq. 8.38: $\delta_v = \dfrac{14.14\nu}{Vf^{1/2}} = \dfrac{14.14(1.059 \times 10^{-5})}{14(0.018)^{1/2}} = 0.0000797$ ft $= 0.000\,957$ inch

Sec. 8.10: $y_t = 70\nu/u_* = 14\delta_v = 14(0.000\,957) = 0.013\,40$ inch ◄

8.10.5 *For the data in Exer. 8.10.3(a), what is the distance from the wall to the assumed limit of the transition region where true turbulent flow begins?*

Exer. 8.10.3(a): Water at 15°C flows in a pipe (f = 0.012) with a mean velocity of 3.2 m/s.

SI

Table A.1 for water at 15°C: $\nu = 1.139 \times 10^{-6}$ m²/s

Eq. 8.38: $\delta_v = \dfrac{14.14\nu}{Vf^{1/2}} = \dfrac{14.14(1.139 \times 10^{-6})}{3.2(0.012)^{1/2}} = 4.59 \times 10^{-5}$ m $= 0.0459$ mm

Sec. 8.10: $y_t = 70\nu/u_* = 14\delta_v = 14(0.0459) = 0.643$ mm ◄

8.10.6 *Water at 50°C flows in a 150-mm-diameter pipe with $V = 6.5$ m/s and $e = 0.14$ mm. Head loss measurements indicate that $f = 0.020$. (a) What is the thickness of the viscous sublayer? (b) Is the pipe behaving as a fully rough pipe?*

SI

Table A.1 for water at 50°C: $\nu = 0.553 \times 10^{-6}$ m²/s

(*a*) Eq. 8.38: $\delta_v = \dfrac{14.14\nu}{Vf^{1/2}} = \dfrac{14.14(0.553 \times 10^{-6}}{6.5(0.020)^{1/2}} = 8.51 \times 10^{-6}$ m $= 8.51$ μm ◄

(*b*) $e/14 = 140$ μm/14 $= 10$ μm, so $\delta_v < e/14$; so, yes, pipe is behaving as fully rough ◄

Sec. 8.10: Viscous Sublayer in Turbulent Flow -- *Problems* 8.14–8.15

8.14 *Water at 50°F enters a pipe with a uniform velocity of $U = 14$ fps. (a) What is the distance at which the transition occurs from a laminar to a turbulent boundary layer? (b) If the thickness of this initial laminar boundary layer is given by $4.91\sqrt{\nu x/U}$ (from Eq. 9.10), what is its thickness at the point of transition?*

BG

Table A.1 for water at 50°F: $\nu = 1.410 \times 10^{-5}$ ft²/sec

(*a*) Sec. 8.10: At transition point (turbulent boundary layer begins): For $\mathbf{R}_x = 500{,}000 = Ux/\nu$

$x = 500{,}000\nu/U = 500{,}000(0.000\,0141)/14 = 0.504$ ft $= 6.04$ inches ◄

(*b*) Given: $\delta = 4.91\sqrt{\nu x/U} = 4.91\sqrt{0.000\,0141 \times 0.504/14} = 0.003\,50$ feet or 0.0420 inches ◄

8.15 *Water at 10°C enters a pipe with a uniform velocity of $U = 3.4$ m/s. (a) What is the distance at which the transition occurs from a laminar to a turbulent boundary layer? (b) If the thickness of this initial laminar boundary layer is given by $4.91\sqrt{\nu x/U}$ (from Eq. 9.10), what is its thickness at the point of transition?*

SI

Table A.1 for water at 10°C: $\nu = 1.306 \times 10^{-6}$ m²/s

(*a*) Sec. 8.10: At transition point (turbulent boundary layer begins): For $\mathbf{R}_x = 500\,000 = Ux/\nu$

$x = 500\,000\nu/U = 500\,000(1.306 \times 10^{-6})/3.4 = 0.1921$ m $= 192.1$ mm ◄

(*b*) Given: $\delta = 4.91\sqrt{\nu x/U} = 4.91\sqrt{1.306 \times 10^{-6} \times 0.1921/3.4} = 0.001\,334$ m or 1.334 mm ◄

Sec. 8.11: Velocity Profile in Turbulent Flow -- *Exercises* (3)

8.11.1 *In a 1.00-m-diameter pipe velocities are measured as 5.35 m/s on the centerline and 4.91 m/s at $r = 70$ mm. Approximately what is the flow rate?*

SI

Eq. 8.40: $4.91 = 5.35 - 5.76u_*\log[500/(500 - 70)]$ from which $u_* = 1.166$ m/s

Eq. 8.42: $V = 5.35 - 3.75(1.166) = 0.977$ m/s

$Q = AV = (\pi/4)1.00^2(0.977) = 0.767$ m³/s ◄

8.11.2 *For turbulent flow in a circular pipe, find r/r_0 at the radial distance from the centerline where the mean velocity occurs.*

N

Turbulent flow. Eq. 8.44 for $u = V$: $V = (1 + 1.326f^{1/2})V - 2.04f^{1/2}V\log[r_0/(r_0 - r)]$

$0 = 1.326f^{1/2}V - 2.04f^{1/2}V\log\dfrac{r_0}{r_0 - r}$; $\log\dfrac{r_0}{r_0 - r} = \dfrac{1.326}{2.04} = 0.650$; $\dfrac{r_0}{r_0 - r} = 4.47$; $\dfrac{r}{r_0} = 0.776$ ◄

8.11.3 *Oil (s = 0.92) with a viscosity of 0.000 65 lb·sec/ft² flows at a rate of 6 cfs through a 4-in-diameter pipe having f = 0.040. Find the friction head loss. Determine the shear stress at the pipe wall and the velocity at 1.5 in from the centerline.*

BG

Eq. 4.7: $V = \dfrac{4(6)}{\pi(4/12)^2} = 68.8$ fps; Eq. 8.1: $\mathbf{R} = \dfrac{DV\rho}{\mu} = \dfrac{(4/12)68.8(1.94 \times 0.92)}{0.000\,65} = 62{,}200$

Eq. 8.13: $h_f = \dfrac{0.040(1)}{(4/12)}\dfrac{68.8^2}{2(32.2)} = 8.81$ feet per foot ◄

Eq. 8.19: $\tau_0 = f\rho V^2/8 = 0.040(0.92 \times 1.940)68.8^2/8 = 42.2$ psf ◄

From Eq. 8.43: $u_{max} = V(1 + 1.326\sqrt{f}) = 68.8(1 + 1.326\sqrt{0.040}) = 87.0$ fps

Eq. 8.40: $u = u_{max} - 5.76\sqrt{\dfrac{\tau_0}{\rho}}\log\dfrac{r_0}{r_0 - r} = 87.0 - 5.76\sqrt{\dfrac{42.2}{0.92(1.940)}}\log\dfrac{2.0}{2.0 - 1.5} = 70.1$ fps ◄

Sec. 8.11: Velocity Profile in Turbulent Flow -- *Problems 8.16–8.23*

8.16 *In a 36-in-diameter pipe velocities are measured as 18.5 fps at r = 0 and 18.0 fps at r = 4.0 in. Approximately what is the flow rate?*

BG

Eq. 8.40: $18.0 = 18.5 - 5.76u_*\log[18/(18 - 4.0)]$ from which $u_* = 0.795$ fps

Eq. 8.37: $0.795 = u_* = V\sqrt{f/8}$. Thus $f^{1/2} = 2.25/V$ (1)

Eq. 8.43: $V/18.5 = 1/(1 + 1.326f^{1/2})$ (2)

Eliminating f between (1) and (2): $\dfrac{18.5}{V} = 1 + 1.326\dfrac{2.25}{V}$, from which $V = 15.52$ fps (so $f = 0.0579$)

Eq. 4.7: $Q = AV = (\pi/4)(36/12)^2 15.52 = 109.7$ cfs ◄

8.17 *Water at 130°F flows in an 0.5-in-diameter copper tube (e = 0.000 005 ft) at 1.2 gpm. Find the head loss per 100 ft, using Eq. (8.29) or (8.52) to find f. What is the centerline velocity, and what is the value of δ_v?*

BG

$Q = 1.2$ gpm $= 0.002\,67$ cfs; Eq. 4.7: $V = Q/A = \dfrac{4Q}{\pi D^2} = \dfrac{4(0.002\,67)}{\pi(0.5/12)^2} = 1.961$ fps

Table A.1 for water at 130°F: $\nu = 5.58 \times 10^{-6}$ ft²/sec

Eq. 8.1: $\mathbf{R} = \dfrac{DV}{\nu} = \dfrac{(0.5/12)1.963}{5.58 \times 10^{-6}} = 14{,}640$ (flow is turbulent) ; $\dfrac{e}{D} = \dfrac{0.000\,005}{0.5/12} = 0.000\,12$

Eq. 8.52: $f = 0.028$; Eq. 8.13: $h_f = 0.028\dfrac{100}{(0.5/12)}\dfrac{1.961^2}{2(32.2)} = 4.01$ feet ◄

Eq. 8.43: $1.961/u_{max} = 1/[1 + 1.326(0.028)^{1/2}]$; $u_{max} = 1.222(1.961) = 2.40$ fps ◄

Eq. 8.38: $\delta_v = \dfrac{14.14(5.58 \times 10^{-6})}{1.961(0.028)^{1/2}} = 0.240 \times 10^{-3}$ ft ◄

8.18 *Water at 60°C flows in a 15-mm-diameter copper tube (e = 0.0015 mm) at 0.06 L/s. Find the head loss per 10 m, using Eq. (8.29) or (8.52) to find f. What is the centerline velocity, and what is the value of δ_v?*

SI

Table A.1: At 60°C, $\nu = 0.474 \times 10^{-6}\ \text{m}^2/\text{s}$

Eq. 4.7: $V = Q/A = \dfrac{4Q}{\pi D^2} = \dfrac{4(0.06 \times 10^{-3})}{\pi(0.015)^2} = 0.340\ \text{m/s}$

Eq. 8.1: $\mathbf{R} = \dfrac{DV}{\nu} = \dfrac{0.015(0.340)}{0.474 \times 10^{-6}} = 10\,740$ (flow is turbulent) ; $\dfrac{e}{D} = \dfrac{0.0015}{15} = 0.000\,100$

Eq. 8.52: $f = 0.0304$; Eq. 8.13: $h_f = 0.031\dfrac{10}{(0.015)}\dfrac{0.340^2}{2(9.81)} = 0.1191\ \text{m}$ ◄

Eq. 8.43: $0.340/u_{max} = 1/[1 + 1.326(0.0304)^{1/2}]$; $u_{max} = 0.418\ \text{m/s}$ ◄

Eq. 8.38: $\delta_v = \dfrac{14.14(0.474 \times 10^{-6})}{0.340(0.0304)^{1/2}} = 0.000\,1132\ \text{m}$ ◄

8.19 *Repeat Prob. 8.17 for flow rates of 0.08 and 18 gpm.*

Prob. 8.17: Water at 130°F flows in an 0.5-in-diameter copper tube (e = 0.000 005 ft) at 1.2 gpm. Find the head loss per 100 ft, using Eq. (8.29) or (8.52) to find f. What is the centerline velocity, and what is the value of δ_v?

BG

Inside cover: cfs = gpm/449. Table A.1 for water at 130°F: $\nu = 5.58 \times 10^{-6}\ \text{ft}^2/\text{sec}$

Q (given)	0.08 gpm	18 gpm
Q	0.000 1782 cfs	0.0401 cfs
$V = Q/A$	0.1307 fps	29.4 fps
e/D	0.000 12	0.000 12
$\mathbf{R} = DV/\nu$	976	2.20×10^5
Flow regime	Laminar	Turbulent
f	0.0655 (Eq. 8.29)	0.0161 (Eq. 8.52)
h_f (Eq. 8.13)	0.0417 ft ◄	525 ft ◄
u_{max} (from Eq. 8.43)	0.1751 fps ◄	34.4 fps ◄
δ_v (Eq. 8.38)	2.36×10^{-3} ft ◄	2.10×10^{-5} ft ◄

8.20 *Repeat Prob. 8.18 for flow rates of 0.004 and 0.9 L/s.*

Prob. 8.18: Water at 60°C flows in a 15-mm-diameter copper tube (e = 0.0015 mm) at 0.06 L/s. Find the head loss per 10 m, using Eq. (8.29) or (8.52) to find f. What is the centerline velocity, and what is the value of δ_v?

SI

Table A.1: At 60°C, $\nu = 0.474 \times 10^{-6}\ \text{m}^2/\text{s}$

Q (given)	0.004 L/s	0.9 L/s
Q	0.000 004 m³/s	0.0009 m³/s
$V = Q/A$	0.0226 m/s	5.09 m/s
e/D	0.0001	0.0001
$\mathbf{R} = DV/\nu$	716	161 200
Flow regime	Laminar	Turbulent
f	0.089 (Eq. 8.29)	0.0168 (Eq. 8.52)
h_f (Eq. 8.13)	0.001 549 m ◄	14.98 m ◄
u_{max} (from Eq. 8.43)	0.0316 m/s ◄	5.97 m/s ◄
δ_v (Eq. 8.38)	0.000 993 m ◄	1.009×10^{-5} m ◄

8.21 *Oil (s = 0.85) with a viscosity of 0.0056 N·s/m² flows at a rate of 80 L/s in a 150-mm-diameter pipe having e = 0.90 mm. Find the head loss, using Eq. (8.29) or (8.52) to find f. Determine the shear stress at the pipe wall. Find the velocity 25 mm from the centerline. Under these conditions is the pipe behaving as a fully rough, transitional, or smooth pipe?*

SI

$V = Q/A = 0.08/[(\pi/4)0.15^2] = 4.53$ m/s; $\rho = 0.85(1000) = 850$ kg/m³

Eq. 8.1: $\mathbf{R} = DV\rho/\mu = (0.15\text{ m})(4.53\text{ m/s})(850\text{ kg/m}^3)/0.0056\text{ N·s/m}^2) = 103\,100$ (flow is turbulent)

$e/D = 0.9/150 = 0.006$; Eq. 8.52: $f = 0.0329$

From Eq. 8.14: $h_f/L = (f/D)(V^2/2g) = (0.0329/0.15)(4.53^2/2(9.81) = 0.229$ m/m ◀

Eq. 8.19: $\tau_0 = f\rho V^2/8 = 0.0329(850)4.53^2/8 = 71.6$ N/m² ◀

Eq. 8.44: $u = (1 + 1.326\sqrt{0.0329})4.53 - 2.04\sqrt{0.0329}(4.53)\log[7.5/(7.5 - 2.5)] = 5.32$ m/s ◀

Eq. 8.38: $\delta_v = \dfrac{14.14\nu}{Vf^{1/2}} = \dfrac{14.14\mu}{\rho Vf^{1/2}} = \dfrac{14.14(0.0056)}{850(4.53)0.0329^{1/2}} = 0.0001134\text{ m} = 0.1134\text{ mm}$

So $e/14 = 0.0643\text{ mm} < \delta_v < e$. Sec. 8.10: pipe is behaving as transitionally rough ◀

8.22 *The flow rate in a 10-in-diameter pipe is 7 cfs. The flow is known to be turbulent, and the centerline velocity is 15.2 fps. Plot the velocity profile, and determine the head loss per foot of pipe.*

BG

Eq. 4.7: $V = Q/A = 7/[\pi(10/12)^2/4] = 12.83$ fps

Eq. 8.43: $V/u_{max} = 12.83/15.2 = 1/(1 + 1.326f^{1/2})$; $f = 0.019\,32$

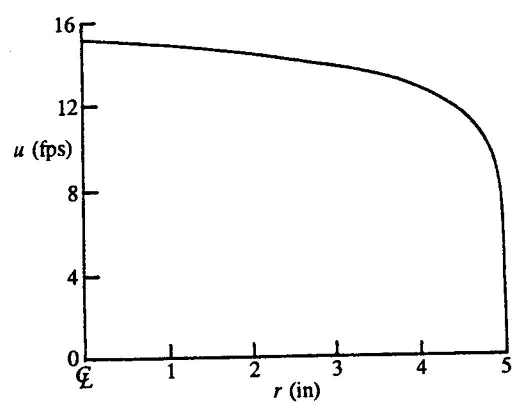
Eq. 8.13: $h_f = 0.019\,32\dfrac{1}{(10/12)}\dfrac{12.83^2}{2(32.2)} = 0.0593$ feet per foot ◀

From Eq. 8.37: $u_* = V(f/8)^{1/2} = 12.83(0.019\,32/8)^{1/2} = 0.631$ fps

Eq. 8.40: $u = 15.2 - 5.76(0.631)\log[r_0/(r_0 - r)] = 15.2 - 3.63\log[r_0/(r_0 - r)]$

r (in)	u (fps)
0	15.20
2	14.39
3	13.75
4	12.66
4.5	11.57
4.9	9.03
4.99	5.39

▲



8.23 *The flow rate in a 250-mm-diameter pipe is 200 L/s. The flow is know to be turbulent, and the centerline velocity is 4.75 m/s. Plot the velocity profile, and determine the head loss per meter of pipe.*

SI

Eq. 4.7: $V = Q/A = 4Q/(\pi D^2) = 4(0.20)/(\pi 0.25^2) = 4.07$ m/s

Eq. 8.43: $V/u_{max} = 4.07/4.75 = 1/(1 + 1.326 f^{1/2})$; $f = 0.015\,64$

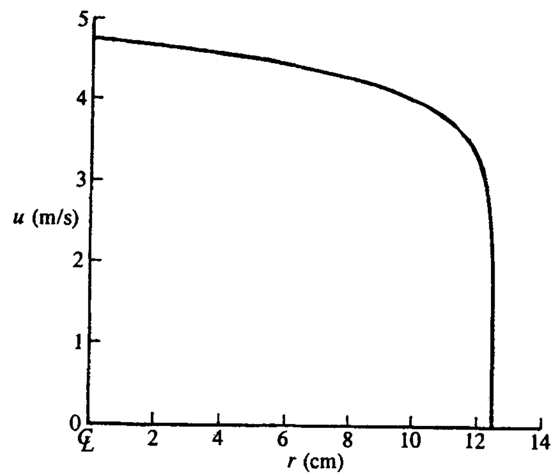
Eq. 8.13: $h_f = 0.015\,64(1/0.25)4.07^2/(2 \times 9.81) = 0.0529$ m per m ◄

From Eq. 8.37: $u_* = V(f/8)^{1/2} = 4.07(0.015\,64/8)^{1/2} = 0.1801$ m/s

Eq. 8.40: $u = 4.75 - 5.76(0.1801)\log[r_0/(r_0 - r)] = 4.75 - 1.038\log[r_0/(r_0 - r)]$

r (mm)	U (m/s)
0	4.75
50	4.52
80	4.29
100	4.02
120	3.30
122.5	2.99
124.9	1.537

▲



Sec. 8.12: Pipe Roughness – ***Exercises (3)***

8.12.1 *Using implicit Eq. (8.46), approximate Eq. (8.47), and Blasius' Eq. (8.48), solve for the smooth-pipe friction factor f using Reynolds numbers of (a) 4000, (b) 20,000, and (c) 10^5. (d) For which of these three values do the equations show the most variation in f?*

N

Part:	(*a*)	(*b*)	(*c*)
R	4000	20,000	10^5
f values, computed from:			
Prandtl's Eq. 8.35:	0.0399	0.0259	0.017 99
Approx Eq. 8.36:	0.0404	0.0257	0.017 82
Blasius Eq. 8.37:	0.0397	0.0266	0.017 77
	▲	▲	▲
Average of 3:	0.0400	0.0261	0.017 86
Absolute variation	0.000 688	0.000 824	0.000 220
% variation	1.719	3.16 ◄	1.231

8.12.2 *Substitute into Eq. (8.51) the given and computed data of Sample Problem 8.5a. How well does the right-hand side of the equation agree with the left-hand side?*

*Sample Prob. 8.5a: Water at 20°C flows in a 500-mm-diameter welded steel pipe, with a friction loss gradient of 0.006. e/D = 0.000 092, by trial and error **R** = 1.059 × 10⁶, and f = 0.0131.*

SI

Eq. 8.51: Left side $= \dfrac{1}{0.0131^{1/2}}$; Right side $= -2\log\left[\dfrac{0.000\,092}{3.7} + \dfrac{2.51}{(1.059\times 10^6)0.0131^{1/2}}\right]$

$= 1/0.1145$ $\qquad = -2\log(0.000\,0248 + 0.000\,0207) = -2\log(0.000\,0455)$

$= 8.74$ ◄ $\qquad = -2(-4.34) = 8.68$ ◄

It checks closely. The right side is 0.625% smaller than the left side. ◄

8.12.3 *Repeat Exer. 8.12.2 using Eq. (8.52).*

Exer. 8.12.2: Substitute in the given and computed data of Sample Prob. 8.5a. How well does the right side of the equation agree with the left side?

*Sample Prob. 8.5a: Water at 20°C flows in a 500-mm-diameter welded steel pipe, with a friction loss gradient of 0.006. e/D = 0.000 092, by trial and error **R** = 1.059 × 10⁶, and f = 0.0131.*

SI

Left side Eq. 8.52: $1/(0.0131)^{1/2} = 1/0.1145 = 8.74$ ◄

Right side Eq. 8.52: $-1.8\log[(e/3.7D)^{1.11} + 6.9/\mathbf{R}] = -1.8\log[(0.000\,092/3.7)^{1.11} + 6.9/(1.059\times 10^6)]$

$= -1.8\log[(0.000\,0248)^{1.11} + 0.000\,006\,52] = -1.8\log[0.000\,007\,75 + 0.000\,006\,52]$

$= -1.8\log(0.000\,014\,26) = -1.8(-4.85) = 8.72$ ◄

It checks very closely. The right side is 0.167% smaller than the left side. ◄

Sec. 8.12: Pipe Roughness -- *Problems 8.24–8.28*

8.24 *If the diameter of a pipe is doubled, what effect does this have on the flow rate for a given head loss? Consider (a) laminar flow; (b) turbulent flow.*

N

(*a*) Laminar flow, Eq. 8.28: $h_L = 32\nu LV/(gD^2) = \text{const.} \times V/D^2$

$V = kD^2$; $Q = AV = (\pi D^2/4)(kD^2) = k'D^4$

Thus doubling the diameter will increase the flow by a factor of $2^4 = 16$ ◄

(*b*) Turbulent flow, Eq. 8.13: $h_f = f(L/D)V^2/(2g)$

(*i*) If f = constant (complete turbulence), $h_f = k_1V^2/D$; $V = k_2D^{1/2}$ and $Q = AV = k_3D^{5/2}$

Thus doubling the diameter will increase the flow rate by a factor of $2^{5/2} = 5.66$ for turbulent flow with complete turbulence. ◄

(*ii*) For smooth pipe with $\mathbf{R} < 100{,}000$, Blasius Eq. 8.48: $f = 0.316/\mathbf{R}^{1/4} = k/(DV)^{1/4}$

$h_f = f(L/D)V^2/(2g) = k_1V^{7/4}/D^{5/4}$ (cf Eq. 8.53 in Sec. 8.14)

$V = k_2D^{5/7}$ and $Q = AV = k_3D^{19/7}$

Thus doubling the diameter will increase the flow rate by a factor of $2^{19/7} = 6.56$ for turbulent flow in smooth pipes. ◄

8.25 *Kerosene (s = 0.813) at a temperature of 70°F flows in a 2-in-diameter smooth brass pipeline at a rate of 6.5 gpm. (a) Find the friction head loss per foot. (b) For the same head loss what would be the flow rate if the temperature of the kerosene were raised to 100°F?*

BG

6.5 gpm = 0.01448 cfs; $V = Q/A = 4(0.01448)/[\pi(2/12)^2] = 0.664$ fps

(*a*) Fig. A.2 for kerosene at 70°F: $\nu = 2.4 \times 10^{-5}$ ft²/sec.

Eq. 8.1: $\mathbf{R} = (2/12)0.664/(2.4 \times 10^{-5}) = 4610$

Blasius' Eq. 8.48 for smooth pipe ($3000 \le \mathbf{R} < 10^5$: $f = 0.316/4610^{0.25} = 0.0384$

Eq. 8.13: $h_f = 0.0384(12/2)0.664^2/[2(32.2)] = 0.001\,574$ ft per ft ◄

(*b*) Same h_f, raised temperature. Fig. A.2 for kerosene at 100°F: $\nu = 1.8 \times 10^{-5}$ ft²/sec.

Substitute Blasius Eq. 8.48 into Eq. 8.13 to eliminate f, using Eq. 8.1 for $\mathbf{R}$, yields Eq. 8.53 or

$$\frac{h_f}{L} = \frac{0.1580\nu^{0.25}}{gD^{1.25}}V^{1.75} \text{ provided } 3000 \le \mathbf{R} < 10^5\,; \quad 0.001574 = \frac{0.1580(1.8 \times 10^{-5})^{0.25}V^{1.75}}{32.2(2/12)^{1.25}}$$

from which $V = 0.692$ fps ; $\mathbf{R} = DV/\nu = 6404$, so the Blasius Eq. 8.48 is valid

and $Q = AV = (\pi/4)(2/12)^2 0.692 = 0.01509$ cfs = 6.77 gpm ◄

8.26 *Kerosene (s = 0.813) at a temperature of 20°C flows in a 75-mm-diameter smooth brass pipeline at a rate of 0.80 L/s. (a) Find the friction head loss per meter. (b) For the same head loss what would be the flow rate if the temperature of the kerosene were raised to 40°C?*

SI

Eq. 4.7: $V = Q/A = 4(0.0008)/(\pi 0.075^2) = 0.1811$ m/s;

(*a*) Fig. A.1 for kerosene at 20°C: $\mu = 1.65 \times 10^{-3}$ N·s/m²

Eq. 8.1: $\mathbf{R} = 0.075(0.1811)(0.813 \times 1000)/(1.65 \times 10^{-3}) = 6690$

Eq. 8.48 for smooth pipe: $f = 0.316/(6690)^{0.25} = 0.0349$

$$\text{Eq. 8.13:} \quad h_f = 0.0349\frac{1}{0.075}\frac{0.1811^2}{2(9.81)} = 0.000\,779 \text{ m/m} \quad ◄$$

(*b*) Same h_f, raised $T°$. Fig. A.2 for kerosene at 40°C: $\nu = 1.7 \times 10^{-6}$ m²/s

Substitute Blasius Eq. 8.48 into Eq. 8.13 to eliminate f, using Eq. 8.1 for $\mathbf{R}$, yields Eq. 8.53 or

$$\frac{h_f}{L} = \frac{0.1580\nu^{0.25}}{gD^{1.25}}V^{1.75} \text{ provided } 3000 \le \mathbf{R} < 10^5\,; \text{ i.e., } 0.000\,779 = \frac{0.1580(1.7 \times 10^{-6})^{0.25}V^{1.75}}{9.81(0.075)^{1.25}}$$

from which $V = 0.1857$ m/s ; $\mathbf{R} = DV/\nu = 8193$, so Blasius Eq. 8.48 is valid

and $Q = 0.1857(\pi/4)(0.075)^2 = 0.000\,821$ m³/s ◄

8.27 *Water at 50°C flows in a 150-mm-diameter pipe with $V = 7.5$ m/s. Head loss measurements indicate that $f = 0.020$. Determine the value of e. Find the shear stress at the pipe wall and at $r = 30$ mm. What is the value of du/dy at $r = 30$ mm?*

SI

Table A.1 for water at 50°C: $\nu = 0.553 \times 10^{-6}$ m²/s, $\rho = 988$ kg/m³

Eq. 8.1: $\mathbf{R} = 0.15(7.5)/(0.553 \times 10^{-6}) = 2.03 \times 10^6$

Eq. 8.51, for $f = 0.020$ and $\mathbf{R} = 2.03 \times 10^6$: $e/D = 0.001046$; $e = 0.001046(150) = 0.1569$ mm ◄

Eq. 8.19: $\tau_0 = f\rho V^2/8 = 0.020(988)7.5^2/8 = 138.9$ N/m² ◄ (using kg·m/s² = N)

Eq. 8.18 at $r = 30$ mm: $\tau = (3/7.5)138.9 = 55.6$ N/m² ◄

Sec. 8.11: $\dfrac{du}{dy} = \dfrac{(\tau_0/\rho)^{1/2}}{Ky} = \dfrac{[138.9\ (\text{kg/m}^3)(\text{m/s}^2)/(988\ \text{kg/m}^3)]^{1/2}}{0.40(0.075 - 0.03)} = 20.8\ \text{sec}^{-1}$ ◄

8.28 *Oil ($s = 0.85$) with an absolute viscosity of 0.0056 N·s/m² flows in a 150-mm-diameter pipe having $e = 0.90$ mm.* (a) Above what flow rate will this pipe behave as a fully rough pipe? (b) Below what flow rate will it behave as a smooth pipe?

SI

$\rho = 0.85(1000) = 850$ kg/m³

(*a*) From Sec. 8.10: Fully rough when $\delta_v < e/14$. In Eq. 8.38 assume f = constant; $\delta_v \propto 1/V$

So $\delta_v = 0.9/14 = 0.0643$ mm when $V = V_{prev}[(\delta_v)_{prev}/\delta_v] = 4.53(0.1133/0.0643) = 7.98$ m/s

and when $Q = 80(0.1133/0.0643) = 141$ L/s ◄

(*b*) From Fig. 8.9: Smooth when $\delta_v > e$. f varies rapidly with **R** for smooth pipe (Fig. 8.11).

Eqs. 8.39 and 8.47 to eliminate f:

$$\delta_v = \frac{14.14\nu}{V\sqrt{f}} = \frac{14.14\mu}{\rho V}1.8\log\left(\frac{\mathbf{R}}{6.9}\right) = \frac{14.14(0.0056)}{850V}1.8\log\left(\frac{\mathbf{R}}{6.9}\right)\text{ m} = \frac{0.1677}{V}\log\frac{\mathbf{R}}{6.9}\text{ mm}$$

where (Eq. 8.1), $\mathbf{R} = 0.15(V)850/0.0056 = 22\,768V$,

so that $\delta_v = \dfrac{0.1677}{V}\log\dfrac{22\,768V}{6.9} = \dfrac{0.1677}{V}\log(3300V)$ mm

Solve by trials (or using an equation solver).

Try V m/s	**R**	δ_v mm	*cf.* e mm
1	22 768	0.590	$< e = 0.9$
0.5	11 384	1.079	$> e$
0.7	15 938	0.806	$< e$
0.65	14 799	0.859	$< e$
0.616	14 025	0.9005	equal!

Then $Q = AV = (\pi/4)(0.15)^2 0.616 = 0.010\,88$ m³/s = 10.88 L/s ◄

Sec. 8.13: Chart for Friction Factor -- *Exercises (4)*

8.13.1 *Oil ($s = 0.90$) with viscosity 1.6×10^{-4} lb-sec/ft² flows in a 4-in-diameter welded-steel pipe (see Table 8.1) at 0.25 cfs. What is the friction head loss per foot of pipe?*

BG

Eq. 4.7: $V = 4Q/(\pi D^2) = 4(0.25)/[\pi(4/12)^2] = 2.86$ fps; Table 8.1: $e = 0.000\,15$ ft

Eq. 8.1: $\mathbf{R} = (4/12)2.86(1.940 \times 0.90)/(1.6 \times 10^{-4}) = 10{,}420$; $e/D = 0.000\,15/(4/12) = 0.000\,45$

Fig. 8.11: $f = 0.0315$; Eq. 8.13: $\dfrac{h_f}{L} = \dfrac{0.0315}{(4/12)}\dfrac{2.86^2}{2(32.2)} = 0.0204$ ft per ft ◄

8.13.2 *Oil (s = 0.94) with viscosity 0.0096 N·s/m² flows in a 90-mm-diameter welded-steel pipe (see Table 8.1) at 7.2 L/s. What is the friction head loss per meter of pipe?*

SI

Eq. 4.7: $V = 4Q/(\pi D^2) = 4(0.0072)/(\pi 0.090^2) = 1.132$ m/s; Table 8.1 for welded steel: $e = 0.046$ mm

Eq. 8.1: $\mathbf{R} = 0.090(1.132)(1000 \times 0.94)/0.0096 = 9974$; $e/D = 0.046/90 = 0.000511$

Fig. 8.11: $f = 0.032$; Eq. 8.13: $\dfrac{h_f}{L} = \dfrac{0.032}{0.090}\dfrac{1.132^2}{2(9.81)} = 0.0232$ m/m ◄

8.13.3 *Water at 20°C flows through a 150-mm-diameter pipe with e = 0.015 mm. (a) If the mean velocity is 5 m/s, what is the nominal thickness δ_v of the viscous sublayer? (b) What will δ_v be if the velocity is increased to 6.2 m/s?*

SI

Table A.1 at 20°C: $\nu = 1.003 \times 10^{-6}$ m²/s

(*a*) Eq. 8.1: $\mathbf{R} = DV/\nu = 0.150(5)/(1.003 \times 10^{-6}) = 7.48 \times 10^5$

$e/D = 0.015/150 = 0.0001$; from Fig. 8.11 (or Eq. 8.51 or 8.52): $f = 0.0138$

Eq. 8.38: $\delta_v = \dfrac{14.14\nu}{Vf^{1/2}} = \dfrac{14.14(1.003 \times 10^{-6})}{5(0.0138)^{1/2}} = 24.1 \times 10^{-6}$ m $= 0.0241$ mm ◄

(*b*) Eq. 8.1: $\mathbf{R} = DV/\nu = 0.150(6.2)/(1.003 \times 10^{-6}) = 9.27 \times 10^5$

$e/D = 0.0001$ as before ; from Fig 8.11 (or Eq. 8.51 or 8.52): $f = 0.0135$

Eq. 8.38: $\delta_v = \dfrac{14.14(1.003 \times 10^{-6})}{6.2(0.0135)^{1/2}} = 19.67 \times 10^{-6} = 0.019\,67$ mm ◄

8.13.4 *A straight, new, 48-in-diameter asphalted cast-iron pipe (see Table 8.1) 700 ft long carries 78°F water at an average velocity of 12 fps. (a) Using the value of f as determined from Fig. 8.11, find the shear force on the pipe. (b) What will be the shear force if the average velocity is reduced to 4.2 fps?*

BG

Table 8.1: $e = 0.0004$ ft; $e/D = 0.004(12)/48 = 0.0001$

Interpolating in Table A.1: $\nu = 0.956 \times 10^{-5}$ ft²/sec

(*a*) Eq. 8.1: $\mathbf{R} = DV/\nu = (48/12)12/(0.956 \times 10^{-5}) = 5.02 \times 10^6$; Fig. 8.11: $f = 0.0123$

Eq. 8.19: $\tau_0 = f\rho V^2/8 = 0.0123(1.940)12^2/8 = 0.430$ lb/ft²

Shear force $= \pi D(L)\tau_0 = \pi(48/12)700(0.431) = 3780$ lb ◄

(*b*) If $V = 4.2$ fps, $\mathbf{R} = 1.76 \times 10^6$, $f = 0.0129$, $\tau_0 = 0.0552$ lb/ft²

Shear force $= \pi D(L)\tau_0 = \pi(48/12)700(0.0522) = 485$ lb ◄

Sec. 8.13: Chart for Friction Factor – *Problems* 8.29–8.37

8.29 *Air at 70°F and atmospheric pressure flows with a velocity of 22 fps through a 3-in-diameter pipe (e = 0.00015 in). Find the friction head loss in 75 ft of pipe.*

BG

Table A.2 for air at 70°F: $\nu = 0.164 \times 10^{-3}$ ft²/sec

Eq. 8.1: $\mathbf{R} = DV/\nu = (3/12)22/(0.164 \times 10^{-3}) = 33{,}537$; $e/D = 0.000\,15/3 = 0.000\,05$

Fig. 8.11: $f = 0.0230$; Eq. 8.13: $h_f = 0.0230[75/(3/12)]22^2/[2(32.2)] = 51.9$ ft ◄

8.30 *Air at 30°C and atmospheric pressure flows with a velocity of 6.5 m/s through a 75-mm-diameter pipe (e = 0.002 mm). Find the friction head loss in 30 m of pipe.*

SI

Table A.2 for air at 30°C: $\nu = 16.0 \times 10^{-6}$ m²/s

Eq. 8.1: $\mathbf{R} = DV/\nu = (0.075)6.5/(16.0 \times 10^{-6}) = 3.05 \times 10^4$; $e/D = 0.002/75 = 0.000\,0267$

Fig. 8.11: $f = 0.0235$; Eq. 8.13: $h_f = 0.0235(30/0.075)6.5^2/[(2)9.81] = 20.2$ m ◄

8.31 *Crude oil (s = 0.855) at 50°C flows at 300 L/s through a 450-mm-diameter pipe (e = 0.054 mm) 1500 m long. Find the kilowatt loss.*

SI

Fig. A.2 for crude oil ($s = 0.855$) at 50°C: $\nu = 3.8 \times 10^{-6}$ m²/s

$V = 0.30/(\pi 0.225^2) = 1.886$ m/s; Eq. 8.1: $\mathbf{R} = DV/\nu = 0.45(1.886)/(3.8 \times 10^{-6}) = 2.23 \times 10^5$

$e/D = 0.054/450 = 0.000\,12$; Fig. 8.11: $f = 0.0163$

Eq. 8.13: $h_f = f\dfrac{L}{D}\dfrac{V^2}{2g} = 0.0163\dfrac{1500}{0.45}\dfrac{1.886^2}{2(9.81)} = 9.85$ m

Eq. 5.41: Power loss $= \gamma Q h_f = (0.885 \times 9807)0.30(9.85)/1000 = 25.7$ kW ◄

8.32 *When water at 50°F flows at 2.5 cfs in a 20-in pipeline, the head loss is 0.0004 ft/ft. What will be the friction head loss when glycerin at 68°F flows through this same pipe at the same rate?*

BG

Table A.1 for water at 50°F: $\nu = 1.410 \times 10^{-5}$ ft²/sec

$V = Q/A = 2.5/[(\pi/4)(20/12)^2] = 1.146$ fps; $\mathbf{R} = (20/12)1.146/(1.410 \times 10^{-5}) = 1.355 \times 10^5$

Eq. 8.13: $0.0004 = f(12/20)1.146^2/2g$; $f = 0.0327$; Thus from Fig. 8.11: $e/D = 0.0060$

Table A.4 for glycerin at 68°F: $s = 1.26$, $\mu = 0.0312$ lb·sec/ft²

$\nu = 0.0312/(1.26 \times 1.940) = 0.012\,76$ ft²/sec; $\mathbf{R} = (20/12)1.146/0.012\,76 = 149.6$; flow is laminar.

Eq. 8.29: $f = \dfrac{64}{149.5} = 0.428$; Eq. 8.13: $\dfrac{h_f}{L} = \dfrac{0.428}{(20/12)}\dfrac{1.146^2}{2(32.2)} = 0.005\,23$ ft per ft ◄

8.33 *Air at 90 psia and 70°F flows through a 4.5-in-diameter welded-steel pipe (see Table 8.1) at 60 lb/min. Find the friction head loss and pressure drop in 200 ft of this pipe. Assume the air to be of constant density.*

BG

Table A.5 for air: $R = 1715$ ft²/(s²·°R)

Eq. 2.5: $\gamma = \rho g = pg/RT = 90(144)32.2/[1715(460 + 70)] = 0.459$ lb/ft³

Eq. 4.5: $G = 60/60 = 1.00$ lb/sec $= \gamma AV = 0.459(\pi/4)(4.5/12)^2 V$; $V = 19.72$ ft/sec;

End of Sec. 2.11: ν depends strongly on density, so we must look up μ.

Table A.2 for air at 70°F: $\mu = 0.382 \times 10^{-6}$ lb·sec/ft²

Eq. 8.1: $\mathbf{R} = \dfrac{DV\rho}{\mu} = \dfrac{DQ\gamma}{(\pi D^2/4)g\mu} = \dfrac{4G}{\pi D g \mu} = \dfrac{4(1)}{\pi(4.5/12)32.2(0.382 \times 10^{-6})} = 276{,}000$

Table 8.1 for welded steel: $e = 0.000\,15$ ft; $e/D = 0.000\,15/(4.5/12) = 0.000\,400$

Fig. 8.11: $f = 0.0177$; Eq. 8.13: $h_f = 0.0177\dfrac{200}{(4.5/12)}\dfrac{19.72^2}{2(32.2)} = 57.0$ feet of air ◄

Per Eq. 3.4: $\Delta p = h_f(\gamma) = 57.0(0.459) = 26.2$ lb/ft² $= 0.1818$ psi ◄

8.34 *Air flows at an average velocity of 0.7 m/s through a long 3.8-m-diameter circular tunnel (e = 1.5 mm). Find the friction head-loss gradient at a point where the air temperature and pressure are 20°C and 102 kPa abs respectively. Find also the shear stress at the pipe wall and the thickness δ_v of the viscous sublayer.*

SI

Table A.2 for air at 20°C: $\mu = 18.1\times10^{-6}$ N·s/m², $\gamma = 11.82$ N/m³

Eq. 8.1: $\mathbf{R} = DV\rho/\mu = DV\gamma/(\mu g) = 3.8(0.7)11.82/(18.1\times10^{-6}\times9.81) = 177\,100$

$e/D = 1.5/3800 = 0.000\,395$; Fig. 8.11: $f = 0.0185$

Eq. 8.14: $S = h_f/L = (0.0185/3.8)0.7^2/[2(9.81)] = 0.000\,1216$ m/m ◄

Eq. 8.19: $\tau_0 = f\rho V^2/8 = 0.0182(11.82/9.81)0.7^2/8 = 0.001\,365$ N/m² ◄

Eq. 8.38: $\delta_v = \dfrac{14.14\nu}{Vf^{1/2}} = \dfrac{14.14\mu g}{Vf^{1/2}\gamma} = \dfrac{14.14(18.1\times10^{-6})9.81}{0.7(0.0185)^{1/2}11.82} = 0.002\,23\text{ m} = 2.23\text{ mm}$ ◄

8.35 *Repeat Prob. 8.34 for the case where the average velocity is 7.0 m/s.*
Prob. 8.34: Air flows through a long 3.8-m-diameter circular tunnel (e = 1.5 mm). Find the head-loss gradient at a point where the air temperature and pressure are 20°C and 102 kPa abs respectively. Find also the shear stress at the pipe wall and the thickness δ_v of the viscous sublayer.

SI

Table A.2 for air at 20°C: $\gamma = 11.82$ N/m³, $\mu = 18.1\times10^{-6}$ N·s/m²,

Eq. 8.1: $\mathbf{R} = DV\rho/\mu = DV\gamma/(\mu g) = 3.8(7.0)11.82/[18.1\times10^{-6}(9.81)] = 1\,771\,000$

$e/D = 1.5/3800 = 0.000\,395$; Fig. 8.11: $f = 0.0161$

Eq. 8.14: $S = h_f/L = (0.0161/3.8)7.0^2/[2(9.81)] = 0.010\,58$ m/m ◄

Eq. 8.19: $\tau_0 = 0.0161(11.82/9.81)7.0^2/8 = 0.1188$ N/m² ◄

Eq. 8.38: $\delta_v = \dfrac{14.14\nu}{Vf^{1/2}} = \dfrac{14.14\mu g}{Vf^{1/2}\gamma} = \dfrac{14.14(18.1\times10^{-6})9.81}{7.0(0.0161)^{1/2}11.82} = 0.000\,239\text{ m} = 0.239\text{ mm}$ ◄

8.36 *For* **R** *ranging from 10^2 to 10^7, make a plot of the values of α and β versus* **R** *for smooth brass pipes. On the same plot also show values of u_{max}/V.*

N

Flow	f from	α from	β from	u_{max}/V from
Laminar	Eq. 8.29	Sec. 5.1	Sec. 6.3	Sec. 8.6
Turbulent	Fig. 8.11	Eq. 8.45*a*	Eq. 8.45*b*	Eq. 8.43

R	Flow	f	α	β	u_{max}/V
10^2	lam.	0.640	2.0	1.333	2.0
10^3	lam.	0.0640	2.0	1.333	2.0
10^4	turb.	0.0307	1.083	1.030	1.233
10^5	turb.	0.0178	1.048	1.017	1.177
10^6	turb.	0.0116	1.031	1.011	1.143
10^7	turb.	0.0081	1.022	1.008	1.120
▲			▲	▲	▲

8.37 *Water at 40°F flows in a 42-in-diameter concrete pipe (e = 0.022 in). Determine **R**, τ_0, u_{max}/V, δ_v, δ_v/e, and the flow regime (hydraulic smoothness) for flow rates of 250, 25, 2.5, 0.25, and 0.025 cfs.*

BG

E.g., for Q = 250 cfs: $V = Q/A = 250/[(\pi/4)(42/12)^2] = 26.0$ fps

Table A.1 for water at 40°F: $\nu = 1.664 \times 10^{-5}$ ft²/sec

Eq. 8.1: $\mathbf{R} = DV/\nu = (42/12)26.0/(1.664 \times 10^{-5}) = 5.47 \times 10^6$; $e/D = 0.022/42 = 0.000\,524$

Fig. 8.11: $f = 0.0170$; Eq. 8.19: $\tau_0 = f\rho V^2/8 = 0.0170(1.940)26.0^2/8 = 2.78$ psf

Determine u_{max}/V from Eq. 8.43 and δ_v from Eq. 8.38. For the other discharges:

Q cfs	V fps	**R**	f	τ_0 psf	u_{max}/V	δ_v ft	δ_v/e	Regime
250	26.0	5.47E6	0.0170	2.78	1.173	0.000 0694	0.0379	Rough
25	2.60	5.47E5	0.0177	0.0290	1.176	0.000 522	0.285	Trans
2.5	0.260	54,700	0.0223	3.54E−4	1.198	0.006 06	3.31	Smooth
0.25	0.0260	5470	0.0371	6.07E−6	1.255	0.0470	25.6	Smooth
0.025	0.00260	547	0.0787	1.29E−7	1.372	0.323	176	Laminar
		▲		▲	▲	▲	▲	▲

Sec. 8.15: Single-Pipe Flow: Solution by Trials -- *Exercises (4)*

8.15.1 *Compute the friction head per 100 ft of 3-in-diameter pipe for a Reynolds number of 50,000 if (a) the flow is laminar (achievable with great care); (b) the flow is turbulent in a smooth pipe; and (c) the flow is turbulent in a rough pipe with e/D = 0.05. Consider two situations, one where the fluid is 70°F water, the other where the fluid is SAE 10 (Western lubricating) oil at 150°F.*

BG

These are Type 1 problems, to find h_f. Table A.1 for water at 70°F: $\nu = 1.059 \times 10^{-5}$ ft²/sec

Eq. 8.1: $\mathbf{R} = (3/12)V/(1.059 \times 10^{-5}) = 50{,}000$; $V = 2.12$ fps

(*a*) For laminar flow, Eq. 8.29: $f = 64/\mathbf{R} = 64/50{,}000 = 0.001\,280$

Eq. 8.13: $h_f = 0.001\,280[100/(3/12)]2.12^2/[2(32.2)] = 0.0357$ ft ◄

(*b*) For turbulent flow in smooth pipe, Fig. 8.11 (smooth pipe curve): $f = 0.0209$

Eq. 8.13: $h_f = 0.0209[100/(3/12)]2.12^2/[2(32.2)] = 0.582$ ft ◄

(*c*) For turbulent flow in rough pipe ($e/D = 0.05$), Fig. 8.11: $f = 0.0720$

Eq. 8.13: $h_f = 0.0720[100/(3/12)]2.12^2/[2(32.2)] = 2.01$ ft ◄

Fig. A.2 for SAE 10 (Western lubricating) oil at 150°F: $\nu = 0.000\,16$ ft²/sec

Eq. 8.1: $\mathbf{R} = (3/12)V/0.000\,16 = 50{,}000$; $V = 32.0$ fps. Friction factors are the same as before.

Eq. 8.13: (*a*) $h_f = 8.15$ ft ◄ (*b*) 133 ft ◄ (*c*) 459 ft ◄

8.15.2 *California crude oil, warmed until its kinematic viscosity is 0.0004 ft²/sec and its specific weight is 53.45 lb/ft³, is pumped through a 3-in pipe (e = 0.001 in). (a) For laminar flow with $\mathbf{R} = \mathbf{R}_{crit} = 2000$, what would be the loss in energy head in psi per 1000 ft of pipe? (b) What would be the loss in head per 1000 ft if the velocity were three times the value in (a)?*

BG

These are Type 1 problems, to find h_f.

(*a*) Eq. 8.1: $\mathbf{R}_{crit} = 2000 = (2/12)V/0.0004$; $V = 3.2$ fps

Eq. 8.29: $f = 64/2000 = 0.032$; $h_f = 0.032[1000/(2/12)]3.2^2/[2(32.2)] = 20.3$ ft per 1000 ft

Pressure loss $= \gamma h_f = 53.45(20.3)/144 = 7.55$ psi per 1000 ft ◄

(*b*) $V = 3(3.2) = 9.6$ fps; $\mathbf{R} = 6000$, $e/D = 0.001/3 = 0.000\,333$; Fig. 8.11: $f = 0.0359$

$h_f = 0.359[1000/(3/12)]9.6^2/[2(32.2)] = 205$ ft per 1000 ft ◄

8.15.3 *Water at 50°F flowing through 80 ft of 4-in-diameter average cast-iron pipe causes a friction head loss of 0.27 ft. Find the flow rate.*

BG

This is a Type 2 problem to find Q.

Table 8.1 for average cast-iron: $e = 0.000\,85$, $e/D = 0.00\,85/(4/12) = 0.002\,55$,

for which (Fig. 8.11) $f_{min} \approx 0.025$

Table A.1 at 50°F: $\nu = 1.410 \times 10^{-5}$ ft²/sec. $h_f = 0.27$ ft in 80 ft, given.

From Eq. 8.13: $h_f = 0.27 = f[80/(4/12)]V^2/[2(32.2)]$; i.e. $V = 0.269/f^{1/2}$

Eq. 8.1: $\mathbf{R} = DV/\nu = (4/12)V/(1.410 \times 10^{-5}) = 23{,}640V$

Try $f = 0.025 \approx f_{min}$. Then $V = 0.269/(0.025)^{1/2} = 1.703$ fps, $\mathbf{R} = 23{,}640(1.703) = 40{,}260$

Eq. 8.51 or Fig. 8.11 with $e/D = 0.002\,55$ and $\mathbf{R} = 4.03 \times 10^4$: $f = 0.0283$

This and subsequent trials:

Try f	V fps	$\mathbf{R}$	Obtained f	
0.025	1.703	4.02×10^4	0.0283	Try again
0.0283	1.601	3.79×10^4	0.0284	Try again
0.0284	1.597	3.78×10^4	0.0284	Converged!

$Q = (\pi/4)(4/12)^2 1.597 = 0.1394$ cfs ◀

8.15.4 *When gasoline with a kinematic viscosity of 5×10^{-7} m²/s flows in a 200-mm-diameter smooth pipe the friction head loss is 0.43 m per 100 m. Find the flow rate.*

SI

This is a Type 2 problem to find Q.

For smooth pipe $e = 0$, for which (Fig. 8.11) initial assumptions: $f \approx 0.02$ (mid-range)

From Eq. 8.13: $h_f = 0.43 = f(100/0.2)V^2/[2(9.81)]$; i.e. $V = 0.1299/f^{1/2}$

Eq. 8.1: $\mathbf{R} = DV/\nu = 0.2V/(5 \times 10^{-7}) = 400\,000V$

Try $f = 0.02$. Then $V = 0.1299/(0.02)^{1/2} = 0.918$ m/s, $\mathbf{R} = 400\,000(0.918) = 3.67 \times 10^5$

Eq. 8.47 or Fig. 8.11 with for smooth pipe ($e = 0$) and $\mathbf{R} = 3.67 \times 10^5$: $f = 0.014$

This and subsequent trials:

Try f	V fps	$\mathbf{R}$	Obtained f	
0.02	0.918	3.67×10^5	0.014	Try again
0.014	1.101	4.4×10^5	0.0134	Try again
0.0134	1.12	4.48×10^5	0.0134	Converged!

$Q = (\pi/4)(0.2)^2 1.12 = 0.0352$ m³/s $= 35.2$ L/s ◀

Sec. 8.15: Single-Pipe Flow: Solution by Trials -- *Problems 8.38–8.45*

8.38 *A steel pipe (e = 0.0002 ft) 13,450 feet long is to convey oil (ν = 0.000 54 ft^2/sec) at 13 cfs from a reservoir with surface elevation 705 ft to one with surface elevation 390 ft (Fig. P8.38). Theoretically, what pipe size is required?*

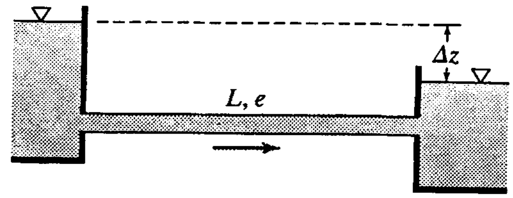


Figure P8.38

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A Type 3 problem, to find D. e = 0.000 065 m (given).

Eq. 8.13: $h_f = (fL/D)(V^2/2g) = (fL/D)[(Q/A)^2/(2g)]$

$$= [fL/(2gD)][Q/(\pi D^2/4)]^2 = 8fLQ^2/(\pi^2 gD^5)$$

$705 - 390 = 315 = 8f(13{,}450)13^2/[\pi^2(32.2)D^5];\ D^5 = 181.6f$ (1)

$V = Q/A = Q/(\pi D^2/4) = 4Q/(\pi D^2) = 4(13)/(\pi D^2) = 16.55/D^2$ (2)

$\mathbf{R} = DV/\nu = DV/0.000\,54$ (3)

Try f	D ft (from 1)	V fps (from 2)	**R** (from 3)	e/D	Chart f	
0.030	1.404	8.40	21,837	0.000 142	≈ 0.0257	Try again
0.026	1.364	8.89	22,468	0.000 147	≈ 0.0257	Converged

D = 1.364 ft = 16.37 in ◄

8.39 *A steel pipe (e = 0.065 mm) 4200 m long is to convey oil (ν = 5.2 × 10^{-5} m^2/s) at 300 L/s from a reservoir with surface elevation 247 m to one with surface elevation 156 m (Fig. P8.38). Theoretically what pipe size is required?*

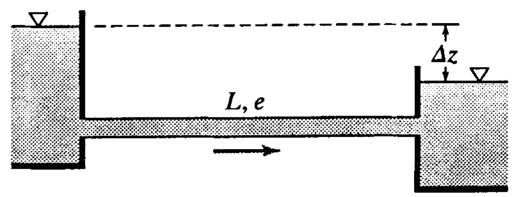

Figure P8.38

SI

This is a Type 3 problem, to find D.

e = 0.000 065 m (given). h_f = 247 − 156 = 91 m.

Eq. 8.13: $h_f = 91\ \text{m} = f[4200/(2 \times 9.81D)][0.3/(\pi D^2/4)]^2$,

i.e. $D^5 = 0.343f$

Try f	D m	$V = 4Q/(\pi D^2)$ m/s	$\mathbf{R} = DV/\nu$	e/D	Obtained f	
0.030	0.400	2.382	1.83×10^4	0.000 162	0.027	Try again
0.027	0.392	2.484	1.87×10^4	0.000 166	0.027	Converged

D = 0.392 m ◄

8.40 *Water at 15°C flowing through 25 m of 100-mm-diameter galvanized iron pipe causes a friction head loss of 75 mm. Find the flow rate.*

SI

This is a Type 2 problem, to find Q.

Table 8.1 for galvanized iron: e = 0.15 mm; e/D = 0.15/100 = 0.0015,

for which (Fig. 8.11) $f_{min} \approx 0.0215$

Eq. 8.13: $h_f = 0.075\ \text{m} = f(25/0.10)V^2/[2(9.81)]$, i.e. $V = 0.0767/f^{1/2}$

Table A.1 for water at 15°C: $\nu = 1.139 \times 10^{-6}$ m^2/s

Try f	V m/s	$\mathbf{R} = DV/\nu$	Chart f	
0.021	0.529	4.65×10^4	0.026	Try again
0.026	0.476	4.18×10^4	0.026	Converged!

$Q = (\pi/4)(0.1)^2 0.476 = 0.003\,74$ m^3/s = 3.74 L/s ◄

8.41 *When gasoline with a kinematic viscosity of 6×10^{-6} ft²/sec flows in a 10-in-diameter smooth pipe the friction head loss is 0.36 ft per 100 ft. Find the flow rate.*

BG

This is a Type 2 problem to find Q.

For smooth pipe $e = 0$, for which (Fig. 8.11) initial assumptions: $f \approx 0.02$ (mid-range)

$\nu = 6 \times 10^{-6}$ ft²/sec (given); $h_f = 0.36$ ft per 100 ft (given).

From Eq. 8.13: $h_f = 0.36 = f\left(\dfrac{100}{10/12}\right)\dfrac{V^2}{2(32.2)}$; i.e. $V = \dfrac{0.44}{f^{1/2}}$; Eq. 8.1: $\mathbf{R} = \dfrac{(10/12)V}{6 \times 10^{-6}} = 138{,}900V$

Try $f = 0.02$. Then $V = 0.44/(0.02)^{1/2} = 3.11$ fps, $\mathbf{R} = 138{,}900(3.11) = 4.32 \times 10^5$

Eq. 8.47 or Fig. 8.11 with for smooth pipe ($e = 0$) and $\mathbf{R} = 4.32 \times 10^5$: $f = 0.0135$

This and subsequent trials:

Try f	V fps	$\mathbf{R}$	Obtained f	
0.02	3.11	4.32×10^5	0.0135	Try again
0.0135	3.78	5.25×10^5	0.0130	Try again
0.0130	3.85	5.35×10^5	0.0130	Converged!

$Q = (\pi/4)(10/12)^2 3.85 = 2.10$ cfs ◀

8.42 *Oil with a kinematic viscosity of 0.000 22 ft²/sec is to flow at 7.4 cfs with a friction head loss of 0.37 ft·lb/lb per 100 ft of pipe length. What size pipe (e = 0.000 13 ft) is theoretically required?*

BG

This is a Type 3 problem, to find D. Assume laminar flow.

Eq. 8.28: $h_f = 32\nu LV/(gD^2) = 32\nu LQ(4)/(g\pi D^4) = 32(0.000\,22)100(7.4)4/(32.2\pi D^4)$

But $h_f = 0.37$ ft·lb/lb (given); Solving for D, $D = 1.158$ ft if flow is laminar.

Check for laminar flow: Eq. 8.1: $\mathbf{R} = DV/\nu = DQ(4)/(\nu\pi D^2) = 7.4(4)/(0.000\,22\pi 1.158) = 36{,}900$

Flow is not laminar, so use Eq. 8.13: $h_f = 0.37 = f[100/(2 \times 32.2D)][7.4/(\pi D^2/4)]^2$

from which $D^5 = 373f$; $e/D = 0.000\,13/D$; $V = Q/A = 7.4/(\pi D^2/4) = 9.42/D^2$

Eq. 8.1: $\mathbf{R} = DV/\nu = D(9.42/D^2)/0.000\,22 = 42{,}800/D$. Start by assuming a mid-range value of f:

Try f	D ft	e/D	$\mathbf{R}$	Obtained f	
0.0300	1.621	0.000 0802	2.64×10^4	0.0245	Try again
0.0245	1.556	0.000 0835	2.75×10^4	0.0242	Close
0.0242	1.552	0.000 0837	2.76×10^4	0.0242	Converged

$D = 1.552$ ft $= 18.62$ in ◀

8.43 *Oil with a kinematic viscosity of 2.0×10^{-5} m²/s is to flow at 210 L/s with a head loss of 0.42 N·m/N per 100 m of pipe length. What size pipe (e = 0.038 mm) is theoretically required?*

SI

This is a Type 3 problem, to find D. $e \approx 0.000\,038$ m (given)

Eq. 8.13: $h_f/L = 0.42/100 = [f/(2\times 9.81D)][0.21/(\pi D^2/4)]^2$, i.e. $D^5 = 0.868f$

Try f	D m	$V = 4Q/(\pi D^2)$ m/s	$\mathbf{R} = DV/\nu$	e/D	Obtained f	
0.030	0.482	1.151	2.77×10^4	0.000 0788	0.0235	Try again
0.235	0.459	1.269	2.91×10^4	0.000 0827	0.0235	Converged

$D = 0.459$ m ◀

8.44 *Water at 150°F flows in a straight 0.863-in-diameter iron pipe (e = 0.000 15 ft) between points A and B 350 ft apart. At A the elevation of the pipe is 112.0 ft and the pressure is 8.35 psi. At B the elevation of the pipe is 105.0 ft and the pressure is 8.80 psi. Compute the flow rate as accurately as you can.*

BG

This is a Type 2 problem, to find Q.

e = 0.000 15 ft (given); $e/D = 0.000\,15/(0.863/12) = 0.002\,09$ for which (Fig. 8.11) $f_{min} \approx 0.024$.

Table A.1 at 150°F: $\gamma = 61.2$ pcf, $\nu = 0.476 \times 10^{-5}$ ft²/sec

Sec. 8.4: $h_f = (z_A + p_A/\gamma) - (z_B + p_B/\gamma) = [112 + 8.35(144)/61.2] - [105 + 8.8(144)/61.2] = 5.94$ ft

Eq. 8.13: $h_f = 5.94 = f[350/(0.863/12)]V^2/[2(32.2)]$, i.e. $V = 0.280/f^{1/2}$

Try $f = f_{min} \approx 0.024$. Then $V = 0.280/(0.024)^{1/2} = 1.810$ fps

Eq. 8.1: $\mathbf{R} = DV/\nu = (0.863/12)1.810/(0.476 \times 10^{-5}) = 2.73 \times 10^5$

Eq. 8.51 or Fig. 8.11 with $e/D = 0.000\,092$ and $\mathbf{R} = 2.73 \times 10^5$: $f = 0.286$

This and subsequent trials:

Try f	V fps	$\mathbf{R}$	Obtained f	
0.024	1.81	2.73×10^5	0.0286	Try again
0.286	1.66	2.5×10^5	0.029	Try again
0.029	1.65	2.49×10^5	0.029	Converged!

$Q = (\pi/4)(0.863/12)^2 1.65 = 0.006\,69$ cfs ◄

8.45 *Water at 60°C flows in a straight 20-mm-diameter pipe (e = 0.060 mm) between points A and B 100 m apart. At A the elevation of the pipe is 54.1 m and the pressure is 88.7 kPa. At B the elevation of the pipe is 52.0 m and the pressure is 91.8 kPa. Compute the flow rate as accurately as you can.*

SI

This is a Type 2 problem, to find Q.

$e/D = 0.060/20 = 0.003$, for which (Fig. 8.11) $f_{min} \approx 0.026$

Table A.1 for water at 60°C: $\nu = 0.474 \times 10^{-6}$ m²/s, $\gamma = 9.642$ kN/m³

Sec 8.4: $h_f = (z + p/\gamma)_A - (z + p/\gamma)_B = [54.1 + (88.7/9.642)] - [52.0 + (91.8/9.642)] = 1.778$ m

Eq. 8.13: $h_f = 1.778 = f(100/0.02)V^2/[2(9.81)]$; i.e. $V = 0.0835/f^{1/2}$

Try f	V m/s	$\mathbf{R} = DV/\nu$	Obtained f	
0.026	0.518	2.19×10^4	0.0307	Try again
0.0307	0.477	2.01×10^4	0.0315	Close
0.0315	0.471	1.99×10^4	0.0315	Converged!

$Q = (\pi/4)(0.02)^2 0.471 = 0.000\,1479$ m²/s = 0.1479 L/s ◄

Sec. 8.16: Single-Pipe Flow: Direct Solutions -- ***Exercises*** *(2)*

8.16.1 *Solve Exer. 8.15.3 without trial and error, using only a basic scientific calculator.*

Exer. 8.15.3: Water at 50°F flowing through 80 ft of 4-in-diameter average cast-iron pipe causes a head loss of 0.27 ft. Find the flow rate.

A basic scientific calculator is here defined to be one that is not programmable and does not have automatic equation solving capabilities.

BG

For Eq. 8.56*a*, the quantity $\sqrt{2gDh_f/L} = \sqrt{2(32.2)4(0.27)/(12 \times 80)} = 0.269$ fps

Assuming turbulent flow:

$$\text{Eq. 8.56}a\text{:}\quad V = -2(0.269)\log\left[\frac{0.000\,85}{(4/12)3.7} + \frac{2.51(1.41 \times 10^{-5})}{(4/12)0.269}\right] = 1.596 \text{ fps}$$

$$\mathbf{R} = \frac{DV}{\nu} = \frac{(4/12)1.596}{1.41 \times 10^{-5}} = 37{,}736,$$ so flow is turbulent and use of Eq. 8.56*a* is appropriate.

$Q = AV = (\pi/4)(4/12)^2 1.596 = 0.1393$ cfs. ◄

8.16.2 *Solve Exer. 8.15.4 without trial and error, using only a basic scientific calculator.*

Exer. 8.15.4: When gasoline with a kinematic viscosity of 5×10^{-7} m²/s flows in a 200-mm-diameter smooth pipe the head loss is 0.43 m per 100 m. Find the flow rate.

A basic scientific calculator is here defined to be one that is not programmable and does not have automatic equation solving capabilities.

SI

For Eq. 8.56*a*, the quantity $\sqrt{2gDh_f/L} = \sqrt{2(9.81)0.2(0.43)/100} = 0.1299$ m/s

Assuming turbulent flow:

Eq. 8.56*a*: $V = -2(0.1299)\log[0 + 2.51(5 \times 10^{-7})/[(0.2)0.1299]] = 1.121$ m/s

$\mathbf{R} = DV/\nu = (0.2)1.121/(5 \times 10^{-7}) = 448\,506$, so flow is turbulent and use of Eq. 8.56*a* is appropriate.

$Q = AV = (\pi/4)(0.2)^2 1.121 = 0.0352\ \text{m}^3/\text{s} = 35.2$ L/s. ◄

Sec. 8.16: Single-Pipe Flow: Direct Solutions -- ***Problems*** *8.46–8.49*

8.46 *Using only a basic scientific calculator, solve Prob. 8.40 without trial and error.*

Prob. 8.40: $L = 25$ m, $D = 0.1$ m, $h_L = 0.075$ m, $e/D = 0.0015$, $\nu = 1.139 \times 10^{-6}$ m²/s, find Q.

A basic scientific calculator is here defined to be one that is not programmable and does not have automatic equation solving capabilities.

SI

This is a type 2 problem to find Q.

For Eq. 8.56*a*, the quantity $\sqrt{2gDh_f/L} = \sqrt{(2)9.81(0.1)0.075/25} = 0.0767$ m/s

Assuming turbulent flow:

$$\text{Eq. 8.56}a\text{:}\quad V = -2(0.0767)\log\left[\frac{0.0015}{3.7} + \frac{2.51(1.139 \times 10^{-6})}{(0.1)0.0767}\right] = 0.477 \text{ m/s}$$

$\mathbf{R} = DV/\nu = 0.1(0.477)/(1.139 \times 10^{-6}) = 4.19 \times 10^4$. ∴ the use of turbulent Eq. 8.56*a* was valid.

$Q = AV = (\pi/4)(0.1)^2 0.477 = 0.003\,75\ \text{m}^3/\text{s} = 3.75$ L/s ◄

8.47 *Using only a basic scientific calculator, solve Prob. 8.45 without trial and error.*

Prob. 8.45: Water at 60°C flows in a 100-m-long 20-mm-diameter pipe (e = 0.060 mm) between points A and B. At A the elevation of the pipe is 54.1 m and the pressure is 88.7 kPa. At B the elevation of the pipe is 52.0 m and the pressure is 91.8 kPa. Compute the flow rate as accurately as you can.

A basic scientific calculator is here defined to be one that is not programmable and does not have automatic equation solving capabilities.

SI

This is a type 2 problem to find Q.

Table A.1 for water at 60°C: $\nu = 0.474 \times 10^{-6}$ m²/s, $\gamma = 9.642$ kN/m³. $e/D = 0.060/20 = 0.003$.

Sec. 8.4: $h_f = (z + p/\gamma)_A - (z + p/\gamma)_B = (54.1 + 88.7/9.64) - (52.0 + 91.8/9.64) = 1.778$ m

For Eq. 8.56*a*, the quantity $(2gDh_L/L)^{1/2} = [(2)9.81(0.02)1.778/100]^{1/2} = 0.0835$ m/s

Assuming turbulent flow: Eq. 8.56*a*: $V = -2(0.0835)\log\left[\dfrac{0.003}{3.7} + \dfrac{2.51(0.474 \times 10^{-6})}{(0.020)0.0835}\right] = 0.471$ m/s

$\mathbf{R} = DV/\nu = 0.020(0.471)/(0.474 \times 10^{-6}) = 1.99 \times 10^4$. ∴ the use of turbulent Eq. 8.56*a* was valid.

$Q = AV = (\pi/4)(0.020)^2 0.471 = 0.000\,1478$ m³/s $= 0.1478$ L/s ◄

8.48 *Using only a basic scientific calculator, solve Prob. 8.43 without trial and error.*

Prob. 8.43: $Q = 210$ L/s, $h_f/L = 0.42/100$ m, $e = 0.000\,038$ m, $\nu = 2.0 \times 10^{-5}$ m²/s, find D.

A basic scientific calculator is here defined to be one that is not programmable and does not have automatic equation solving capabilities.

SI

This is a Type 3 problem, to find D.

Eq. 8.57: $\mathbf{N}_1 = \dfrac{128gh_fQ^3}{\pi^3 L\nu^5} = \dfrac{128(9.81)0.42}{(2.0 \times 10^{-5})100}\left(\dfrac{0.21}{\pi}\right)^3 = 4.92 \times 10^{20}$

Eq. 8.58: $\mathbf{N}_2 = \dfrac{\pi e\nu}{4Q} = \left(\dfrac{\pi}{4}\right)\dfrac{0.000\,038}{0.21}2.0 \times 10^{-5} = 2.84 \times 10^{-9}$

If laminar, Eq. 8.59: $\mathbf{R} = (4.92 \times 10^{20}/64)^{0.25} = 5.27 \times 10^4$ (not laminar!)

If turbulent, Eq. 8.61: $\mathbf{R}^{2.5} = -2\sqrt{4.92 \times 10^{20}}\,\log\left[\dfrac{2.84 \times 10^{-9}(4.92 \times 10^{20})^{0.208}}{2.59} + \dfrac{4.29}{(4.92 \times 10^{20})^{0.188}}\right]$

$= 1.438 \times 10^{11}$ so $\mathbf{R} = 2.90 \times 10^4$

Comparing both $\mathbf{R}$ values with $\mathbf{R}_{crit} = 2000$ (Eq. 8.2), the flow is turbulent, not laminar.

Eq. 8.62: $D = 4Q/(\pi\nu\mathbf{R}) = 4(0.21)/[\pi(2.0 \times 10^{-5})2.90 \times 10^4] = 0.461$ m ◄

8.49 *Using only a basic scientific calculator, solve Prob. 8.39 without trial and error.*

Prob. 8.39: A steel pipe (e = 0.065 mm) 4200 m long is to convey oil ($\nu = 5.2 \times 10^{-5}$ m²/s) at 300 L/s from a reservoir with surface elevation 247 m to one with surface elevation 156 m. Theoretically what pipe size is required?

A basic scientific calculator is here defined to be one that is not programmable and does not have automatic equation solving capabilities.

SI

This is a Type 3 problem, to find D. $h_f = 247 - 156 = 91$ m, $e = 0.000\,065$ m.

Eq. 8.57: $\mathbf{N}_1 = \dfrac{128(9.81)91}{(5.2 \times 10^{-5})^5 4200}\left(\dfrac{0.3}{\pi}\right)^3 = 6.23 \times 10^{19}$

Eq. 8.58: $\mathbf{N}_2 = \left(\dfrac{\pi}{4}\right)\dfrac{0.000\,065}{0.3}5.2 \times 10^{-5} = 8.85 \times 10^{-9}$

If laminar, Eq. 8.59: $\mathbf{R} = (6.23 \times 10^{19}/64)^{0.25} = 3.14 \times 10^4$ (not laminar!)

/cont...

If turbulent,

Eq. 8.61: $\mathbf{R}^{2.5} = -2\sqrt{6.23 \times 10^{19}} \log\left[\dfrac{8.85 \times 10^{-9}(6.23 \times 10^{19})^{0.208}}{2.59} + \dfrac{4.29}{(6.23 \times 10^{19})^{0.188}}\right] = 4.84 \times 10^{10}$

so $\mathbf{R} = 1.87 \times 10^4$

Comparing both **R** values with $\mathbf{R}_{crit} = 2000$ (Eq. 8.2), the flow is turbulent, not laminar.

Eq. 8.62: $D = 4Q/(\pi \nu \mathbf{R}) = 4(0.3)/[\pi(5.2 \times 10^{-5})1.87 \times 10^4] = 0.393$ m ◄

Sec. 8.17: Single-Pipe Flow: Automated Solutions -- *Exercises (3)*

8.17.1 *Solve Sample Prob. 8.5 using an equation solver on (a) a programmable scientific calculator; (b) computer software.*

Sample Prob. 8.5: Water at 20°C flows in a 500-mm-diameter welded steel pipe. If the friction loss gradient is 0.006, determine the flow rate.

SI

(*a*) Given: $D = 0.5$ m, $h_f = 0.006$ m, $L = 1$ m, $g = 9.81$ m/s^2, $e = 0.000\,046$ m,

$\nu = 1.003 \times 10^{-6}$ m^2/s. Estimate $Q = 1$ m^3/s.

Solve Colebrook-based combination Eq. 8.56*b* for Q, then Eq. 4.7 for V, then Eq. 8.1 for Reynolds number **R**, and then Colebrook Eq. 8.51 for f if desired.

Results are: $Q = 0.414$ m^3/s ◄ $V = 2.11$ m/s, $\mathbf{R} = 1\,049\,989$, $f = 0.013\,27$

(*b*) Use the same given and estimated data as for part (*a*). Place in an equation solver Eqs. 8.13, 4.7, 8.1, and 8.51, and solve them simultaneously. Results are the same as for part (*a*). ◄

8.17.2 *Solve Exer. 8.15.3 using an equation solver on (a) a programmable scientific calculator; (b) computer software.*

Exer. 8.15.3: Water at 50°F flowing through 80 ft of 4-in-diameter average cast-iron pipe causes a head loss of 0.27 ft. Find the flow rate.

BG

(*a*) Given: $h_f = 0.27$ ft, $L = 80$ ft, $D = 4/12$ ft, $g = 32.2$ ft/s^2, $e = 0.000\,85$ ft,

$\nu = 1.410 \times 10^{-5}$ ft^2/s. Estimate $Q = 1$ cfs.

Solve Colebrook-based combination Eq. 8.56*b* for Q, then Eq. 4.7 for V, then Eq. 8.1 for Reynolds number **R**, and then Colebrook Eq. 8.51 for f if desired.

Results are: $Q = 0.1393$ cfs ◄ $V = 1.596$ fps, $\mathbf{R} = 37{,}736$, $f = 0.0284$.

(*b*) Use the same given and estimated data as for part (*a*). Place in an equation solver Eqs. 8.13, 4.7, 8.1, and 8.51, and solve them simultaneously. Results are the same as for part (*a*). ◄

8.17.3 *Solve Exer. 8.15.4 using an equation solver on (a) a programmable scientific calculator; (b) computer software.*

Exer. 8.15.4: When gasoline with a kinematic viscosity of 5×10^{-7} m^2/s flows in a 200-mm-diameter smooth pipe the head loss is 0.43 m per 100 m. Find the flow rate.

SI

(*a*) Given: $h_f = 0.43$ m, $L = 100$ m, $D = 0.2$ m, $g = 9.81$ m/s^2, $e = 0$, $\nu = 5 \times 10^{-7}$ m^2/s.

Estimate $Q = 0.1$ m^3/s.

Solve Colebrook-based combination Eq. 8.56*b* for Q, then Eq. 4.7 for V, then Eq. 8.1 for Reynolds number **R**, and then Colebrook Eq. 8.51 for f if desired.

Results are: $Q = 0.0352$ m^3/s ◄ $V = 1.121$ m/s, $\mathbf{R} = 448\,506$, $f = 0.013\,42$.

(*b*) Use the same given and estimated data as for part (*a*). Place in an equation solver Eqs. 8.13, 4.7, continuity, 8.1, and 8.51, and solve them simultaneously. Results are the same as for part (*a*). ◄

Sec. 8.17: Single-Pipe Flow: Automated Solutions -- *Problems* 8.50−8.51

8.50 *Using an equation solver on a programmable scientific calculator, or in computer software, solve the following without manual trial and error: (a) Prob. 8.40; (b) Prob. 8.41; (c) Prob. 8.44; (d) Prob. 8.45. [An instructor may assign any one of hese four independent parts.]*

B

These are Type 2 problems, to find Q.

(*i*) Using an equation solver on a programmable scientific calculator, first solve Colebrook-based Eq. 8.56*b* for Q, then the continuity equation for V, then Eq. 8.1 for Reynold's number **R**, and then Colebrook Eq. 8.51 for f if desired.

(*ii*) Using computer software such as in Excel or Mathcad, insert Eqs. 8.13, 4.7, 8.1, and 8.51, and solve them simultaneously.

Part:	(*a*)	(*b*)	(*c*)	(*d*)	
Solve Prob:	8.40	8.41	8.44	8.45	
Input data					
D	0.1 m	10/12 ft	0.863/12 ft	0.02 m	
L	25 m	100 ft	350 ft	100 m	
h_f	0.075 m	0.36 ft	5.94 ft	1.778 m	
e	0.000 15 m	0	0.000 15 ft	0.000 060 m	
ν	6×10^{-6} m²/s	6×10^{-6} ft²/s	0.476×10^{-5} ft²/s	0.474×10^{-6} m²/s	
Results					
Q	0.003 75 m³/s	2.10 cfs	0.006 69 cfs	0.000 15 m³/s	◄◄
V	0.477 m/s	3.86 fps	1.647 fps	0.471 m/s	
R	41 883	535,512	24,881	19 861	
f	0.0259	0.013	0.0290	0.0315	

8.51 *Using an equation solver on a programmable scientific calculator, or in computer software, solve the following without manual trial and error: (a) Prob. 8.38; (b) Prob. 8.39; (c) Prob. 8.42; (d) Prob. 8.43. [An instructor may assign any one of these four independent parts.]*

B

These are Type 3 problems, to find D.

(*i*) Using an equation solver on a programmable scientific calculator, first solve Colebrook-based Eq. 8.56*b* for D, then Eq. 4.7 for V, then Eq. 8.1 for Reynold's number **R**, and then Colebrook Eq. 8.51 for f if desired.

(*ii*) Using computer software such as in Excel or Mathcad, insert Eqs. 8.13, 4.7, 8.1, and 8.51, and solve them simultaneously.

Part:	(*a*)	(*b*)	(*c*)	(*d*)	
Solve Prob:	8.38	8.39	8.42	8.43	
Input data					
Q	13 cfs	0.3 m³/s	7.4 cfs	0.21 m³/s	
L	13,450 ft	4200 m	100 ft	100 m	
h_f	315 ft	91 m	0.37 ft	0.42 m	
e	0.0002 ft	0.000 065 m	0.000 13 ft	0.000 038 m	
ν	0.000 54 ft²/s	5.2×10^{-5} m²/s	0.000 22 ft²/s	2.0×10^{-5} m²/s	
Results					
D	1.359 ft	0.391 m	1.552 ft	0.461 m	◄◄
V	8.97 fps	2.50 m/s	3.91 fps	1.261 m/s	
R	22,563	18 786	27,595	29 028	
f	0.0255	0.0266	0.0242	0.0239	

Sec. 8.18: Empirical Equations for Single-Pipe Flow -- *Exercises* (3)

8.18.1 *When water flows at 2.5 cfs through a 20-in-diameter pipeline, the head loss is 0.0004 ft/ft. Find the value of the Hazen-Williams coefficient.*

BG

Eq. 8.63*a*: $V = 1.318 C_{HW} R_h^{0.63} S^{0.54}$; Sec. 8.3: $R_h = (20/12)/4 = 0.417$ ft

$2.5/[\pi(10/12)^2] = 1.318 C_{HW}(0.417)^{0.63}(0.0004)^{0.54}$; $C_{HW} = 103.2$ ◄

8.18.2 *Water flows at 0.32 m³/s through a 600-mm-diameter pipeline with a head loss of 0.0029 m/m. Find the value of the Hazen-Williams coefficient.*

SI

Eq. 8.63*a*: $V = 0.849 C_{HW} R_h^{0.63} S^{0.54}$; Sec. 8.3: $R_h = D/4 = 0.150$ m

$0.32/(\pi 0.30^2) = 0.849 C_{HW}(0.150)^{0.63}(0.0029)^{0.54}$; $C_{HW} = 103.3$ ◄

8.18.3 *When water flows at 70 cfs through a 5-ft-diameter pipeline the head loss is 3.0 ft per mile. Find the value of Manning's n.*

BG

Eq. 4.7: $V = Q/A = 4Q/(\pi D^2) = 4(70)/(\pi 5^2) = 3.57$ fps

Eq. 8.64*a*: $3.57 = \dfrac{1.486}{n}\left[\dfrac{5}{4}\right]^{2/3}\left[\dfrac{3}{5280}\right]^{1/2}$; $n = 0.011\,53$ ◄

Sec. 8.18: Empirical Equations for Single-Pipe Flow -- *Problems* 8.52–8.53

8.52 *In a field test of the 16-ft-diameter Colorado River aqueduct flowing full, Manning's n was found to have a value of 0.0132 when 50°F water was flowing at a Reynolds number of 10.5 ×10⁶. Determine the average value of e for this conduit.*

BG

Write Eqs. 8.14 and 8.64*a* in terms of *S*: $S = \dfrac{h_f}{L} = \dfrac{f}{D}\dfrac{V^2}{2g} = \left(\dfrac{nV}{1.486 R_h^{2/3}}\right)^2$ where $R_h = \dfrac{D}{4}$ (Sec. 8.3)

Thus $f = \left(\dfrac{n}{1.486}\right)^2 \dfrac{D2g}{(D/4)^{4/3}} = \left(\dfrac{0.0132}{1.486}\right)^2 \dfrac{16(2)32.2}{(16/4)^{4/3}} = 0.012\,80$

Fig. 8.11 or Eq. 8.51 or 8.52 for $\mathbf{R} = 10.5 \times 10^6$ (given) and $f = 0.012\,80$: $e/D = 0.000\,133$

$e = 0.000\,133(16 \text{ ft}) = 0.002\,13$ ft ◄

[*Note:* Calculations for V (= 9.25 fps) and S (= 0.001 06) are unnecessary as they can be cancelled out.]

8.53 *Measurements were taken on pressure flow through a 2.2-m-diameter aqueduct built of concrete. When the water temperature was 10°C and the Reynolds number was 2.0 ×10⁶, Manning's n was determined to be 0.0130. Find the average value of e for this aqueduct.*

SI

Write Eqs. 8.14 and 8.64*b* in terms of *S*: $S = \dfrac{h_L}{L} = \dfrac{f}{D}\dfrac{V^2}{2g} = \dfrac{n^2 V^2}{R_h^{4/3}}$ where $R_h = \dfrac{D}{4}$ (Sec. 8.3)

Thus $f = n^2 D2g/(D/4)^{4/3} = (0.013)^2 2.2(2)9.81/(2.2/4)^{4/3} = 0.016\,19$

Fig. 8.11 or Eq. 8.51 or 8.52 for $\mathbf{R} = 2 \times 10^6$ (given) and $f = 0.016\,19$: $e/D = 0.000\,398$

$e = 0.000\,398(2.2) = 0.000\,876$ m $= 0.876$ mm ◄

[*Note:* Calculations for V (= 1.187 m/s) and S (= 0.000 529) are unnecessary, they can be cancelled out.]

Sec. 8.22: Loss of Head at Submerged Discharge -- *Exercises (5)*

8.22.1 *A 12-in-diameter pipe (f = 0.02) carries fluid at 8 fps between two tanks. The entrance and exit conditions to and from the pipe are square-edged and flush with the wall of the tank. Find the ratio of the minor losses divided by the pipe friction loss if the length of the pipe is (a) 4 ft; (b) 80 ft; (c) 1600 ft.*

BG

Pipe friction loss: $0.02(L/D)(V^2/2g)$

Minor losses: $0.5(V^2/2g) + (V^2/2g) = 1.5(V^2/2g)$

L	Minor	Fiction	Minor/friction
4 ft	1.491	0.0795	18.75:1
80 ft	1.491	1.590	0.938:1
1600 ft	1.491	31.8	0.0469:1

▲

8.22.2 *A 375-mm-diameter pipe (f = 0.017) carries fluid at 3.6 m/s between two tanks. The entrance and exit conditions to and from the pipe are reentrant. Find the ratio of the minor losses divided by the pipe friction loss if the length of the pipe is (a) 2 m; (b) 50 m; (c) 1000 m.*

SI

Pipe friction loss: $0.017\frac{L}{D}\frac{V^2}{2g} = 0.017\frac{L}{0.375}\frac{3.6^2}{(2)9.81} = 0.0299L$

Minor losses: $0.8(V^2/2g) + (V^2/2g) = 1.8(V^2/2g) = 1.8(3.6)^2/(2 \times 9.81)$

L	Minor (m)	Fiction (m)	Minor/friction
2 m	1.189	0.0599	19.85:1
50 m	1.189	1.497	0.794:1
1000 m	1.189	29.9	0.0397:1

▲

8.22.3 *Water leaves a turbine ay 18.5 fps and enters a tailrace having an average velocity of 1.5 fps. (a) What is the submerged discharge head loss? (b) By what percentage is this loss reduced if the provision of a draft tube increases the discharge flow area to six times the size?*

BG

(a) Eq. 8.74: $h_d' = \frac{18^2 - 1.5^2}{2(32.2)} = 5.28$ ft ◄

(b) Continuity: $V_2 = V_1(A_1/A_2) = 18.5/6 = 3.08$ fps

Eq. 8.74: $h_d' = \frac{3.08^2 - 1.5^2}{2(32.2)} = 0.1127$ ft

% reduction = $\frac{(5.28 - 0.1127)}{5.28}100\% = 97.9\%$ ◄

8.22.4 *A smooth 300-mm-diameter pipe is 90 m long and has a flush entrance and a submerged discharge. It carries 15°C water at a velocity of 3 m/s. What is the total head loss?*

SI

Eqs. 8.13, 8.70, 8.71: $\Sigma h_L = h_{Lf} + h_e' + h_d' = [f(L/D) + k_e + k_d]V^2/2g$

Table A.1 for water at 20°C: $\nu = 1.003 \times 10^{-6}$ m²/s

Eq. 8.1: $\mathbf{R} = DV/\nu = 0.4(2.4)/(1.003 \times 10^{-6}) = 9.57 \times 10^5$

Fig. 8.11 or Eq. 8.46 or Eq. 8.47 for smooth pipe: $f = 0.011\,73$; Fig. 8.13: $k_e = 0.5$

$\Sigma h_L = [0.011\,73(120/0.4) + 0.5 + 1.0]2.4^2/[2(9.81)] = (5.02)0.294 = 1.474$ m ◄

8.22.5 *Oil with a kinematic viscosity of 0.001 ft²/sec and a specific gravity of 0.92 flows at 8 fps through a smooth 15-in-diameter pipe which is 400 ft long with a flush entrance and submerged discharge. What is the head loss in feet of oil and in psi?*

BG

Eq. 8.1 for oil: $\mathbf{R} = DV/\nu = 1.25(8)/0.001 = 1 \times 10^4$

Fig. 8.11 or Eq. 8.46 or Eq. 8.47 for smooth pipe: $f = 0.0309$; Fig. 8.13: $k_e = 0.5$

$\Sigma h_L = h_{Lf} + h_e' + h_d' = [0.0309(400/1.25) + 0.5 + 1.0]8^2/[2(32.2)] = (11.38)0.994 = 11.31$ ft oil ◄

From Eq. 3.4: $\Delta p = \gamma \Sigma h_L = 0.92(62.4/144)11.31 = 4.51$ psi ◄

Sec. 8.22: Loss of Head at Submerged Discharge – ***Problems*** *8.54–8.55*

8.54 *A smooth 15-in-diameter pipe is 400 ft long and has a flush entrance and a submerged discharge. It carries 70°F water at a velocity of 8 fps. What is the total head loss?*

BG

Eqs. 8.13, 8.70, 8.71: $\Sigma h_L = h_{Lf} + h_e' + h_d' = [f(L/D) + k_e + k_d]V^2/2g$

Table A.1 for water at 70°F: $\nu = 1.059 \times 10^{-5}$ ft²/sec

Eq. 8.1: $\mathbf{R} = DV/\nu = 1.25(8)/(1.059 \times 10^{-5}) = 9.44 \times 10^5$

Fig. 8.11 or Eq. 8.46 or Eq. 8.47 for smooth pipe: $f = 0.011\,76$; Fig. 8.13: $k_e = 0.5$

$\Sigma h_L = [0.011\,76(400/1.25) + 0.5 + 1.0]8^2/[2(32.2)] = (5.26)0.994 = 5.23$ ft ◄

8.55 *Oil with a kinematic viscosity of 9.7 × 10⁻⁵ m²/s and a specific gravity of 0.94 flows at 3 m/s through a smooth 300-mm-diameter pipe which is 90 m long with a flush entrance and submerged discharge. What is the head loss in meters of oil and in kPa?*

SI

Eq. 8.1 for oil: $\mathbf{R} = DV/\nu = 0.40(2.4)/(9.7 \times 10^{-5} = 9897$

Fig. 8.11 or Eq. 8.46 or Eq. 8.47 for smooth pipe: $f = 0.0310$; Fig. 8.13: $k_e = 0.5$

$\Sigma h_L = h_{Lf} + h_e' + h_d' = [0.0310(120/0.40) + 0.5 + 1.0]2.4^2/[2(9.81)] = (10.79)0.294 = 3.17$ m oil ◄

From Eq. 3.4: $\Delta p = \gamma \Sigma h_L = 0.94(9.81)3.17 = 29.2$ kN/m² $= 29.2$ kPa ◄

Sec. 8.24: Loss Due to Expansion – ***Exercises*** *(2)*

8.24.1 *Two pipes with a diameter ratio of 1:2 are connected in series (Fig. X8.24.1). With a velocity of 6.8 m/s in the smaller pipe, find the loss of head due to (a) sudden contraction; (b) sudden enlargement; (c) expansion in a conical diffuser with a total angle of 30°, and of 10°.*

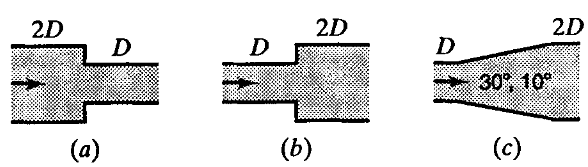

Figure X8.24.1

SI

(*a*) Table 8.2: For $D_2/D_1 = 0.5$, $k_c = 0.33$

Eq. 8.75: $h_c = k_c V_2^2/2g = 0.33(6.8)^2/[2(9.81)] = 0.778$ m ◄

(*b*) Eq. 8.76: $h_x' = (V_1 - V_2)^2/2g$ where $V_2 = (0.5)^2 V_1 = (0.5)^2 6.8 = 1.700$ m/s

$h_x' = (6.8 - 1.700)^2/2g = 5.100^2/(2 \times 9.81) = 1.326$ m ◄

(*c*) Eq. 8.78: $h' = k'(V_1 - V_2)^2/2g = k'(1.613 \text{ m})$

Fig. 8.20 for $\alpha = 30°$: $k' = 0.65$, $\therefore h' = 0.65(1.326) = 0.862$ m ◄

Fig. 8.20 for $\alpha = 10°$: $k' = 0.175$, $\therefore h' = 0.175(1.326) = 0.232$ m ◄

8.24.2 *A 5-in-diameter pipe (f = 0.033) 110 ft long connects two reservoirs whose water-surface elevations differ by 12 ft (Fig. X8.24.2). The pipe entrance is flush, and the discharge is submerged. (a) Compute the flow rate. (b) If the last 10 ft of pipe were replaced with a conical diffuser with a cone angle of 10°, compute the flow rate.*

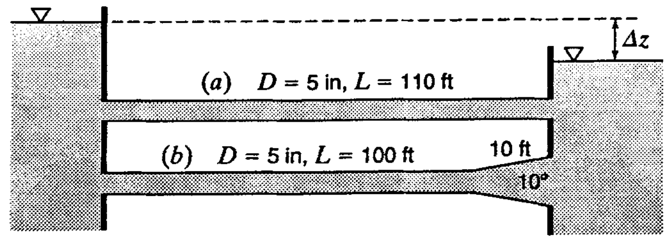

Figure X8.24.2

BG

(*a*) Energy:

$$12 - 0.5\frac{V^2}{2g} - 0.033\frac{110}{(5/12)}\frac{V^2}{2g} - \frac{V^2}{2g} = 0$$

$12 = V^2/2g(0.5 + 8.71 + 1.0) = 10.21V^2/2g$; $V^2/2g = 1.175$ ft, $V = 8.70$ fps

$Q = AV = (\pi/4)(5/12)^2 8.70 = 1.186$ cfs ◄

(*b*) Fig. 8.20 for $\alpha = 10°$: $k' = 0.175$

$D_d = D + 2L\tan(10°/2) = (5/12) + 2(10)\tan 5° = 2.17$ ft

Energy: $12 - 0.5V^2/2g - 0.033[100/(5/12)](V^2/2g) - 0.175(V - V_d)^2/2g = 0$

$V_d = V(D/D_d)^2 = V[(5/12)/2.17]^2 = 0.0370V$; $V - V_d = 0.963V$

$12 = (V^2/2g)(0.5 + 7.92 + 0.1623) = 8.58V^2/2g$; $V = 9.49$ fps

$Q = AV = (\pi/4)(5/12)^2 9.49 = 1.294$ cfs ◄

Sec. 8.24: Loss Due to Expansion – *Problems* 8.56–8.57

8.56 *A smooth pipe consists of 52 ft of 9-in pipe followed by 310 ft of 18-in pipe with an abrupt change of cross section at the junction (Fig. P8.56). The entrance is flush and the discharge is submerged. If it carries water at 60°F, with a velocity of 19 fps in the smaller pipe, what is the total head loss?*

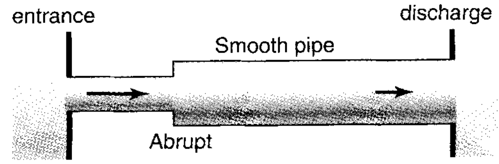

Figure P8.56

BG

We note (Eqs. 8.76 and 8.77) that:

$$h_x' = \frac{(V_1 - V_2)^2}{2g} = \left(1 - \frac{V_2}{V_1}\right)^2\frac{V_1^2}{2g} = \left(1 - \frac{A_1}{A_2}\right)^2\frac{V_1^2}{2g}$$

$$\Sigma h_L = h_e' + h_{Lf1} + h_x' + h_{Lf2} + h_d' = \left[k_e + f_1\frac{L_1}{D_1} + \left(1 - \frac{A_1}{A_2}\right)^2\right]\frac{V_1^2}{2g} + \left[f_2\frac{L_2}{D_2} + k_d\right]\frac{V_2^2}{2g}$$

Table A.1 for water at 60°F: $\nu = 1.217 \times 10^{-5}$ ft²/sec; Fig. 8.13: $k_e = 0.5$

Eq. 8.1 for 9-inch pipe: $\mathbf{R} = DV/\nu = (9/12)19/(1.217 \times 10^{-5}) = 1.171 \times 10^6$

Fig. 8.11 or Eq. 8.46 or Eq. 8.47 for smooth pipe: $f_1 = 0.011\,34$

Eq. 8.1 for 18-inch pipe: $\mathbf{R} = DV/\nu = (18/12)19(9/12)^2/(1.217 \times 10^{-5}) = 3.29 \times 10^5$

Fig. 8.11 or Eq. 8.46 or Eq. 8.47 for smooth pipe: $f_2 = 0.014\,21$

$$\Sigma h_L = \left\{0.5 + 0.011\,34\frac{52(12)}{9} + \left[1 - \left(\frac{9}{24}\right)^2\right]^2\right\}\frac{19^2}{2(32.2)} + \left(0.014\,21\frac{310(12)}{18} + 1.0\right)\left(\frac{9}{24}\right)^4\frac{19^2}{2(32.2)}$$

$= 12.03 + 0.436 = 12.46$ ft ◄

8.57 *A smooth pipe consists of 12 m of 180-mm-diameter pipe followed by 75 m of 550-mm-diameter pipe with an abrupt change of cross section at the junction (Fig. P8.56). The entrance is flush and the discharge is submerged. If it carries water at 15°C, with a velocity of 5.7 m/s in the smaller pipe, what is the total head loss?*

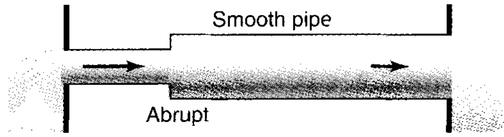

Figure P8.56

SI

Eq. 4.3: $Q = A_1V_1 = (\pi/4)D_1^2V_1 = (\pi/4)(0.18)^2 5.7 = 0.1451\ \text{m}^3/\text{s}$

$Q = A_2V_2 = (\pi/4)(0.55)^2V_2;\quad V_2 = (0.18/0.55)^2 5.7 = 0.611\ \text{m/s}$

Table A.1 for water at 15°C: $\nu = 1.139 \times 10^{-6}\ \text{m}^2/\text{s}$

Eq. 8.1: $\mathbf{R}_1 = D_1V_1/\nu = 0.18(5.7)/(1.139 \times 10^{-6}) = 9.01 \times 10^5$; $\mathbf{R}_2 = D_2V_2/\nu = 2.95 \times 10^5$

Eq. 8.46 by T and E (or approx from Fig. 8.11 or Eq. 8.47): $f_1 = 0.011\,86$, $f_2 = 0.014\,51$

$$\Sigma h_L = [k_e + (f_1L_1/D_1)]V_1^2/(2g) + (V_1 - V_2)^2/(2g) + [(f_2L_2/D_2) + 1]V_2^2/(2g)$$

$$= \left[0.5 + \frac{0.011\,86(12)}{0.18}\right]\frac{5.7^2}{2(9.81)} + \frac{(5.7 - 0.611)^2}{2(9.81)} + \left[\frac{0.014\,51(75)}{0.55} + 1\right]\frac{0.611^2}{2(9.81)}$$

$= 2.14 + 1.320 + 0.0566 = 3.51$ m ◄

Sec. 8.26: Loss in Bends and Elbows -- *Exercises* (2)

8.26.1 *Water at 72°F flows through a 110-ft-long, 5-in-diameter wrought-iron pipe that contains the following fittings: one open globe valve, one medium-radius elbow, and one 90° pipe bend (k_b = 0.13) with a radius of curvature of 45 in. The length of the bend is not included in the 110 ft. There are no entrance or discharge losses. Find the total head loss if the flow velocity is 4.8 fps.*

BG

From Table A.1 for water at 72°F, by linear interpolation: $\nu = 1.033 \times 10^{-5}\ \text{ft}^2/\text{s}$

Eq. 8.1: $\mathbf{R} = (5/12)4.8/(1.033 \times 10^{-5}) = 1.936 \times 10^5$; Table 8.1: $e = 0.000\,15$ ft

$e/D = 0.000\,15/(5/12) = 0.000\,360$; Fig. 8.11 or Eq. 8.51: $f = 0.018\,13$ (or 0.017 92 using Eq. 8.52)

$\Sigma h_L = [f(L/D) + k_v + k_{b1} + k_{b2}]V^2/(2g)$ where L = pipe + bend (Sec. 8.26)

Using Table 8.3: $\Sigma h_L = \left(0.018\,13\frac{[(110)12 + (\pi/2)45]}{5} + 10 + 0.75 + 0.13\right)\frac{4.8^2}{2(32.2)} = 5.70$ ft ◄

<u>Alternatively</u>: using Eq. 8.52: $\Sigma h_L = (0.017\,92[278] + 10.88)0.358 = 5.68$ ft ◄

8.26.2 *Water at 18°C flows through a 25-m-long, 75-mm-diameter commercial steel pipe that contains the following fittings: one open angle valve, one short-radius elbow, and one 90° pipe bend (k_b = 0.12) with a radius of curvature of 600 mm. The length of the bend is not included in the 25 m. There are no entrance or discharge losses. Find the total head loss if the flow velocity is 2 m/s.*

SI

From Table A.1 for water at 18°C, by linear interpolation: $\nu = 1.057 \times 10^{-6}\ \text{m}^2/\text{s}$

Eq. 8.1: $\mathbf{R} = 0.075(2)/(1.057 \times 10^{-6}) = 141\,857$

Table 8.1 for commercial steel: $e = 0.046$ mm; $e/D = 0.046/75 = 0.000\,613$

Fig. 8.11 or Eq. 8.51: $f = 0.019\,99$ (or 0.019 78 using Eq. 8.52)

$\Sigma h_L = [f(L/D) + k_v + k_{b1} + k_{b2}]V^2/(2g)$ where L = pipe + bend (Sec. 8.26)

Using Table 8.3: $\Sigma h_L = \left(0.019\,99\frac{[25 + (\pi/2)0.6]}{0.075} + 5 + 0.9 + 0.12\right)\frac{2^2}{2(9.81)} = 2.64$ m ◄

<u>Alternatively</u>: using Eq. 8.52: $\Sigma h_L = (0.019\,78[346] + 6.02)0.204 = 2.62$ m ◄

Sec. 8.27: Single-Pipe Flow with Minor Losses – *Exercises* *(6)*

8.27.1 *An 8-in-diameter pipeline (f = 0.028) 500 ft long discharges a 3-in-diameter water jet into the atmosphere at a point which is 250 ft below the water surface at intake (Fig. X8.27.1). The entrance to the pipe is reentrant with k_e = 0.9, and the nozzle loss coefficient is 0.045. Find the flow rate and the pressure head at the base of the nozzle.*

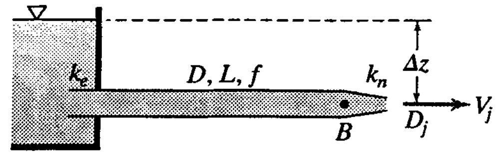

Figure X8.27.1

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$$250 - 0.9\frac{V^2}{2g} - 0.028\frac{500}{8/12}\frac{V^2}{2g} - 0.045\frac{V_j^2}{2g} = \frac{V_j^2}{2g}$$

$V_j = V(8/3)^2 = 7.11V$

$250 - (V^2/2g)(0.9 + 21 + 2.28) = 50.6(V^2/2g)$; $250 = (V^2/2g)74.7$; $V = 14.68$ fps

$Q = AV = 0.267(14.68) = 5.12$ cfs ◀

$(V^2/2g) = p/\gamma - 2.28(V^2/2g) = 50.6(V^2/2g)$

$p/\gamma = 51.9(V^2/2g) = 51.9(14.68)^2/(2 \times 32.2) = 173.6$ ft; $p = 173.6(62.4)/144 = 75.3$ psi ◀

8.27.2 *A 180-mm-diameter pipeline (f = 0.032) 150 m long discharges a 60-mm-diameter water jet into the atmosphere at a point which is 80 m below the water surface at intake (Fig. X8.27.1). The entrance to the pipe is reentrant with k_e = 0.9, and the nozzle loss coefficient is 0.055. Find the flow rate and the pressure head at the base of the nozzle.*

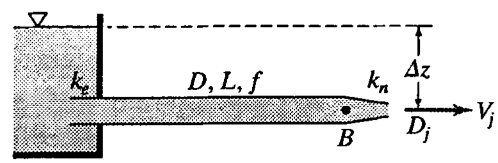

Figure X8.27.1

SI

$$80 - 0.9\frac{V^2}{2g} - 0.032\frac{150}{0.18}\frac{V^2}{2g} - 0.055\frac{V_j^2}{2g} = \frac{V_j^2}{2g}$$

$V_j = 9V$; $80 = [V^2/(2 \times 9.81)]113.0$; $V = 3.73$ m/s

$Q = (\pi 0.09^2)3.73 = 0.0948\ m^3/s = 94.8$ L/s ◀

$(V^2/2g) + p/\gamma - 4.46(V^2/2g) = 81(V^2/2g)$

$p/\gamma = 84.5(V^2/2g) = 84.5(3.73^2)/(2 \times 9.81) = 59.8$ m; $p = 59.8(9.81) = 586\ kN/m^2$ ◀

8.27.3 *A horizontal 4-in-diameter pipe (f = 0.028) projects into a body of water 2.5 ft below the surface (Fig. X8.27.3). Considering all losses, find the pressure at a point 15 ft from the end of the pipe if the velocity is 12 fps and the flow is (a) into the body of water; (b) out of the body of water.*

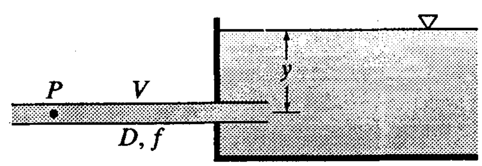

Figure X8.27.3

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(*a*) Outflow from pipe. Sec. 8.22: $k_d = 1.0$

$$h_L = \left(k_d + f\frac{L}{D}\right)\frac{V^2}{2g} = \left(1 + 0.028\frac{15}{4/12}\right)\frac{12^2}{2(32.2)} = 2.26(2.236) = 5.05 \text{ ft}$$

Energy Eq: $p/\gamma + 0 + 2.236 = 2.5 + 5.05$; $p/\gamma = 5.31$ ft, $p = 2.30$ psi ◀

(*b*) Inflow into pipe. Fig. 8.13: $k_e = 0.8$

$$h_L = \left(k_e + f\frac{L}{D}\right)\frac{V^2}{2g} = \left(0.8 + 0.028\frac{15}{4/12}\right)\frac{12^2}{2g} = 2.06(2.236) = 4.61 \text{ ft}$$

Energy Eq.: $p/\gamma + 0 + 2.236 = 2.5 - 4.61$; $p/\gamma = -4.34$ ft, $p = -1.882$ psi ◀

8.27.4 *A horizontal 100-mm diameter pipe (f = 0.027) projects into a body of water 1 m below the surface (Fig. x8.27.3). Considering all losses, find the pressure at a point 5 m from the end of the pipe if the velocity is 4 m/s and the flow is (a) into the body of water; (b) out of the body of water.*

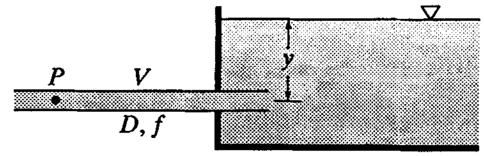


Figure X8.27.3

SI

(*a*) Outflow from pipe. Sec. 8.22: k_d = 1.0

$$h_L = \left(k_d + f\frac{L}{D}\right)\frac{V^2}{2g} = \left(1 + 0.027\frac{5}{0.10}\right)\frac{4^2}{2(9.81)} = 2.35(0.815) = 1.916 \text{ m}$$

Energy Eq: $p/\gamma + 0 + 0.815 = 1 + 1.916$; $p/\gamma = 2.10$ m, $p = 20.6$ kN/m^2 ◄

(*b*) Inflow into pipe. Fig. 8.13: k_e = 0.8

$$h_L = \left(k_e + f\frac{L}{D}\right)\frac{V^2}{2g} = \left(0.8 + 0.027\frac{5}{0.10}\right)\frac{4^2}{2(9.81)} = 2.15(0.815) = 1.753 \text{ m}$$

Energy Eq: $p/\gamma + 0 + 0.815 = 1 - 1.753$; $p/\gamma = -1.569$ m, $p = -15.39$ kN/m^2 ◄

8.27.5 *A 450-ft-long pipeline runs between two reservoirs, both ends being under water, and the intake end is square-edged and nonprojecting. The difference between the water surface levels of the two reservoirs is 150 ft. (a) What is the discharge if the pipe diameter is 12 in and f = 0.028? (b) When this same pipe is old, assume that the growth of tubercles has reduced the diameter to 11.25 in and that f = 0.06. What will the rate of discharge be then?*

BG

(*a*) $150 = [1.5 + 0.028(450/1)]V^2/(2g) = (1.5 + 12.60)V^2/(2g)$; $V^2/(2g) = 150/14.1 = 10.64$

$V = [2(32.2)10.64]^{1/2} = 26.2$ fps ; $Q = AV = (0.7854)26.2 = 20.6$ cfs ◄

(*b*) $150 = [1.5 + 0.06(450)12/11.25]V^2/(2g) = (1.5 + 28.8)V^2/(2g)$; $V^2/(2g) = 150/30.3 = 4.95$

$V = [2(32.2)4.95]^{1/2} = 17.86$ fps ; $Q = AV = (0.690)17.86 = 12.33$ cfs ◄

8.27.6 *Solve Exer. 8.27.5a when f is unknown but given that e = 0.005 ft and the water temperature is 60°F. What, then, is the value of f?*

Exer. 8.27.5a: L = 450 ft, D = 12 in, f = 0.028, both pipe ends are submerged, k_e = 0.5. Find Q when h_L = 150 ft.

BG

Table A.1 for water at 60°F: $\nu = 1.217 \times 10^{-5}$ ft^2/sec. $e/D = 0.005/1 = 0.005$.

Energy: $h_L = [\Sigma k + (fL/D)]V^2/(2g)$, i.e. $150 = [1.5 + (450/1)f]V^2/(2 \times 32.2)$ (1)

Continuity: $Q = AV = (\pi/4)D^2V$, i.e. $Q = (\pi/4)(1^2)V$ (2)

Eq. 8.1: $\mathbf{R} = DV/\nu$, i.e. $\mathbf{R} = 1V/(1.217 \times 10^{-5})$ (3)

Eq. 8.51: $\dfrac{1}{\sqrt{f}} = -2\log\left[\dfrac{e/D}{3.7} + \dfrac{2.51}{\mathbf{R}\sqrt{f}}\right]$, i.e. $\dfrac{1}{\sqrt{f}} = -2\log\left[\dfrac{0.005}{3.7} + \dfrac{2.51}{\mathbf{R}\sqrt{f}}\right]$ (4)

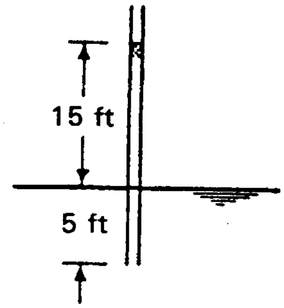
In these four equations the four unkowns are f, V, Q, and $\mathbf{R}$. Solve for them as in Sample Prob. 8.9, (*a*) by making trials for f in (1) and solving for V, then with this V solving for $\mathbf{R}$ in (3) and f in (4) or more conveniently in Eq. 8.52, and repeating these steps until the resulting f equals the assumed f, when Q may be found from (2); or (*b*) by using an equation solver on a programmable calculator to solve Eq. 8.81 for Q and then find V, $\mathbf{R}$, and f from (2), (3), and Eq. 8.52; or (*c*) by using an equation solver in computer software to solve (1) - (4) simultaneously.

Results: Q = 19.81 cfs ◄ V = 25.2 fps, $\mathbf{R}$ = 2,072,372, f = 0.0304 ◄

Sec. 8.27: Single-Pipe Flow with Minor Losses -- *Problems* 8.58−8.70

8.58 *Water flows at 12 fps through a vertical 4-in-diameter pipe standing in a body of water with its lower end 5 ft below the surface. Considering all losses and with $f = 0.026$, find the pressure in the pipe at a point 15 ft above the surface of the water when the flow is (a) downward; (b) upward.*

BG



(*a*) Downward flow.

$$h_L = h_d' + f\frac{L}{D}\frac{V^2}{2g} = \left(1 + 0.026\frac{(15 + 5)}{4/12}\right)\frac{12^2}{2(32.2)} = (2.56)2.24 = 5.72 \text{ ft}$$

Energy: $(p/\gamma + 15 + 2.24) - 0 = 5.72$ ft

$p/\gamma = -11.51$ ft; $p = -11.52(62.4/144) = -4.99$ psi ◄

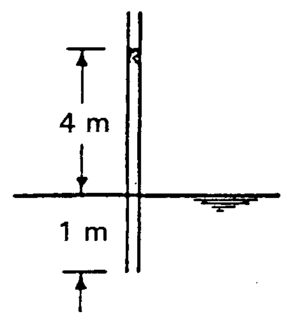
(*b*) Upward flow. Fig. 8.13: $k_e = 0.8$

$$h_L = \left(k_e + f\frac{L}{D}\right)\frac{V^2}{2g} = \left(0.8 + 0.026\frac{(15 + 5)}{4/12}\right)\frac{12^2}{2(32.2)} = (2.36)2.24 = 5.28 \text{ ft}$$

Energy: $0 - (p/\gamma + 15 + 2.24) = 5.28$; $p/\gamma = -22.5$ ft; $p = -22.5(62.4/14.4) = -9.76$ psi ◄

8.59 *Water flows at 2.5 m/s through a vertical 100-mm-diameter pipe standing in a body of water with its lower end 1 m below the surface. Considering all losses and with $f = 0.024$, find the pressure in the pipe at a point 4 m above the surface of the water when the flow is (a) downward; (b) upward.*

SI



(*a*) Downward flow.

$$h_L = \left(k_d + f\frac{L}{D}\right)\frac{V^2}{2g} = \left(1 + 0.024\frac{(4 + 1)}{0.10}\right)\frac{2.5^2}{2(9.81)} = (2.20)0.319 = 0.701 \text{ m}$$

Energy: $(p/\gamma + 4 + 0.319) - 0 = 0.701$ m

$p/\gamma = -3.62$ m; $p = -3.62(9.81) = -35.5$ kN/m^2 ◄

(*b*) Upward flow. Fig. 8.13: $k_e = 0.8$

$$h_L = \left(k_e + f\frac{L}{D}\right)\frac{V^2}{2g} = \left(0.8 + 0.024\frac{(46 + 1)}{0.10}\right)\frac{2.5^2}{2(9.81)}$$

$$= (2.0)0.319 = 0.637 \text{ m}$$

Energy: $0 - (p/\gamma + 4 + 0.319) = 0.637$; $p/\gamma = -4.96$ m; $p = -4.96(9.81) = -48.6$ kN/m^2 ◄

8.60 *A 12-in-diameter pipe ($f = 0.028$) 450 ft long runs from one reservoir to another, both ends of the pipe being under water (Fig. P8.60). The intake is square-edged. The difference between the water surface levels of the two reservoirs is 150 ft. Find (a) the flow rate, and (b) the pressure in the pipe at a point 320 ft from the intake, where the elevation is 135 ft lower than the surface of the water in the upper reservoir.*

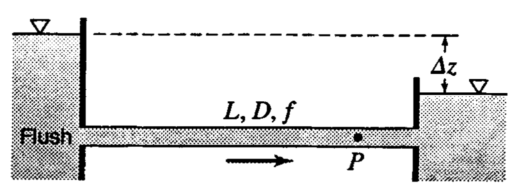


Figure P8.60

BG

(*a*) $h_L = 150 = [0.5 + 1 + 0.028(450/1)]V^2/(2g) = 14.10V^2/2g$

$V^2/2g = 150/14.10 = 10.64$ ft; $V = 26.2$ fps; $Q = 20.6$ cfs ◄

(*b*) $h_L' = [0.5 + 0.028(320/1)]10.64 = 100.6$ ft

Energy: $135 - (p/\gamma + V^2/2g) = 100.6$; $p/\gamma = 23.7$ ft; $p = 23.7(62.4/144) = 10.28$ psi ◄

8.61 *A 320-mm-diameter pipe (f = 0.025) 140 m long runs from one reservoir to another, both ends of the pipe being under water (Fig. P8.60). The intake is square-edged. The difference between the water surface levels of the two reservoirs is 36 m. Find (a) the flow rate, and (b) the pressure in the pipe at a point 95 m from the intake, where the elevation is 39 m lower than the surface of the water in the upper reservoir.*

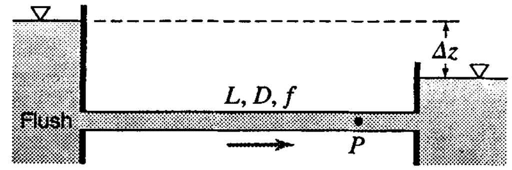


Figure P8.60

SI

(a) $h_L = 36 \text{ m} = [0.5 + 1 + 0.025(140/0.32)]V^2/(2g) = 12.44V^2/2g$

$V^2/2g = 36/12.44 = 2.89$ m; $V = 7.42$ m/s; $Q = 0.606 \text{ m}^3/\text{s}$ ◄

(b) $h_L' = [0.5 + 0.025(95/0.32)]2.89 = 22.9$ m

Energy: $39 - (p/\gamma + V^2/2g) = 22.9$; $p/\gamma = 13.18$ m; $p = 13.18(9.81) = 129.3 \text{ kN/m}^2$ ◄

8.62 *A pump delivers water through 300 ft of 4-in fire hose (f = 0.025) to a nozzle which throws a 1-in-diameter jet. The loss coefficient of the nozzle is 0.04. The nozzle is 20 ft higher than the pump, and a jet velocity of 70 fps is required. What must be the pressure in the hose at the pump?*

BG

Nozzle loss $= 0.04V_j^2/(2g)$; continuity: $V_j = (D_1/D_j)^2V_1 = 16V_1$

Hose friction $= 0.025(300 \times 12/4)V_1^2/(2g) = 22.5V_1^2/(2g) = 0.0879V_j^2/(2g)$

Energy: $p_1/\gamma + V_1^2/(2g) = 20 + V_j^2/(2g) + (0.04 + 0.0879)V_j^2/(2g)$

where $V_j^2/(2g) = 70^2/(2 \times 32.2) = 76.1$ ft

$p_1/\gamma = 20 + 76.1 + 0.1279(76.1) - 76.1/16^2 = 105.5$ ft; $p_1 = 105.5(62.4/144) = 45.7$ psi ◄

8.63 *A pump delivers water through 100 m of 100-mm fire hose (f = 0.025) to a nozzle which throws a 25-mm-diameter jet. The loss coefficient of the nozzle is 0.04. The nozzle is 6 m higher than the pump, and a jet velocity of 20 m/s is required. What must be the pressure in the hose at the pump?*

SI

Nozzle loss $= 0.04V_j^2/(2g)$; continuity: $V_j = (D_1/D_j)^2V_1 = 16V_1$

Hose friction $= 0.025(100/0.10)V_1^2/(2g) = 25V_1^2/(2g) = 0.0977V_j^2/(2g)$

Energy: $p_1/\gamma + V_1^2/(2g) = 6 + V_j^2/(2g) + (0.04 + 0.0977)V_j^2/(2g)$

where $V_j^2/(2g) = 20^2/(2 \times 9.81) = 20.4$ m

$p_1/\gamma = 6 + 20.4 + 0.1377(20.4) - 20.4/16^2 = 29.1$ m; $p_1 = 29.1(9.81) = 286 \text{ kN/m}^2$ ◄

8.64 *A 6-in-diameter horizontal jet of water is discharged into air through a nozzle (loss coefficient 0.15) at a point 150 ft lower than the water surface above the intake (Fig. P8.64). The 12-in-diameter pipeline (f = 0.015) is 600 ft long, with a square-edged nonprojecting entrance. What is (a) the jet velocity, and (b) the pressure at the base of the nozzle?*

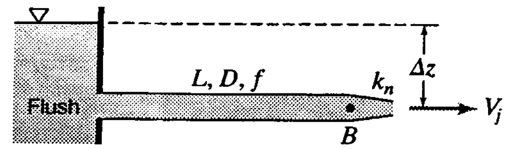


Figure P8.64

BG

(*a*) Nozzle loss = $0.15\dfrac{V_j^2}{2g}$; continuity: $V_j = \left(\dfrac{D}{D_j}\right)^2 V = 4V$

$h_L = [0.5 + 0.015(600/1)]V^2/(2g) + 0.15V_j^2/(2g) = 9.5V^2/(2g) + 0.15V_j^2/(2g)$

$= (9.5/4^2)V_j^2/(2g) + 0.15V_j^2/(2g) = 0.744V_j^2/(2g)$

Energy: $150 - 0.744V_j^2/(2g) = V_j^2/(2g)$; $V_j^2/(2g) = 86.0$ ft

$V_j = [2(32.2)86.0]^{1/2} = 74.4$ fps ◄

(*b*) (*i*) Energy, from water surface to nozzle base:

$(0 + 150 + 0) - 9.5V^2/(2g) = [p_{\text{base}}/\gamma + 0 + V^2/(2g)]$

$p_{\text{base}}/\gamma = 150 - 10.5V^2/2g = 150 - (10.5/4^2)V_j^2/2g = 150 - (10.5/16)86.0 = 93.5$ ft.

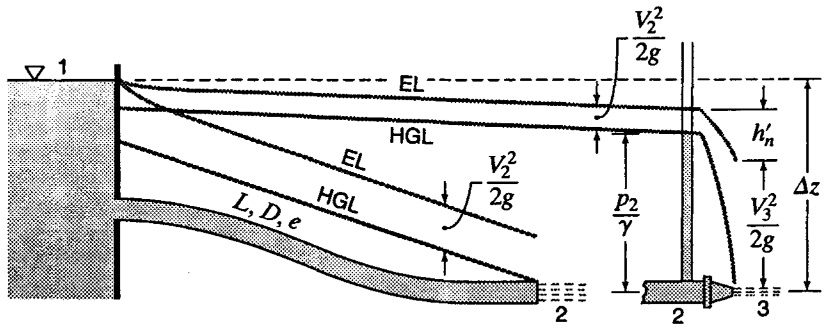
(*ii*) Energy, from nozzle base to jet:

$[p_{\text{base}}/\gamma + z + V^2/(2g)] - 0.15V_j^2/(2g) = [0 + z + V_j^2/(2g)]$

$p_{\text{base}}/\gamma = 1.15V_j^2/(2g) - V^2/(2g) = 1.15V_j^2/(2g) - (1/4^2)V_j^2/(2g) = 1.088V_j^2/(2g)$ ft

$= 1.088(86.0) = 93.5$ ft ; $p_{\text{base}} = 93.5(62.4/144) = 40.5$ psi ◄

8.65 *Refer to Fig. S8.11b. Suppose $\Delta z = 50$ ft, the line is 600 ft of 8-in-diameter pipe ($f = 0.025$), and the nozzle loss coefficient is 0.05. Find the jet diameter that will result in the greatest jet horsepower.*



(*a*) Free discharge (*b*) With nozzle

Figure S8.11

BG

Energy: $50 - 0.5V_8^2/(2g) - 0.025(600 \times 8/12)V_8^2/(2g) - 0.05V_j^2/(2g) = V_j^2/(2g)$

$50 - 23(V_8^2/(2g) = 1.05(V_j^2/2g)$

Continuity: $D_j^2V_j = D_8^2V_8;\quad V_8 = V_j(D_j/D_8)^2;\quad D_8 = 8$ inches

$50 = 1.05V_j^2/(2g) + 23(D_j/D_8)^4(V_j^2/2g);\quad V_j^2/(2g) = 50/[1.05 + 23(D_j/D_8)^4]$

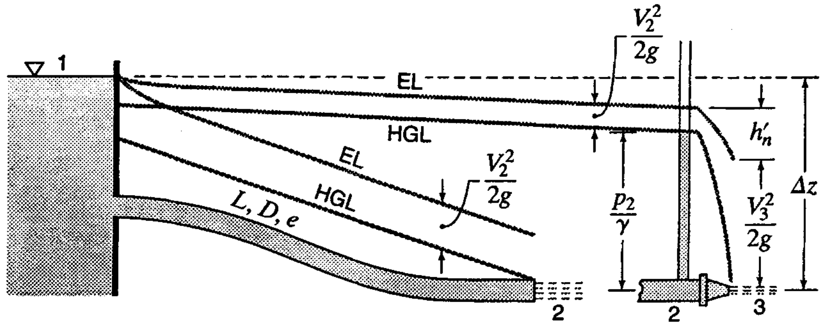
Eq. 5.40: $hp = \gamma Q(V_j^2/2g)/550 = \gamma A_jV_j(V_j^2/2g)/550 = 0.001\,762A_jV_j^3$

D_j (in)	A_j (ft^2)	$V_j^2/2g$	V_j	hp
0	0	--	--	0
1.0	0.005 45	47.6	55.3	1.62
2.0	0.0218	44.0	53.0	5.70
3.0	0.0491	33.2	46.2	8.50
4.0	0.0873	20.1	36.0	7.15
6.0	0.196	6.04	19.7	2.61
8.0	0.349	2.08	11.5	0.93

Maximum power occurs with a jet diameter of approximately 3.0 inches. ◀

(An exact answer may be obtained by using differential calculus.)

8.66 *Refer to Fig. S8.11b. Suppose $\Delta z = 15$ m, the line is 180 m of 200-mm-diameter pipe ($f = 0.025$), and the nozzle loss coefficient is 0.05. Find the jet diameter that will result in the greatest jet power.*



(*a*) Free discharge (*b*) With nozzle

Figure S8.11

SI

Energy: $15 - 0.5V_8^2/(2g) - 0.025(180/0.20)V^2/(2g) - 0.05V_j^2/(2g) = V_j^2/(2g)$

$15 - 23(V^2/(2g) = 1.05(V_j^2/2g)$; continuity: $D_j^2 V_j = D^2 V$; $V = V_j(D_j/D)^2$; D = 200 mm

$15 = 1.05V_j^2/(2g) + 23(D_j/D)^4(V_j^2/2g)$; $V_j^2/(2g) = 15/[1.05 + 23(D_j/D)^4]$

Eq. 5.41: $kW = \gamma Q(V_j^2/2g)/1000 = \gamma A_j V_j(V_j^2/2g)/1000 = 0.5A_j V_j^3$

D_j (mm)	A_j (m²)	$V_j^2/2g$ (m)	V_j (m/s)	kW
0	0	--	--	--
20	0.000 314	14.25	16.72	0.734
40	0.001 257	13.80	16.46	2.80
60	0.002 83	12.13	15.43	5.20
80	0.005 03	9.15	13.40	6.05
100	0.007 85	6.03	10.88	5.05
120	0.011 31	3.72	8.54	3.53
140	0.015 39	2.28	6.69	2.31
160	0.0201	1.433	5.30	1.498

Maximum power occurs with a jet diameter of approximately 80 mm. ◀

(An exact answer may be obtained by using differential calculus.)

8.67 *Water at 60°F flows through 800 ft of 12-in-diameter pipe between two reservoirs whose water-surface elevation difference is 16 ft. The pipe entrance is flush and square-edged, and there is a half-open gate valve in the line. Using only a basic scientific calculator, find the flow rate (a) if e = 0.0018 in, and (b) if e is twenty times larger.*

A basic scientific calculator is here defined to be one that is not programmable and does not have automatic equation solving capabilities.

BG

$L/D = 800 < 1000$, so minor losses are not negligible.

Energy: $\Delta z = 16\text{ ft} = \Sigma h_L = [0.5 + 2.06 + 1.0 + f(800/1)]V^2/2g$

from which $V = [32(32.2)/(800f + 3.56)]^{1/2}$ (1)

Table A.1 for water at 60°F: $\nu = 1.217 \times 10^{-5}\text{ ft}^2/\text{sec}$

Eq. 8.1: $\mathbf{R} = DV/\nu = (1)V/(1.217 \times 10^{-5}) = 8.22 \times 10^4 V$ (2)

(*a*) $e/D = 0.0018/12 = 0.000\,15$

Solve by trial satisfying (1), (2), and Fig. 8.11 or Eq. 8.52:

Try f	V from (1)	**R** from (2)	Obtained f	
0.03	6.11 fps	5.02×10^5	0.0149	Try again
0.0149	8.15	6.70×10^5	0.0145	Close enough
0.0145	8.23			

$Q = (\pi/4)(1^2)8.23 = 6.47\text{ cfs}$ ◀

(*b*) $e = 20(0.0018) = 0.036\text{ in}$; $e/D = 0.036/12 = 0.003$

Try f	V from (1)	**R** from (2)	Obtained f	
0.03	6.11 fps	5.02×10^5	0.0264	Try again
0.0264	6.46	5.31×10^5	0.0264	Converged.

$Q = (\pi/4)(1^2)6.46 = 5.07\text{ cfs}$ ◀

Note: Increasing e by 20 times reduces Q by only 22%.

Check minor losses: Minor losses / friction losses = $3.56/(fL/D)$

(*a*) $3.56/(0.0145 \times 800/1) = 0.307 = 30.7\%$

(*b*) $3.56/(0.0264 \times 800/1) = 0.1686 = 16.9\%$

These minor losses are large fractions of the friction loss, so may not be neglected.

8.68 *Solve Prob. 8.67 without manual trial and error, by using an equation solver (i) on a programmable calculator or (ii) in computer software.*

Prob. 8.67: 60°F water flows through a pipeline, L = 800 ft, D = 12 in, Σk = 3.56, h_L = 16 ft. Find Q (a) if e = 0.0018 in, and (b) if e is 20 times larger.

BG

$L/D = 800 < 1000$, so minor losses are not negligible. $\Sigma k = 0.5 + 2.06 + 1.0 = 3.56$

Table A.1 for water at 60°F: $\nu = 1.217 \times 10^{-5}$ ft²/sec

Energy: $h_L = [\Sigma k + (fL/D)]V^2/2g$, i.e. $16 = [3.56 + (800/1)f]V^2/(2 \times 32.2)$ (1)

Continuity: $Q = AV = (\pi/4)D^2V$, i.e. $Q = (\pi/4)(1^2)V$ (2)

Eq. 8.1: $\mathbf{R} = DV/\nu$, i.e. $\mathbf{R} = (1)V/(1.217 \times 10^{-5})$ (3)

Eq. 8.51: $1/\sqrt{f} = -2\log[(e/D)/3.7 + 2.51/(\mathbf{R}\sqrt{f})]$ (4)

(*a*) $e/D = 0.0018/12 = 0.000\,15$; Substitute this into Eq. (4)

(*i*) Use an equation solver on a programmable calculator to solve Eq. 8.81 for Q.

Then find V, $\mathbf{R}$, and f from (2), (3) and Eq. 8.52 respectively.

(*ii*) Use an equation solver in computer software to solve Eqs. (1)–(4) simultaneously for Q, V, $\mathbf{R}$, f.

Results: $Q = 6.47$ cfs ◄, $V = 8.24$ fps, $\mathbf{R} = 676{,}757$, $f = 0.014\,54$.

(*b*) $e/D = 20(0.0018/12) = 0.003$; Substitute this into Eq. (4). Repeat (*i*) and (*ii*) as for (*a*).

Results: $Q = 5.07$ cfs ◄, $V = 6.46$ fps, $\mathbf{R} = 530{,}709$, $f = 0.0264$.

Note: Increasing e by 20 times reduces Q by only 22%.

Check minor losses: Minor losses / friction losses $= 3.56/(fL/D)$

(*a*) $3.56/(0.01454 \times 800/1) = 0.306 = 30.6\%$

(*b*) $3.56/(0.0264 \times 800/1) = 0.1686 = 16.9\%$

These minor losses are large fractions of the friction loss, so may not be neglected.

8.69 *A 150-m-long commercial steel pipe is to convey 30 L/s of oil (s = 0.9, μ = 0.038 N·s/m²) from one tank to another (submerged discharge) where the difference in elevation of the free liquid surfaces is 2 m (Fig. P8.69). The pipe entrance is flush, and there is a fully-open gate valve in the line. Using only a basic scientific calculator, find the diameter theoretically required. (Hint: It may be easier to try for D than for f in this case.)*

A basic scientific calculator is here defined to be one that is not programmable and does not have automatic equation solving capabilities.

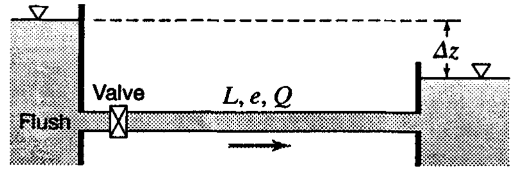


Figure P8.69

SI

Eq. 2.11: $\nu = \mu/\rho = 0.038/(1000 \times 0.9) = 0.000\,0422$ m²/s

Eq. 8.80: $h_L = (fL/D + \Sigma k)V^2/(2g) = (fL/D + \Sigma k)[4Q/(\pi D^2)]^2/2g$

from which $f = [2gh_L(\pi D^2/4Q)^2 - \Sigma k](D/L)$ (1)

Eq. 8.1: $\mathbf{R} = DV/\nu = (D/\nu)[4Q/(\pi D^2)] = 4Q/(\pi \nu D)$ (2)

Sec. 8.21 for flush entrance: $k_e = 0.5$; Sec. 8.22 for submerged discharge: $k_d = 1.0$

Table 8.3 for fully open gate valve: $k_v = 0.19$. ∴ $\Sigma k = 0.5 + 1.0 + 0.19 = 1.69$

Substituting this and the given values into (1) and (2) we get $f = 179.3D^5 - D/88.8$ (3); $\mathbf{R} = 905/D$ (4)

From Table 8.1 for commercial steel pipe: $e = 0.000\,046$ m

If we try values of f, we must solve (3) by T & E. ∴, it is easier in this case to try values of D (than f); but we must check for f_{min}.

For a first approx: Neglect minor losses Σk. Then $179.3D^5 = f$

Assuming a mid-range value of $f = 0.03$: $D = 0.1757$ m

With minor losses, from (3) we expect the required D to be a little larger.

Try D (m)	e/D	f_{min} (Eq.8.54)	f (Eq. 3)	$\mathbf{R}$ (Eq. 4)	f (Fig. 8.11 or Eq. 8.52)
0.18	0.000 255	0.014 44	0.0318	5028	0.0379
0.185	0.000 249	0.014 36	0.0368	4892	0.0382
0.186	0.000 247	0.014 34	0.0378	4866	0.0382 Close enough

Eq. 8.52 is valid because $\mathbf{R} > 2000$.

$L/D = 150/0.186 = 806 < 1000$, so minor losses <u>must</u> be included. $D = 0.186$ m ◀

8.70 *Solve Prob. 8.69 without manual trial and error, by using an equation solver (a) on a programmable calculator or (b) in computer software.*

Prob. 8.69: Oil (s = 0.9, μ = 0.038 N·s/m^2) is to flow at 30 L/s through a commercial steel pipe, L = 150 m, Σk = 1.69 with h_L = 2 m. Find the D theoretically required.

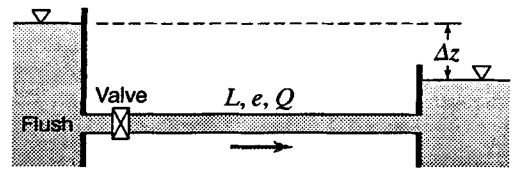


Figure P8.69

SI

$\Sigma k = 0.5 + 0.19 + 1.0 = 1.69$

Eq. 2.11: $\nu = \mu/\rho = 0.038/(1000 \times 0.9) = 0.000\,0422$ m^2/s

Table 8.1 for comercial steel: e = 0.046 mm = 0.000 046 m

Knowns: L = 150 m, g = 9.81 m^2/s, h_L = 2 m, Q = 0.03 m^3/s, Σk = 1.69, e = 0.000 046 m, ν = 0.0000422 m^2/s

Find: D, V, **R**, f.

Energy: $h_L = [\Sigma k + (fL/D)]V^2/(2g) = 2 = [1.69 + f(150/D)]V^2/(2 \times 9.81)$ (1)

Continuity: $Q = AV = (\pi/4)D^2V = 0.03$ (2)

Eq. 8.1: $\mathbf{R} = DV/\nu = DV/0.000\,0422$ (3)

Eq. 8.51: $1/\sqrt{f} = -2\log[0.000\,046/(3.7D) + 2.51/(\mathbf{R}\sqrt{f})]$ (4)

(*a*) Use an equation solver on a programmable calculator to solve Eq. 8.81 for D.

Then find V, **R**, and f from (2), (3) and Eq. 8.52 respectively.

(*b*) Use an equation solver in computer software to solve Eqs. (1)-(4) simultaneously for D, V, **R**, f.

Results: D = 0.1861 m ◄ V = 1.102 m/s, **R** = 4863, f = 0.0380.

Eqs. 8.51, 8.52, and 8.81 are valid because **R** > 2000.

L/D = 150/0.1861 = 806 < 1000, so minor losses <u>must</u> be included.

Sec. 8.28: Pipeline with Pump or Turbine – *Exercises (10)*

8.28.1 *An 80-in-diameter pipe (f = 0.025) 7252 ft long delivers water to a powerhouse at a point 1500 ft lower in elevation than the water surface at intake. When the flow is 450 cfs, what is the horsepower delivered to the plant?*

BG

$A = (\pi/4)(80/12)^2 = 34.9$ ft^2; $V = 450/34.9 = 12.89$ fps; $V^2/(2g) = 2.58$ ft

$h_L = \left[0.5 + 0.025\dfrac{7252}{80/12}\right]2.58 = 71.5$ ft. Head delivered to plant, $h = \Delta z - h_L = 1500 - 71.5 = 1429$ ft

Eq. 5.40: Power delivered = 62.4(450)1429/550 = 72,900 hp ◄

8.28.2 *A 10-in-diameter pipeline (f = 0.020) is 3 miles long (Fig. X8.28.2). when pumping 4 cfs of water through it, with a total actual lift of 25 ft, how much power is required? The pump efficiency is 72 percent.*

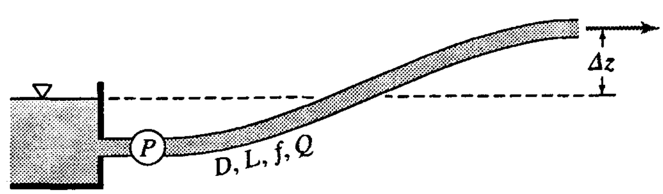


Figure X8.28.2

BG

$V = Q/A = 4/[\pi(5/12)^2] = 7.33$ fps

$V^2/(2g) = 0.835$ ft

Eq. 8.13: $h_f = 0.020\dfrac{3(5280)}{10/12}0.835 = 318$ ft; $h_p = 25 + 318 = 343$ ft

Using Eq. 5.40: Power required = 62.4(4)343/(550 × 0.72) = 216 hp ◄

8.28.3 *A 250-mm pipeline (f = 0.025) is 4.7 km long (Fig. X8.28.2). When pumping 100 L/s of water through it, with a total actual lift of 10.5 m, how much power is required? The pump efficiency is 75 percent.*

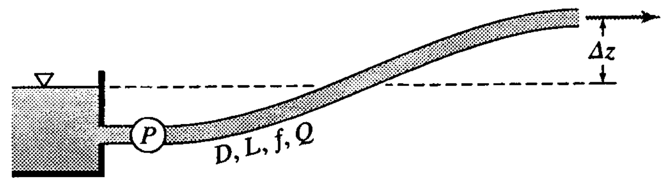


Figure X8.28.2

SI

$V = Q/A = 0.100/(\pi 0.125^2) = 2.04$ m/s

$V^2/(2g) = 0.212$ m

Eq. 8.13: $h_f = 0.025(3)(4700/0.25)0.212 = 99.4$ m; $h_p = 10.5 + 99.4 = 109.9$ m

Using Eq. 5.41: Power required = 9810(0.1)109.9/(1000 × 0.75) = 143.8 kW ◄

8.28.4 *If in Sample Problem 8.13 the vapor pressure of the liquid is 1.9 psia and the atmospheric pressure is 14.5 psia, what is the maximum theoretical flow rate?*

Sample Prob. 8.13: The tanks, pump, and pipelines have the characteristics indicated in Fig. S8.13. The suction line entrance is flush, and the pump adds 2.0 hp to the liquid.

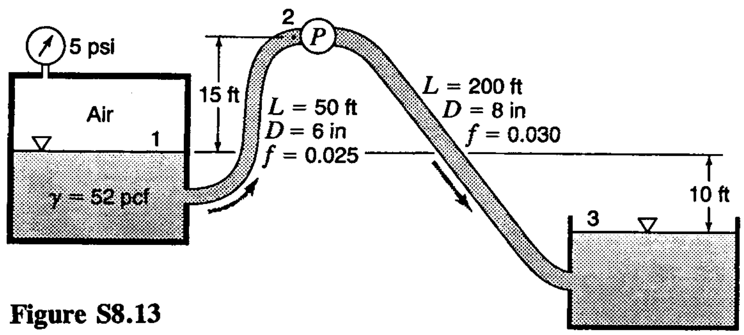


Figure S8.13

BG

From Eq. 5.43: $p_c/\gamma = -(p_{at}/\gamma - p_v/\gamma) = -(144/52)(14.5 - 1.9) = -34.9$ ft

Energy, tank to pump inlet: $\left(\frac{5(144)}{52} + 0 + 0\right) - 0.5\frac{V_6^2}{2g} - 0.025\left(\frac{50}{6/12}\right)\frac{V_6^2}{2g} = \left(-34.9 + 15 + \frac{V_6^2}{2g}\right)$

$V_6^2/(2g) = 8.43$ ft, $V_6 = 23.3$ fps; $Q_{max} = (0.1964)23.3 = 4.58$ cfs ◄

8.28.5 *In Fig. 8.26 assume the pipe diameter is 12 in, f = 0.021, BC = 200 ft, and Δz = 120 ft. The entrance to the pipe at the intake is flush with the wall, and discharge losses are negligible. (a) If Q = 8 cfs of water, what head does it supply to the turbine? (b) What power does the turbine deliver if its efficiency is 75 per cent?*

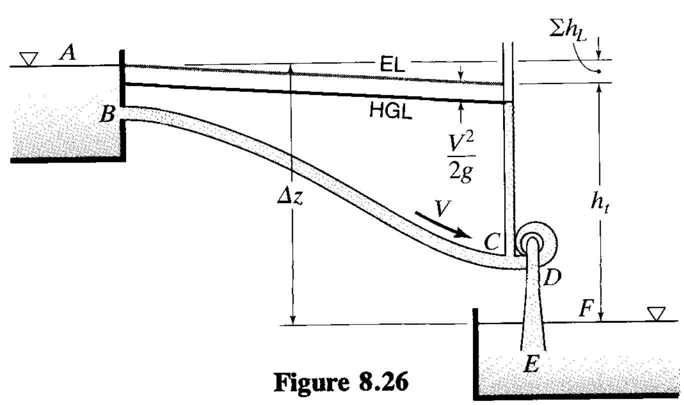


Figure 8.26

BG

(*a*) $V = Q/A = 8/0.7854 = 10.19$ fps

$V^2/2g = 1.611$ ft

Fig. 8.13: $k_e = 0.5$

$\therefore\ h_L = [0.5 + 0.021(200/1)]1.611 = 7.57$ ft ; Eq. 8.85: $h_t = 120 - 7.57 = 112.4$ ft ◄

(*b*) Eq. 5.40: Power delivered by turbine = (0.75)62.4(8)112.4/550 = 76.5 hp ◄

8.28.6 *In Fig. 8.26 assume the pipe diameter is 300 mm, $f = 0.021$, $BC = 60$ m, and $\Delta z = 36.5$ m. The entrance to the pipe at the intake is flush with the wall, and discharge losses are negligible. (a) If $Q = 225$ L/s of water, what head does it supply to the turbine? (b) What power does the turbine deliver if its efficiency is 75 per cent?*

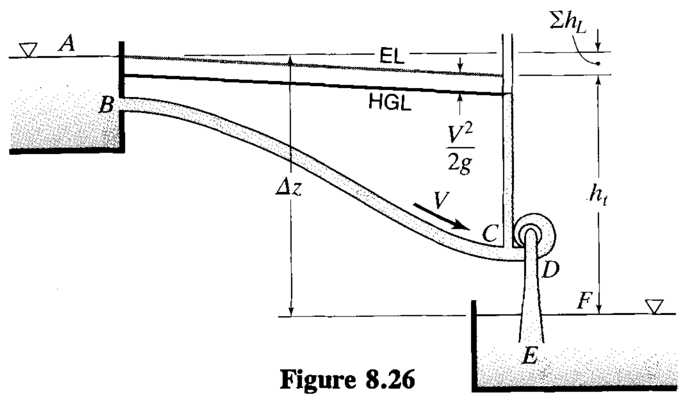


Figure 8.26

SI

(a) $V = \frac{Q}{A} = \frac{0.225}{\pi 0.15^2} = 3.18$ m/s

$\frac{V^2}{2g} = 0.516$ m

Fig. 8.13: $k_e = 0.5$, $\therefore$ $h_L = [0.5 + 0.021(60/0.3)]0.516 = 2.43$ m

Eq. 8.85: $h_t = 36.5 - 2.43 = 34.1$ m ◄

(b) Eq. 5.41: Power delivered by turbine $= (0.75)9810(0.225)34.1/1000 = 56.4$ kW ◄

8.28.7 *A 12-in-diameter pipe 9200 ft long ($f = 0.024$) discharges freely into the air at an elevation 18 ft below the surface of the water at intake (Fig. X8.28.7). It is necessary to double the flow by inserting a pump. If the efficiency of the pump is 73 per cent, how much power will be required?*

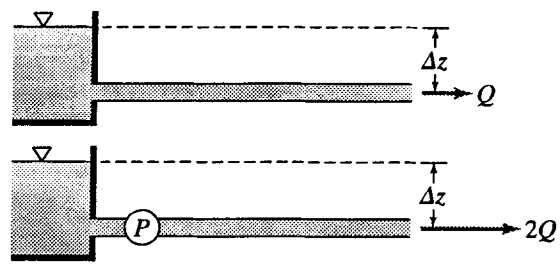


Figure X8.28.7

BG

Energy: $18 = 0.024(9200/1)V^2/(2g) + V^2/(2g)$

$V^2/(2g) = 0.0812$ ft. $V = \sqrt{2(32.2)0.0812} = 2.29$ fps

$Q_1 = AV = (\pi 1^2/4)2.29 = 1.796$ cfs

If the flow is doubled, $Q_2 = 2(1.796) = 3.59$ cfs

In the above $h_L = 18$ ft, and for double the flow (constant f), $h_L = 2^2(18) = 72$ ft

However gravity supplies 18 ft and so the pump must provide $h_p = 72 - 18 = 54$ ft

Eq. 5.40: Power required $= 62.4(3.59)54/(550 \times 0.73) = 30.1$ hp ◄

8.28.8 *A 300-mm-diameter pipe 3400 m long ($f = 0.022$) discharges freely into the air at an elevation 5.6 m below the surface of the water at intake (Fig. X8.28.7). It is necessary to double the flow by inserting a pump. If the efficiency of the pump is 76 per cent, how much power will be required?*

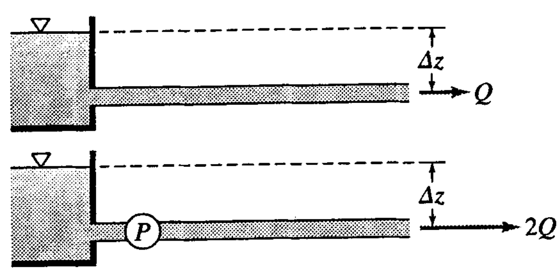


Figure X8.28.7

SI

Energy: $5.6 = 0.022(3400/0.3)V^2/(2g) + V^2/(2g)$

$V^2/(2g) = 0.0224$ m. $V = \sqrt{2(9.81)0.0224} = 0.662$ m/s

$Q_1 = AV = (\pi 0.15^2)0.662 = 0.0468$ m³/s

If the flow is doubled, $Q_2 = 2(0.0468) = 0.0937$ m³/s

In the above $h_L = 5.6$ m, and for double the flow (constant f), $h_L = 2^2(5.6) = 22.4$ m

However gravity supplies 5.6 m and so the pump must provide $h_p = 22.4 - 5.6 = 16.8$ m

Eq. 5.41: Power required $= 9810(0.0937)16.8/(1000 \times 0.76) = 20.3$ kW ◄

8.28.9 *Refer to Fig. 8.25. When the pump is delivering 1.2 cfs of water, a pressure gage at D reads 25 psi, while a vacuum gage at C reads 10 inHg. The pressure gage at D is 2 ft higher than the vacuum gage at C. The pipe diameters are 4 in for the suction pipe and 3 in for the discharge pipe. Find the power delivered to the water.*

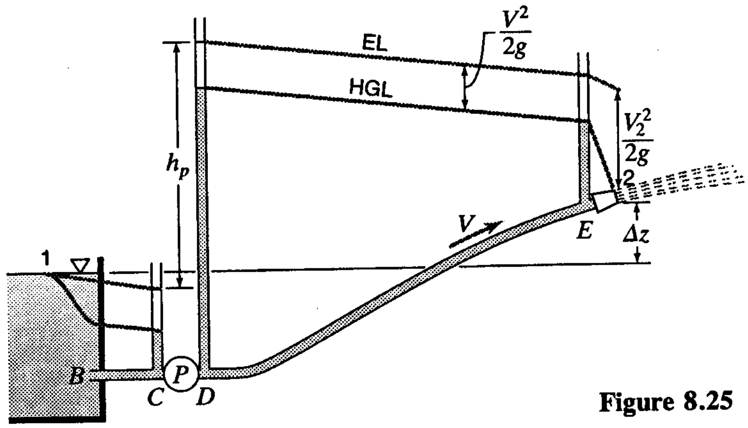


Figure 8.25

BG

$V_C = Q/A = 1.2/0.0873 = 13.75$ fps

$V_C^2/(2g) = 2.94$ ft

(25 psi)144/62.4 = 57.7 ft water

$(p/\gamma)_D = 57.7 + d + 2$

$V_D = Q/A = 1.2/0.491 = 24.4$ fps; $V_D^2/(2g) = 9.28$ ft.

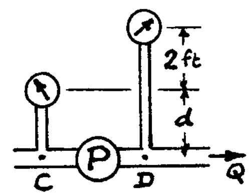


Table A.4: $s_{Hg} = 13.56$; 10 inHg vac ≡ (−10/12)13.56 = −11.3 ft water

$(p/\gamma)_C = -11.3 + d$. $\quad (z_C = z_D = z)$

$h_p = H_D - H_C = (57.7 + d + 2 + z + 9.28) - (-11.3 + d + z + 2.94) = 77.3$ ft

Eq. 5.40: Power delivered = 62.4(1.2)77.3/550 = 10.53 hp ◄

8.28.10 *Refer to Fig. 8.25. When the pump is delivering 35 L/s of water, a pressure gage at D reads 175 kPa, while a vacuum gage at C reads 250 mmHg. The pressure gage at D is 600 mm higher than the vacuum gage at C. The pipe diameters are 100 mm for the suction pipe and 75 mm for the discharge pipe. Find the power delivered to the water.*

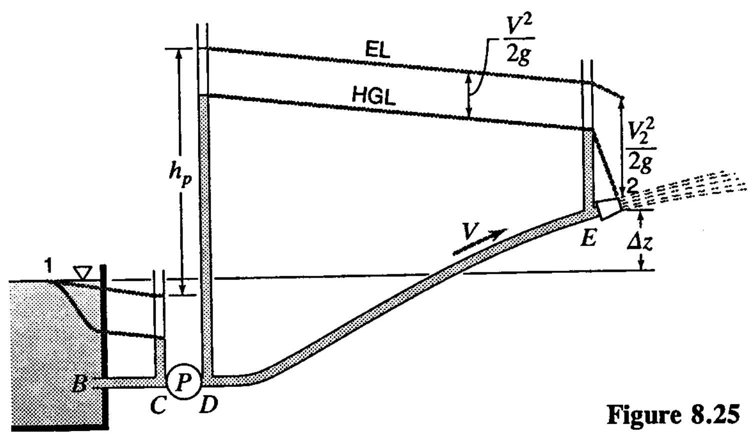


Figure 8.25

SI

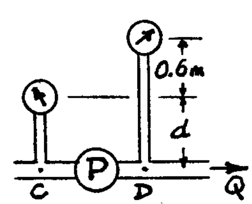
$$V_C = \frac{Q}{A} = \frac{4(0.035)}{\pi 0.10^2} = 4.46 \text{ m/s}$$

$V_C^2/(2g) = 1.012$ m

$V_D = Q/A = 4(0.035)/(\pi 0.075^2) = 7.92$ m/s; $V_D^2/(2g) = 3.20$ m

175 kN/m² ≡ 17.84 m water; 250 mmHg ≡ 3.39 m water. $(z_C = z_D = z)$

$h_p = H_D - H_C = (17.84 + d + 0.60 + z + 3.20) - (-3.39 + d + z + 1.012) = 24.0$ m



Eq. 5.41: Power delivered = 9810(0.035)24.0/1000 = 8.25 kW ◄

Sec. 8.28: Pipeline with Pump or Turbine -- *Problems 8.71–8.78*

8.71 *In Fig. 8.24 assume a pipe diameter of 350 mm, f = 0.016, BC = 12 m, DE = 920 m, and Δz = 48 m. Find the maximum theoretical flow rate if 15°C water is being pumped at an altitude of 1000 m above sea level. Point C is 6.0 m above the lower water surface.*

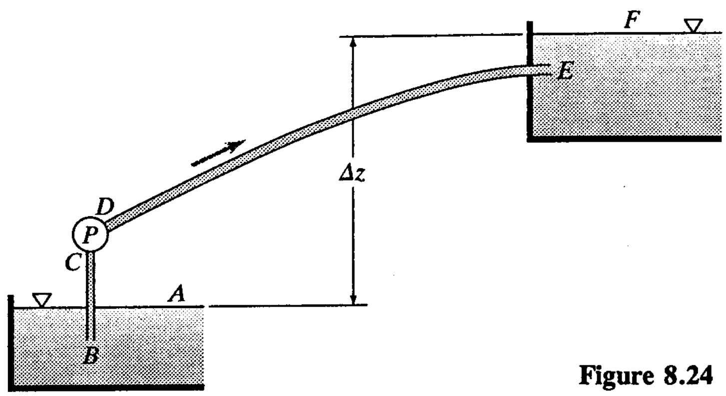


Figure 8.24

SI

Table A.1, 15°C H_2O: $\dfrac{p_v}{\gamma}$ = 0.17 m abs

Table A.3 for 1000 m elevation:

p = 89.876 kPa abs;

$$\frac{p_{atm}}{\gamma} = \frac{89.9}{9.81} = 9.16 \text{ m of water}$$

Energy equation from water surface A to pump intake C using absolute pressure heads, when $p_C = p_v$:

$$9.16 - 0.8(V^2/2g) - 0.016(12/0.35)V^2/2g = 0.17 + 6.0 + V^2/2g$$

$$2.99 = (V^2/2g)(0.8 + 0.549 + 1) = 2.35(V^2/2g)$$

$V^2/2g$ = 2.99/2.35 = 1.274; V = 5.00 m/s, Q_{max} = 0.481 m³/s ◄

8.72 *In Fig. 8.24 assume that the pipe diameter is 4 in, f = 0.035, BC = 15 ft, DE = 180 ft, and Δz = 60 ft. The elevation of C is 10 ft above the lower water surface. (a) If the pressure head at C is to be no less than −20 ft, what is the maximum rate at which the water can be pumped? (b) If the efficiency of the pump is 65 percent, what is the horsepower required?*

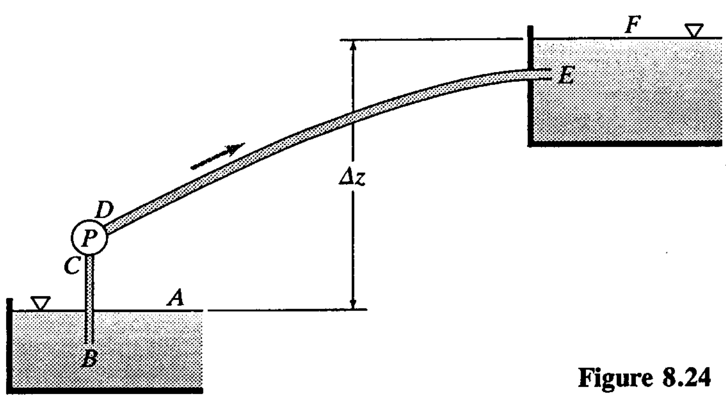


Figure 8.24

BG

$$h_L = [0.8 + 0.035(15)12/4]V^2/2g$$
$$= (0.8 + 1.575)V^2/2g = 2.38V^2/2g$$

Energy from A to C: $(0 + 0 + 0) = (-20 + 10 + V^2/2g) + 2.38V^2/2g$

$V^2/2g$ = 10/3.38 = 2.96 ft; $V = [2(32.2)2.96]^{1/2}$ = 13.81 fps

(*a*) $Q_{max} = AV$ = 0.0491(13.81) = 1.205 cfs

(*b*) h_p = 60 + [0.8 + 1.575 + 0.035(180)12/4]2.96 = 123.0 fps

Eq. 5.40: hp delivered to the water = 62.4(1.205)123.0/500 = 16.83 ◄

From Eq. 5.42: hp delivered to the pump (excluding motor) = 16.83/0.65 = 25.9 ◄

8.73 *In a testing laboratory, a certain turbine has been found to discharge 10 cfs under a head h_t of 45 ft. In the field, it is to be installed near the end of a pipe 360 ft long (Fig. P8.73). The supply line (flush entrance) and discharge line (submerged exit) will both have diameters of 10 inches with f = 0.025. The total fall from the surface of the water at intake to the surface of the tailwater will be 52 ft. What will be the head on the turbine, the rate of discharge, and the power delivered to the flow? Note that for turbines, $Q \propto \sqrt{h_t}$ (from Eq. 16.17).*

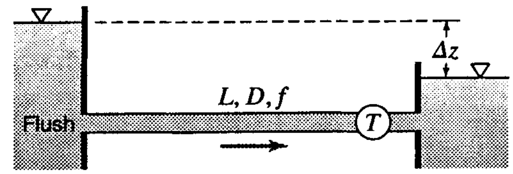


Figure P8.73

BG

$V = Q/A = Q/[(\pi/4)(10/12)^2] = 1.833Q$

$h_L = [0.5 + 0.025(360)(12/10) + 1]V^2/2g = 12.30V^2/2g$

Substituting for V: $h_L = 12.30(1.833Q)^2/(2 \times 32.2) = 0.642Q^2$

Thus, in Eq. 8.85: $h_t = 52 - 0.642Q^2$

Also, from Eq. 16.17: $Q_1/Q = 10/Q = [45/(52 - 0.642Q^2)]^{1/2}$, i.e. $Q^2/100 = (52 - 0.642Q^2)/45$

from which $Q = 6.90$ cfs ◄ so that $h_t = 52 - 0.642(6.90)^2 = 21.4$ ft ◄

Eq. 5.40: Power = 62.4(6.90)21.4/550 = 16.78 HP ◄

<u>Alternative solution method</u>:

Lab: $Q = K\sqrt{h_t}$; $10 = K\sqrt{45}$ from which $K = 1.491$

Field: $Q = K\sqrt{h_t} = 1.491\sqrt{\Delta z - h_L} = 1.491\sqrt{52 - 0.642Q^2}$; solve for Q

8.74 *In a testing laboratory, a certain turbine has been found to discharge 285 L/s under a head h_t of 13.5 m. In the field, it is to be installed near the end of a pipe 110 m long (Fig. P8.73). The supply line (flush entrance) and discharge line (submerged exit) will both have diameters of 250 mm with f = 0.024. The total fall from the surface of the water at intake to the surface of the tailwater will be 15.5 m. What will be the head on the turbine, the rate of discharge, and the power delivered to the flow? Note that for turbines, $Q \propto \sqrt{h_t}$ (from Eq.16.17).*

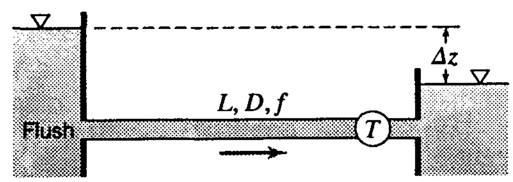


Figure P8.73

SI

$V = Q/A = 4Q/(\pi D^2) = 4Q/(\pi 0.25^2) = 20.4Q$

$h_L = [0.5 + 0.024(110/0.25) + 1]V^2/2g = 12.06V^2/2g$

Substituting for V: $h_L = 12.06(20.4Q)^2/(2 \times 9.81) = 255Q^2$

Thus, in Eq. 8.85: $h_t = 15.5 - 255Q^2$

Also, from Eq. 16.17: $Q_1/Q = 0.285/Q = [13.5/(15.5 - 255Q^2)]^{1/2}$

i.e. $Q^2/0.285^2 = (15.5 - 255Q^2)/13.5$ from which $Q = 0.1918\ m^3/s = 191.8$ L/s ◄

so that $h_t = 15.5 - 255(0.1918)^2 = 6.11$ m ◄

Eq. 5.41: Power = 9810(0.1918)6.11/1000 = 11.51 kW ◄

<u>Alternative solution method</u>:

Lab: $Q = K\sqrt{h_t}$; $0.285 = K\sqrt{13.5}$ from which $K = 0.0776$

Field: $Q = K\sqrt{h_t} = 0.0776\sqrt{\Delta z - h_L} = 0.0776\sqrt{15.1 - 255Q^2}$; solve for Q

8.75 *Assume the total fall from the surface of one body of water to another is 130 ft. The water is conveyed by 250 ft of 15-in pipe (f = 0.020) which has its entrance flush with the wall (Fig. P8.75). At the end of the pipe is a turbine and draft tube which discharged 6 cfs of water when tested under a head of 63.1 ft in another location. Discharge losses are negligible. What would be the rate of discharge through the turbine and the head on it under the present conditions? Note that for turbines, $Q \propto \sqrt{h_t}$ (from Eq. 16.17).*

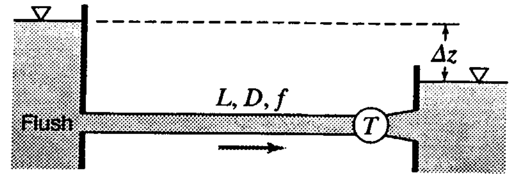


Figure P8.75

BG

$V = Q/A = Q/[(\pi/4)(15/12)^2] = 0.815Q$; $h_L = [0.5 + 0.020(250)(12/15) + 0]V^2/2g = 4.50V^2/2g$

Substituting for V: $h_L = 4.50(0.815Q)^2/(2 \times 32.2) = 0.0464Q^2$; $\therefore$ Eq. 8.85: $h_t = 130 - 0.0464Q^2$

Also, from Eq. 16.17: $Q_1/Q = 6/Q = [63.1/(130 - 0.0464Q^2)]^{1/2}$

i.e. $Q^2/36 = (130 - 0.0464Q^2)/63.1$ from which $Q = 8.50$ cfs ◄

so that $h_t = 130 - 0.0464(8.50)^2 = 126.6$ ft ◄

<u>Alternative solution method</u>: Test: $Q = K\sqrt{h_t}$; $6 = K\sqrt{63.1}$ from which $K = 0.755$

Present: $Q = K\sqrt{h_t} = 0.755\sqrt{\Delta z - h_L} = 0.755\sqrt{130 - 0.0464Q^2}$; solve for Q

8.76 *A pump is installed to deliver water from a reservoir of surface elevation zero to another of elevation 200 ft (Fig. P8.76). The 12-in-diameter suction pipe (f = 0.020) is 40 ft long, and the 10-in-diameter discharge pipe (f = 0.032) is 4500 ft long. The pump head may be defined as $h_p = 300 - 20Q^2$, where the pump head h_p is in feet and Q is in cubic feet per second. Compute the rate at which this pump will deliver the water. Also, what is the horsepower input to the water?*

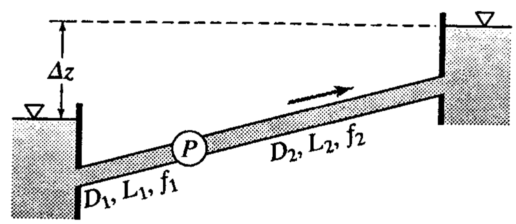


Figure P8.76

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Continuity: $V_{10} = (12/10)^2V_{12} = 1.440V_{12}$ and $Q = (\pi/4)1^2V_{12}$

Eq. 8.63: $h_p = 200 + 0.02(40/1)V_{12}^2/2g + 0.032(4500)/(10/12)V_{10}^2/2g$

$= 200 + 0.8V_{12}^2/2g + 172.8(1.440V_{12})^2/2g = 200 + 359V_{12}^2/2g = 200 + 9.04Q^2$

But also $h_p = 300 - 20Q^2$ (given). Equating these, $300 - 20Q^2 = 200 + 9.04Q^2$; $Q = 1.856$ cfs ◄

$h_p = 300 - 20(1.856)^2 = 231$ ft. Eq. 5.40: Power input $= 62.4(1.856)231/550 = 48.7$ hp ◄

Note: Problems 8.108 – 110 also involve pumps.

8.77 *Refer to Fig. 8.25. Suppose that the water-surface elevation, elevation of the pump, and elevation of the nozzle tip are 100, 90, and 120 ft, respectively. Pipe BC is 40 ft long, has a diameter of 8 in, with f = 0.025; pipe DE is 200 ft long, has a diameter of 10 in, with f = 0.030; the jet diameter is 6 in, and the nozzle loss coefficient is 0.04. Assume the pump is 80 percent efficient under all conditions of operation. Make a plot of flow rate and p_C/γ versus pump horsepower input. At what flow rate will cavitation occur in the pipe at C if the water temperature is 50°F and the atmospheric pressure is 13.9 psia?*

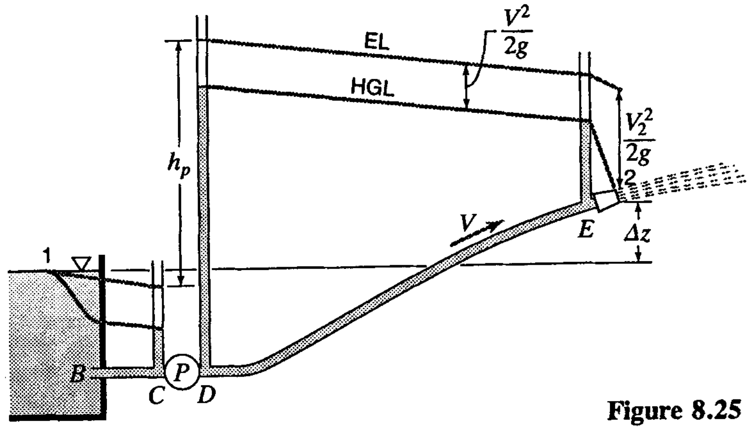

Figure 8.25

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Table A.1 for water at 50°F: p_v/γ = 0.41 ft

Cavitation begins (Eq. 5.43) when $p_c/\gamma = -(p_{at}/\gamma - p_v/\gamma) = -[33.9(13.9/14.7) - 0.41] = -31.6$ ft

Continuity: $V_8 = (6/8)^2V_6 = 0.563V_6$; $V_{10} = (6/10)^2V_6 = 0.360V_6$

Energy equation from 1 to 2:

$$100 - \left[0.8 + 0.025\frac{40}{8/12}\right]\frac{(0.563V_6)^2}{2g} + h_p - 0.030\frac{200}{10/12}\frac{(0.360V_6)^2}{2g} - 0.04\frac{V_6^2}{2g} = 120 + \frac{V_6^2}{2g}$$

i.e., $h_p = 20 + 2.70(V_6^2/2g)$. But $V_6 = Q/0.1963 = 5.09Q$; $V_6^2 = 25.9Q^2$; so $h_p = 20 + 1.088Q^2$

Eq. 5.40: $HP_{input} = \gamma Q h_p/[550(0.8)] = 0.1418Q(20 + 1.088Q^2) = 2.84Q + 0.1543Q^3$

Energy Eq. from 1 to C: $100 - [0.8 + 0.025(40)/(8/12)](V_8^2/2g) = 90 + p_C/\gamma + (V_8^2/2g)$

i.e., $p_C/\gamma = 10 - 3.3(V_8^2/2g)$. But $V_8 = Q/0.349 = 2.86Q$; $V_8^2 = 8.21Q^2$; so $p_C/\gamma = 10 - 0.421Q^2$

Let $p_C/\gamma = -31.6 = 10 - 0.421Q^2$; Q = 9.95 cfs when cavitation begins ◄

For plot:

Q (cfs)	h_p (ft)	HP_{input}	p_C/γ (ft)
0	20	0	10.0
2	24.4	5.91	8.32
4	37.4	21.2	3.27
6	59.2	50.3	−5.14
8	89.6	101.7	−16.91
10	128.8	182.6	−32.1
▲		▲	▲

8.78 *In Fig. 8.24 assume the pipe diameter is 9 in, $f = 0.025$, $BC = 20$ ft, $DE = 2800$ ft, and $\Delta z = 125$ ft. Water is pumped at 5.5 cfs, the pump efficiency is 78 per cent. (a) What horsepower is required? (b) If the elevation of C above the lower water surface is 12 ft, that of D is 15 ft, and that of E is 100 ft, compute the pressure heads at B, C, D, and E. (c) Sketch the energy line and the hydraulic grade line.*

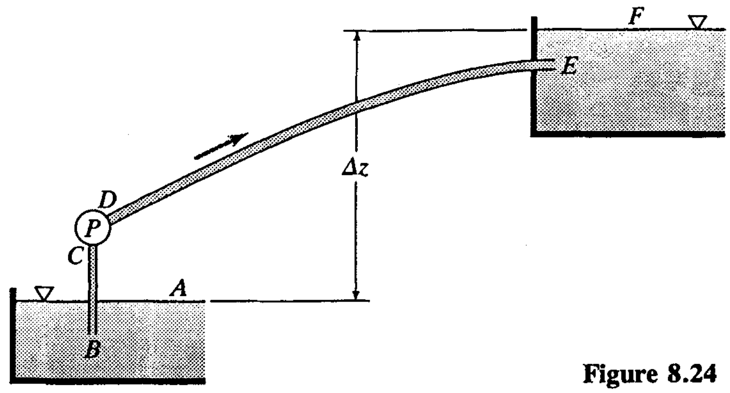

Figure 8.24

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(*a*) $V = \dfrac{Q}{A} = \dfrac{5.5}{0.442} = 12.45$ fps

$V^2/(2g) = 2.41$ ft

Suction pipe, $h_L = [0.8 + 0.025(20)12/9]2.41 = 3.53$ ft

Discharge pipe, $h_L = [0.025(2800)12/9 + 1]2.41 = 227$ ft

Total $h_L = 3.53 + 227 = 231$ ft; Pump head, $h_p = 125 + 231 = 356$ ft

Eq. 5.40: hp delivered to the water = 62.4(5.5)356/550 = 222 hp

From Eq. 5.42: hp delivered to the pump (excluding motor) = 222/0.78 = 284 hp ◄

(*b*) Eq. 5.35: $H_A = 0 + 0 + 0 = 0$; $H_B = p_B/\gamma - 8 + 2.41$; $H_C = p_C/\gamma + 12 + 2.41$

$H_D = p_D/\gamma + 15 + 2.41$; $H_E = p_E/\gamma + 100 + 2.41$; $H_F = 0 + 125 + 0$

Energy, *AB*: $H_A - H_B = 0 - (p_B/\gamma - 5.59) = 0$; $\therefore\ p_B/\gamma = 5.59$ ft ◄

Energy, *AC*: $H_A - H_C = 0 - (p_C/\gamma + 12 + 2.41) = 3.53$; $\therefore\ p_C/\gamma = -17.94$ ft ◄

Energy, *DF*: $H_D - H_F = (p_D/\gamma + 17.41) - 125 = 227$ ft; $\therefore\ p_D/\gamma = 335$ ft ◄

Energy, *EF*: $H_E - H_F = (p_E/\gamma + 102.41) - 125 = 2.41$ or $p_E/\gamma = 25.0$ ft ◄ (either inside pipe or just outside.)

(*c*) For sketch: Positions of lines relative to *A* (datum, assumed at elevation 0 ft):

Point:	*A*	*B*	*C*	*D*	*E*	*F*	
p/γ, ft	0	5.59	−17.94	334.62	25.00	0	
z, ft	0	−8.00	12.00	15.0	100.00	125.00	
$V^2/(2g)$, ft	0	2.41	2.41	2.41	2.41	0	
HGL, ft	0	−2.41	−5.94	349.62	125.00	125.00	◄◄
EL, ft	0	0	−3.53	352.03	127.41	125.00	◄◄

Sec. 8.29: Branching Pipes – *Problems* 8.79–8.91

8.79 *In Fig. 8.27, suppose that pipe 1 is 36-in smooth concrete, 5000 ft long; pipe 2 is 24-in cast iron, 3000 ft long; and pipe 3 is 20-in cast iron, 1300 ft long. The elevations of the water surfaces in reservoirs A and B are 225 and 200 ft, respectively, and the discharge through pipe 1 is 42 cfs. The water temperature is 60°F. Using a basic scientific calculator only, find the elevation of the surface of reservoir C. Neglect minor losses and assume the energy line and hydraulic grade line are coincident.*

A basic scientific calculator is here defined to be one that is not programmable and does not have automatic equation solving capabilities.

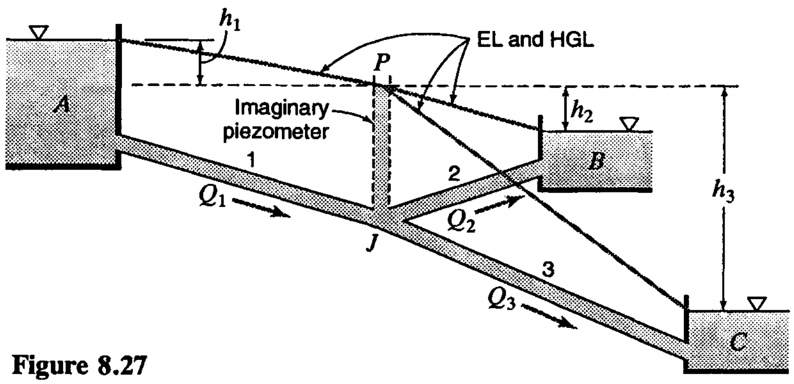

Figure 8.27

BG

This is an example of Case 1. Table A.1 for water at 60°F: $\nu = 1.217 \times 10^{-5}$ ft²/sec

Pipe 1: $V_1 = Q_1/A_1 = 42/(\pi 3^2/4) = 5.94$ fps

Table 8.1 for smooth concrete: $e_1 = 0.001$ ft; $e/D = 0.001/3 = 0.000\,333$

$\mathbf{R}_1 = D_1V_1/\nu = 3(5.94)/(1.217 \times 10^{-5}) = 1{,}464{,}700$

Haaland Eq. 8.52: $1/\sqrt{f_1} = -1.8\log[(0.000\,333/3.7)^{1.11} + 6.9/1{,}464{,}700]$; $f_1 = 0.015\,72$

Eq. 8.13: $h_1 = 0.015\,72(5000/3)5.94^2/(2 \times 32.2) = 14.36$ ft

So Elev P = Elev $A - h_1 = 225 - 14.36 = 210.64$ ft

Pipe 2: h_2 = Elev P – Elev B = 210.64 - 200 = 10.64 ft

Table 8.1 for cast iron: $e_2 = 0.000\,85$ ft; $e/D = 0.000\,85/2 = 0.000\,425$

In Eq. 8.56*b* for Pipe 2: The quantity $\sqrt{2gDh_f/L} = \sqrt{2(32.2)2(10.64)/3000} = 0.676$ fps

Eq. 8.56*b*: $Q_2 = -2(\pi 2^2/4)0.676\log[0.000\,425/3.7 + (2.51/2)1.217 \times 10^{-5}(0.676)] = 16.40$ cfs

Continuity at J: $Q_3 = Q_1 - Q_2 = 42 - 16.40 = 25.60$ cfs

Pipe 3: $V_3 = Q_3/A_3 = 25.6/[(\pi/4)(20/12)^2] = 11.73$ fps

$\mathbf{R}_3 = (20/12)11.73(1.217 \times 10^{-5}) = 1{,}607{,}100$; $e/D = 0.000\,85/(20/12) = 0.000\,51$

Haaland Eq. 8.52: $f_3 = 0.017\,08$; Eq. 8.13: $h_3 = 0.017\,08(1300)(12/20)11.73^2/(2 \times 32.2) = 28.49$ ft

∴ Elev C = Elev $P - h_3 = 210.64 - 28.49 = 182.15$ ft ◄

8.80 *In Fig. 8.27, suppose that pipe 1 is 900-mm smooth concrete, 1500 m long; pipe 2 is 600-mm cast iron, 900 m long; and pipe 3 is 500-mm cast iron, 400 m long. The elevations of the water surfaces in reservoirs A and B are 75 and 67 m, respectively, and the discharge through pipe 1 is 1.2 m³/s. The water temperature is 15°C. Using a basic scientific calculator only, find the elevation of the surface of reservoir C. Neglect minor losses and assume the energy line and hydraulic grade line are coincident.*

A basic scientific calculator is here defined to be one that is not programmable and does not have automatic equation solving capabilities.

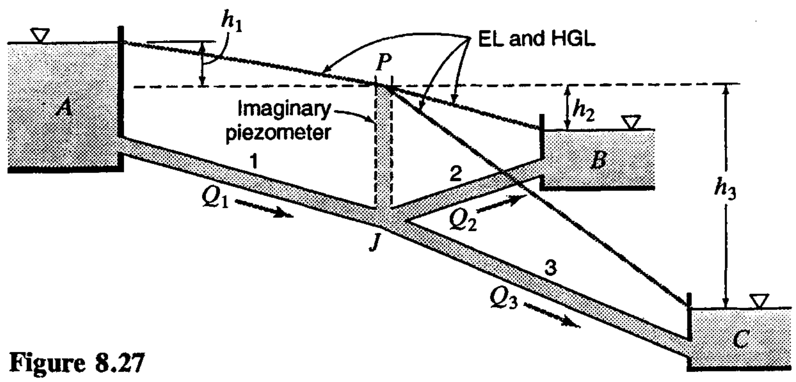

Figure 8.27

SI

This is an example of Case 1.

Table A.1 for water at 15°C: $\nu = 1.139 \times 10^{-6}$ m²/s

Pipe 1: $V_1 = Q_1/A_1 = 1.2/(\pi 0.9^2/4) = 1.886$ m/s

Table 8.1 for smooth concrete: $e_1 = 0.0003$ m; $e/D = 0.0003/0.9 = 0.000\,333$

$\mathbf{R}_1 = D_1V_1/\nu = 0.9(1.886)/(1.139 \times 10^{-6}) = 1\,490\,500$

Haaland Eq. 8.52: $1/\sqrt{f_1} = -1.8\log[(0.000\,333/3.7)^{1.11} + 6.9/1\,490\,500]$; $f_1 = 0.015\,71$

Eq. 8.13: $h_1 = 0.015\,71(1500/0.9)1.886^2/(2 \times 9.81) = 4.75$ m

So Elev P = Elev $A - h_1 = 75 - 4.75 = 70.25$ m

Pipe 2: h_2 = Elev P − Elev B = 70.25 - 67 = 3.25 m

Table 8.1 for cast iron: $e_2 = 0.000\,25$ m; $e/D = 0.000\,25/0.6 = 0.000\,417$

In Eq. 8.56*b* for Pipe 2: The quantity $\sqrt{2gDh_f/L} = \sqrt{2(9.81)0.6(3.25)/900} = 0.206$ m/s

Eq. 8.56*b*: $Q_2 = -2(\pi 0.6^2/4)0.206\log[0.000\,417/3.7 + (2.51/0.6)1.139 \times 10^{-6}(0.206)] = 0.460$ m³/s

Continuity at *J*: $Q_3 = Q_1 - Q_2 = 1.2 - 0.460 = 0.740$ m³/s

Pipe 3: $V_3 = Q_3/A_3 = 0.740/[(\pi/4)(0.5)^2] = 3.77$ m/s

$\mathbf{R}_3 = (0.5)3.77(1.139 \times 10^{-6}) = 1654\,400$; $e/D = 0.000\,25/0.5 = 0.0005$

Haaland Eq. 8.52: $f_3 = 0.017\,00$; Eq. 8.13: $h_3 = 0.017\,00(400/0.5)3.77^2/(2 \times 9.81) = 9.85$ m

∴ Elev C = Elev $P - h_3 = 70.25 - 9.85 = 60.40$ m ◄

8.81 *Solve Prob. 8.80 by using an equation solver in computer software.*

Prob. 8.80: In Fig. 8.27, suppose that pipe 1 is 900-mm smooth concrete, 1500 m long; pipe 2 is 600-mm cast iron, 900 m long; and pipe 3 is 500-mm cast iron, 400 m long. The elevations of the water surfaces in reservoirs A and B are 75 and 67 m, respectively, and the discharge through pipe 1 is 1.2 m³/s. The water temperature is 15°C. Find the elevation of the surface of reservoir C. Neglect minor losses and assume the energy line and hydraulic grade line are coincident.

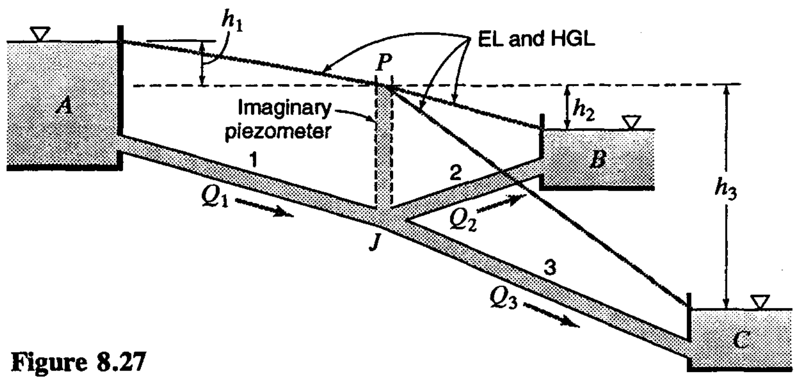

Figure 8.27

SI

Table A.1 for water at 15°C: $\nu = 1.139 \times 10^{-6}$ m²/s

Enter the following 12 known values, without units, into an equation solver in computer software:

$g = 9.81$	$D1 = 0.90$	$L1 = 1500$	$e1 = 0.0003$
$\nu = 1.139 \cdot 10^{-6}$	$D2 = 0.60$	$L2 = 900$	$e2 = 0.000\,25$
$Q1 = 1.2$	$D3 = 0.50$	$L3 = 400$	$e3 = 0.000\,25$

Also enter guessed values for the following 14 unkowns:

$Q2 = 2$	$V1 = 3$	$h1 = 10$	$R1 = 800\,000$	$f1 = 0.01$
$Q3 = 2$	$V2 = 3$	$h2 = 2$	$R2 = 800\,000$	$f2 = 0.01$
	$V3 = 3$	$h3 = 10$	$R3 = 800\,000$	$f3 = 0.01$

Also enter the following four governing equations for Pipe 1:

$$\frac{1}{\sqrt{f1}} = -2 \cdot \log\left(\frac{e1}{3.7 \cdot D1} + \frac{2.51}{R1 \cdot \sqrt{f1}}\right); \quad R1 = \frac{D1 \cdot V1}{\nu}$$

$$h1 = \frac{f1 \cdot L1 \cdot (V1)^2}{D1 \cdot 2 \cdot g}; \quad Q1 = \frac{V1 \cdot \pi \cdot (D1)^2}{4}$$

Enter four similar equations for Pipe 2, and four for Pipe 3, replacing the 1's with 2's and 3's. Also enter the equations $h1 + h2 = 8 \qquad Q1 = Q2 + Q3$

Thus there are 14 simultaneous equations, corresponding to the 14 unknowns.

Instruct the solver to find $V1$, $V2$, $V3$, $h1$, $h2$, $h3$, R1, R2, R3, $f1$, $f2$, $f3$, $Q2$, $Q3$

Results are:

	V	h	R	f	Q
Pipe 1:	1.886	4.76	1490 500	0.015 74	(given)
Pipe 2:	1.593	3.24	839 000	0.016 72	0.450
Pipe 3:	3.818	10.11	1676 100	0.017 01	0.750

So Elev C = Elev $A - h_1 - h_3 = 75 - 4.76 - 10.11 = 60.13$ m ◄

8.82 *In Fig. 8.27, suppose that pipe 1 is 36-in smooth concrete, 5000 ft long; pipe 2 is 24-in cast iron, 3000 ft long; and pipe 3 is 20-in cast iron, 1300 ft long. The surface elevations of reservoirs A and C are 250 and 180 ft, respectively, and the discharge through pipe 2 is 10 cfs of water into reservoir B. The water temperature is 60°F. Using a basic scientific calculator only, find the surface elevation of reservoir B.*

A basic scientific calculator is here defined to be one that is not programmable and does not have automatic equation solving capabilities.

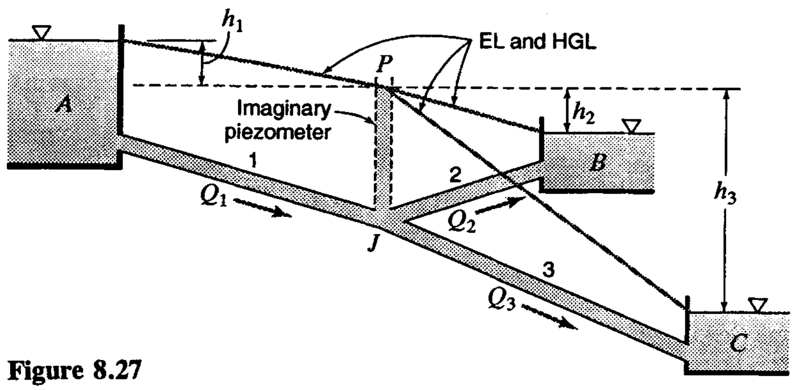

Figure 8.27

BG

This is a Case 2 problem. Table A.1 for water at 60°F: $\nu = 1.217 \times 10^{-5}$ ft²/sec

	Pipe:	1	2	3
Given:	L, ft	5000	3000	1300
	D, ft	3	2	20/12
	e, ft (Table 8.1)	0.001	0.000 85	0.000 85
Calc:	L/D	1667	1500	780
	A, ft²	7.07	3.14	2.18
	e/D	0.000 333	0.000 425	0.000 51

Find the elevation of P (Fig. 8.27) by trial and error. /cont...

Elev. P lies between 180 and 250 ft. Find V from Eq. 8.56*a* assuming turbulent flow. Trials:

El. P	h_1	h_3	V_1	V_3	Q_1	Q_3	ΣQ	Move P ?
200	50	20	11.15	9.81	78.8	19.97	+38.9	Up
	20	50	7.02	15.58	49.6	34.0	+5.64	Up

Extrapolation (Fig. 8.28): (El. P − 230)/(230 − 200) = 5.64/(38.9 − 5.64); El. P = 235.09

235	15	55	6.07	16.34	42.9	35.7	−2.76	Down

Interpolation: (235 − El. P)/(235 − 230) = 2.76/(2.76 + 5.64); Elev P = 233.36, close enough!

$\mathbf{R}_1 = D_1V_1/\nu = 2.75 \times 10^6$, $\mathbf{R}_3 = 2.13 \times 10^6$, both <u>are</u> turbulent, so Eq. 8.56*a* and results are valid.

$V_2 = Q_2/A_2 = 10/(\pi 2^2/4) = 3.18$ fps; $\mathbf{R}_2 = D_2V_2/\nu = 2(3.18)/(1.217 \times 10^{-5}) = 523{,}100$

Eq. 8.52: $f_2 = 0.017\,04$; Eq. 8.13: $h_2 = 4.02$ ft

Elev. B = Elev. $P - h_2$ = 233.36 − 4.02 = 229.34 ft ◄

8.83 *Repeat Prob. 8.82, except that the 10 cfs discharge through pipe 2 is now from (not into) reservoir B.*

Prob. 8.82: In Fig. 8.27, suppose that pipe 1 is 36-in smooth concrete, 5000 ft long; pipe 2 is 24-in cast iron, 3000 ft long; and pipe 3 is 20-in cast iron, 1300 ft long. The surface elevations of reservoirs A and C are 250 and 180 ft, respectively, and the discharge through pipe 2 is 10 cfs of water. The water temperature is 60°F. Using a basic scientific calculator only, find the surface elevation of reservoir B.

A basic scientific calculator is here defined to be one that is not programmable and does not have automatic equation solving capabilities.

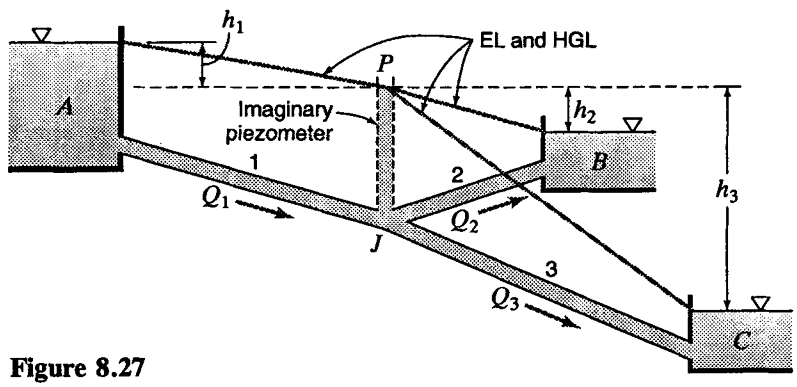

Figure 8.27

BG

This is a Case 2 problem.

Table A.1 for water at 60°F: $\nu = 1.217 \times 10^{-5}$ ft²/sec

	Pipe:	1	2	3
Given:	L, ft	5000	3000	1300
	D, ft	3	2	20/12
	e, ft (Table 8.1)	0.001	0.000 85	0.000 85
Calc:	L/D	1667	1500	780
	A, ft²	7.07	3.14	2.18
	e/D	0.000 333	0.000 425	0.000 51

Find the elevation of P (Fig. 8.27) by trial and error.

Elev. P lies between 180 and 250 ft. Find V from Eq. 8.56a, assuming turbulent flow. Trials:

El. P	h_1	h_3	V_1	V_3	Q_1	Q_3	ΣQ	Move P ?
230	20	50	7.02	15.58	49.6	34.0	+25.6	Up
245	5	65	3.47	17.77	24.5	38.8	−4.26	Down
Interpolation (Fig. 8.28): (245 − El. P)/(245 − 230) = 4.26/(4.26 − 25.6); El. P = 242.86								
243	7	63	4.12	17.50	29.1	38.2	+0.945	Up
Interpolation: (245 − El. P)/(245 − 243) = 4.26/(4.26 + 0.945); Elev P = 243.36, close enough!								

$\mathbf{R}_1 = D_1V_1/\nu = 1.02 \times 10^6$, $\mathbf{R}_3 = 2.40 \times 10^6$, both are turbulent, so Eq. 8.56a and results are valid.

$V_2 = Q_2/A_2 = 10/(\pi 2^2/4) = 3.18$ fps; $\mathbf{R}_2 = D_2V_2/\nu = 2(3.18)/(1.217 \times 10^{-5}) = 523{,}100$

Eq. 8.52: $f_2 = 0.017\,04$; Eq. 8.13: $h_2 = 4.02$ ft

Elev. B = Elev. $P + h_2 = 243.36 + 4.02 = 247.38$ ft ◀

8.84 *Solve Prob. 8.82 without manual trial and error, by using an equation solver in computer software.*

Prob. 8.82: In Fig. 8.27, suppose that pipe 1 is 36-in smooth concrete, 5000 ft long; pipe 2 is 24-in cast iron, 3000 ft long; and pipe 3 is 20-in cast iron, 1300 ft long. The surface elevations of reservoirs A and C are 250 and 180 ft, respectively, and the discharge through pipe 2 is 10 cfs of water into reservoir B. The water temperature is 60°F. Find the surface elevation of reservoir B.

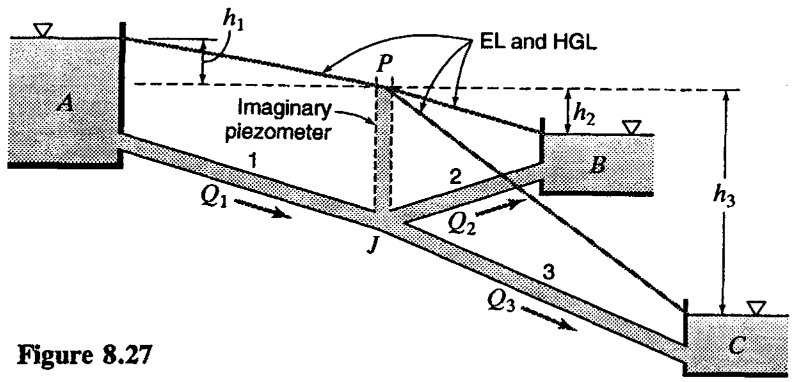

Figure 8.27

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Table A.1 for water at 60°F: $\nu = 1.217 \times 10^{-5}$ ft²/sec

Enter the following 12 known values, without units, into an equation solver in computer software:

$g = 32.2$	$D1 = 36/12$	$L1 = 5000$	$e1 = 0.001$
$\nu = 1.217 \cdot 10^{-5}$	$D2 = 24/12$	$L2 = 3000$	$e2 = 0.00085$
$Q2 = 10$	$D3 = 20/12$	$L3 = 1300$	$e3 = 0.00085$

Also enter guessed values for the following 14 unkowns:

$Q1 = 20$	$V1 = 10$	$h1 = 10$	$\mathrm{R1} = 800000$	$f1 = 0.01$
$Q3 = 20$	$V2 = 10$	$h2 = 10$	$\mathrm{R2} = 800000$	$f2 = 0.01$
	$V3 = 10$	$h3 = 30$	$\mathrm{R3} = 800000$	$f3 = 0.01$

Enter the following four governing equations for Pipe 1:

$$\frac{1}{\sqrt{f1}} = -2\cdot\log\left(\frac{e1}{3.7\cdot D1} + \frac{2.51}{\mathrm{R1}\cdot\sqrt{f1}}\right); \quad \mathrm{R1} = \frac{D1\cdot V1}{\nu}; \quad h1 = \frac{f1\cdot L1\cdot(V1)^2}{D1\cdot 2\cdot g}; \quad Q1 = \frac{V1\cdot\pi\cdot(D1)^2}{4}$$

Note that the first (Colebrook) equation assumes flow is turbulent.

Enter four similar equations for Pipe 2, and four for Pipe 3, replacing the 1's with 2's and 3's. Also enter the equations $h1 + h3 = 70 \quad Q1 = Q2 + Q3$

Thus these are 14 simultaneous equations, corresponding to the 14 unknowns.

Instruct the solver to find $Q1, Q3, V1, V2, V3, h1, h2, h3, \mathrm{R1}, \mathrm{R2}, \mathrm{R3}, f1, f2, f3$

Results are:

	Q	V	h	**R**	f
Pipe 1:	45.1	6.38	16.58	1,573,900	0.015 72
Pipe 2:	(given)	3.18	4.04	523,100	0.017 14
Pipe 3:	35.1	16.10	53.42	2,205,300	0.017 01

All three **R** > 2000 (Eq. 8.2) confirms that the flows are turbulent, so the use of the Colebrook Eq. 8.51 was valid, and the results are valid.

So Elev B = Elev $A - h_1 - h_2 = 250 - 16.58 - 4.04 = 229.37$ ft ◄

8.85 *In Fig. 8.27, suppose that pipe 1 is 900-mm smooth concrete, 1500 m long; pipe 2 is 600-mm cast iron, 900 m long; and pipe 3 is 500-mm cast iron, 400 m long. The surface elevations of reservoirs A and C are 60 and 38 m, respectively, and the discharge through pipe 2 is 0.3 m³/s of water into reservoir B. The water temperature is 15°C. Using a basic scientific calculator only, find the surface elevation of reservoir B.*

A basic scientific calculator is here defined to be one that is not programmable and does not have automatic equation solving capabilities.

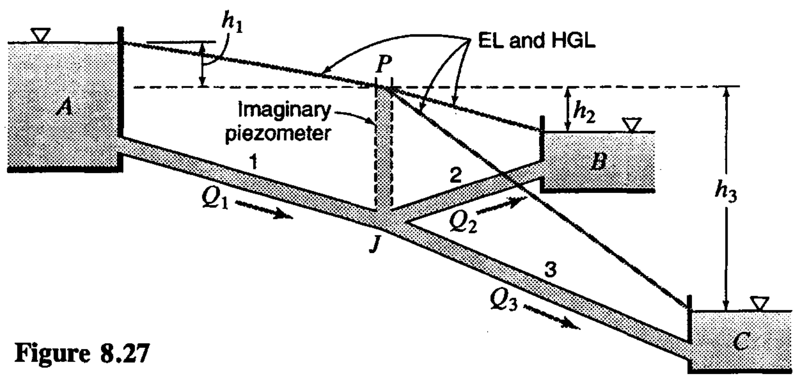

Figure 8.27

SI

This is a Case 2 problem.

Table A.1 for water at 15°C: $\nu = 1.139 \times 10^{-6}$ m²/s

	Pipe:	1	2	3
Given:	L, m	1500	900	400
	D, m	0.90	0.60	0.50
	e, m (Table 8.1)	0.0003	0.000 25	0.000 25
Calc:	L/D	1667	1500	800
	A, m²	0.636	0.283	0.1963
	e/D	0.000 333	0.000 417	0.0005

Find the elevation of P (Fig. 8.27) by trial and error.

Elev. P lies between 38 and 60 m. Find V from Eq. 8.56a assuming turbulent flow. Trials:

El. P	h_1	h_3	V_1	V_3	Q_1	Q_3	ΣQ	Move P ?
50	10	12	2.75	4.16	1.747	0.817	+0.630	Up
55	5	17	1.934	4.96	1.231	0.974	−0.044	Down

Interpolation (Fig. 8.28): $\dfrac{55 - \text{El. } P}{55 - 50} = \dfrac{0.044}{0.044 + 0.630}$; El. P = 54.37, close enough!

$\mathbf{R}_1 = D_1V_1/\nu = 1.53 \times 10^6$, $\mathbf{R}_3 = 2.18 \times 10^6$, both are turbulent, so Eq. 8.56a and results are valid.

$V_2 = Q_2/A_2 = 0.3/(\pi 0.6^2/4) = 1.061$ m/s ; $\mathbf{R}_2 = D_2V_2/\nu = 0.6(1.061)/(1.139 \times 10^{-6}) = 558\,900$

Eq. 8.52: $f_2 = 0.01693$; Eq. 8.13: $h_2 = 1.457$ m

Elev. B = Elev. $P - h_2 = 54.67 - 1.46 = 53.21$ m ◄

8.86 *Repeat Prob. 8.85, except that the 0.3 m^3/s discharge through pipe 2 is now from (not into) reservoir B.*

Prob. 8.85: In Fig. 8.27, suppose that pipe 1 is 900-mm smooth concrete, 1500 m long; pipe 2 is 600-mm cast iron, 900 m long; and pipe 3 is 500-mm cast iron, 400 m long. The surface elevations of reservoirs A and C are 60 and 38 m, respectively, and the discharge through pipe 2 is 0.3 m^3/s of water. The water temperature is 15°C. Using a basic scientific calculator only, find the surface elevation of reservoir B.

A basic scientific calculator is here defined to be one that is not programmable and does not have automatic equation solving capabilities.

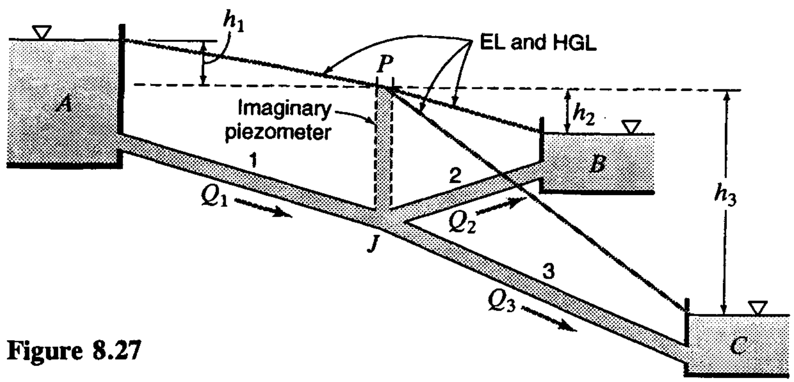

Figure 8.27

SI

This is a Case 2 problem.

Table A.1 for water at 15°C: $\nu = 1.139 \times 10^{-6}$ m²/s

	Pipe:	1	2	3
Given:	L, m	1500	900	400
	D, m	0.90	0.60	0.50
	e, m (Table 8.1)	0.0003	0.000 25	0.000 25
Calc:	L/D	1667	1500	800
	A, m²	0.636	0.283	0.1963
	e/D	0.000 333	0.000 417	0.0005

Find the elevation of P (Fig. 8.27) by trial and error.

Elev. P lies between 38 and 60 m. Find V from Eq. 8.56a, assuming turbulent flow. Trials:

El. P	h_1	h_3	V_1	V_3	Q_1	Q_3	ΣQ	Move P ?
55	5	17	1.934	4.96	1.231	0.974	+0.556	Up
58	2	20	1.214	5.38	0.772	1.057	+0.015	Up (close)

Extrapolation (Fig. 8.28): (El. P − 58)/(58 − 55) = 0.015/(0.556 − 0.015);

El. P = 58.08, close enough!

$\mathbf{R}_1 = D_1V_1/\nu = 9.59 \times 10^5$, $\mathbf{R}_3 = 2.36 \times 10^6$, both <u>are</u> turbulent, so Eq. 8.56 and results are valid.

$V_2 = Q_2/A_2 = 0.3/(\pi 0.6^2/4) = 1.061$ m/s; $\mathbf{R}_2 = D_2V_2/\nu = 0.6(1.061)/(1.139 \times 10^{-6}) = 558\,900$

Eq. 8.52: $f_2 = 0.016\,93$; Eq. 8.13: $h_2 = 1.457$ m

Elev. B = Elev. $P - h_2$ = 58.08 + 1.46 = 59.54 m ◀

8.87 *Suppose, in Fig. 8.27, that pipes 1, 2, and 3 are 900 m of 600 mm, 300 m of 450 mm, and 1200 m of 400 mm, respectively, of new welded-steel pipe. The surface elevations of reservoirs A, B, and C are 36, 22 and 0 m, respectively. The water temperature is 15°C. Using a basic scientific calculator only, find the flow in all pipes.*

A basic scientific calculator is here defined to be one that is not programmable and does not have automatic equation solving capabilities.

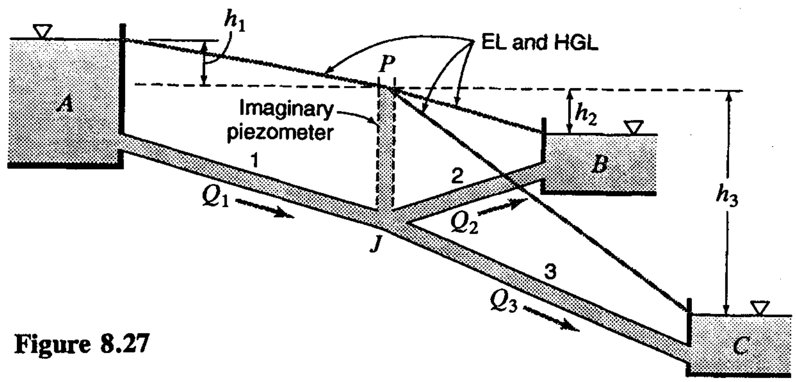

Figure 8.27

SI

This is a Case 3 problem. Solve by T and E.

Table A.1 for water at 15°C: ν = 1.139 × 10^{-6} m/s. Assume turbulent flow (for Eq. 8.56).

Pipe:	1	2	3
L, m (given)	900	300	1200
D m (given)	0.60	0.45	0.40
e, m (Table 8.1)	0.000 046	0.000 046	0.000 046
Trial 1: Try P at El of reservoir surface B = 22 m:			
h, m	14	0	22
$\sqrt{2gDh_L/L}$, m/s	0.428	0	0.379
Q, m³/s (Eq. 8.56b)	1.088	0	0.410

At J, ΣQ = in − out = 1.088 − 0.410 = 0.678 m³/s. This must be zero, so P must be raised; then water will flow into reservoir B. Mid way between El A and El B = (36 + 22)/2 = 29 m.

Trial 2: Try P at El. 29 m:

h, m	7	7	29
Q, m³/s	0.759	0.632	0.473

At J, ΣQ = in − out = 0.759 − 0.632 − 0.473 = −0.346 m³/s. P is too high.

By interpolation (Fig. 8.28): (29 − El. P)/(29 − 22) = 0.346/(0.346 + 0.678); El. P = 26.63 m.

Trial 3: Try P at El. 26.6 m:

h, m	9.4	4.6	26.6
Q, m³/s	0.885	0.509	0.453

At J, ΣQ = 0.885 - 0.509 - 0.453 = -0.077 m³/s. Error = 0.077/0.885 = 8.7% (so try again).

By extrapolation: (26.6 − El. P)/(29 − 26.6) = 0.077/(0.346 − 0.077); El. P = 25.91 m

/cont...

Trial 4: Try P at El. 25.9 m:

h, m	10.1	3.9	25.9
V, m/s (Eq. 8.56a)	3.25	2.94	3.55
Q, m^3/s = $AV = \pi D^2 V/4$	0.919	0.467	0.446

At J, $\Sigma Q = 0.919 - 0.467 - 0.446 = +0.00557$ m^3/s. Error = 0.00557/0.919 = 0.61%, close enough!

Check Reynolds numbers:

$\mathbf{R} = DV/\nu$	1.71×10^6	1.16×10^6	1.25×10^6

All three flows are turbulent (as assumed), so use of Eq. 8.56 and results are valid.

$Q_1 = 0.919$ m^3/s, $Q_2 = 0.467$ m^3/s, $Q_3 = 0.446$ m^3/s ◄◄

8.88 *Solve Prob. 8.87 without manual trial and error, by using an equation solver in computer software.*
Prob. 8.87: Suppose, in Fig. 8.27, that pipes 1, 2, and 3 are 900 m of 600 mm, 300 m of 450 mm, and 1200 m of 400 mm, respectively, of new welded-steel pipe. The surface elevations of reservoirs A, B, and C are 36, 22 and 0 m, respectively. The water temperature is 15°C. Find the flow in all pipes.

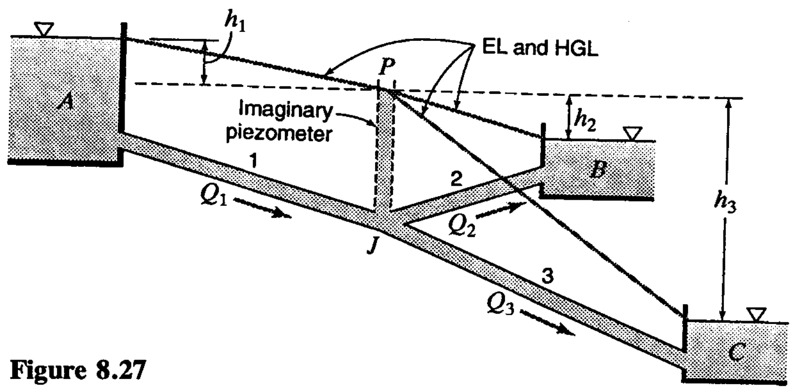

Figure 8.27

SI

Table A.1 for water at 15°C: $\nu = 1.139 \times 10^{-6}$ m²/sec

Enter the following 11 known values, without units, into an equation solver in computer software:

$g = 9.81$	$D1 = 0.60$	$L1 = 900$	$e1 = 0.000\,046$
$\nu = 1.139 \cdot 10^{-6}$	$D2 = 0.45$	$L2 = 300$	$e2 = 0.000\,046$
	$D3 = 0.40$	$L3 = 1200$	$e3 = 0.000\,046$

Also enter guessed values for the following 15 unkowns:

$Q1 = 2$	$V1 = 3$	$h1 = 10$	$R1 = 800\,000$	$f1 = 0.01$
$Q2 = 2$	$V2 = 3$	$h2 = 5$	$R2 = 800\,000$	$f2 = 0.01$
$Q3 = 2$	$V3 = 3$	$h3 = 10$	$R3 = 800\,000$	$f3 = 0.01$

Enter the following four governing equations for Pipe 1:

$$\frac{1}{\sqrt{f1}} = -2 \cdot \log\left(\frac{e1}{3.7 \cdot D1} + \frac{2.51}{R1 \cdot \sqrt{f1}}\right); \quad R1 = \frac{D1 \cdot V1}{\nu}; \quad h1 = \frac{f1 \cdot L1 \cdot (V1)^2}{D1 \cdot 2 \cdot g}; \quad Q1 = \frac{V1 \cdot \pi \cdot (D1)^2}{4}$$

Note that the first (Colebrook) equation assumes flow is turbulent.

Enter four similar equations for Pipe 2, and four for Pipe 3, replacing the 1's by 2's and 3's.

Also enter the equations $h1 + h2 = 14$ $\quad h1 + h3 = 36$ and $Q1 = Q2 + Q3$

Thus there are 15 simultaneous equations, corresponding to the 15 unknowns.

Instruct the solver to find $Q1, Q2, Q3, V1, V2, V3, h1, h2, h3, R1, R2, R3, f1, f2, f3$

Results are:

	Q	V	h	R	f
Pipe 1:	0.917	3.24	10.05	1707 800	0.012 51
Pipe 2:	0.470	2.95	3.95	1167 400	0.013 31
Pipe 3:	0.447	3.55	25.95	1248 400	0.013 43

▲

All three **R** > 2000 (Eq. 8.2) confirms that the flows are turbulent, so the use of Colebrook Eq. 8.51 was valid, and the results are valid.

8.89 *Suppose, in Fig. 8.27, that pipes 1, 2, and 3 are 3000 ft of 24 in, 1000 ft of 18 in, and 4000 ft of 16 in, respectively, of new welded-steel pipe. The surface elevations of reservoirs A, B, and C are 120, 75 and 0 ft, respectively. The water temperature is 60°F. Using a basic scientific calculator only, find the flow in all pipes.*

A basic scientific calculator is here defined to be one that is not programmable and does not have automatic equation solving capabilities.

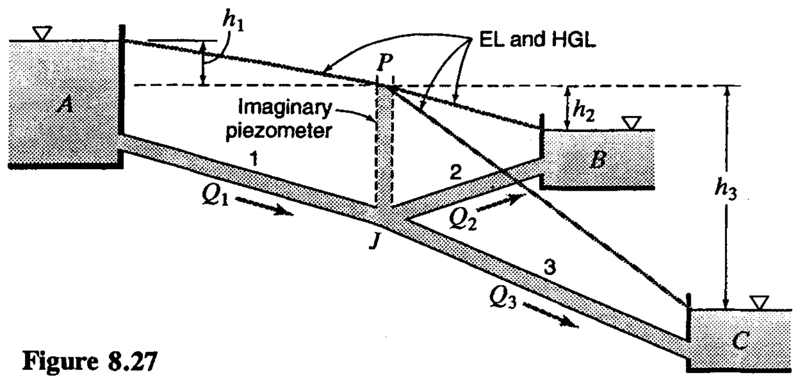

Figure 8.27

BG

Water surface Els.: A = 120 ft, B = 75 ft, C = 0 ft ; $T°$ = 60°F

To find the 3 Q's this is Case 3. Solve by T and E.

Table A.1 for water at 60°F: $\nu = 1.217 \times 10^{-5}$ ft²/sec. Assume turbulent flow (for Eq. 8.56).

Pipe:	1	2	3
L, ft (given)	3000	1000	4000
D ft (given)	2.0	1.5	16/12
e, ft (Table 8.1)	0.000 15	0.000 15	0.000 15
e/D	0.000 075	0.0001	0.000 1125

<u>Trial 1</u>: Try P at El of reservoir surface B = 75 ft:

h, ft	45	0	75
$\sqrt{2gDh_L/L}$, fps	1.390	0	1.269
Q, cfs (Eq. 8.56b)	39.3	0	15.29

At J, ΣQ = in − out = 39.3 − 15.29 = 24.1 cfs. This must be zero, so P must be raised; then water will flow <u>into</u> reservoir B. Mid way between El A and El B = (120 + 75)/2 = 97.5 ft.

<u>Trial 2</u>: Try P at El. 100 ft:

h, ft	20	25	100
Q, cfs	25.8	24.1	17.74

At J, ΣQ = in − out = 25.8 − 24.1 − 17.74 = −16.07 cfs.

By interpolation (Fig. 8.28): (100 − El. P)/(100 − 75) = 16.07/(16.07 + 24.1); El. P = 90.0 ft.

<u>Trial 3</u>: Try P at El. 90.0 ft:

h, ft	30	15	90
Q, cfs	31.89	18.54	16.80

At J, ΣQ = in - out = = 31.89 - 18.54 - 16.80 = -3.45 cfs.

Error = 3.45/31.89 = 10.8% (so try again). /cont...

By extrapolation: (90 − El. P)/(100 − 90) = 3.45/(16.07 − 3.45); El. P = 87.3 ft.

Trial 4: Try P at El. 87.3 ft:

h, ft	32.7	12.3	87.3
V, fps (Eq. 8.56a)	10.61	9.35	11.85
Q, cfs = $AV = \pi D^2V/4$	33.348	16.523	16.540

At J, ΣQ = in − out = 33.348 − 16.523 − 16.540 = 0.285 cfs

Error = 0.285/33.348 = 0.85%, close enough! Check Reynolds numbers:

$\mathbf{R} = DV/\nu$	1.744×10^6	1.152×10^6	1.298×10^6

All 3 flows are turb., so using Eq. 8.56 and results are valid. Q_1 = 33.3 cfs, $Q_2 = Q_3$ = 16.5 cfs ◄

8.90 ***Solve Prob. 8.89 without manual trial and error, by using an equation solver in computer software.***

***Prob. 8.89:** Suppose, in Fig. 8.27, that pipes 1, 2, and 3 are 3000 ft of 24 in, 1000 ft of 18 in, and 4000 ft of 16 in, respectively, of new welded-steel pipe. The surface elevations of reservoirs A, B, and C are 120, 75 and 0 ft, respectively. The water temperature is 60°F. Find the flow in all pipes.*

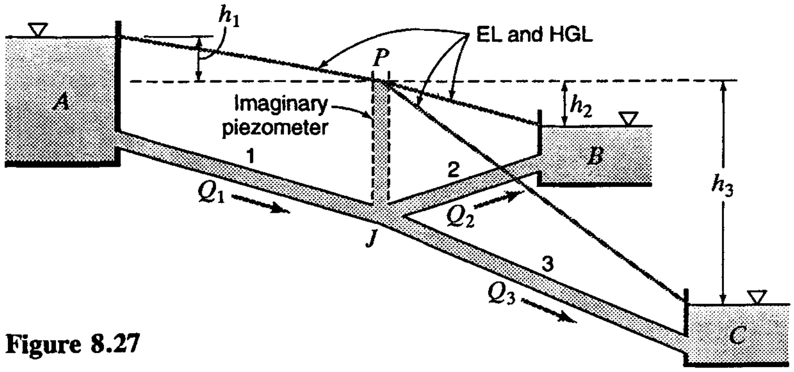

Figure 8.27

BG

Table A.1 for water at 60°F: $\nu = 1.217 \times 10^{-5}$ ft²/sec

Enter the following 12 known values, without units, into an equation solver in computer software:

$g = 32.2$	$D1 = 24/12$	$L1 = 3000$	$e1 = 0.000\,15$
$\nu = 1.217 \cdot 10^{-5}$	$D2 = 18/12$	$L2 = 1000$	$e2 = 0.000\,15$
	$D3 = 16/12$	$L3 = 4000$	$e3 = 0.000\,15$

Also enter guessed values for the following 15 unkowns:

$Q1 = 70$	$V1 = 10$	$h1 = 30$	$\mathrm{R1} = 800\,000$	$f1 = 0.01$
$Q2 = 70$	$V2 = 10$	$h2 = 15$	$\mathrm{R2} = 800\,000$	$f2 = 0.01$
$Q3 = 70$	$V3 = 10$	$h3 = 30$	$\mathrm{R3} = 800\,000$	$f3 = 0.01$

Enter the following four governing equations for Pipe 1:

$$\frac{1}{\sqrt{f1}} = -2 \cdot \log\left(\frac{e1}{3.7 \cdot D1} + \frac{2.51}{\mathrm{R1} \cdot \sqrt{f1}}\right) \; ; \quad \mathrm{R1} = \frac{D1 \cdot V1}{\nu} \; ; \quad h1 = \frac{f1 \cdot L1 \cdot (V1)^2}{D1 \cdot 2 \cdot g} \; ; \quad Q1 = \frac{V1 \cdot \pi \cdot (D1)^2}{4}$$

Note that the first (Colebrook) equation assumes flow is turbulent.

Enter four similar equations for Pipe 2, and four for Pipe 3, replacing the 1's with 2's and 3's.

Also enter the equations $h1 + h2 = 45$ $\quad h1 + h3 = 120$ and $Q1 = Q2 + Q3$

Thus there are 15 simultaneous equations, corresponding to the 15 unknowns.

Instruct the solver to find $Q1$, $Q2$, $Q3$, $V1$, $V2$, $V3$, $h1$, $h2$, $h3$, R1, R2, R3, $f1$, $f2$, $f3$

Results are:

	Q	V	h	**R**	f
Pipe 1:	33.31	10.60	32.63	1,742,600	0.012 46
Pipe 2:	16.77	9.49	12.37	1,169,400	0.013 27
Pipe 3:	16.55	11.85	87.37	1,298,300	0.013 35

▲

All three **R** > 2000 (Eq. 8.2) confirms that the flows are turbulent, so the use of Colebrook Eq. 8.51 was valid, and the results are valid.

8.91 *Suppose that, in Fig. 8.27, pipe 1 is 1500 ft of 12-in new cast-iron pipe, pipe 2 is 800 ft of 6-in wrought-iron pipe, and pipe 3 is 1200 ft of 8-in wrought-iron pipe. The water surface elevation of reservoir B is 20 ft less that of A, while the junction J is 35 ft lower than the surface of A. In place of reservoir C, pipe 3 leads away to some other destination but its elevation at C is 60 ft below A. (a) Find the flow of 60°F water in each pipe. (b) Find the pressure head at C, when the pressure head at J is 25 ft. Neglect velocity heads.*

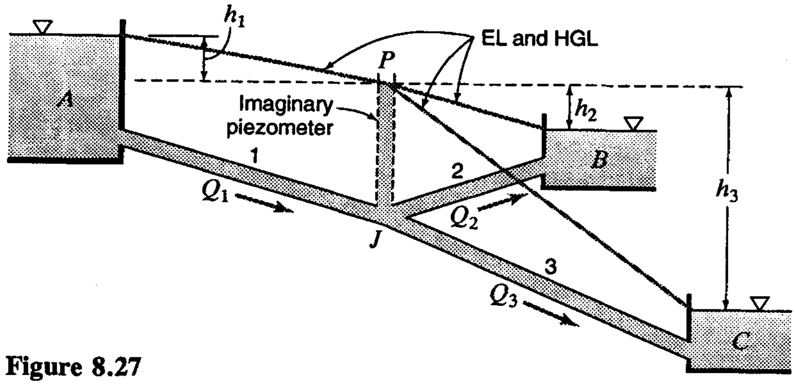

Figure 8.27

BG

(*a*) Table A.1 for water at 60°F: $\nu = v = 1.217 \times 10^{-5}$ ft²/sec. Assume turbulent flow (for Eqs. 8.56).

	Pipe:	1	2	3
Given:	L, ft	1500	800	1200
	D, ft	1	0.5	8/12
	e, ft	0.000 85	0.000 15	0.000 15
Calc:	h_L, ft	35 − 25 = 10	20 − h_1 = 10	
	$\sqrt{2gDh_L/L}$, fps	0.655	0.634	
	V, fps (Eq. 8.56*a*)	4.66	4.76	
	Q, cfs (Eq. 8.56*b*)	3.66 ◄	0.934 ◄	

$Q_3 = Q_1 - Q_2 = 3.66 - 0.934 = 2.73$ cfs ◄ $V_3 = Q_3/A_3 = 2.73/[(\pi/4)(8/12)^2] = 7.82$ fps

$\mathbf{R}_1 = D_1V_1/\nu = 383{,}100$, $\mathbf{R}_2 = 195{,}500$, $\mathbf{R}_3 = 428{,}100$

All the flows are turbulent ($\mathbf{R} > 2000$), so the equations used and answers obtained are valid.

(*b*) Haaland Eq. 8.52: $f_3 = 0.015\,72$; Eq. 8.13: $h_3 = 26.8$ ft

$H_J - H_C = h_3$. Neglecting velocity heads: $(p/\gamma + z)_J - (p/\gamma + z)_C = h_3$

So $(25 - 35) - (p_C/\gamma - 60) = 26.8$; $p_C/\gamma = 23.2$ ft ◄

Sec. 8.30: Pipes in Series – *Problems 8.92–8.101*

8.92 *A pipeline 900 ft long discharges freely at a point 200 ft lower than the water surface at intake (Fig. P8.92). The pipe intake projects into the reservoir. The first 600 ft is of 10-in-diameter, and the remaining 300 ft is of 6-in diameter. (a) Find the rate of discharge, assuming f = 0.06. If the junction point C of the two sizes of pipe is 150 ft below the intake water surface level, find the pressure head (b) just upstream of C and (c) just downstream of C. Assume a sudden contraction at C.*

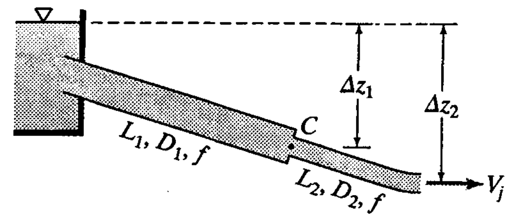

Figure P8.92

BG

(*a*) Sudden contraction at *C*, $D_2/D_1 = 0.6$; Table 8.2: $k_c = 0.28$

Contraction loss $= 0.28V_2^2/2g$. Fig. 8.13: $k_e = 0.8$ for reentrant entrance.

$h_L = [0.8 + 0.06(600)(12/10)]V_1^2/2g + [0.28 + 0.06(300/0.50)]V_2^2/2g = 44.0V_1^2/2g + 36.3V_2^2/2g.$

Continuity: $V_1 = (6/10)^2V_2 = 0.36V_2$. $\therefore\ h_L = 44.0(0.36V_2)^2/2g + 36.3V_2^2/2g = 42.0V_2^2/2g$

Energy: $200 - h_L = V_2^2/2g$, $\therefore\ 200 = (42.0 + 1)V_2^2/2g$, $V_2^2/2g = 4.65$ ft

$V_2 = \sqrt{2(32.2)4.65} = 17.31$ fps; $Q = A_2V_2 = (\pi/4)(0.5^2)17.31 = 3.40$ cfs ◄

(*b*) Energy from water surface to point 1 just upstream of contraction *C*:

$(0 + 150 + 0) - 44.0(0.36V_2)^2/2g = [p_2/\gamma + 0 + (0.36V_2)^2/2g]$; $p_1/\gamma = 122.86$ ft ◄

(*c*) Energy from point 2 just downstream of contraction *C* to jet:

$(p_2/\gamma + 50 + V_2^2/2g) - 0.06(300/0.5)V_2^2/2g = (0 + 0 + V_2^2/2g)$; $p_2/\gamma = 117.51$ ft ◄

Check head loss across contraction *C*:

(*a*) $h_L = [122.86 + z + (0.36V_2)^2/2g] - [117.51 + z + V_2^2/2g] = 1.303$ ft

(*b*) $h_L = 0.28V_2^2/2g = 0.28(4.65) = 1.303$ ft. Agrees!

8.93 *A pipeline 300 m long discharges freely at a point 50 m lower than the water surface at intake (Fig. P8.92). The pipe intake projects into the reservoir. The first 200 m is of 350 mm diameter, and the remaining 100 m is of 250 mm diameter. (a) Find the rate of discharge, assuming f = 0.06. If the junction point C of the two sizes of pipe is 40 m below the intake water surface level, find the pressure head (b) just upstream of C and (c) just downstream of C. Assume a sudden contraction at C.*

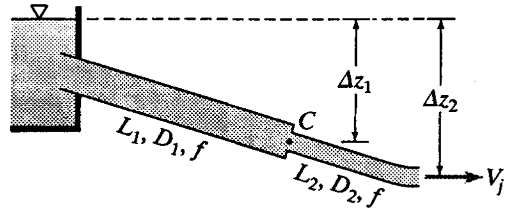

Figure P8.92

SI

(*a*) Sudden contraction at *C*, $D_2/D_1 = 25/35 = 0.714$. Interpolating in Table 8.2: $k_c = 0.21$

Contraction loss $= 0.21V_2^2/2g$. Fig. 8.13: $k_e = 0.8$ for reentrant entrance.

$h_L = [0.8 + 0.06(600)(200/0.35)]V_1^2/2g + [0.21 + 0.06(100/0.25)]V_2^2/2g = 35.1V_1^2/2g + 24.2V_2^2/2g.$

Continuity: $V_1 = (25/35)^2V_2 = 0.510V_2$. $\therefore\ h_L = 35.1(0.510V_2)^2/2g + 24.2V_2^2/2g = 33.3V_2^2/2g$

Energy: $50 - h_L = V_2^2/2g$, $\therefore\ 50 = (33.3 + 1)V_2^2/2g$, $V_2^2/2g = 1.456$ m

$V_2 = \sqrt{2(9.81)1.456} = 5.34$ m/s; $Q = A_2V_2 = (\pi/4)(0.25^2)5.34 = 0.262\ \text{m}^3/\text{s}$ ◄

(*b*) Energy from water surface to point 1 just upstream of contraction *C*:

$(0 + 150 + 0) - 35.1(0.510V_2)^2/2g = [p_1/\gamma + 0 + (0.510V_2)^2/2g]$; $p_1/\gamma = 26.32$ m ◄

(*c*) Energy from point 2 just downstream of contraction *C* to jet:

$(p_2/\gamma + 10 + V_2^2/2g) - 0.06(100/0.25)V_2^2/2g = (0 + 0 + V_2^2/2g)$; $p_2/\gamma = 24.94$ m ◄

Check head loss across contraction *C*:

(*a*) $h_L = [26.32 + z + (0.510V_2)^2/2g] - [24.94 + z + V_2^2/2g] = 0.306$ m

(*b*) $h_L = 0.21V_2^2/2g = 0.21(1.456) = 0.306$ m. Agrees!

8.94 *Repeat Prob. 8.92 neglecting minor losses.*

Prob. 8.92: A pipeline 900 ft long discharges freely at a point 200 ft lower than the water surface at intake (Fig. P8.92). The pipe intake projects into the reservoir. The first 600 ft is of 10-in-diameter, and the remaining 300 ft is of 6-in diameter. (a) Find the rate of discharge, assuming f = 0.06. If the junction point C of the two sizes of pipe is 150 ft below the intake water surface level, find the pressure head (b) just upstream of C and (c) just downstream of C. Assume a sudden contraction at C.

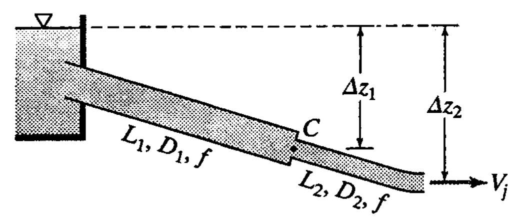

Figure P8.92

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(*a*) $h_L = 0.06(600)(12/10)V_1^2/2g + 0.06(300/0.5)V_2^2/2g = 43.2V_1^2/2g + 36V_2^2/2g$

Continuity: $V_1 = (6/10)^2V_2 = 0.36V_2$. $\therefore\ h_L = 43.2(0.36V_2)^2/2g + 36V_2^2/2g = 41.6V_2^2/2g$

Energy: $200 - h_L = V_2^2/2g$, $\therefore\ 200 = (41.6 + 1)V_2^2/2g$, $V_2^2/2g = 4.69$ ft

$V_2 = \sqrt{2(32.2)4.69} = 17.39$ fps; $Q = A_2V_2 = (\pi/4)(0.5^2)17.39 = 3.41$ cfs ◄

(*b*) Energy from water surface to point 1 just upstream of contraction *C*:

$(0 + 150 + 0) - 43.2(0.36V_2)^2/2g = [p_1/\gamma + 0 + (0.36V_2)^2/2g]$; $p_1/\gamma = 123.11$ ft ◄

(*c*) Energy from point 2 just downstream of contraction *C* to jet:

$(p_2/\gamma + 50 + V_2^2/2g) - 0.06(300/0.5)V_2^2 = (0 + 0 + V_2^2/2g)$; $p_2/\gamma = 119.02$ ft ◄

8.95 *Repeat Prob. 8.93 neglecting minor losses.*

Prob. 8.93: A pipeline 300 m long discharges freely at a point 50 m lower than the water surface at intake (Fig. P8.92). The pipe intake projects into the reservoir. The first 200 m is of 350 mm diameter, and the remaining 100 m is of 250 mm diameter. (a) Find the rate of discharge, assuming f = 0.06. If the junction point C of the two sizes of pipe is 40 m below the intake water surface level, find the pressure head (b) just upstream of C and (c) just downstream of C. Assume a sudden contraction at C.

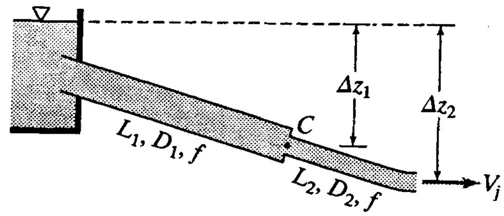

Figure P8.92

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(*a*) $h_L = 0.06(200/0.35)V_1^2/2g + 0.06(100/0.25)V_2^2/2g = 34.3V_1^2/2g + 24V_2^2/2g$

Continuity: $V_1 = (25/35)^2V_2 = 0.510V_2$. $\therefore\ h_L = 34.3(0.510V_2)^2/2g + 24V_2^2/2g = 32.9V_2^2/2g$

Energy: $50 - h_L = V_2^2/2g$, $\therefore\ 50 = (32.9 + 1)V_2^2/2g$, $V_2^2/2g = 1.474$ m

$V_2 = \sqrt{2(9.81)1.474} = 5.38$ m/s; $Q = A_2V_2 = (\pi/4)(0.25^2)5.38 = 0.264\ \text{m}^3/\text{s}$ ◀

(*b*) Energy from water surface to point 1 just upstream of contraction *C*:

$(0 + 40 + 0) - 34.3(0.510V_2)^2/2g = [p_1/\gamma + 0 + (0.510V_2)^2/2g]$; $p_1/\gamma = 26.46$ m ◀

(*c*) Energy from point 2 just downstream of contraction *C* to jet:

$(p_2/\gamma + 10 + V_2^2/2g) - 0.06(100/0.25)V_2^2/2g = (0 + 0 + V_2^2/2g)$; $p_2/\gamma = 25.37$ m ◀

8.96 *Suppose that in Fig. 8.29 pipes 1, 2, and 3 are 750 ft of 4-in, 250 ft of 2-in, and 300 ft of 3-in asphalt-dipped cast-iron pipe. With a total head loss of 25 ft between A and B, find the flow of 60°F water.*

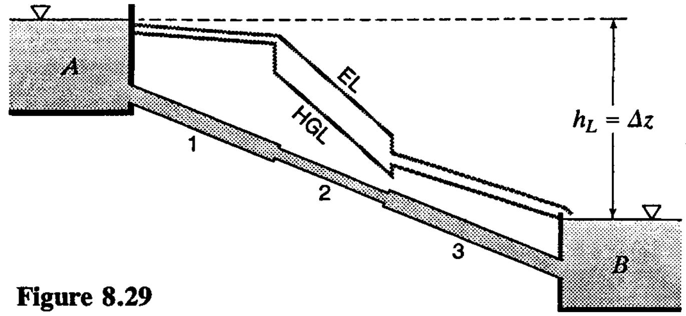

Figure 8.29

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Table 8.1: e = 0.0004 ft.

Trial 1: Initially assume $*f = f_{min}$ (fully rough value) from Eq. 8.54 or Fig. 8.11.

Pipe	L, ft	D, in	L/D	e/D	*Try f	D_1/D	$(D_1/D)^4$
1	750	4	2250	0.0012	0.0205	1.00	1.00
2	250	2	1500	0.0024	0.0246	2.00	16.00
3	300	3	1200	0.0016	0.0221	1.333	3.16

All L/D are > 1000, so we can neglect minor losses.

Eq. 8.89: $25 = [0.0205(2250) + 0.0246(1500)16 + 0.0221(1200)3.16]V_1^2/2g$

$V_1^2/2g = 25/720 = 0.0347$ ft; $V_1 = 1.495$ fps

$D_1''V_1 = 4(1.495) = 5.98$; $\therefore f_1 = 0.0249$ (Fig. 8.11)

$D_2''V_2 = 2(1.495)2^2 = 11.96$; $\therefore f_2 = 0.0263$

$D_3''V_3 = 3(1.495)1.333^2 = 7.97$; $\therefore f_3 = 0.0251$

Trial 2: Repeating Eq. 8.89 with improved f values: $25 = 782V_1^2/2g$, $V_1 = 1.434$ fps

$D_1''V_1 = 4(1.434) = 5.74$; $\therefore f_1 = 0.0251$

$D_2''V_2 = 2(1.434)2^2 = 11.48$; $\therefore f_2 = 0.0263$

$D_3''V_3 = 3(1.434)1.333^2 = 7.65$; $\therefore f_3 = 0.0252$

Trial 3: Repeating Eq. 8.89: $25 = 783V_1^2/2g$, $V_1 = 1.434$ fps (no change, converged)

$Q = A_1V_1 = (\pi/4)(4/12)^2 1.434 = 0.1251$ cfs ◄

Note: f values may instead be obtained from Haaland Eq. 8.52, using $\nu = 12.17 \times 10^{-6}$ ft²/sec.

Alternative solution, by equation solver in computer software:

Establish the four standard equations for each pipe (see Sample Prob. 8.14*b*) plus Eq. 8.87, for a total of 13 governing equations. Assign known and guessed values to the variables (see, e.g., the solution to Prob. 8.81). Solve for the following 13 unknowns: Three values each of V, h, **R**, and f, and one value of Q.

8.97 *Suppose that in Fig. 8.29 pipes 1, 2, and 3 are 150 m of 80-mm, 60 m of 50-mm, and 120 m of 60 mm wrought-iron pipe. With a total head loss of 6 m between A and B, find the flow of 15°C water.*

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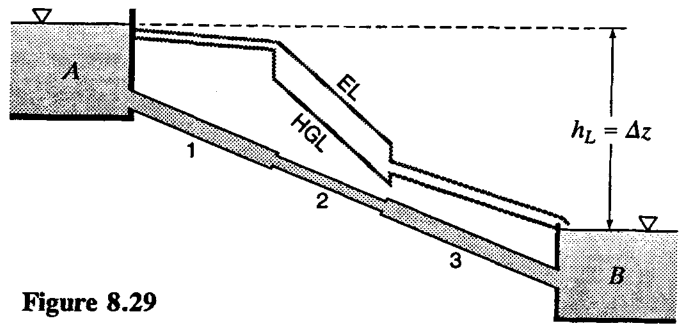

Figure 8.29

Table 8.1: e = 0.000 046 m

Table A.1 at 15°C: ν = 1.139×10^{-6} m²/s.

Trial 1: *assume $f = f_{min}$ (fully rough value) from Eq. 8.54.

Pipe	L, m	D, m	L/D	e/D	*Try f	D_1/D	$(D_1/D)^4$
1	150	0.08	1875	0.000 575	0.017 24	1.00	1.00
2	60	0.05	1200	0.000 920	0.019 24	1.60	6.55
3	120	0.06	2000	0.000 767	0.018 43	1.333	3.16

All L/D are > 1000, so we can neglect minor losses.

Eq. 8.89: $6 = [0.01724(1875) + 0.01924(1200)6.55 + 0.01843(2000)3.16]V_1^2/2g = 300V_1^2/2g$

$V_1^2/2g = 6/300 = 0.0200$ m; $V_1 = 0.626$ m/s

$\mathbf{R}_1 = D_1V_1/\nu = 43\,988$; $\therefore f_1 = 0.022\,97$ (from Eq. 8.41)

$\mathbf{R}_2 = D_2V_2/\nu = D_2V_1(D_1/D_2)^2/\nu = D_1^2V_1/(D_2\nu) = \mathbf{R}_1(D_1/D_2) = 70\,381$; $\therefore f_2 = 0.022\,50$

$\mathbf{R}_3 = \mathbf{R}_1(D_1/D_3) = 58\,651$; $\therefore f_3 = 0.022\,54$

Trial 2 (Eq. 8.89): $6 = 362V_1^2/2g$; $V_1 = 0.570$ m/s (9.8% change)

Trial 3 (Eq. 8.89): $6 = 367V_1^2/2g$; $V_1 = 0.566$ m/s (0.63% change, close enough)

$Q = A_1V_1 = (\pi/4)(0.08^2)0.566 = 0.002\,85$ m³/s = 2.85 L/s ◄

Alternative solution, by equation solver in computer software:

Establish the four standard equations for each pipe (see Sample Prob. 8.14*b*) plus Eq. 8.87, for a total of 13 governing equations. Assign known and guessed values to the variables (see, e.g., the solution to Prob. 8.81). Solve for the following 13 unknowns: Three values each of V, h, $\mathbf{R}$, and f, and one value of Q.

8.98 *Two pipes connected in series are respectively 150 ft of 2-in (e = 0.000 006 ft) and 450 ft of 8-in (e = 0.0009 ft) (Fig. P8.98). With a total head loss of 30 ft, find the flow of 60°F water, using a basic scientific calculator only. Neglect minor losses.*

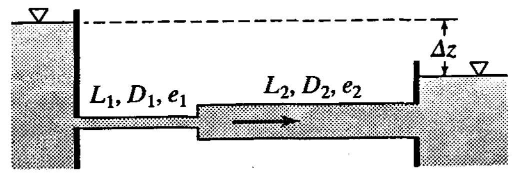

Figure P8.98

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Start with min (fully turbulent) trial values of f (Eq. 8.54):

Pipe	D, in	L, ft	e, ft	L/D	e/D	D_2/D	$(D_2/D)^4$	f_{min}
1	2	150	0.000 006	900	0.000 036	4	256	0.009 95
2	8	450	0.0009	675	0.001 35	1	1	0.0211

<u>Trial 1</u>: Eq. 8.89: $30 = (V_2^2/2g)[0.009\,95(900)256 + 0.0211(675)] = 2310V_2^2/2g$; $V_2 = 0.915$ fps

$D_1''V_1 = 2(0.915)4^2 = 29.3$; Fig. 8.11: $f_1 = 0.0165$ (40% change).

$D_2''V_2 = 8(0.915) = 7.32$; $f_2 = 0.025$ (15% change)

<u>Trial 2</u>: $30 = (V_2^2/2g)[0.0165(900)256 + 0.025(675]$; $V_2 = 0.711$ fps

$D_1''V_1 = 22.8$; $f_1 = 0.017$ (3% change)

$D_2''V_2 = 5.69$; $f_2 = 0.026$ (2% change)

Both of the $D''V$ (and so **R**) values are in the turbulent range.

<u>Trial 3</u>: $V_2 = 0.701$ fps (1.4% change, close enough).

$Q = A_2V_2 = (\pi/4)(8/12)^2 0.701 = 0.245$ cfs ◄

8.99 *Repeat Prob. 8.96 for the case where the fluid is an oil with $s = 0.92$, $\mu = 0.000\,96$ lb·sec/ft^2.*

Prob. 8.96: Suppose that in Fig. 8.29 pipes 1, 2, and 3 are 750 ft of 4-in, 250 ft of 2-in, and 300 ft of 3-in asphalt-dipped cast-iron pipe. With a total head loss of 25 ft between A and B, find the flow rate.

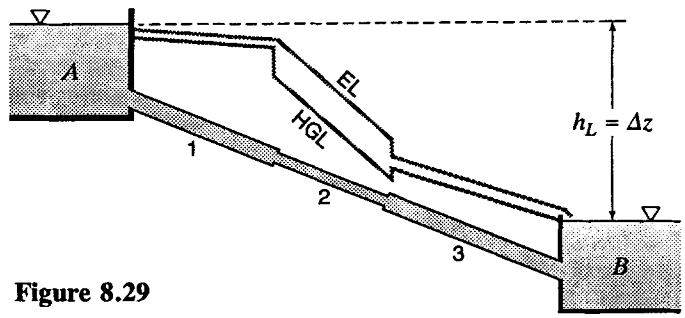

Figure 8.29

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$$\nu = \frac{\mu}{\rho} = \frac{0.000\,96}{0.92 \times 1.940} = 5.38 \times 10^{-4} \text{ ft}^2/\text{s}$$

Table 8.1: $e = 0.0004$ ft. Initially assume $f = f_{min}$ (fully rough value) from Eq. 8.54.

Pipe	L, ft	D, in	L/D	e/D	Try f	D_1/D	$(D_1/D)^4$
1	750	4	2250	0.0012	0.020 54	1.00	1.00
2	250	2	1500	0.0024	0.024 60	2.00	16.00
3	300	3	1200	0.0016	0.022 09	1.333	3.16

All L/D are > 1000, so we can neglect minor losses.

Eq. 8.89: $25 = [0.020\,54(2250) + 0.0246(1500)16 + 0.022\,09(1200)3.16]V_1^2/2g$

$V_1^2/2g = 25/720 = 0.0347$ ft; $V_1 = 1.495$ fps

$\mathbf{R}_1 = D_1V_1/\nu = 927$

$\mathbf{R}_2 = D_2V_2/\nu = D_2V_1(D_1/D_2)^2/\nu = D_1^2V_1/(D_2\nu) = \mathbf{R}_1(D_1/D_2) = 1853$

$\mathbf{R}_3 = \mathbf{R}_1(D_1/D_3) = 1236$

These estimates suggest that all three flows may be laminar, in which case a trial solution is unnecessary.

Use laminar flow Eq. 8.28: $h_f = (32\nu/g)[(L_1V_1/D_1^2) + (L_2V_2/D_2^2) + L_3V_3/D_3^2)]$

i.e. $25 = (32/32.2)(5.38 \times 10^{-4})[(L_1V_1/D_1^2) + (L_2V_1/D_2^2)(D_1/D_2)^2 + (L_3V_1/D_3^2)(D_1/D_3)^2]$

Substituting for the L's and D's we find: $25 = 27.4V_1$; $V_1 = 0.912$ fps

$Q = A_1V_1 = (\pi/4)(4/12)^2 0.912 = 0.0796$ cfs ◄

Check: $\mathbf{R}_1 = 565$, $\mathbf{R}_2 = 1131$, $\mathbf{R}_3 = 754$ (all laminar, so OK).

8.100 *Repeat Prob. 8.97 for the case where the fluid is an oil with s = 0.94, μ = 0.04 N·s/m².*

Prob. 8.97: Suppose that in Fig. 8.29 pipes 1, 2, and 3 are 150 m of 80-mm, 60 m of 50-mm, and 120 m of 60 mm wrought-iron pipe. With a total head loss of 6 m between A and B, find the flow rate.

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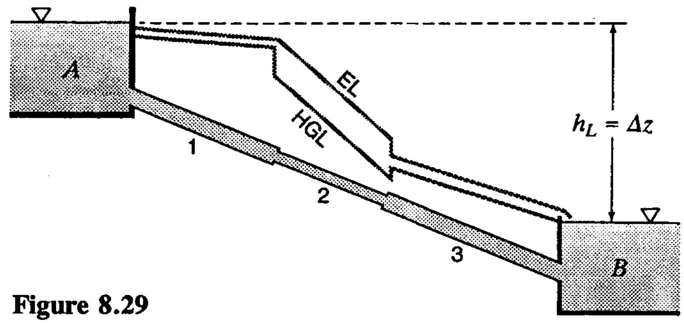

Figure 8.29

$$\nu = \frac{\mu}{\rho} = \frac{0.04}{0.94 \times 999} = 4.26 \times 10^{-5}\ \text{m}^2/\text{s}.$$

Table 8.1: e = 0.000 046 m. Initially assume $f = f_{min}$ (fully rough value) from Eq. 8.54.

Pipe	L, m	D, m	L/D	e/D	Try f	D_1/D	$(D_1/D)^4$
1	150	0.08	1875	0.000 575	0.017 24	1.00	1.00
2	60	0.05	1200	0.000 920	0.019 24	1.60	6.55
3	120	0.06	2000	0.000 767	0.018 43	1.333	3.16

All L/D are > 1000, so we can neglect minor losses.

Eq. 8.89: $6 = [0.017\,24(1875) + 0.019\,24(1200)6.55 + 0.018\,43(200)3.16]V_1^2/2g$

$V_1^2/2g = 6/300 = 0.0200$ m; $V_1 = 0.626$ m/s

$\mathbf{R}_1 = D_1V_1/\nu = 1176$

$\mathbf{R}_2 = D_2V_2/\nu = D_2V_1(D_1/D_2)^2/\nu = D_1^2V_1/(D_2\nu) = \mathbf{R}_1(D_1/D_2) = 1882$

$\mathbf{R}_3 = \mathbf{R}_1(D_1/D_3) = 1568$

These estimates suggest that all three flows may be laminar, in which case a trial solution is unnecessary.

Use laminar flow Eq. 8.28: $h_f = (32\nu/g)[(L_1V_1/D_1^2) + (L_2V_2/D_2^2) + (L_3V_3/D_3^2)]$

i.e. $6 = (32/9.81)(4.26 \times 10^{-4})[(L_1V_1/D_1^2) + (L_2V_1/D_2^2)(D_1/D_2)^2 + (L_3V_1/D_3^2)(D_1/D_3)^2]$

Substituting for the L's and D's we find: $6 = 20.0V_1$; $V_1 = 0.300$ m/s

$Q = A_1V_1 = (\pi/4)(0.08^2)0.300 = 0.001\,506\ \text{m}^3/\text{s} = 1.506$ L/s ◄

Check: $\mathbf{R}_1 = 563$, $\mathbf{R}_2 = 900$, $\mathbf{R}_3 = 750$ (all laminar, so OK).

8.101 *Three new cast-iron pipes, having diameters of 24, 21, and 18 in, respectively, each 450 ft long, are connected in series. The 24-in pipe leads from a reservoir (flush entrance), and the 18-in pipe discharges into the air at a point 15 ft lower than the reservoir water surface level. Assuming all changes in section to be abrupt, find the rate of discharge of water at 60°F.*

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Table 8.1: for new cast iron: $e = 0.000\,85$ ft.

Pipe	D (in)	L/D	e/D	Trial f	D_1/D	$(D_1/D)^4$
1	24	225	0.000 425	0.016	1.00	1.00
2	21	257	0.000 486	0.017	1.143	1.706
3	18	300	0.000 567	0.018	1.333	3.16
Total		782				

$\Sigma(L/D) = 782 < 1000$, so include minor losses.

Flush entrance, Fig. 8.13: $k_e = 0.5$

Sudden contractions, Table 8.2: $k_c = 0.0825$ for $D_2/D_1 = 0.875$; $k_c = 0.0986$ for $D_3/D_2 = 0.857$

Discharge: $k_d = 1.0$ or $h_L = V_3^2/2g$

Energy: $(0 + 15 + 0) = (0 + 0 + V_3^2/2g) + \Sigma h_L$, i.e.,

$15 = [0.5 + 0.016(225) + (0.0825 + 0.017 \times 257)1.706 + (0.0986 + 0.018 \times 300)3.16 + 1(3.16)]V_1^2/2g$

$V_1^2/2g = 15/32.2 = 0.465$ ft; $V_1 = \sqrt{2(32.2)0.465} = 5.47$ fps

$D_1''V_1 = 24(5.47) = 131.4$; $\therefore f_1 = 0.0167$ (from Fig. 8.11)

$D_2''V_2 = 21(5.47)1.143^2 = 150.1$; $\therefore f_2 = 0.0171$

$D_3''V_3 = 18(5.47)1.133^2 = 175.2$; $\therefore f_3 = 0.0175$

$15 = [0.5 + 0.0167(225) + (0.0825 + 0.0171 \times 257)1.706 + (0.0986 + 0.0175 \times 300)3.16 + 1(3.16)]V_1^2/2g$

$V_1^2/2g = 15/32.0 = 0.469$; $V_1 = \sqrt{2(32.2)0.469} = 5.50$ fps (little change)

$Q = A_1V_1 = (\pi/4)(2^2)5.50 = 17.27$ cfs ◀

Sec. 8.31: Pipes in Parallel -- *Problems* 8.102–8.107

8.102 *Suppose that in Fig. 8.30 pipes 1, 2, and 3 are of smooth brass as follows: 500 ft of 2-in, 350 ft of 3-in; and 750 ft of 4-in respectively. When the total flow of 70°F crude oil (s = 0.855) is 0.7 cfs, find the head loss from A to B and the flow in each pipe. Note: This problem can be solved without trial and error, using a basic scientific calculator only.*

A basic scientific calculator is here defined to be one that is not programmable and does not have automatic equation solving capabilities.

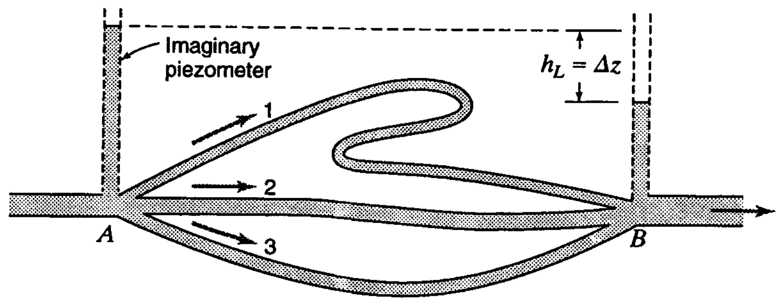

Figure 8.30

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Fig. A.2 for crude oil (s = 0.855) at 70°F: $\nu = 7.6 \times 10^{-5}$ ft²/sec

Assume the three Reynolds numbers are in the range ($3000 < \mathbf{R} < 10^5$) where Eq. 8.53 (from Blasius) for smooth pipe may be used.

Eq. 8.53: $h_f = 0.1580\nu^{0.25}\dfrac{LV^{1.75}}{gD^{1.25}} = (\text{const})\dfrac{LV^{7/4}}{D^{5/4}}$, where const $= \dfrac{0.1580\nu^{1/4}}{g}$

Hence Eq. 8.91: $(h_f)_1 = (h_f)_2 = (h_f)_3$ yields $\dfrac{L_1V_1^{7/4}}{D_1^{5/4}} = \dfrac{L_2V_2^{7/4}}{D_2^{5/4}} = \dfrac{L_3V_3^{7/4}}{D_3^{5/4}}$

Substituting for the L's and D's: $210V_1^{7/4} = 88.6V_2^{7/4} = 132.6V_3^{7/4}$; so $V_2 = 1.638V_1$, $V_3 = 1.301V_1$

Eq. 8.90: $Q = 0.70 = 0.0218V_1 + 0.0491(1.638V_1) + 0.0873(1.301V_1) = 0.216V_1$; $V_1 = 3.24$ fps.

$\mathbf{R}_1 = 7114$, $\mathbf{R}_2 = 17{,}478$, $\mathbf{R}_3 = 18{,}516$, all in the required range, so the assumed Eq. 8.53 is valid.

$Q_1 = (0.0218)3.24 = 0.0707$ cfs ◄

$Q_2 = (0.0491)1.638V_1 = 0.261$ cfs ◄

$Q_3 = (0.0873)1.301V_1 = 0.368$ cfs ◄ Check: $\Sigma Q = 0.700$ cfs, right on!

$h_f = h_{f1} = \dfrac{0.1580\nu^{1/4}}{g}\dfrac{500(3.24)^{7/4}}{(2/12)^{5/4}} = 16.87$ ft ◄

Answers may vary somewhat due to values read from Fig. A.2.

8.103 *Repeat Prob. 8.102 for the case where the total flow rate is 0.07 cfs. Does the "Note" still apply?*

Prob. 8.102: Suppose that in Fig. 8.30 pipes 1, 2, and 3 are of smooth brass as follows: 500 ft of 2-in, 350 ft of 3-in; and 750 ft of 4-in respectively. When 70°F crude oil (s = 0.855) is flowing, find the head loss from A to B and the flow in each pipe. Note: This problem can be solved without trial and error, using a basic scientific calculator only.

A basic scientific calculator is here defined to be one that is not programmable and does not have automatic equation solving capabilities.

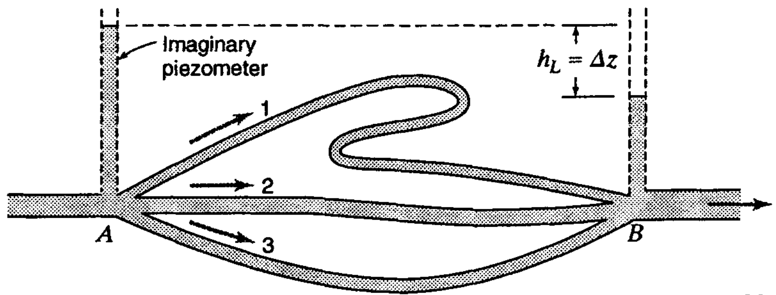

Figure 8.30

BG

Fig. A.2 for crude oil (s = 0.855) at 70°F: $\nu = 7.6 \times 10^{-5}$ ft²/sec

$\bar{V} = \Sigma Q/\Sigma A = 0.07/(0.0218 + 0.0491 + 0.0873) = 0.442$ fps

With this $\bar{V}$ in the largest pipe, $\mathbf{R}_{max} = (4/12)\bar{V}/\nu = 1940$

Thus all three flows may be laminar, so use laminar Eq. 8.28 with Eq. 8.91.

From Eq. 8.28: $gh_f/(32\nu) = L_1V_1/D_1^2 = L_2V_2/D_2^2 = L_3V_3/D_3^2 =$ const (thus eliminating f)

$500V_1/2^2 = 350V_2/3^2 = 750V_3/4^2$; $V_2 = 3.21V_1$, $V_3 = 2.67V_1$

Eq. 8.90: $0.07 = 0.0218V_1 + 0.0491(3.21V_1) + 0.0873(2.67V_1)$; $V_1 = 0.1698$ fps;

$\mathbf{R}_1 = 372$, $\mathbf{R}_2 = 1795$, $\mathbf{R}_3 = 1986$. All the flows are laminar, so the equations used are valid.

Since we can eliminate f from laminar flow, we can solve without T & E, so the "Note" does apply ◄

$Q_1 = A_1V_1 = 0.003\,70$ cfs ◄ $Q_2 = 0.0268$ cfs ◄ $Q_3 = 0.0395$ cfs ◄

Eq. 8.28: $h_L = h_{L1} = h_{f1} = 32(7.6 \times 10^{-5})500(0.1698)/[32.2(2/12)^2] = 0.231$ ft ◄

Answers may vary somewhat due to values read from Fig. A.2.

8.104 *Repeat Prob. 8.102 for the case where all the pipes are of galvanized iron. Does the "Note" still apply?*

Prob. 8.102: In Fig. 8.30, pipe 1 is 500 ft of 2-in, pipe 2 is 350 ft of 3-in, and pipe 3 is 750 ft of 4-in diameter, all of smooth brass. Crude oil (s = 0.855) at 70°F is flowing at 0.7 cfs. Find the head loss from A to B, and the flow in each pipe. Note: This problem can be solved without trial and error, using a basic scientific calculator only.

A basic scientific calculator is here defined to be one that is not programmable and does not have automatic equation solving capabilities.

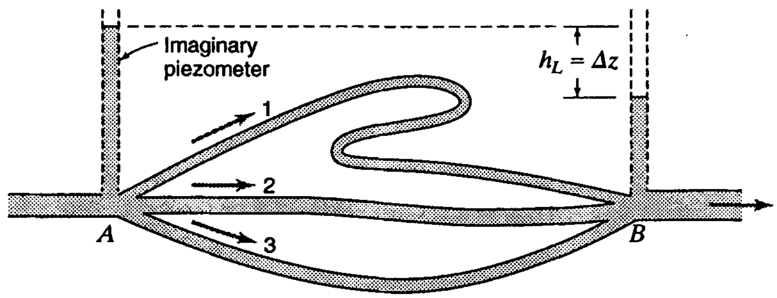

Figure 8.30

BG

Fig. A.2 for crude oil (s = 0.855) at 70°F: $\nu = 7.6 \times 10^{-5}$ ft²/sec. Table 8.1: e = 0.0005 ft.

$$V = \frac{Q}{\Sigma A} = \frac{4Q}{\pi(D_1^2 + D_2^2 + D_3^2)} = \frac{4(0.7)144}{\pi(2^2 + 3^2 + 4^2)} = 4.43 \text{ fps}$$

for which $\mathbf{R}_2 = \dfrac{(3/12)4.43}{7.6 \times 10^{-5}} = 13{,}973$ so laminar flow seems very unlikely.

Eq. 8.91 using Eq. 8.13: $f_1(L_1/D_1)V_1^2/2g = f_2(L_2/D_2)V_2^2/2g = f_3(L_3/D_3)V_3^2/2g$

For the first trial assume the f's are the same, so cancel. Substituting for L's and D's:

$3000V_1^2 = 1400V_2^2 = 2250V_3^2$; so $V_2 = 1.464V_1$, $V_3 = 1.155V_1$

Eq. 8.90: $Q = 0.7 = 0.0218V_1 + 0.0491(1.464V_1) + 0.0873(1.155V_1) = 0.1944V_1$; $V_1 = 3.60$ fps

$\mathbf{R}_1 = D_1V_1/\nu = 7895$, $\mathbf{R}_2 = 17{,}336$, $\mathbf{R}_3 = 18{,}233$

As $e > 0$, Blasius Eq. 8.48 may not be used, and from the **R** values the flows are not laminar. So the "Note" does not apply. ◀

For 2nd trial: From Eq. 8.52: $f_1 = 0.036\,57$, $f_2 = 0.030\,18$, $f_3 = 0.029\,05$

Eq. 8.91: $0.036\,57(3000)V_1^2 = 0.030\,18(1400)V_2^2 = 0.029\,05(2250)V_3^2$; so $V_2 = 1.611V_1$, $V_3 = 1.296V_1$

Eq. 8.90: $0.7 = 0.218V_1 + 0.0491(1.611V_1) + 0.0873(1.296V_1) = 0.214V_1$

$V_1 = 3.27$ fps (10.0% change); $\mathbf{R}_1 = 7174$, $\mathbf{R}_2 = 17{,}341$, $\mathbf{R}_3 = 18{,}589$

For 3rd trial: From Eq. 8.52: $f_1 = 0.037\,33$, $f_2 = 0.030\,18$, $f_3 = 0.028\,95$

Eq. 8.91: $0.037\,33(3000)V_1^2 = 0.030\,18(1400)V_2^2 = 0.028\,95(2250)V_3^2$; so $V_2 = 1.628V_1$, $V_3 = 1.311V_1$

Eq. 8.90: $0.7 = 0.0218V_1 + 0.0491(1.628V_1) + 0.0873(1.311V_1) = 0.216V_1$

$V_1 = 3.24$ fps (1.02% change, close enough!). $Q_1 = A_1V_1 = 0.0218(3.24) = 0.0707$ cfs ◀

$Q_2 = 0.0491(1.628 \times 3.24) = 0.259$ cfs ◀ $Q_3 = 0.0873(1.311 \times 3.24) = 0.371$ cfs ◀

Eq. 8.13: $h_L = h_{L1} = h_{f1} = 0.037\,33(3000)(3.24)^2/2g = 18.24$ ft ◀

Answers may vary somewhat due to values read from Fig. A.2.

8.105 *Suppose that, in Fig. 8.30, pipes 1, 2, and 3 are of smooth plastic tubing as follows: 90 m of 20 mm, 150 m of 40 mm and 80 m of 60 mm, respectively. When the total flow of 50°C crude oil (s = 0.855) is 7 L/s, find the head loss from A to B and the flow in each pipe. Note: This problem can be solved without trial and error, using a basic scientific calculator only.*

A basic scientific calculator is here defined to be one that is not programmable and does not have automatic equation solving capabilities.

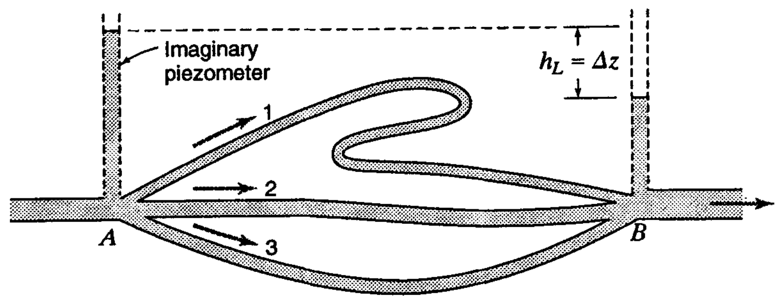

Figure 8.30

SI

Fig. A.2 for crude oil (s = 0.855) at 50°C: $\nu = 3.8 \times 10^{-6}$ m²/s

Assume Blasius' Eq. 8.48 applies for smooth plastic pipe and then check if $3000 < \mathbf{R} < 10^5$.

Based on Eq. 8.48, Eq. 8.53: $h_f = 0.1580\nu^{0.25}LV^{1.75}/(gD^{1.25})$ i.e., $h_f \propto LV^{1.75}/D^{1.25}$.

So in Eq. 8.91: $90V_1^{1.75}/2^{1.75} = 150V_2^{1.75}/4^{1.25} = 80V_3^{1.75}/6^{1.25}$

$37.8V_1^{1.75} = 26.5V_2^{1.75} = 8.52V_3^{1.75}$; so $V_1 = 0.816V_2$, $V_3 = 1.913V_2$

Eq. 8.90: $Q = 0.007 = (\pi/4)[0.02^2(0.816V_2) + 0.04^2V_2 + 0.06^2(1.913V_2)]$; $V_2 = 1.011$ m/s

Check: $\mathbf{R}_2 = 0.04(1.011)/(3.8 \times 10^{-6}) = 10\,644$; $\mathbf{R}_1 = 4343$; $\mathbf{R}_3 = 30\,547$

These **R** are all in the required range, so assumed Eq. 8.48 <u>does</u> apply.

Thus $V_1 = 0.825$ m/s, $V_2 = 1.011$ m/s, $V_3 = 1.935$ m/s

$Q_1 = 0.259$ L/s ◄ $Q_2 = 1.271$ L/s ◄ $Q_3 = 5.47$ L/s ◄

$h_L = h_{L1} = h_{f1} = 0.316(3.8 \times 10^{-6})^{0.25}150(1.011)^{1.75}/[(0.04)^{1.25}2(9.81)] = 6.08$ m ◄

Answers may vary somewhat due to values read from Fig. A.2.

8.106 *Repeat Prob. 8.105 for the case where the total flow rate is 0.35 L/s. Does the "Note" still apply?*

Prob. 8.105: In Fig. 8.30 pipes 1,2, and 3 are of smooth plastic tubing as follows: 90 m of 20-mm, 150 m of 40-mm, and 80 m of 60-mm, respectively. When crude oil (s = 0.855) at 50°C is flowing, find the head loss from A to B and the flow in each pipe. Note: This problem can be solved without trial and error, using a basic scientific calculator only.

A basic scientific calculator is here defined to be one that is not programmable and does not have automatic equation solving capabilities.

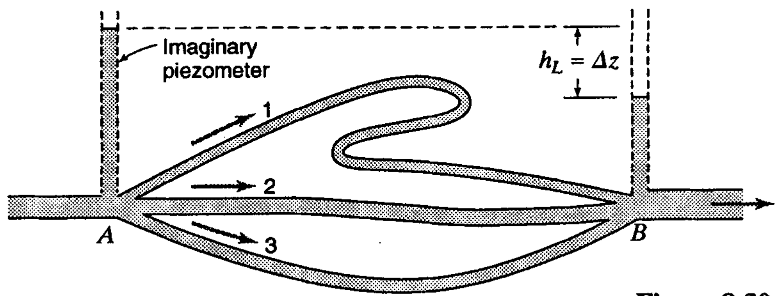

Figure 8.30

SI

Fig. A.2 for crude oil (s = 0.855) at 50°C: ν = 3.8×10^{-6} m²/s

$\bar{V} = Q/\Sigma A = (0.000\,35\text{ m}^3/\text{s})/[(\pi/4)(0.02^2 + 0.04^2 + 0.06^2)\text{ m}^2] = 0.0796$ m/s (low)

With this $\bar{V}$ in the largest pipe, $\mathbf{R}_{max} = 0.06(0.0796)/(3.86 \times 10^{-6}) = 1256$

So assume all the flows are laminar, use Eq. 8.28, and then check **R** values.

Eq. 8.28: $h_f = 32\nu(L/gD^2)V$; $h_L \propto LV/D^2$

From Eq. 8.91: $90V_1/2^2 = 150V_2/4^2 = 80V_3/6^2$; $V_1 = 0.417V_2$, $V_3 = 4.22V_2$

Eq. 8.90: $Q = 0.000\,35 = \Sigma(AV) = (\pi/4)[0.02^2(0.417V_2) + 0.04^2V_2 + 0.06^2(4.22V_2)] = (\pi/4)0.016\,95V_2$

$V_2 = 0.0263$ m/s, $V_1 = 0.010\,95$ m/s, $V_3 = 0.1109$ m/s

Eq. 8.1: $\mathbf{R} = DV/\nu$: $\mathbf{R}_1 = 57.7$, $\mathbf{R}_2 = 277$, $\mathbf{R}_3 = 1751$

All 3 flows <u>are</u> laminar, so (*a*) Eq. 8.28 and the results are valid, and
(*b*) we can eliminate f and solve without T&E, so the "Note" <u>does</u> still apply ◄

$(\pi/4)D_1^2V_1 = Q_1 = 0.003\,44$ L/s ◄ $Q_2 = 0.0330$ L/s ◄ $Q_3 = 0.314$ L/s ◄

Check: $\Sigma Q = 0.350$ L/s (as required)

Eq. 8.28: $h_L = h_{L1} = h_{f1} = 32(3.8 \times 10^{-6})\dfrac{90}{9.81 \times 0.02^2}0.010\,95 = 0.0305$ m ◄

Answers may vary somewhat due to values read from Fig. A.2.

8.107 *Repeat Prob. 8.105 for the case where all the pipes are of galvanized iron. Does the "Note" still apply?*

Prob. 8.105: In Fig. 8.30 pipe 1 is 90 m of 20-mm, pipe 2 is 150 m of 40-mm, and pipe 3 is 80 m of 60-mm diameter, all of drawn copper tubing. Crude oil (s = 0.855) at 50°C is flowing at 7 L/s. Find the head loss from A to B, and the flow in each pipe. Note: This problem can be solved without trial and error, using a basic scientific calculator only.

A basic scientific calculator is here defined to be one that is not programmable and does not have automatic equation solving capabilities.

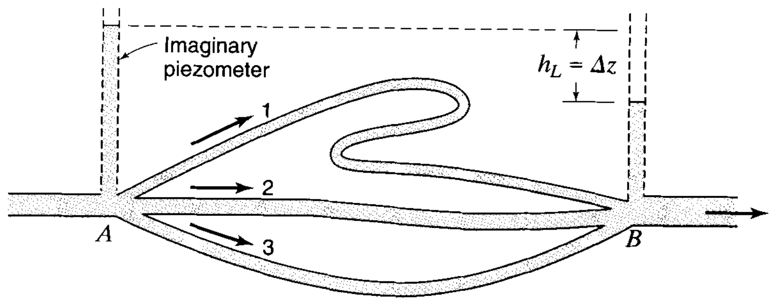

Figure 8.30

SI

Fig. A.2 for crude oil (s = 0.855) at 50°C: $\nu = 3.8 \times 10^{-6}$ m²/s. Table 8.1: e = 0.000 15 m

Eq. 8.91 using Eq. 8.13: $f_1(L_1/D_1)V_1^2/2g = f_2(L_2/D_2)V_2^2/2g = f_3(L_3/D_3)V_3^2/2g$

<u>For the first trial</u> assume the f's are the same, so cancel. Substituting for L's and D's:

$4500V_1^2 = 3750V_2^2 = 1333V_3^2$; so $V_2 = 1.095V_1$, $V_3 = 1.837V_1$

Eq. 8.90: $Q = 0.007 = (\pi/4)[0.02^2V_1 + 0.04^2(1.095V_1) + 0.06^2(1.837V_1) = 0.006\,89V_1$

V_1 = 1.017 m/s; $\mathbf{R}_1 = D_1V_1/\nu = 5351$, $\mathbf{R}_2 = 11\,723$, $\mathbf{R}_3 = 29\,491$

As $e > 0$, Blasius Eq. 8.48 may not be used, and from the **R** values the flows are not laminar.
So the "Note" does <u>not</u> apply. ◄

<u>For 2nd trial</u>: Eq. 8.52: $f_1 = 0.044\,43$, $f_2 = 0.034\,91$, $f_3 = 0.028\,86$

Eq. 8.91: $0.044\,43(4500)V_1^2 = 0.034\,91(3750)V_2^2 = 0.028\,86(1333)V_3^2$; so $V_2 = 1.236V_1$, $V_3 = 2.28V_1$

Eq. 8.90: $0.007 = (\pi/4)[0.02^2V_1 + 0.04^2(1.236V_1) + 0.06^2(2.28V_1)] = 0.008\,31V_1$

V_1 = 0.842 m/s (21% change); $\mathbf{R}_1 = 4432$, $\mathbf{R}_2 = 10\,955$, $\mathbf{R}_3 = 30\,310$

<u>For 3rd trial:</u> Eq. 8.52: $f_1 = 0.046\,08$, $f_2 = 0.035\,30$, $f_3 = 0.028\,77$

Eq. 8.91: $0.046\,08(4500)V_1^2 = 0.035\,30(3750)V_2^2 = 0.028\,77(1333)V_3^2$; so $V_2 = 1.252V_1$, $V_3 = 2.33V_1$

Eq. 8.90: $0.007 = (\pi/4)[0.02^2V_1 + 0.04^2(1.252V_1) + 0.06^2(2.33V_1) = 0.008\,46V_1$

V_1 = 0.827 m/s (1.8% change, close enough!)

$Q_1 = A_1V_1 = (\pi/4)0.02^2(0.827) = 0.000\,260$ m³s = 0.260 L/s ◄

$Q_2 = (\pi/4)0.04^2(1.252 \times 0.827) = 0.001\,301$ m³/s = 1.301 L/s ◄

$Q_3 = (\pi/4)0.06^2(2.33 \times 0.827) = 0.005\,44$ m³/s = 5.44 L/s ◄

Eq. 8.13: $h_L = h_{L1} = h_{f1} = 0.046\,08(4500)(0.827)^2/2g = 7.23$ m ◄

Answers may vary somewhat due to values read from Fig. A.2.

Sec. 8.32: Pipe Networks -- *Problems 8.108–8.120*

8.108 *Referring to Fig. P8.108, A is at elevation 30 ft, and the pipe characteristics are as follows: pipe B is 5000 ft long, of 3 ft diameter, with f = 0.035; pipe E is 4500 ft long, of 2 ft diameter, with f = 0.035; and pipe C is 5000 ft long, of 3 ft diameter, with f = 0.025. When the pump develops 30 ft of head, the velocity of flow in pipe C is 5 fps. Neglecting minor losses, find (a) the flow rates in cubic feet per second in pipes B and E under these conditions, and (b) the elevation of the discharge end of pipe E.*

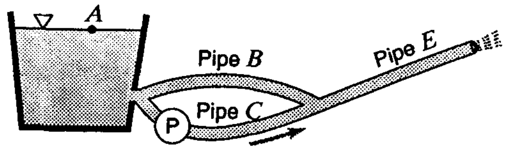

Figure P8.108

BG

(*a*) Energy Drop across *A*, Eq. 8.13: $(h_L)_A = 0.035(5000/3)V_A^2/2g = 58.3V_A^2/2g$

Given $V_C = 5$ fps, energy drop across $C = (h_L)_C - h_p = 0.025(5000/3)5^2/2g - 30 = -13.83$ ft

Hence energy is greater at right end of pipe *C* and flow will be to the left in pipe *A*.

$58.3(V_A^2/2g) = 13.83$; $V_A = 3.91$ fps; $Q_A = (\pi 3^2/4)3.91 = 27.6$ cfs to left ◄

$Q_C = (\pi 3^2/4)5 = 35.3$ cfs to right; $Q_C - Q_A = Q_B = 7.73$ cfs ◄

(*b*) $V_B = Q_B/A_B = 7.73/(\pi 2^2/4) = 2.46$ fps; $V_B^2/2g = 0.0940$ ft

$h_{LB} = f(L/D)V^2/2g = 0.035(4500/2)0.0940 = 7.40$ ft

Energy, water surface to jet *J*: $H_S + h_{LA} - h_{LB} = H_J$

$(0 + 30 + 0) + 13.83 - 7.40 = (0 + z_J + 0.0940)$; $z_J = 36.3$ ft ◄

8.109 *Repeat Prob. 8.108 for the case where the velocity in pipe C is 6 fps with all other data remaining the same.*

Prob. 8.108: Referring to Fig. P8.108, A is at elevation 30 ft, and the pipe characteristics are as follows: pipe B is 5000 ft long, of 3 ft diameter, with f = 0.035; pipe E is 4500 ft long, of 2 ft diameter, with f = 0.035; and pipe C is 5000 ft long, of 3 ft diameter, with f = 0.025. The pump develops 30 ft of head. Neglecting minor losses, find (a) the flow rates in cubic feet per second in pipes B and E under these conditions, and (b) the elevation of the discharge end of pipe E.

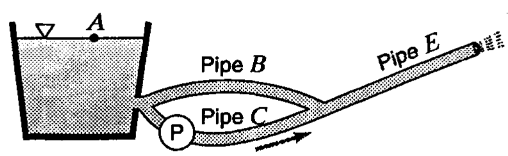

Figure P8.108

BG

(*a*) Eq. 8.13: $(h_L)_A = 58.3V_A^2/2g$; $(\Delta E)_C = (h_L)_C - h_p = (41.7)6^2/2g - 30 = -6.71$ ft

Energy is greater at right end of pipe *C* thus flow will be to the left in pipe *A*

$58.3(V_A^2/2g) = 6.71$; $V_A = 2.72$ fps, $Q_A = 19.24$ cfs to the left ◄

$V_C = 6.0$ fps (given), $Q_C = 42.4$ cfs to the right

$Q_B = Q_C - Q_A = 42.4 - 19.24 = 23.2$ cfs ◄

(*b*) $V_B = Q_B/A_B = 23.2/(\pi 2^2/4) = 7.38$ fps; $V_B^2/2g = 0.845$ ft

$h_{LB} = f(L/D)V^2/2g = 0.035(4500/2)0.845 = 66.5$ ft

Energy, water surface to jet *J*: $H_S + h_{LA} - h_{LB} = H_J$

$(0 + 30 + 0) + 6.71 - 66.5 = (0 + z_J + 0.845)$; $z_J = -30.7$ ft ◄

8.110 *Refer to Fig. P8.108. Assume the water surface in the reservoir is instead at elevation 200 m. Pipes B, C, and E are all 600 m long, and they all have a diameter of 500 mm with $f = 0.030$. When the pump develops 15 m of head, the velocity in pipe C is 5.0 m/s. Neglecting minor losses, find (a) the flow rate in all pipes and (b) the elevation of the discharge end of pipe B.*

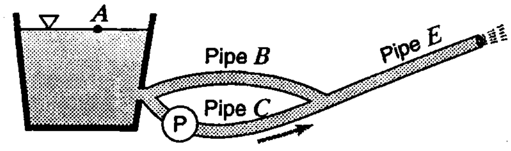

Figure P8.108

BG

(*a*) Δ Energy head across pipe $C = h_p - h_{L_C} = 15 - 0.030(600/0.50)5^2/(2 \times 9.81) = -30.9$ m

Thus flow in pipe A is to the right.

$h_{LA} = 30.9 \text{ m} = 0.030(600/0.50)V_A^2/(2 \times 9.81)$; $V_A = 4.10$ m/s

$Q_A = \pi 0.25^2(4.10) = 0.805 \text{ m}^3/\text{s}$ ◄

$Q_C = \pi 0.25^2(5) = 0.982 \text{ m}^3/\text{s}$ ◄

$Q_B = Q_A + Q_C = 1.787 \text{ m}^3/\text{s}$ ◄

(*b*) $V_B = \dfrac{Q_B}{A_B} = \dfrac{1.787}{\pi 0.5^2/4} = 9.10$ m/s; $\dfrac{V_B^2}{2g} = 4.22$ m. $h_{LB} = f\dfrac{L}{D}\dfrac{V^2}{2g} = 0.030\dfrac{600}{0.50}4.22 = 152.0$ m

Energy, water surface to jet J: $H_S - h_{LA} - h_{LB} = H_J$

$(0 + 200 + 0) - 30.9 - 152.0 = (0 + z_J + 4.22)$; $z_J = 12.90$ m ◄

8.111 *In Fig. P8.111, pipe AB is 1200 ft long, of 8 in diameter, with $f = 0.035$; pipe BC (upper) is 800 ft long, of 6 in diameter, with $f = 0.025$; pipe BC (lower) is 900 ft long, of 4 in diameter, with $f = 0.045$; and pipe CD is 500 ft long, of 6 in diameter, with $f = 0.025$. The elevations are reservoir water surface = 150 ft, A = 120 ft, B = 70 ft, C = 60 ft, and D = 30 ft. There is free discharge to the atmosphere at D. Neglecting velocity heads, (a) compute the flow in each pipe and (b) determine the pressures at B and C.*

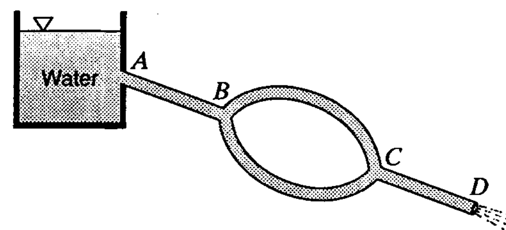

Figure P8.111

BG

(*a*) B to C, Eq. 8.91: $0.025(1600)V_u^2/2g = 0.045(2700)V_L^2/2g$; $V_u = 1.743V_L$

Continuity: $Q = 0.349V_1 = 0.1963(1.743V_L) + 0.0873V_L = 0.1963V_2$

from which $V_1 = 1.230V_L$, $V_2 = 2.19V_L$. All $L/D > 1000$, therefore neglect minor losses.

Energy Eq. (neglecting velocity heads) from water surface to D:

$$(0 + 150) = (0 + 30) + h_{L1} + h_{LL} + h_{L2}. \quad \therefore \; h_L = 150 - 30 = f_1\frac{L_1}{D_1}\frac{V_1^2}{2g} + f_L\frac{L_L}{D_L}\frac{V_L^2}{2g} + f_2\frac{L_2}{D_2}\frac{V_2^2}{2g}$$

$$120 = 0.035(1800)(1.230V_L)^2/2g + 0.045(2700)V_L^2/2g + 0.025(1100)(2.19V_L)^2/2g = 348V_L^2/2g$$

$V_L = 4.71$ fps, $Q_L = 0.411$ cfs ◄ $V_u = 8.21$ fps, $Q_u = 1.612$ cfs ◄

$V_1 = 5.79$ fps, $Q_1 = 2.02$ cfs ◄ $V_2 = 10.30$ fps, $Q_2 = 2.02$ cfs ◄

(*b*) Energy, water surface to B: $(0 + 150) = (p_B/\gamma + 70) + (h_L)_{AB}$

$p_B/\gamma = 80 - 0.035(1800)(5.79)^2/2g = 47.16$ ft; $p_B = 47.16(62.4/144) = 20.4$ psi ◄

Energy, C to D: $(p_C/\gamma + 60) = (0 + 30) + (h_L)_{CD}$

$p_C/\gamma = 0.025(1100)(10.30^2/2g) - 30 = 15.31$ ft; $p_C = 15.31(62.4/144) = 6.64$ psi ◄

8.112 *In Fig. P8.111, pipe AB is 600 m long, of 180 mm diameter, with f = 0.035; pipe BC (upper) is 500 m long, of 120 mm diameter, with f = 0.025; pipe BC (lower) is 400 m long, of 160 mm diameter, with f = 0.030; and pipe CD is 900 m long, of 320 mm diameter, with f = 0.020. The elevations are: reservoir water surface = 150 m, A = 100 m, B = 60 m, C = 50 m, D = 20 m. There is free discharge to the atmosphere at D. Neglecting velocity heads, (a) compute the flow in each pipe and (b) determine the pressures at B and C. Comment on the practicality of this system.*

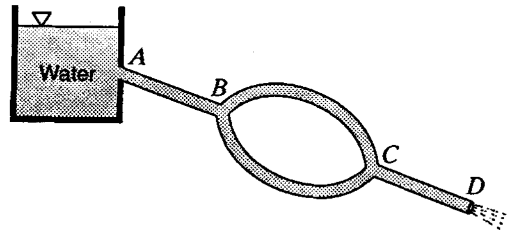

Figure P8.111

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(*a*) *B* to *C*, Eq. 8.91: $0.025(4167)V_u^2/2g = 0.030(2500)V_L^2/2g$; $V_u = 0.849V_L$

Continuity: $Q = 0.0254V_1 = 0.011\,31(0.849V_L) + 0.0201V_L = 0.0804V_2$

from which $V_1 = 1.167V_L$, $V_2 = 0.369V_L$

Energy Eq. (neglecting velocity heads) from water surface to *D*:

All $L/D > 1000$, therefore neglect minor losses

$(0 + 150) = (0 + 20) + h_{L1} + h_{LL} + h_{L2}$

$$\therefore\ h_L = 150 - 20 = f_1\frac{L_1}{D_1}\frac{V_1^2}{2g} + f_L\frac{L_L}{D_L}\frac{V_L^2}{2g} + f_2\frac{L_2}{D_2}\frac{V_2^2}{2g}$$

$130 = 0.035(3333)(1.167V_L)^2/2g + 0.030(2500)V_L^2/2g + 0.020(2813)(0.369V_L)^2/2g = 242V_L^2/2g$

$V_L = 3.25$ m/s, $Q_L = 0.0653\ \text{m}^3/\text{s} = 65.3$ L/s ◄

$V_u = 2.76$ m/s, $Q_u = 0.0312\ \text{m}^3/\text{s} = 31.2$ L/s ◄

$V_1 = 3.79$ m/s, $Q_1 = 0.0965\ \text{m}^3/\text{s} = 96.5$ L/s ◄

$V_2 = 1.200$ m/s, $Q_2 = 0.0965\ \text{m}^3/\text{s} = 96.5$ L/s ◄

(*b*) Energy, water surface to *B*: $(0 + 150) = (p_B/\gamma + 60) + (h_L)_{AB}$

$p_B/\gamma = 90 - 0.035(3333)(3.79)^2/2g = 4.48$ m; $p_B = 4.48(9.81) = 43.9$ kPa ◄

Energy, *C* to *D*: $(p_C/\gamma + 50) = (0 + 20) + (h_L)_{CD}$

$p_C/\gamma = 0.020(2813)(1.200^2/2g) - 30 = -25.9$ m (impossible) ◄

Comment: The head loss in the parallel pipes is so great that it wants to pull the HGL down below zero absolute pressure. ∴ The system will not function. ◄

8.113 *A 10-in cast-iron pipe 1500 ft long forms one link in a pipe network. If the velocities to be encountered are assumed to fall within the range of 4 to 10 fps, derive an equation for the flow of water at 60°F in this pipe in the form* $h_f = KQ^n$. Hint: *Using information from Fig. 8.11 and Table 8.1, set up two simultaneous equations corresponding to the ends of the desired velocity range, then solve for the unknowns K and n.*

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Table 8.1: e = 0.000 85 ft; e/D = 0.000 85(12/10) = 0.001 02

(*i*) At 4 fps: $D''V = 40$. Fig. 8.11 or Eq. 8.52: f = 0.020 77

Eq. 8.13: $h_f = 0.020\,77[1500/(10/12)]4^2/(2 \times 32.2) = 9.29$ ft; $Q = AV = 2.18$ cfs

Given $h_f = KQ^n$. $\therefore$ $9.29 = K(2.18)^n$; $\log 9.29 = \log K + n \log 2.18$ (1)

(*ii*) At 10 fps: $D''V = 100$. Fig. 8.11 or Eq. 8.52: f = 0.020 17

Eq. 8.13: $h_f = 0.020\,17[1500/(10/12)]10^2/(2 \times 32.2) = 56.4$ ft; $Q = 5.45$ cfs

$\therefore$ $56.4 = K(5.45)^n$; $\log 56.4 = \log K + n \log 5.45$ (2)

(2) − (1): $\log(56.4/9.29) = n \log(5.45/2.18)$; $n = 1.968$; $K = 56.4/5.45^{1.968} = 2.00$

So required equation is $h_f = 2.00Q^{1.968}$ ◀

8.114 *A 250 mm cast-iron pipe 400 m long forms one link in a pipe network. If the velocities to be encountered are assumed to fall within the range of 0.75 to 3 m/s, derive an equation for the flow of water at 15°C in this pipe in the form* $h_f = KQ^n$. Hint: *See Prob. 8.113.*

Prob. 8.113 hint: Using information from Fig. 8.11 and Table 8.1, set up two simultaneous equations corresponding to the ends of the desired velocity range, then solve for the unknowns K and n.

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Table A.1 for water at 15°C: $\nu = 1.139 \times 10^{-6}$ m²/s

Table 8.1: e = 0.25 mm; e/D = 0.25/250 = 0.001

(*i*) At 0.75 m/s: $\mathbf{R} = DV/\nu = 164\,600$. Fig. 8.11 or Eq. 8.52: f = 0.021 30

Eq. 8.13: $h_L = 0.021\,30(400/0.25)0.75^2/(2 \times 9.81) = 0.977$ m; $Q = AV = 0.0368$ m³/s

Given $h_L = KQ^n$. $\therefore$ $0.977 = K(0.0368)^n$; $\log 0.977 = \log K + n \log 0.0368$ (1)

(*ii*) At 2.3 m/s: $\mathbf{R} = 504\,800$. Fig. 8.11 or Eq. 8.52: f = 0.020 23

Eq. 8.13: $h_L = 0.020\,23(400/0.25)2.3^2/(2 \times 9.81) = 8.73$ m; $Q = 0.1129$ m³/s

$\therefore$ $8.73 = K(0.1129)^n$; $\log 8.73 = \log K + n \log 0.1129$ (2)

(2) − (1): $\log(8.73/0.977) = n \log(0.1129/0.0368)$; $n = 1.954$; $K = 8.73/0.1129^{1.954} = 619$

So the required equation is $h_L = 619Q^{1.954}$ ◀

8.115 *The pipes in the system shown in Fig. P8.115 are all galvanized iron. (a) With a flow of 15 cfs, find the head loss from A to D. (b) What should be the diameter of a single pipe from B to C such that it replaces pipes 2, 3, and 4 without altering the capacity for the same head loss from A to D?*

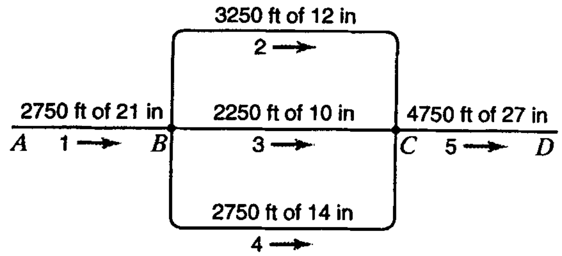

Figure P8.115

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Table 8.1: $e = 0.0005$ ft.

(*a*) Trial 1: Assume $f = f_{min}$ (complete turbulence), from Eq. 8.54.

Pipe	*D* (in)	*L/D*	*e/D*	Try *f*	*A* (ft²)
2	12	3250	0.0005	0.0167	0.785
3	10	2700	0.0004	0.0174	0.545
4	14	2357	0.000 429	0.0161	1.069

All *L/D* are > 1000, so neglect minor losses. Then in Eq. 8.93, $C = A\sqrt{2gD/fL}$

Pipe	*C*	% discharge	*Q* cfs	*V* fps	*D''V*	*f* (Fig. 8.11 or Eq. 8.52)	New *C*
2	0.856	29.6	4.45	5.66	67.9	0.017 64	0.832
3	0.639	22.1	3.32	6.09	60.9	0.018 32	0.622
4	1.393	48.2	7.24	6.77	94.8	0.016 91	1.359
Σ	2.888	99.9	15.01				2.814

ΣC has changed by only 2.6% so these values are very close to correct.

Eq. 8.93: $Q = \Sigma C h_L^{1/2}$; $15 = 2.814 h_L^{1/2}$; $\therefore\ h_L = 28.4$ ft between *B* and *C*

Pipe	*D* (in)	*L/D*	*e/D*	*V*	*D''V*	*f*	h_L ft (Eq. 8.13)
1	21	1571	0.000 286	6.24	131.0	0.015 54	14.75
5	27	2111	0.000 222	3.77	101.9	0.015 14	7.06

Then h_L (*A* to *D*) = 14.75 + 28.4 + 7.06 = 50.2 ft ◄

(*b*) This is a Type 3 problem, to find *D*. Given $L = 2250$ ft.

Eq. 8.57: $\mathbf{N}_1 = 2.12 \times 10^{28}$; Eq. 8.58: $\mathbf{N}_2 = 3.19 \times 10^{-10}$

Eq. 8.61: $\mathbf{R}^{2.5} = 1.146 \times 10^{15}$, so $\mathbf{R} = 1.056 \times 10^{6}$

Eq. 8.62: $D = 1.486$ ft = 17.83 in ◄

Alternative: Using a solver with Eq. 8.56*b*: $D = 1.482$ ft = 17.79 in

8.116 *(a) With the same pipe lengths, sizes, and connections as in Prob. 8.115, find the flow in each pipe if the head loss from A to D is 150 ft and if all pipes have f = 0.020. Also find the head losses from A to B, B to C, and C to D. (b) Find the new head loss distributions and the percentage increase in the capacity of the system achieved by adding another 12-in pipe 3250 ft long between B and C.*

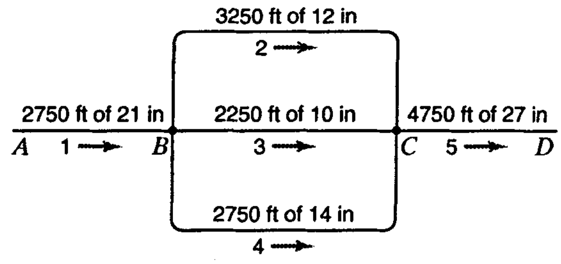

Figure P8.115

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(*a*) Eq. 8.13: $h_L = f\frac{L}{D}\frac{V^2}{2g} = f\frac{L}{D}\frac{Q^2}{2gA^2}$

$$= f\frac{L}{2gD}\left(\frac{Q}{\pi D^2/4}\right)^2 = \frac{8f}{\pi^2 g}\left(\frac{LQ^2}{D^5}\right)$$

Eq. 8.91: $h_{L2} = h_{L3} = h_{L4}$; f = const, so 3 values of LQ^2/D^5 are equal

i.e., $3250Q_2^2/1^5 = 2250Q_3^2/(10/12)^5 = 2750Q_4^2/(14/12)^5$ from which $Q_2 = 1.313Q_3$, $Q_4 = 2.10Q_3$

Eq. 8.90: $Q_1 = Q_5 = Q_2 + Q_3 + Q_4 = (1.313 + 1 + 2.10)Q_3 = 4.41Q_3$

Eq. 8.87: $(h_L)_{AD} = 150 = h_{L1} + h_{L3} + h_{L5} = \frac{8(0.020)}{\pi^2 g}\left(\frac{L_1Q_1^2}{D_1^5} + \frac{L_3Q_3^2}{D_3^5} + \frac{L_5Q_5^2}{D_5^5}\right)$

$$= 0.000\,503\left(\frac{2750(4.41Q_3)^2}{(21/12)^5} + \frac{2250Q_3^2}{(10/12)^5} + \frac{4750(4.41Q_3)^2}{(27/12)^5}\right) = 5.27Q_3^2$$

So $Q_3 = 5.34$ cfs ◄ $Q_1 = Q_5 = 23.5$ cfs ◄ $Q_2 = 7.00$ cfs ◄ $Q_4 = 11.20$ cfs ◄

$(h_L)_{AB} = h_{L1} = 0.000\,503(2750)23.5^2/(21/12)^5 = 46.73$ ft ◄

$(h_L)_{BC} = h_{L3} = 0.000\,503(2250)5.34^2/(10/12)^5 = 80.29$ ft ◄

$(h_L)_{CD} = h_{L5} = 0.000\,503(4750)23.5^2/(27/12)^5 = 22.98$ ft ◄ Check: $\Sigma h_L = 150.00$ ft -- correct!

(*b*) The additional pipe from *B* to *C* is identical to pipe 2.

As above, $Q_2 = 1.313Q_3$, $Q_4 = 2.10Q_3$. But now $Q_1 = Q_5 = 2Q_2 + Q_3 + Q_4 = 5.72Q_3$, so now

$$(h_L)_{AD} = 150 = 0.000\,503\left[\frac{2750(5.72Q_3)^2}{(21/12)^5} + \frac{2250Q_3^2}{(10/12)^2} + \frac{4750(5.72Q_3)^2}{(27/12)^5}\right] = 6.94Q_3^2$$

So $Q_3 = 4.65$ cfs, $Q_1 = Q_5 = 26.6$ cfs

$(h_L)_{AB} = h_{L1} = 0.000\,503(2750)26.6^2/(21/12)^5 = 59.71$ ft ◄

$(h_L)_{BC} = h_{L3} = 0.000\,503(2250)4.65^2/(10/12)^5 = 60.93$ ft ◄

$(h_L)_{CD} = h_{L5} = 0.000\,503(4750)26.6^2/(27/12)^5 = 29.36$ ft ◄ Check: $\Sigma h_L = 150.00$ ft -- correct!

$Q_b/Q_a = 26.6/23.5 = 1.130$, or 13.0% increase in capacity ◄

8.117 *Find the magnitude and direction of the flow in network lines ab and bc (Fig. P8.117) after making two sets of corrections. The numbers on the figure are the K values of each line; take n = 2.0. Start by assuming initial flows as follows: 9 cfs in lines ab and cd, 6 cfs in lines ac and bd, and 3 cfs in line bc.*

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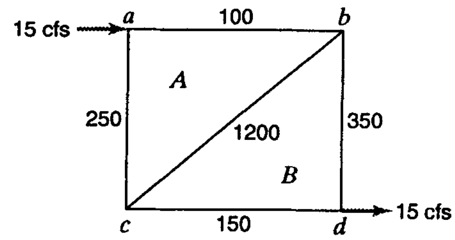


Figure P8.117

Given initial assumptions:

Loop A

First approx:

Pipe	K	Q	KQ^2	$\lvert 2KQ \rvert$
ab	100	+9	+8100	1800
bc	1200	+3	+10,800	7200
ac	250	−6	−9000	3000
			+9900	12,000

ΔQ = -(+9900/12,000) = -0.825

Second approx. (after first corrections):

ab	100	+8.175	+6683	1635
bc	1200	+1.441	+2492	3458
ac	250	−6.825	−11,645	3413
			−2470	8506

ΔQ = -(-2470/8506) = +0.290

Loop B

Pipe	K	Q	KQ^2	$\lvert 2KQ \rvert$
bd	350	+6	+12,600	4200
bc	1200	−3	−10,800	7200
cd	150	−9	−12,150	2700
			−10,350	14,100

ΔQ = -(-10,350)/14,100) = +0.734

bd	350	+6.734	+15,871	4714
bc	1200	−1.441	−2492	3458
cd	150	−8.266	−10,249	2480
			+3130	10,652

ΔQ = -(+3130/10,652) = -0.294

After second corrections:

Flow in line *ab* = +8.175 + 0.290 = 8.465 cfs from *a* to *b* ◀

Flow in line *bc* = +1.441 + 0.290 − (−0.294) = 2.025 cfs from *b* to *c* ◀

8.118 *Find the magnitude and direction of the flow in network lines ab and bc (Fig. P8.118) after making two sets of corrections. The numbers on the figure are the K values of each line; take n = 2.0. Start by assuming initial flows as follows: 0.3 m³/s in lines ab and cd, 0.2 m³/s in lines ac and bd, and 0.1 m³/s in line bc.*

SI

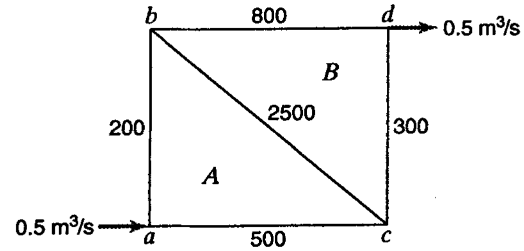


Figure P8.118

Given initial assumptions:

First approx:

Loop A

Pipe	K	Q	KQ^2	$\lvert 2KQ \rvert$
ab	200	+0.3	+18.0	120
bc	2500	+0.1	+25.0	500
ac	500	−0.2	−20.0	200
			+23.0	820

$\Delta Q = -(+23/820) = -0.028$

Loop B

Pipe	K	Q	KQ^2	$\lvert 2KQ \rvert$
bd	800	+0.2	+32.0	320
bc	2500	−0.1	−25.0	500
cd	300	−0.3	−27.0	180
			−20.0	1000

$\Delta Q = -(-20.0/1000) = +0.020$

Second approx. (after first corrections):

ab	200	+0.272	+14.80	109
bc	2500	+0.052	+6.76	260
ac	500	−0.228	−25.99	228
			−4.43	597

$\Delta Q = -(-4.43/597) = +0.007$

bd	800	+0.22	+38.72	352
bc	2500	−0.052	−6.76	260
cd	300	−0.28	−23.52	168
			+8.44	780

$\Delta Q = -(+8.44/780) = -0.011$

After second corrections: Flow in line ab = +0.272 + 0.007 = 0.279 m³/s from a to b ◄

Flow in line bc = +0.052 + 0.007 − (−0.011) = 0.070 m³/s from b to c ◄

8.119 *Carry the solution for the pipe network of Fig. P8.119 through four trials, to find the flow in each pipe. For simplicity, take n = 2.0 and use the value of f for complete turbulence, as given by Eq. (8.54). All pipes are cast iron, and are at the same elevation. For initial flows, assume only values of 30, 15, and 0 L/s (the zeros in dg and fh. If the pressure head at a is 40 m, find the pressure head at d (which might represent a fire demand, for example) neglecting velocity heads.*

SI

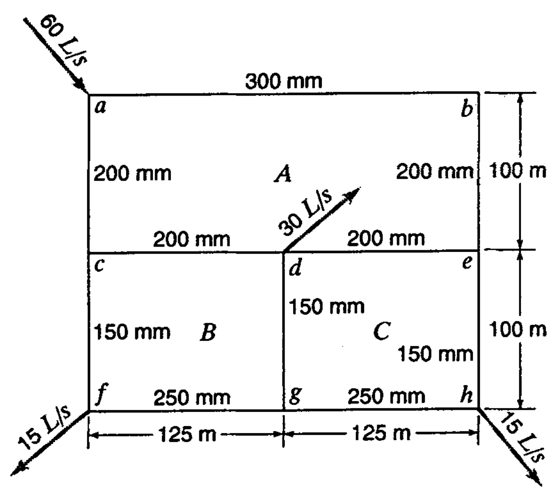


Figure. P8.119

Per Eq. 8.65 and Sec. 8.19 (nonrigorous solutions):

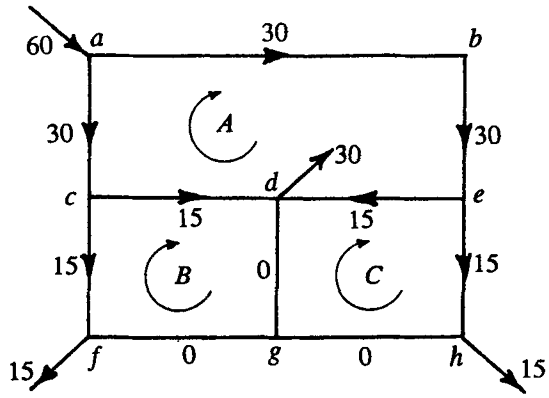
$h_L = KQ^2$ with $K = 8fL/(\pi^2 g D^5)$.

Table 8.1 for cast iron: e = 0.000 25 m

Using $f = f_{min}$ for complete turbulence from Eq. 8.54 (or Fig. 8.11), the pipe characteristics are:

Pipes	L, m	D, m	e/D	f_{min}	K
ab	250	0.30	0.000 833	0.018 79	160
ac, be	100	0.20	0.001 25	0.020 75	536
cd, ed	125	0.20	0.001 25	0.020 75	670
cf, dg, eh	100	0.15	0.001 667	0.022 33	2430
gf, gh	125	0.25	0.001 00	0.019 64	208

Initial flow assumptions (L/s):



<u>First approximation and correction</u>
Note: +/− signs are used to indicate clockwise/counter-clockwise flows in the loops indicated.

Loop A

Pipe	Q	K	KQ^n	$\|KnQ^{n-1}$
ab	+30	160	+144 000	9600
be	+30	536	+482 400	32 160
ed	+15	670	+150 750	20 100
cd	−15	670	−150 750	20 100
ac	−30	536	−482 400	32 160
			+144 000	114 120

ΔQ = −(+144 000)/114 120 = −1.26 L/s

Loop B

Pipe	Q	K	KQ^n	$\|KnQ^{n-1}\|$
cd	+15	670	+150 750	20 100
dg	0	2430	0	0
gf	0	208	0	0
cf	−15	2430	−546 750	72 900
			−396 000	93 000

ΔQ = −(−396 000)/93 000 = +4.26 L/s

Loop C

Pipe	Q	K	KQ^n	$\|KnQ^{n-1}\|$
ed	−15	670	−150 750	20 100
eh	+15	2430	+546 750	72 900
gh	0	208	0	0
dg	0	2430	0	0
			+396 000	93 000

ΔQ = −(+396 000)/93 000 = −4.26 L/s

Pipe	In Loop	Q, L/s	(2nd Approximation)			
ab	A	+30	−1.26			= +28.74
be	A	+30	−1.26			= +28.74
ed	A	+15	−1.26		+4.26	= +18.00
ed	C	−15	+1.26		−4.26	= −18.00
cd	A	−15	−1.26	−4.26		= −20.52
cd	B	+15	+1.26	+4.26		= +20.52
ac	A	−30	−1.26			= −31.26
dg	B	0		+4.26	+4.26	= +8.52
dg	C	0		−4.26	−4.26	= −8.52
gf	B	0		+4.26		= +4.26
cf	B	−15		+4.26		= −10.74
eh	C	+15			−4.26	= +10.74
gh	C	0			−4.26	= −4.26

Repeating the same procedure, the subsequent loop corrections ΔQ in L/s are as follows:

Loop	1st	2nd	3rd correction
A	−1.26	+0.11	−0.13
B	+4.26	−1.48	+0.28
C	+4.26	+0.98	−0.41

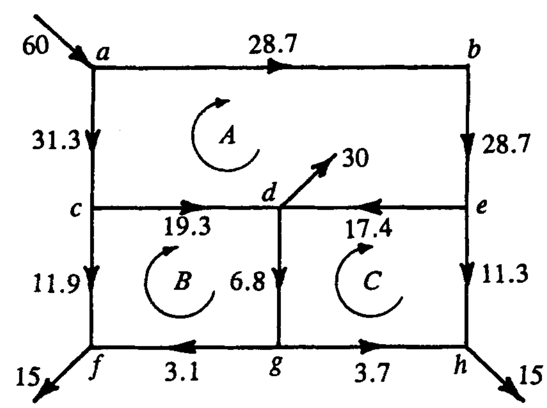
The loop corrections are quite rapidly becoming small.

/cont...

The approximations of the flows Q, in L/s, were as follows:

Pipe	In Loop	1st	2nd	3rd	4th
ab	A	+30	+28.74	+28.85	+28.72
be	A	+30	+28.74	+28.85	+28.72
ed	A	+15	+18.00	+17.13	+17.41
ed	C	−15	−18.00	−17.13	−17.41
cd	A	−15	−20.52	−18.93	−19.34
cd	B	+15	+20.52	+18.93	+19.34
ac	A	−30	−31.26	−31.15	−31.28
dg	B	0	+8.52	+6.06	+6.75
dg	C	0	−8.52	−6.06	−6.75
gf	B	0	+4.26	+2.78	+3.06
cf	B	−15	−10.74	−12.22	−11.94
eh	C	+15	+10.74	+11.72	+11.31
gh	C	0	−4.26	−3.28	−3.69

This table shows the process of converging, which occurs more slowly in pipes that are common to two loops. Resulting final flows, L/s:



Given all pipes (junctions) are at the same elevation. Final approximated flows from *a* to *d* are: 31.26 L/s from *a* to *c*, and 19.34 L/s from *c* to *d*.

The head losses are given by $h_L = KQ^2$.

$\therefore$ neglecting velocity heads:

$$p_d/\gamma = p_a/\gamma - (h_L)_{ac} - (h_L)_{cd} = 40 - 536(32.28/1000)^2 - 670(19.34/1000)^2 = 40 - 0.523 - 0.251$$

$$= 39.23 \text{ m} \quad \blacktriangleleft$$

8.120 *Carry the solution for the pipe network of Fig. P8.120 through five trials, to find the flow in each pipe. The 12-in and 16-in pipes are of average cast iron, while the 18-in and 24-in sizes are of average concrete (e = 0.003 ft). Assume n = 2.0, and use the values of f from Eq. (8.54) for complete turbulence. If the pressure at h is 80 psi, find the pressure at f.*

BG

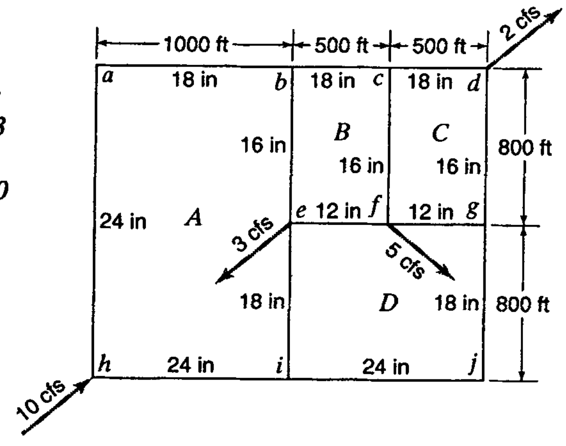


Figure P8.120

Per Eq. 8.66 and Sec. 8.19 (nonrigorous solutions):

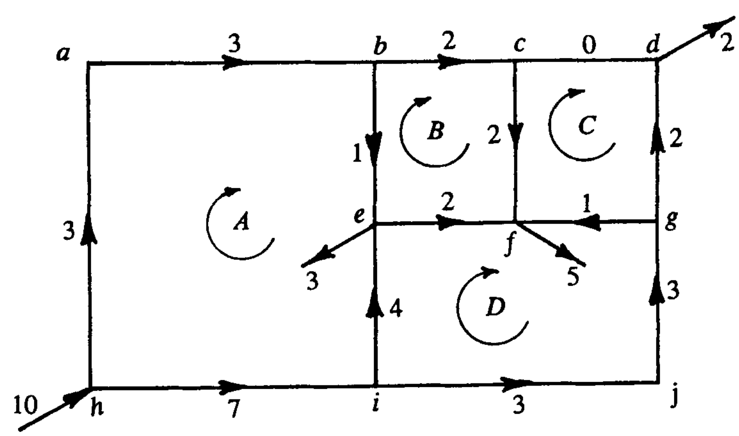
$h_L = KQ^2$ with $K = 8fL/(\pi^2 g D^5) = fL/(39.7D^5)$

Table 8.1 for average cast iron: $e = 0.000\,85$ ft

Using $f = f_{min}$ for complete turbulence from Eq. 8.54 (or Fig. 8.11), the pipe characteristics are:

Pipe	Mat'l	L, ft	D, ft	e/D	f	D^5	$K = \frac{fL}{39.7D^5}$
ab	avg conc	1000	1.50	0.002 00	0.0233	7.59	0.0772
bc	avg conc	500	1.50	0.002 00	0.0233	7.59	0.0386
cd	avg conc	500	1.50	0.002 00	0.0233	7.59	0.0386
ef	avg C.I.	500	1.00	0.000 85	0.0188	1.00	0.237
fg	avg C.I.	500	1.00	0.000 85	0.0188	1.00	0.237
hi	avg conc	1000	2.00	0.001 50	0.0218	32.00	0.017 15
ij	avg conc	1000	2.00	0.001 50	0.0218	32.00	0.017 15
ah	avg conc	1600	2.00	0.001 50	0.0218	32.00	0.0274
be	avg C.I.	800	1.33	0.000 62	0.0175	4.21	0.0836
ei	avg conc	800	1.50	0.002 00	0.0233	7.59	0.0618
cf	avg C.I.	800	1.33	0.000 62	0.0175	4.21	0.0836
dg	avg C.I.	800	1.33	0.000 62	0.0175	4.21	0.0836
gi	avg conc	800	1.50	0.002 00	0.0233	7.59	0.0618

Initial flow assumptions (cfs):



/cont...

First approximation (note that +/− signs are used to indicate clockwise/counter-clockwise flows in the loops indicated):

Loop A

Pipe	Q_0	KQ_0^2	$\lvert h_L/Q_0 \rvert$
ha	+3	+0.247	0.0823
ab	+3	+0.695	0.232
be	+1	+0.0836	0.0836
hi	−7	−0.840	0.1200
ie	−4	−0.989	0.247
Σ		−0.803	0.765

$\Delta Q = -(-0.803)/(2 \times 0.765) = +0.525$ cfs

Loop B

Pipe	Q_0	KQ_0^2	$\lvert h_L/Q_0 \rvert$
bc	+2	+0.1545	0.0772
cf	+2	+0.335	0.1673
be	−1	−0.0836	0.0836
ef	−2	−0.947	0.473
Σ		−0.541	0.801

$\Delta Q = -(-0.541)/(2 \times 0.801) = +0.338$ cfs

Loop C

Pipe	Q_0	KQ_0^2	$\lvert h_L/Q_0 \rvert$
cd	0	0.000	0.000
gf	+1	+0.237	0.237
cf	−2	−0.335	0.1673
gd	−2	−0.335	0.1673
Σ		−0.571	0.571

$\Delta Q = -(-0.571)/(2 \times 0.571) = +0.379$ cfs

Loop D

Pipe	Q_0	KQ_{02}	$\lvert h_L/Q_0 \rvert$
ie	+4	+0.989	0.247
ef	+2	+0.947	0.473
ij	−3	−0.1543	0.0514
jg	−3	−0.556	0.1854
gf	−1	−0.237	0.237
Σ		+0.989	1.194

$\Delta Q = -(+0.989)/(2 \times 1.194) = -0.414$ cfs

After 5 trials, the flows (cfs) within approx 1% are:

ab = 3.54; *bc* = 2.48; *cd* = 0.40; *ef* = 1.47; *gf* = 1.45; *hi* = 6.46; *ij* = 3.05; *ha* = 3.54

be = 1.06; *ie* = 3.41; *cf* = 2.08; *gd* = 1.60; *jg* = 3.05 ◄◄

Head loss (Eq. 8.95) from *h* to *f* (any path): *hi* = 0.716 ft; *ie* = 0.719 ft; *ef* = 0.511 ft;

Total = 1.945 ft. $p_f = 80 - 1.945(62.4)/144 = 80 - 0.843 = 79.2$ psi ◄

Chapter 9

Forces on Immersed Bodies

PROBLEM SELECTION GUIDE

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
9.3	*Laminar Boundary Layer for Incompressible Flow Along a Smooth Flat Plate*						
	X[1]9.3.1	N	Easy	Short	1		
	9.3.2	BG	Easy	Short	1	P9.3	
	9.3.3	SI	Medium	Medium	3	P9.4	
	P 9.1	N	Medium	Medium	1		Derivation; differentiation.
	9.2	N	Medium	Medium	1		Proof; integration, differentiation.
	9.3	SI	Easy	Short	1	X9.3.2	
	9.4	BG	Medium	Medium	3	X9.3.3	
9.4	*Turbulent Boundary Layer for Incompressible Flow Along a Smooth Flat Plate*						
	X 9.4.1	BG	V Easy	V Short	1	P9.8	The beach
	9.4.2	N	V Easy	V Short	1		
	9.4.3	BG	Easy	Short	3	S9.2	
	9.4.4	SI	Medium	Medium	3	P9.9	Must solve, or have Ans to, Exer 9.3.3
	9.4.5	BG	Medium	Medium	1	P9.10	Power
	P 9.5	N	Medium	Short	1		Derivation; integration.
	9.6	N	Medium	Short	1		Derivation; integration.
	9.7	N	Medium	Short	1		Derivation.
	9.8	BG	Medium	Medium	3	X9.4.4	Must solve, or have Ans to, Prob 9.4
	9.9	SI	Medium	Medium	1	X9.4.5	Power
	9.10	SI	V Easy	V Short	1	X9.4.1	The beach
9.5	*Friction Drag for Incompressible Flow Along a Smooth Flat Plate with a Transition Regime*						
	X 9.5.1	BG	Easy	Short	1		
	9.5.2	BG	Medium	Medium	2	P9.12	
	9.5.3	SI	Medium	Medium	2	P9.13	
	P 9.11	N	V Easy	V Short	1		Proof
	9.12	SI	Medium	Medium	2	X9.5.2	
	9.13	BG	Medium	Medium	2	X9.5.3	
	9.14	BG	Medium	Long	1		Plots
	9.15	BG	Medium	Medium	1	9.14	Plots
	9.16	BG	Medium	V Long	2	9.17	See note with solution. Plots
	9.17	SI	Medium	V Long	2	9.16	See note with solution; interpol'n. Plots

/cont...

[1] For all Exercises (identified by "X"), answers are given in Appendix F of the textbook.
† Answers are sensitive to values (C_d, C_f, τ, σ) that must be read from a graph.
X = Exercise, P = (end-of-chapter) Problem, S = Sample Problem.

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
9.7	***Drag on Three-Dimensional Bodies (Incompressible Flow)***						
	X 9.7.1	BG	Easy	Medium	1	9.7.2	Assume **R** < 1 and confirm
	9.7.2	SI	Easy	Medium	1	9.7.1	Assume **R** < 1 and confirm
	9.7.3	SI	Medium	Long	1	P9.19	†
	9.7.4	BG	Medium	Medium	1	P9.23	† Make assumption. Power
	9.7.5	BG	Easy	Short	1	P9.24	†
	9.7.6	BG	Easy	Medium	1	9.7.7	Ass. **R** > 1000, confirm. Parachute
	9.7.7	SI	Easy	Medium	1	9.7.6	Ass. **R** > 1000, confirm. Parachute
	9.7.8	SI	V Easy	Short	1		
	P 9.18	BG	Medium	Long	1		† T & E
	9.19	BG	Medium	Long	1	9.20,X9.7.3	† Plot
	9.20	BG	Medium	Long	1	9.19	† Plot
	9.21	BG	Medium	Medium	1	9.22	† T & E
	9.22	SI	Medium	Medium	1	9.21	† T & E
	9.23	SI	Medium	Medium	1	X9.7.4	† Make assumption. Power
	9.24	SI	V Easy	V Short	1	X9.7.5	†
	9.25	BG	Medium	Medium	2	9.26	† Uses Fig. 9.13 of (next) Sec. 9.8
	9.26	SI	Medium	Medium	2	9.25	† Uses Fig. 9.13 of (next) Sec. 9.8
	9.27	BG	Medium	Long	1	9.28	† T & E, interpolation.
	9.28	SI	Medium	Long	1	9.27	† T & E
	9.29	SI	Medium	Long	3		† T & E
	9.30	BG	Medium	Medium	2		† Uses $pv = RT$ (Sec. 2.7). Football
	9.31	BG	Hard	Long	3		† Rowing
9.8	***Drag on Two-Dimensional Bodies (Incompressible Flow)***						
	X 9.8.1	BG	Easy	Short	1		†
	9.8.2	BG	Easy	Short	2		
	P 9.32	SI	Medium	Medium	2		†
	9.33	SI	Medium	Medium	2		†
9.9	***Lift and Circulation***						
	X 9.9.1	BG	Medium	Medium	1		
9.10	***Ideal Flow About a Cylinder***						
	X 9.10.1	BG	V Easy	V Short	1	9.10.2	
	9.10.2	SI	V Easy	V Short	1	9.10.1	
	P 9.34	N	Medium	Medium	1		Proof; integration.
	9.35	BG	Medium	Medium	1		† Interpolation
	9.36	BG	Medium	Medium	1		
	9.37	BG	Medium	Medium	2	9.38	Plot
	9.38	SI	Medium	Medium	2	9.37	Plot
	9.39	BG	Easy	Short	2	9.40	
	9.40	SI	Easy	Short	2	9.39	
	9.41	BG	Medium	Medium	1	9.42	Baseball
	9.42	SI	Medium	Medium	1	9.41	Baseball

/cont...

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
9.12	***Induced Drag on Airfoil of Finite Length***						
	X 9.12.1	SI	Medium	Medium	3	P9.43	
	P 9.43	BG	Medium	Medium	3	X9.12.1	
9.13	***Lift and Drag Diagrams***						
	X 9.13.1	BG	Easy	V Short	1	9.13.2	Sailplane
	9.13.2	SI	Easy	V Short	1	9.13.1	Sailplane
	9.13.3	BG	Medium	Medium	4		Power
	P 9.44	SI	Medium	Medium	1	9.45	†
	9.45	BG	Medium	Medium	1	9.44	†
	9.46	N	Medium	Medium	1		†
	9.47	BG	Medium	Long	1		† Interpolation. Kite flying.
	9.48	SI	Medium	Medium	3		†
	9.49	BG	Medium	Long	1		Interpolation, T & E. Sailplane
9.14	***Effects of Compressibility on Drag and Lift***						
	X 9.14.1	BG	Medium	Medium	2	9.14.2	†
	9.14.2	SI	Medium	Medium	2	9.14.1	†
	P 9.50	BG	Hard	Long	1	9.51	
	9.51	SI	Hard	Long	1	9.50	
	9.52	BG	Medium	Long	1	9.53	† T & E
	9.53	SI	Medium	Long	1	9.52	† T & E

Chapter 9
FORCES ON IMMERSED BODIES

Sec. 9.3: Laminar Boundary Layer for Incompressible Flow Along a Smooth Flat Plate -- *Exercises (3)*

9.3.1 *For the critical Reynolds number of 500,000 for transition from laminar to turbulent flow in the boundary layer, find the corresponding critical Reynolds number for flow in a circular pipe. How does this compare with the value given in Chap. 8? (Hint: Let the boundary-layer thickness correspond to the radius of the pipe in laminar flow, and let the undisturbed velocity U of the boundary layer theory represent the centerline velocity u_{max} of the pipe flow.)*

N

Eq. 9.10: $\dfrac{\delta}{x} = \dfrac{4.91}{500{,}000^{1/2}} = 0.006\,94, \quad \therefore\ \mathbf{R} = 500{,}000 = \dfrac{Ux}{\nu} = \dfrac{U\delta}{0.006\,94\,\nu}$

Since: δ corresponds to pipe radius, $D/2$, U corresponds to $U_{max} = 2V$ for laminar flow.

Then for pipe flow, $500{,}000 = \dfrac{2V(D/2)}{0.006\,94\,\nu}$ so that $\dfrac{DV}{\nu} = \mathbf{R}_{pipe} = 500{,}000(0.006\,94) = 3470$ ◄

This is a fair check with the value of 2000 given in Chapter 8. ◄

9.3.2 *Determine the shear stress at 9 in and 18 in back from the leading edge of the plate in Sample Prob. 9.1.*
Sample Prob. 9.1: The plate is 6 in wide × 18 in long, placed longitudinally in oil ($s = 0.925$, $\nu = 0.001\,05$ ft²/sec) flowing with undisturbed $U = 2$ fps.

BG

Eq. 9.11: $\tau_0 = 0.332(\mu U/x)(\mathbf{R}_x)^{0.5}$

where (Sec. 9.3) $\mathbf{R}_x = xU/\nu$, which increases linearly in the downstream direction.

$\therefore\ \mu = \nu\rho = 0.001\,05(0.925)(62.4/32.2) = 0.001\,882$ lb·sec/ft²

x	$\mathbf{R}_x$	τ_0
0.75 ft	1429	0.0630 lb/ft²
1.5 ft	2860	0.0445 lb/ft²

▲

9.3.3 *Find the shear stress and the thickness of the boundary layer (a) at the center and (b) at the trailing edge of a smooth, flat plate 3.0 m wide and 0.6 m long parallel to the flow, immersed in 15°C water flowing at an undisturbed velocity of 0.9 m/s. Assume a laminar boundary layer over the whole plate. Also, (c) find the total friction drag on one side of the plate.*

SI

Table A.1 for water at 15°C: $\rho = 999.1$ kg/m³, $\nu = 1.139 \times 10^{-6}$ m²/s, $\mu = 0.001\,139$ N·s/m²

(*a*) At center, $x = 0.3$ m, $\mathbf{R}_x = \dfrac{xU}{\nu} = \dfrac{0.3(0.9)}{1.139 \times 10^{-6}} = 237\,000$

For $\mathbf{R}_x < 500\,000$ the boundary layer will be laminar if undisturbed.

From Eq. 9.10: $\delta = \dfrac{4.91(0.3)}{237\,000^{1/2}} = 0.003\,03$ m $= 3.03$ mm ◄

Eq. 9.11: $\tau_0 = 0.332\dfrac{(0.001\,139)0.9}{0.3}(237\,000)^{1/2} = 0.552$ N/m² ◄

(*b*) At trailing edge $x = 0.6$ m, $\mathbf{R}_x = 474\,000$, $\delta = 0.004\,28$ m $= 4.28$ mm ◄

$\tau_0 = 0.391$ N/m² ◄

(*c*) Drag on one side of plate: Eq. 9.14: $C_f = 1.328/474\,000^{1/2} = 0.001\,929$

Eq. 9.2: $F_f = (0.001\,929)999.1(0.9^2/2)3.0(0.6) = 1.405$ kg·m/s² $= 1.405$ N ◄

Sec. 9.3: Laminar Boundary Layer for Incompressible Flow Along a Smooth Flat Plate -- *Problems 9.1−9.4*

9.1 *Given the general equation for a parabola, $u = ay^2 + by + c$, derive the dimensionless velocity distribution of Eq. (9.9).*

N

(*a*) $u = ay^2 + by + c$; differentiating: $du/dy = 2ay + b$

Boundary conditions: (1) At $y = 0$, $u = 0$

(2) At $y = \delta$, $u = U$

(3) At $y = \delta$, $du/dy = 0$

(*b*) From (1): $c = 0$

From (2): $U = a\delta^2 + b\delta$

From (3): $(du/dy)_{y=\delta} = 2a\delta + b = 0$; $\therefore\ b = -2a\delta$

Substitute for b into (2): $U = a\delta^2 - 2a\delta^2 = -a\delta^2$

Then, $a = -U/\delta^2$; $b = 2U/\delta$, and $\therefore\ u = -(U/\delta^2)y^2 + (2U/\delta)y$

With $\eta = y/\delta$, $u/U = f(\eta) = -\eta^2 + 2\eta$ which is Eq. (9.9) Q.E.D. ◄

9.2 *Given the parabolic velocity distribution of Eq. (9.9), prove that $\alpha = 0.1333$ (in Eq. 9.5) and $\beta = 2.0$ (in Eq. 9.7).*

Eq. (9.9): $u/U = f(\eta) = 2\eta - \eta^2$

N

From Eq. 9.5: $\alpha = \int_0^1 f(\eta)[1 - f(\eta)]d\eta$; substituting for $f(\eta)$ from Eq. 9.9:

$$\alpha = \int_0^1 (2\eta - \eta^2)[1 - 2\eta + \eta^2]d\eta = \int_0^1 (2\eta - 5\eta^2 - 4\eta^3 - \eta^4)d\eta$$

$$= [\eta^2 - (5/3)\eta^3 + \eta^4 - (1/5)\eta^5]_0^1 = 0.1333 \quad ◄$$

From Eq. 9.7: $\beta = \dfrac{\tau_0\delta}{\mu U} = \left[\dfrac{df(\eta)}{d\eta}\right]_{\eta=0}$ Substituting for $f(\eta)$ from Eq. 9.9:

$$\beta = \frac{d}{d\eta}(2\eta - \eta^2)_{\eta=0} = [2 - 2\eta]_{\eta=0} = 2.0 \quad ◄$$

9.3 *Determine the shear stress at 150 mm and 300 mm back from the leading edge of the plate in Sample Prob. 9.1.*

Sample Prob. 9.1: The plate is 150 mm wide × 500 mm long, placed longitudinally in oil ($s = 0.925$, $\nu = 0.73 \times 10^{-4}$ m²/s) flowing with undisturbed $U = 0.60$ m/s.

SI

Sec. 9.3: $\mathbf{R}_x = xU/\nu = x(0.60)/(0.73 \times 10^{-4}) = 8220x$, which increases linearly downstream.

$\mu = \nu\rho = 0.73 \times 10^{-4}(0.925)1000$ N·s²/m² $= 0.0675$ N·s/m²

Eq. 9.11: $\tau_0 = 0.332(\mu U/x)(\mathbf{R}_x)^{0.5} = 0.332(0.0675 \times 0.60/x)(8220x)^{1/2} = 1.219/x^{1/2}$

x	$\mathbf{R}_x = 8220x$	$\tau_0 = 1.219/x^{1/2}$
0.15 m	1233	3.15 N/m²
0.30 m	2470	2.23 N/m²

▲

9.4 *Find the shear stress and the thickness of the boundary layer (a) at the center and (b) at the trailing edge of a smooth, flat plate 10 ft wide and 2 ft long parallel to the flow, immersed in 60°F water flowing at an undisturbed velocity of 3 fps. Assume a laminar boundary layer over the whole plate. Also, (c) find the total friction drag on one side of the plate.*

BG

Table A.1 for water at 60°F: $\rho = 1.938$ slug/ft^3, $\nu = 1.217 \times 10^{-5}$ ft^2/s, $\mu = 2.359 \times 10^{-5}$ lb·sec/ft^2

(*a*) At center, $x = 1$ ft, $\mathbf{R}_x = \dfrac{xU}{\nu} = \dfrac{1(3)}{1.217 \times 10^{-5}} = 247{,}000$

For $\mathbf{R}_x < 500{,}000$ boundary layer will be laminar if undisturbed.

Eq. 9.10: $\delta = \dfrac{4.91(1)}{247{,}000^{1/2}} = 0.009\,89 \text{ ft} = 0.1187 \text{ inches}$ ◄

Eq. 9.11: $\tau_0 = 0.332\dfrac{(2.359 \times 10^{-5})3}{1}(247{,}000)^{1/2} = 0.011\,67 \text{ lb/ft}^2$ ◄

(*b*) At trailing edge $x = 2$ ft, $\mathbf{R}_x = \dfrac{2(3)}{1.217 \times 10^{-5}} = 493{,}000$

$\delta = \dfrac{4.91(2)}{493{,}000^{1/2}} = 0.013\,99 \text{ ft or } 0.1678 \text{ inches}$ ◄

$\tau_0 = 0.332(2.359 \times 10^{-5})(3/2)493{,}000^{1/2} = 0.008\,25 \text{ lb/ft}^2$ ◄

(*c*) Drag on one side of plate: Eq. 9.14: $C_f = \dfrac{1.328}{493{,}000^{1/2}} = 0.001\,891$

Eq. 9.2: $F_f = 0.001\,891(1.938)(3^2/2)(10)2 = 0.330 \text{ lb}$ ◄

Sec. 9.4: Turbulent Boundary Layer for Incompressible Flow Along a Smooth Flat Plate -- ***Exercises (5)***

9.4.1 *A lifeguard determines the wind velocity 6 ft above the beach to be 25 fps. If one wishes to get out of the wind by lying down, what would be the velocity at (a) 0.5 ft, and (b) at 1.0 ft above beach level?*

BG

Use the seventh-root law to approximate the velocity profile. Eq. 9.20 (and 8.49): $u = 25(y/6)^{1/7}$

(*a*) $u_{0.5} = 25(0.5/6)^{1/7} = 17.53$ fps ◄ (*b*) $u_{1.0} = 25(1/6)^{1/7} = 19.35$ fps ◄

9.4.2 *Compute C_f for $\mathbf{R} = 10^7$ using Eqs. (9.25) and (9.26), and compare the two values.*

N

Eq. 9.25: $C_f = \dfrac{0.0735}{\mathbf{R}^{1/5}} = \dfrac{0.0735}{(10^7)^{1/5}} = 0.002\,93$ ◄

Eq. 9.26: $C_f = \dfrac{0.455}{(\log 10^7)^{2.58}} = \dfrac{0.455}{7^{2.58}} = 0.003\,00$ ◄

The two values of C_f agree closely (within 2.6%). ◄

9.4.3 *Find the shear stress on the sides of the van in Sample Prob. 9.2 at (a) 2 ft, (b) 12 ft, and (c) 22 ft back from the leading edge of the sides.*

Sample Prob. 9.2: U = 88 fps, ν = 0.000 152 ft²/sec, ρ = 0.002 42 slug/ft³.

BG

Eq. 9.22: $\tau_0 = 0.0587\rho\dfrac{U^2/2}{\mathbf{R}_x^{1/5}}$; Sec. 9.3: $\mathbf{R}_x = \dfrac{xU}{\nu}$

Given: U = 88 fps, ν = 0.000 152 ft²/sec, ρ = 0.002 42 lb·sec²/ft⁴

$\therefore\ 0.0587\rho(U^2/2) = 0.0587(0.002\,42\ \text{lb}\cdot\text{sec}^2/\text{ft}^4)(88^2/2)\ \text{ft}^2/\text{sec}^2 = 0.550\ \text{lb/ft}^2$ and $\tau_0 = 0.550/\mathbf{R}_x^{1/5}\ \text{lb/ft}^2$

	x	$\mathbf{R}_x = \dfrac{xU}{\nu}$	$\mathbf{R}_x^{1/5}$	τ_0
(*a*)	2 ft	1,158,000	16.32	0.0337 lb/ft²
(*b*)	12 ft	6,950,000	23.35	0.0236 lb/ft²
(*c*)	22 ft	12,740,000	26.36	0.0209 lb/ft²

▲

9.4.4 *Assume that the boundary layer of Exer. 9.3.3 is disturbed near the leading edge. Compute the corresponding quantities for the turbulent boundary layer covering the entire plate, and compare the results.*

Exer. 9.3.2: A smooth, flat plate 3.0 m wide × 0.6 m long parallel to the flow is immersed in 15°C water (ρ = 999.1 kg/m³) flowing at undisturbed U = 0.9 m/s. Find τ_0 and δ at (a) the center (where x = 0.3 m and $\mathbf{R}_x$ = 237 000) and (b) the trailing edge (where x = 0.6 m and $\mathbf{R}_x$ = 474 000). Also (c) find F_f on one side.

SI

(*a*) At center of plate: x = 0.3 m, $\mathbf{R}_x$ = 237 000 ; Eq. 9.21: $\dfrac{\delta}{x} = \dfrac{0.377}{237\,000^{1/5}} = \dfrac{0.377}{11.88} = 0.0317$

$\therefore\ \delta = 0.3(0.0317) = 0.009\,52\ \text{m} = 9.52\ \text{mm}$ ◄

Eq. 9.22: $\tau_0 = 0.0587(999.1)\left(\dfrac{0.9^2}{2}\right)\left(\dfrac{1}{11.88}\right) = 1.999\ \text{N/m}^2$ ◄

(*b*) At trailing edge: x = 0.6 m, $\mathbf{R}_x$ = 474 000

$\delta = 0.016\,57\ \text{m} = 16.57\ \text{mm}$ ◄ $\tau_0 = 1.740\ \text{N/m}^2$ ◄

(*c*) Total drag on one side of plate, for $\mathbf{R} < 10^7$: Eq. 9.25: $C_f = \dfrac{0.0735}{474\,000^{1/5}} = \dfrac{0.0735}{13.65} = 0.005\,38$

Eq. 9.2: $F_f = 0.005\,38(999.1)(0.9^2/2)(3.0)0.6 = 3.92\ \text{N}$ ◄

The shear stresses, boundary-layer thicknesses, and total friction drag for this turbulent boundary layer are 2.8−4.5 times greater than those for a laminar boundary layer (Exer. 9.3.2). ◄

9.4.5 *A 280-ft-long streamlined train has 8.5-ft-high sides and an 8-ft-wide top. Compute the power required to overcome the skin-friction drag when the train is traveling at 90 mph through the ICAO standard atmosphere at sea level, assuming the drag on the sides and top to be equal to that on one side of a flat plate 25 ft wide and 280 ft long.*

BG

Table A.3 for air at sea level: $\rho = 0.002\,377$ slug/ft^3, $\nu = 0.000\,1572$ ft^2/sec.

With $V = 90(44/30) = 132.0$ fps, Eq. 7.6: $\mathbf{R} = 280(132.0)/0.000\,1572 = 2.35 \times 10^8$

$\mathbf{R} > 500{,}000$, so boundary layer is turbulent.

As $\mathbf{R} > 10^7$, Eq. 9.26: $C_f = \dfrac{0.455}{[\log(2.35 \times 10^8)]^{2.58}} = 0.001\,893$

Eq. 9.2: $F_f = C_f\rho(V^2/2)BL = 0.001\,893(0.002\,377)(132^2/2)(25)280 = 274$ lb

Power $= FV/550 = 274(132)/550 = 65.9$ hp ◀

Sec. 9.4: Turbulent Boundary Layer for Incompressible Flow Along a Smooth Flat Plate -- *Problems 9.5–9.10*

9.5 *Derive Eq. (9.19) along the lines suggested in the text.*

N

Rearranging Eq. 9.18: $\delta^{1/4}\, d\delta = \dfrac{0.023\, dx}{\alpha(U/\nu)^{1/4}}$

Integrating, with $\delta = 0$ at $x = 0$: $\dfrac{4}{5}\delta^{5/4} = \dfrac{0.023x}{\alpha(U/\nu)^{1/4}}$

Multiply by $\dfrac{5}{4}$ and take $\dfrac{4}{5}$ root: $\delta = \left(\dfrac{0.0288}{\alpha}\right)^{4/5}\left(\dfrac{\nu}{Ux}\right)^{1/5}x$ which is Eq. (9.19) Q.E.D. ◀

9.6 *For the turbulent boundary layer, derive the value of $\alpha = 0.0972$ from the seventh-root law given in Eq. (9.20).*

N

Eq. 9.20: $u = U\eta^{1/7}$ or $f(\eta) = \eta^{1/7}$

Then $\alpha = \int_0^1 f(\eta)[1 - f(\eta)]d\eta = \int_0^1 \eta^{1/7}(1 - \eta^{1/7})d\eta = \left[\dfrac{7}{8}\eta^{8/7} - \dfrac{7}{9}\eta^{9/7}\right]_0^1 = 0.0972$ Q.E.D. ◀

9.7 *From the information given, derive Eq. (9.22).*

N

Eq. 9.17: $\tau_0 = 0.023\rho U^2[\nu/(\delta U)]^{1/4}$

Eq. 9.19 with $\alpha = 0.0972$: $\delta = 0.377[\nu/(Ux)]^{1/5}x$ which is Eq. 9.21.

Substituting this δ into Eq. 9.17: $\tau_0 = 0.023\rho U^2\left(\dfrac{\nu}{U}\right)^{1/4}\left[\dfrac{1}{0.377x}\left(\dfrac{\nu}{Ux}\right)^{-1/5}\right]^{1/4}$

$$= \frac{0.023}{0.377^{1/4}}\rho U^2\frac{\nu^{1/4-1/20}}{(Ux)^{1/4-1/20}} = 0.0587\rho(U^2/2)\left(\frac{\nu}{Ux}\right)^{1/5} = 0.0587\rho\frac{U^2/2}{\mathbf{R}_x^{1/5}} \quad \text{Q.E.D.} \blacktriangleleft$$

9.8 *Assume that the boundary layer of Prob. 9.4 is disturbed near the leading edge. Compute the corresponding quantities for the turbulent boundary layer covering the entire plate, and compare the results.*

Prob. 9.4: A smooth flat plate 10 ft wide × 2 ft long parallel to the flow is immersed in 60°F water (ρ = 1.938 slug/ft^3) flowing at undisturbed U = 3 fps. Find τ_0 and δ at (a) the center (where x = 1 ft and $\mathbf{R}_x$ = 247,000) and (b) the trailing edge (where x = 2 ft and $\mathbf{R}_x$ = 493,000). Also (c) find F_f on one side.

BG

(*a*) At center of plate: $x = 1.0$ ft, $\mathbf{R}_x = 247{,}000$; Eq. 9.21: $\frac{\delta}{x} = \frac{0.377}{247{,}000^{1/5}} = \frac{0.377}{11.98} = 0.0315$

$\therefore\ \delta = 1.0(0.0315) = 0.0315$ ft $= 0.378$ inches ◄

Eq. 9.22: $\tau_0 = 0.0587(1.938)(3^2/2)(1/11.98) = 0.0427$ lb/ft^2 ◄

(*b*) At trailing edge: $x = 2.0$ ft, $\mathbf{R}_x = 493{,}000$

Eq. 9.21: $\delta = 2.0\frac{0.377}{493{,}000^{1/5}} = \frac{(2.0)0.377}{13.76} = 0.0548$ ft $= 0.658$ in ◄

Eq. 9.22: $\tau_0 = 0.0587(1.938)(3^2/2)(1/13.76) = 0.0372$ lb/ft^2 ◄

(*c*) Total drag on one side, for $\mathbf{R} < 10^7$: Eq. 9.25: $C_f = \frac{0.0735}{493{,}000^{0.2}} = \frac{0.0735}{13.76} = 0.005\,34$

Eq. 9.2: $F_f = 0.005\,34(1.938)(3^2/2)(10)2.0 = 0.932$ lb ◄

The shear stresses, boundary-layer thicknesses, and total friction drag for this turbulent boundary layer are 2.8–4.5 times greater than those for a laminar boundary layer (Prob. 9.4).

9.9 *A 85-m-long streamlined train has 2.5-m-high sides and a 2.5-m-wide top. Compute the power required to overcome the skin-friction drag when the train is traveling at 40 m/s through the ICAO standard atmosphere at sea level, assuming the drag on the sides and top to be equal to that on one side of a flat plate 7.5 m wide and 85 m long.*

SI

Table A.3 for air at sea level: $\rho = 1.225$ kg/m^3, $\nu = 14.61 \times 10^{-6}$ m^2/s

Eq. 7.6: $\mathbf{R} = \frac{40(85)}{14.61 \times 10^{-6}} = 2.33 \times 10^8$; Eq. 9.26: $C_f = \frac{0.455}{[\log(2.33 \times 10^8)]^{2.58}} = 0.001\,896$

Eq. 9.2: $F_f = 0.001\,896(1.225)(40^2/2)(7.5)85 = 1184$ N

Power $= 1184(40) = 47\,400$ W $= 47.4$ kW ◄

9.10 *A lifeguard determines the wind velocity 2 m above the beach to be 8 m/s. If one wishes to get out of the wind by lying down, what would be the velocity at (a) 0.15 m; (b) at 0.3 m above beach level?*

SI

Use the seventh-root law to approximate the velocity profile. Eq. 9.20 (and 8.49): $u = 8(y/2)^{1/7}$

(*a*) $u_{0.15} = 8(0.15/2)^{1/7} = 5.53$ m/s ◄ (*b*) $u_{0.30} = 8(0.3/2)^{1/7} = 6.10$ m/s ◄

Sec. 9.5: Friction Drag for Incompressible Flow Along a Smooth Flat Plate with a Transition Regime -- ***Exercises***

9.5.1 *A 7.5-ft by 1.5-ft smooth, thin, flat plate with sharpened edges is submerged in 60°F water moving with a velocity of 1.4 fps in the direction of the 7.5 ft length. What is the total drag?*

BG

Table A.1 for 60°F water: $\rho = 1.938$ slugs/ft^3; $\nu = 1.217 \times 10^{-5}$ ft^2/sec

Eq. 7.6: $\mathbf{R} = \dfrac{7.5(1.4)}{1.217 \times 10^{-5}} = 863{,}000$

Eq. 9.27: $C_f = \dfrac{0.455}{(\log 863{,}000)^{2.58}} - \dfrac{1700}{863{,}000} = \dfrac{0.455}{99.0} - 0.001\,970 = 0.002\,63$

Eq. 9.2: $F_f = 2 \times 0.002\,63(1.938)(1.4^2/2)(1.5)7.5 = 0.1124$ lb for 2 sides ◀

9.5.2 *An 1-in-diameter harpoon 6 ft long, with a sharp tip, is launched at 20 fps into 60°F water. Find (a) the friction drag; (b) the maximum thickness of the boundary layer.*

BG

(*a*) Table A.1 for 60°F water: $\rho = 1.938$ slugs/ft^3; $\nu = 1.217 \times 10^{-5}$ ft^2/sec

Secs. 9.3 and 9.5: $x_c = \dfrac{\mathbf{R}_c \nu}{V} = \dfrac{500{,}000(1.217 \times 10^{-5})}{20} = 0.304$ ft

$\therefore\ x_c < L = 6$ ft, and a transition region must exist.

Eq. 7.6: $\mathbf{R} = \dfrac{VL}{\nu} = \dfrac{20(6)}{1.217 \times 10^{-5}} = 9.86 \times 10^6$

Eq. 9.27: $C_f = \dfrac{0.455}{[\log(9.86 \times 10^6)]^{2.58}} - \dfrac{1700}{9.86 \times 10^6} = 0.002\,84$

Eq. 9.2: $F_f = 0.002\,84(1.938)(20^2/2)[\pi(1)/12]6 = 1.728$ lb ◀

(*b*) From Eq. 9.21: $\delta = \dfrac{0.377 \times 6}{(9.86 \times 10^6)^{1/5}} = 0.0903$ ft or 1.084 in ◀

9.5.3 *An airplane wing with a chord length of 3 m parallel to the flow moves through standard atmospheric air at an altitude of 6 km with a speed of 350 km/h. Find (a) the critical roughness for a point one-tenth the chord length back from the leading edge; (b) the surface drag on an 8-m-span section of this wing.*

SI

Table A.3 for air at 6 km altitude: $\rho = 0.660$ kg/m^3, $\nu = 24.16 \times 10^{-6}$ m^2/s

(*a*) For $x = c/10 = 3/10 = 0.3$ m, and $U = 350$ km/h $= 350(1000/3600)$ m/s $= 97.2$ m/s,

Eq. 7.6: $\mathbf{R}_x = \dfrac{97.2(0.3)}{2.416 \times 10^{-5}} = 1.207 \times 10^6$

Eq. 9.28: $e_c = 26.0(2.416 \times 10^{-5}/97.2)(1.207 \times 10^6)^{1/4} = 0.000\,214$ m $= 0.214$ mm ◀

(*b*) For entire wing: $\mathbf{R} = \dfrac{97.2(3)}{2.416 \times 10^{-5}} = 1.207 \times 10^7$

Eq. 9.27: $C_f = \dfrac{0.455}{[\log(1.207 \times 10^7)]^{2.58}} - \dfrac{1700}{1.207 \times 10^7} = 0.002\,77$

Eq. 9.2 for both upper and lower surfaces: $F_f = 0.002\,77(0.660)(97.2^2/2)(8)3(2) = 415$ N ◀

Sec. 9.5: Friction Drag for Incompressible Flow Along a Smooth Flat Plate with a Transition Regime -- ***Problems***

9.11 *Verify that the two expressions of Eq. (9.28) are equal.*

N

Eq. 9.11: $\tau_0 = 0.332(\mu U/x)(Ux/\nu)^{1/2}$

Substituting into the first expression of Eq. 9.28: $e_c = \dfrac{15\nu}{(\tau_0/\rho)^{1/2}} = \dfrac{15\nu}{[0.322(\mu U/\rho x)(Ux/\nu)^{1/2}]^{1/2}}$

$$= \frac{15\nu x^{1/2}\nu^{1/4}}{0.332^{1/2}\nu^{1/2}U^{1/2}U^{1/4}x^{1/4}} = 26.0\frac{\nu}{U}\left(\frac{Ux}{\nu}\right)^{1/4} = 26.0\frac{\nu}{U}\mathbf{R}_x^{1/4} \quad \text{Q.E.D.} \quad ◄$$

9.12 *A 20-mm-diameter harpoon 1.8 m long, with a sharp tip, is launched at 6 m/s into 15°C water. Find (a) the friction drag; (b) the maximum thickness of the boundary layer.*

SI

(*a*) Table A.1 for 15°C water: $\nu = 1.139 \times 10^{-6}$ m²/s, $\rho = 999$ kg/m³

$$x_c = \frac{\mathbf{R}_c \nu}{V} = \frac{500\,000(1.139 \times 10^{-6})}{6} = 0.0949 \text{ m}$$

$\therefore\ x_c < L = 1.8$ m, and a transition region must exist.

Eq. 7.6: $\mathbf{R} = \dfrac{VL}{\nu} = \dfrac{6(1.8)}{1.139 \times 10^{-6}} = 9.48 \times 10^6$

Eq. 9.27: $C_f = \dfrac{0.455}{[\log(9.48 \times 10^6)]^{2.58}} - \dfrac{1700}{9.48 \times 10^6} = 0.002\,85$

Eq. 9.2: $F_f = 0.002\,85(999)(6^2/2)(0.020\pi)1.8 = 5.80$ N ◄

(*b*) Eq. 9.21: $\delta = \dfrac{0.377 \times 1.8}{(9.48 \times 10^6)^{1/5}} = 0.0273$ m or 27.3 mm ◄

9.13 *An airplane wing with a chord length of 7 ft parallel to the flow moves through standard atmospheric air at an altitude of 15,000 ft with a speed of 300 mph (Fig. P9.13). Find (a) the critical roughness for a point one-tenth the chord length back from the leading edge; (b) the surface drag on a 25-ft span section of this wing.*

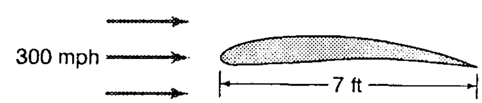


Figure P9.13

BG

Table A.3 for air at 15,000 ft altitude: $\rho = 0.001\,49$ slug/ft³, $\nu = 0.229 \times 10^{-3}$ ft²/sec

(*a*) For $x = c/10 = 7/10 = 0.70$ ft, and $U = 300(44/30) = 440$ fps,

Eq. 7.6: $\mathbf{R}_x = \dfrac{440(0.70)}{2.29 \times 10^{-4}} = 1.345 \times 10^6$

Eq. 9.28: $e_c = 26.0(2.29 \times 10^{-4}/440)(1.345 \times 10^6)^{1/4} = 0.000\,461$ ft or 0.005 53 in ◄

(*b*) For entire wing: $\mathbf{R} = \dfrac{440(7)}{2.29 \times 10^{-4}} = 1.345 \times 10^7$

Eq. 9.27: $C_f = \dfrac{0.455}{[\log(1.345 \times 10^7)]^{2.58}} - \dfrac{1700}{1.345 \times 10^7} = 0.002\,74$

Eq. 9.2 for both upper and lower surfaces: $F_f = 0.002\,74(0.001\,496)(440^2/2)25(7)2 = 138.8$ ◄

9.14 *A flat plate 25 ft long and 1.5 ft wide is towed at 8 fps through a liquid (γ = 50 lb/ft^3, μ = 0.000 26 lb·sec/ft^2). Determine the total drag on the plate. Plot the thickness of the boundary layer and the local shear stress τ_0 as functions of x along the plate. Determine the area under the stress curve, and compare it with the previously computed value of the drag. Assume that the boundary layer changes from laminar to turbulent at a Reynolds number of 300,000.*

BG

$\rho = 1.940(50/62.4) = 1.554$ slug/ft^3 ; Given: $300{,}000 = \mathbf{R}_c = x_c \dfrac{U\rho}{\mu} = x_c \dfrac{(8)1.554}{0.000\,26}$

$\therefore\ x_c = 6.27$ ft, where the turbulent boundary layer begins.

$x_c < L = 25$ ft, so a transition region must exist.

For entire plate: $\mathbf{R} = LU\rho/\mu = (25)8(1.554)/0.000\,26 = 1.196 \times 10^6$

Sec. 9.5: $F_f = \rho \dfrac{U^2}{2} B \left[\dfrac{1.328 x_c}{\mathbf{R}_c^{1/2}} + \dfrac{0.455 L}{(\log \mathbf{R})^{2.58}} - \dfrac{0.0735 x_c}{\mathbf{R}_c^{1/5}} \right]$

$F_f = 1.554(8^2/2)1.5[0.0152 + 0.1081 - 0.0370] = 6.44$ lb/side

$\therefore$ Total drag = 2 × 6.44 = 12.88 lb ◄

For x < 6.27 ft (laminar B.L.), Eqs. 9.10 and 9.11 are applicable:

$\delta = 4.91 \dfrac{x}{\mathbf{R}_x^{1/2}};\quad \tau_0 = 0.332 \dfrac{\mu U}{x} \mathbf{R}_x^{1/2} = 0.000\,691 \dfrac{\mathbf{R}_x^{1/2}}{x}$ where $\mathbf{R}_x = \dfrac{x(8)1.554}{0.000\,26} = 47{,}800x$

x (ft)	2	4	6	6.27	
$\mathbf{R}_x$	95,660	191,320	287,000	300,000	
δ (ft)	0.0318	0.0449	0.0550	0.0562	◄◄
τ_0 (psf)	0.1068	0.0755	0.0617	0.0603	◄◄

For x > 6.27 ft (turbulent B.L.), Eqs. 9.21 and 9.22 apply: $\delta = 0.377 \dfrac{x}{\mathbf{R}_x^{1/5}};\quad \tau_0 = 0.0587 \dfrac{\rho U^2}{2\mathbf{R}_x^{1/5}} = \dfrac{2.92}{\mathbf{R}_x^{1/5}}$

x (ft)	6.27	12	16	25	
$\mathbf{R}_x$	300,000	574,000	765,300	1,195,800	
δ (ft)	0.1898	0.319	0.402	0.574	◄◄
τ_0 (psf)	0.234	0.206	0.1944	0.1778	◄◄

Area A under stress curve = $\int \tau_0\, dx$ = 4.47 lb/ft (by integration),

so drag = AB = 4.47(1.5) = 6.71 lb/side ◄

This compares very well with the previously above computed drag of F_f = 6.44 lb/side. ◄
Actually the δ's and the τ_0's in the second table are too large because they represent values for the case of a turbulent boundary layer over the entire length.

9.15 *For velocities of 0, 10, 20, 30, 40, 50, and 60 fps, make the necessary calculations to plot drag vs. velocity for the plate and data of Prob. 9.14.*

Prob. 9.14: $L = 25$ *ft*, $B = 1.5$ *ft*, $\rho = 1.554$ *slug/ft*^{3}, $\mu = 0.000\,26$ *lb·sec/ft*^{2}, $\mathbf{R}_c = 300{,}000$.

BG

Sec. 9.5: $F_f = \rho\dfrac{U^2}{2}B\left[\dfrac{1.328x_c}{\mathbf{R}_c^{1/2}} + \dfrac{0.455L}{(\log \mathbf{R})^{2.58}} - \dfrac{0.0735x_c}{\mathbf{R}_c^{1/5}}\right]$

Given $\mathbf{R}_c = \dfrac{U\rho x_c}{\mu} = 300{,}000$, $\therefore\ x_c = \dfrac{50.2}{U}$, $\mathbf{R}_c^{1/2} = 548$, $\mathbf{R}_c^{1/5} = 12.46$

$$\mathbf{R} = \frac{LU\rho}{\mu} = \frac{25(U)1.554}{0.00026} = 149{,}500U\ ;\quad \frac{\rho B}{2} = 1.166$$

$$\therefore\ F_f = 1.166U^2\left(\frac{1.328x_c}{548} + \frac{0.455(25)}{(\log \mathbf{R})^{2.58}} - \frac{0.0735x_c}{12.46}\right) = 1.166U^2\left(\frac{11.38}{(\log \mathbf{R})^{2.58}} - 0.003\,48x_c\right)$$

U	0	10	20	30	40	50	60 (fps)
x_c	–	5.02	2.51	1.673	1.254	1.004	0.836 (ft)
$\mathbf{R}$	0	1.495	2.99	4.48	5.98	7.47	8.97 ($\times 10^6$)
F_f	–	10.07	38.7	83.8	144.2	219	309 (lb/side)

◄◄

9.16 *Determine the drag on the harpoon of Exer. 9.5.2 for velocities of 0, 10, 30, and 50 fps, through (a) 60°F water; (b) 60°F air at standard sea-level pressure. Calculate the length x_c of the laminar zone, and plot curves of drag versus velocity for each case.*

Exer. 9.5.2: Harpoon diameter = 1 in, L = 6 ft.

BG

**Note to instructor:* At the lowest V in air, $\mathbf{R} < 500{,}000$, so the boundary layer is only laminar.

Eq. 9.2: $F_f = C_f \rho \dfrac{V^2}{2} BL$

With transition regime (i.e., $x_c < L$), Eq. 9.27: $C_f = \dfrac{0.455}{(\log \mathbf{R})^{2.58}} - \dfrac{1700}{\mathbf{R}}$

With laminar flow only (i.e., $x_c > L$), Eq. 9.14: $C_f = 1.318/\mathbf{R}^{1/2}$

(*a*) Table A.1 for 60°F water: $\rho = 1.938$ slugs/ft³, $\nu = 1.217 \times 10^{-5}$ ft²/sec

$$x_c = \frac{\mathbf{R}_c \nu}{V} = \frac{500{,}000}{V}(1.217 \times 10^{-5}) = \frac{6.09}{V}\ \text{ft}$$

Eq. 7.6: $\mathbf{R} = \dfrac{LV}{\nu} = \dfrac{6V}{1.217 \times 10^{-5}} = 493{,}000V$; Eq. 9.2: $F_f = C_f 1.938 \dfrac{V^2}{2}\left(\dfrac{1\pi}{12}\right)6 = 1.522 C_f V^2$

V (fps)	0	10	30	50	
x_c (ft)	–	0.609	0.203	0.1217	◄◄
$\mathbf{R}$	0	4.93×10^6	1.479×10^7	2.47×10^7	
Eq. for C_f	–	9.27	9.27	9.27	
C_f	–	0.003 03	0.002 71	0.002 54	
F_f (lb)	0	0.461	3.71	9.67	◄◄

(*b*) Table A.2 for air at 60°F: $\rho = 0.002\,37$ slugs/ft³, $\nu = 0.000\,158$ ft²/sec

$$x_c = \frac{\mathbf{R}_c \nu}{V} = \frac{500{,}000}{V}(0.000\,158) = \frac{79.0}{V}\ \text{ft}$$

Eq. 7.6: $\mathbf{R} = \dfrac{LV}{\nu} = \dfrac{6V}{0.000\,158} = 38{,}000V$; Eq. 9.2: $F_f = C_f 0.002\,37 \dfrac{V^2}{2}\left(\dfrac{1\pi}{12}\right)6 = 0.001\,861 C_f V^2$

V (fps)	0	10	30	50	
x_c (ft)	–	$7.90 > L$	2.63	1.580	◄◄
$\mathbf{R}$	0	*3.80×10^5	1.139×10^6	1.899×10^6	
Eq. for C_f	–	9.14	9.27	9.27	
C_f	–	0.002 14	0.002 87	0.003 08	
F_f (lb)	0	0.000 398	0.004 81	0.014 34	◄◄

9.17 *Determine the drag on the harpoon of Prob. 9.12 for velocities of 0, 3, 9, and 15 m/s, through (a) 15°C water; (b) 15°C air at standard sea-level pressure. Calculate the length x_c of the laminar zone, and plot curves of drag versus velocity for each case.*

Prob. 9.12: Harpoon diameter = 20 mm, L = 1.8 m.

SI

**Note to instructor:* At the lowest V in air, $\mathbf{R} < 500{,}000$, so the boundary layer is only laminar.

Eq. 9.2: $F_f = C_f \rho \dfrac{V^2}{2} BL$

With transition regime (i.e., $x_c < L$), Eq. 9.27: $C_f = \dfrac{0.455}{(\log \mathbf{R})^{2.58}} - \dfrac{1700}{\mathbf{R}}$

With laminar flow only (i.e., $x_c > L$), Eq. 9.14: $C_f = 1.318/\mathbf{R}^{1/2}$

(*a*) Table A.1 for 15°C water: $\rho = 999.1$ kg/m³, $\nu = 1.139 \times 10^{-6}$ m²/s

$$x_c = \frac{\mathbf{R}_c \nu}{V} = \frac{500\,000}{V}(1.139 \times 10^{-6}) = \frac{0.570}{V}\ \text{m}$$

$$\text{Eq. 7.6:}\quad \mathbf{R} = \frac{LV}{\nu} = \frac{1.8V}{1.139 \times 10^{-6}} = 1580\,000V$$

$$\text{Eq. 9.2:}\quad F_f = C_f 999.1(V^2/2)(0.020\pi)1.8 = 56.5C_f V^2$$

V (m/s)	0	3	9	15	
x_c (m)	–	0.1898	0.0633	0.0380	◄◄
R	0	4.74×10^6	1.422×10^7	2.37×10^7	
Eq. for C_f	–	9.27	9.27	9.27	
C_f	–	0.003 04	0.002 72	0.002 55	
F_f (N)	0	1.544	12.45	32.5	◄◄

(*b*) Table A.2 for air at 15°C, by interpolation: $\rho = 1.227$ kg/m³, $\nu = 1.455 \times 10^{-5}$ m²/s

$$x_c = \frac{\mathbf{R}_c \nu}{V} = \frac{500\,000}{V}(1.455 \times 10^{-5}) = \frac{7.28}{V}\ \text{N};\quad \text{Eq. 7.6:}\quad \mathbf{R} = \frac{LV}{\nu} = \frac{1.5V}{1.455 \times 10^{-5}} = 103\,100V$$

$$\text{Eq. 9.2:}\quad F_f = C_f 1.227(V^2/2)(0.020\pi)1.8 = 0.0694C_f V^2$$

V (m/s)	0	3	9	15	
x_c (m)	–	$2.43 > L$	0.808	0.485	◄◄
R	0	$*3.09 \times 10^5$	9.28×10^5	1.546×10^6	
Eq. for C_f	–	9.14	9.27	9.27	
C_f	–	0.002 37	0.002 70	0.003 03	
F_f (N)	0	0.001 480	0.015 18	0.0473	◄◄

Sec. 9.7: Drag on Three-Dimensional Bodies (Incompressible Flow) -- *Exercises (8)*

9.7.1 *An 0.3-in-diameter steel sphere (s = 8.0) is released in a large tank of oil (s = 0.80). The terminal velocity of the sphere is determined to be 2.5 fpm. Calculate the viscosity of the oil?*

BG

Assume $\mathbf{R} < 1$, ∴ Use Stoke's Law (Eq. 9.33): $F_D = 3\pi\mu VD$. Table A.8: $\forall_{\text{sphere}} = \pi D^3/6$

At terminal velocity $\sum F_z = 0 = W - F_B - F_D$; ∴ $(\gamma_s - \gamma_l)\pi D^3/6 = 3\pi\mu VD$

from which $\mu = (\gamma_s - \gamma_l)(D^2/18V) = (8 - 0.8)62.4\dfrac{(0.3/12)^2}{18(2.5/60)} = 0.374\ \text{lb·sec/ft}^2$ ◄

Check: $\mathbf{R} = \dfrac{VD\rho}{\mu} = \dfrac{2.5}{60}\left(\dfrac{0.3}{12}\right)\dfrac{1.940 \times 0.8}{0.374} = 0.004\,32 < 1$, so the use of Stoke's Law here is valid.

9.7.2 *A 5-mm-diameter steel sphere (s = 8.0) is released in a large tank of oil (s = 0.80). The terminal velocity of the sphere is determined to be 0.7 m/min. Calculate the viscosity of the oil?*

SI

Assume **R** < 1, ∴ Use Stoke's Law (Eq. 9.33): $F_D = 3\pi\mu VD$. Table A.8: $\mathcal{V}_{sphere} = \pi D^3/6$

At terminal velocity $\sum F_z = 0 = W - F_B - F_D$; ∴ $(\gamma_s - \gamma_l)\pi D^3/6 = 3\pi\mu VD$

from which $\mu = (\gamma_s - \gamma_l)(D^2/18V) = (8 - 0.8)9810\dfrac{0.005^2}{18(0.7/60)} = 8.41\ \text{N·s/m}^2$ ◄

Check: $\mathbf{R} = VD\rho/\mu = (0.7/60)(0.005)(1000 \times 0.8)/8.41 = 0.005\,55$

As **R** < 1, the use of Stoke's Law here is valid.

9.7.3 *For a 380-mm-diameter sphere, compute the drag from wind under sea level conditions in a standard ICAO atmosphere. Plot drag versus velocity for 0, 10, 20, and 30 m/s.*

SI

Table A.3 for air at sea level: $T = 15°\text{C}$, $\rho = 1.225\ \text{kg/m}^3$, $\nu = 1.461 \times 10^{-5}\ \text{m}^2/\text{s}$

$A = \pi r^2 = \pi 0.19^2 = 0.1134\ \text{m}^2$. Eq. 9.32: $F_D = C_D 1.225(V^2/2)0.1134 = 0.0695V^2C_D$

Eq. 7.6: $\mathbf{R} = \dfrac{DV}{\nu} = \dfrac{0.38V}{1.461 \times 10^{-5}} = 26\,000V$. Solve for F_D by finding C_D from Fig. 9.10

V (m/s)	0	10	20	30
R	–	260 000	520 000	780 000
C_D	0	0.18	0.19	0.20
F_D (N)	0	1.250	5.28	12.50

◄◄

9.7.4 *A poorly-streamlined automobile has a body form corresponding roughly to the 1:0.75 oblate ellipsoid of Fig. 9.10, while a well-streamlined car has a body approximating the streamlined body in the same figure, each with a diameter of 5 ft. If the velocity is 50 mph through standard air at sea level (Appendix A, Table A.3), find the horsepower required to overcome air resistance in each case.*

BG

Table A.3: $\rho = 0.002\,377\ \text{slug/ft}^3$, $\nu = 0.000\,1572\ \text{ft}^2/\text{sec}$. $V = 50(44/30) = 73.3$ fps.

$\mathbf{R} = \dfrac{5(73.3)}{0.000\,1572} = 2.33 \times 10^6$. Assuming in Fig. 9.10 that C_D remains constant for $\mathbf{R} > 10^6$:

(*a*) C_D (poorly-streamlined = 1 : 0.75 oblate ellipsoid) = 0.20 (*b*) C_D (streamlined body) = 0.04

(*a*) Eq. 9.32: $F_D = 0.2 \times 0.002\,377(73.3^2/2)(\pi/4)5^2 = 25.1$ lb (*b*) $F_D = (0.04/0.2)25.1 = 5.02$ lb

Power (*a*) $F_DV/550 = (25.1)73.3/550 = 3.35$ hp ◄

(*b*) $F_DV/550 = (5.02)73.3/550 = 0.669$ hp ◄

9.7.5 *For the streamlined train in Exer. 9.4.5, calculate the pressure drag. As a rough approximation, assume that the nose and tail of the train are of the shape of the two halves of the prolate ellipsoid of Fig. 9.10, of 8.5 ft diameter. Find the drag on the ellipsoid (pressure drag on the train), and compare this with the skin-friction drag on the train determined earlier.*

Exer. 9.4.3: $\rho = 0.002\,377\ \text{slug/ft}^3$, $\nu = 0.000\,1572\ \text{ft}^2/\text{sec}$, $V = 132.0$ fps, skin-friction drag F_f = 274 lb.

BG

Eq. 7.6: $\mathbf{R} = \dfrac{8.5(132.0)}{0.000\,1572} = 7.14 \times 10^6$; Fig. 9.10 (prolate ellipsoid): $C_D = 0.08$

Eq. 9.1: $F_p = (0.08)0.002\,377(132^2/2)(\pi/4)8.5^2 = 94.0$ lb ◄

This is about 1/3 of the friction drag of 274 lb determined in Exer. 9.4.3 ◄

9.7.6 *A hemispherical shell with its concave side upstream has a drag coefficient of approximately 1.33 if* **R** > 10^3*. If the total load is 250 lb, find the diameter of a hemispherical parachute needed to provide a fall velocity no greater than that caused by jumping from a height of 10 ft. Assume standard air at sea level.*

BG

In jump from 10 ft height, final $V = \sqrt{2gh} = \sqrt{(2)32.2(10)} = 25.4$ fps.

Table A.3 for standard atmosphere at sea level: $\rho = 0.002\,377$ slug/ft³, $\nu = 0.000\,1572$ ft²/s.

Then, by Eq. 9.32, if **R** > 1000: $250 = (1.33)0.002\,377(25.4^2/2)(\pi/4)D^2$; $D = 17.68$ ft ◀

Check: $\mathbf{R} = \dfrac{17.68(25.4)}{0.000\,1572} = 2.85 \times 10^6 > 1000$, ∴ O.K.

9.7.7 *A hemispherical shell with its concave side upstream has a drag coefficient of approximately 1.33 if* **R** > 10^3*. If the total load is 1000 N, find the diameter of a hemispherical parachute needed to provide a fall velocity no greater than that caused by jumping from a height of 3 m. Assume standard air at sea level.*

SI

In jump from 3 m height, final $V = \sqrt{2gh} = \sqrt{(2)9.81(3)} = 7.67$ m/s.

Table A.3 for standard atmosphere at sea level: $\rho = 1.225$ kg/m³, $\nu = 1.461 \times 10^{-5}$ m²/s.

Then, by Eq. 9.32, if **R** > 1000: $1000 = (1.33)1.225(7.67^2/2)(\pi/4)D^2$; $D = 5.15$ m ◀

Check: $\mathbf{R} = \dfrac{5.15(7.67)}{1.461 \times 10^{-5}} = 2.71 \times 10^6 > 1000$, ∴ O.K.

9.7.8 *Assuming $C_D = 1.25$ and a speed of 30 m/s, find the drag force exerted by a 3.5-m-diameter braking parachute at sea level? At what speed will the same braking force be exerted by this parachute at an elevation of 2000 m? Assume C_D remains constant.*

SI

Table A.3 for air: at sea level, $\rho = 1.225$ kg/m³; at 2000 m, $\rho = 1.007$ kg/m³

Eq. 9.32 at sea level: $F_D = C_D A\rho V^2/2 = 1.25\pi(1.75)^2 1.225(30^2/2) = 6630$ N ◀

At 2000 m: $6630 = 1.25\pi(1.75)^2 1.007(V^2/2)$; $V = 33.1$ m/s ◀

Sec. 9.7: Drag on Three-Dimensional Bodies (Incompressible Flow) -- *Problems* 9.18–9.31

9.18 *What will be the terminal velocity of the sphere of Exer. 9.7.1 in 120°F water?*
Exer. 9.7.1: D = 0.3 in, s = 8.0.

BG

Table A.1 for 120°F water: $\rho = 1.918$ slug/ft³, $\gamma = 61.71$ pcf, $\nu = 6.09 \times 10^{-6}$ ft²/sec

Select a value of C_D from Fig. 9.10. After several trials, assume $C_D = 0.43$

At terminal V, $W - F_B = F_D$ using Eqs. 9.32 and 9.33: $(\gamma_s - \gamma)(\pi D^3/6) = C_D\rho(A/2)V^2$

$(8 \times 62.4 - 61.71)(\pi/6)(0.30/12)^3 = 0.43(1.918)(\pi/2)(0.15/12)^2V^2$; $V = 4.20$ fps

Eq. 7.6: $\mathbf{R} = \dfrac{(0.3/12)4.20}{6.09 \times 10^{-6}} = 17{,}300$

This yields $C_D = 0.43$ for a sphere in Fig. 9.10, therefore $V = 4.20$ fps ◀

9.19 *For an 18-in-diameter sphere, compute the drag from wind under sea level conditions in a standard ICAO atmosphere. Plot drag versus velocity for 0, 25, 50, and 100 fps.*

BG

Table A.3 for air at sea level: $T = 59.0°\text{F}$, $\rho = 0.002\,377$ slug/ft^3, $\nu = 0.000\,1572$ ft^2/sec,

$A = \pi r^2 = \pi(9/12)^2 = 1.767$ ft^2

Eq. 9.32: $F_D = C_D\rho(A/2)V^2 = C_D 0.002\,377(1.767/2)V^2 = 0.002\,10V^2C_D$

Eq. 7.6: $\mathbf{R} = \dfrac{DV}{\nu} = \dfrac{(18/12)V}{0.000\,1572} = 9540V$. Solve for F_D by finding C_D from Fig. 9.10

V (fps)	0	25	50	100
R	–	239,000	477,000	954,000
C_D	0	0.17	0.19	0.20
F_D (lb)	0	0.223	0.998	4.20

◄◄

9.20 *Repeat Prob. 9.19 for wind at a 10,000-ft elevation in a standard ICAO atmosphere.*
Prob. 9.19: Sphere D = 18 in. Plot drag vs. velocity for 0, 25, 50, and 100 fps.

BG

Table A.3 for air at 10,000 ft altitude: $\rho = 0.001\,756$ slug/ft^3, $\nu = 0.000201$ ft^2/sec.

$A = \pi r^2 = \pi(9/12)^2 = 1.767$ ft^2 ; Eq. 9.32: $F_D = C_D 0.001\,756(1.767/2)V^2 = 0.001\,552V^2C_D$

Eq. 7.6: $\mathbf{R} = \dfrac{DV}{\nu} = \dfrac{(18/12)V}{0.000\,201} = 7460V$; Solve by finding C_D from Fig. 9.10

V (fps)	0	25	50	100
R	–	186,600	373,000	746,000
C_D	0	0.44	0.185	0.195
F_D (lb)	0	0.427	0.718	3.03

◄◄

9.21 *A 1.5-ft-diameter metal ball weighing 150 lb is dropped into the ocean from a boat. Determine the maximum velocity of the ball as it falls through sea water with $\rho = 2.0$ slugs/ft^3 and $\mu = 3.3 \times 10^{-5}$ lb·sec/ft^2.*

BG

$W - F_B = F_D = C_D\rho AV^2/2$; $W - \gamma(\pi D^3/6) = C_D\rho(\pi D^2/4)V^2/2$

$$150 - \frac{(62.4)2.0}{1.940}\left(\frac{\pi}{6}\right)1.5^3 = C_D(2.0)\frac{\pi}{8}1.5^2V^2;\quad 20.6 = C_DV^2 \qquad (1)$$

$$\text{Eq. 7.6:}\quad \mathbf{R} = \frac{DV\rho}{\mu} = \frac{1.5V2.0}{3.3 \times 10^{-5}} = 90{,}900V \qquad (2)$$

Solve (1) and (2) by trial and error:

(*i*) Try $C_D = 0.4$: $V^2 = 20.6/0.4 = 51.4$

$V = 7.17$ fps, $\mathbf{R} = 652{,}000$; from Fig. 9.10, $C_D = 0.19$

(*ii*) Try $C_D = 0.19$: $V^2 = 20.6/0.19 = 108.2$

$V = 10.40$ fps, $\mathbf{R} = 946{,}000$; from Fig. 9.10, $C_D = 0.20$

Hence $V = (20.6/0.20)^{1/2} = 10.14$ fps ◄

9.22 *A 400-mm-diameter metal ball weighing 500 N is dropped into the ocean from a boat. Determine the maximum velocity of the ball as it falls through sea water with ρ = 1030 kg/m^3 and μ = 0.0016 N·s/m^2.*

SI

$500 - 9810(1030/1000)(\pi/6)0.4^3 = C_D 1030(\pi/8)0.4^2V^2;\ 2.49 = C_D V^2$ (1)

$\mathbf{R} = DV\rho/\mu = 0.4V(1030)/0.0016 = 257\,500V$ (2)

Solve (1) and (2) by T and E:

(*i*) Try $C_D = 0.4$: $V = (2.49/C_D)^{1/2} = 2.497$ m/s ; $\mathbf{R} = 643\,000$; from Fig. 9.10: $C_D = 0.19$

(*ii*) Try $C_D = 0.2$: $V = 3.62$ m/s; $\mathbf{R} = 933\,000$; From Fig. 9.10, $C_D = 0.20$

Hence $V = (2.49/0.20)^{1/2} = 3.53$ m/s ◄

9.23 *A poorly-streamlined automobile has a body form corresponding roughly to the 1:0.75 oblate ellipsoid of Fig. 9.10, while a well-streamlined car has a body approximating the streamlined body in the same figure, each with a diameter of 1.5 m. If the velocity is 30 m/s through standard air at sea level (Appendix A, Table A.3), find the power in kW required to overcome air resistance in each case.*

SI

Table A.3 for air at sea level: $\rho = 1.225$ kg/m^3, $\nu = 1.461 \times 10^{-5}$ m^2/s.

$\mathbf{R} = 1.5(30)/(1.461 \times 10^{-5}) = 3.08 \times 10^6$. Assuming in Fig. 9.10 that C_D remains constant for $\mathbf{R} > 10^6$:

(*a*) C_D (poorly-streamlined = 1:0.75 oblate ellipsoid) = 0.20

(*b*) C_D (streamlined body) = 0.04

(*a*) Using Eq. 9.32: Power = $F_D V = C_D \rho A(V^3/2) = 0.20(1.225)(\pi 1.5^2/4)30^3/2 = 5.84$ kW ◄

(*b*) Power = (0.04/0.20)5.84 = 1.169 kW ◄

9.24 *For the streamlined train in Prob. 9.9, calculate the pressure drag. As a rough approximation, assume that the nose and tail of the train are of the shape of the two halves of the prolate ellipsoid of Fig. 9.10, of 2.5 m diameter. Find the drag on the ellipsoid (pressure drag on the train), and compare this with the skin-friction drag on the train determined earlier.*

Prob. 9.9: ρ = 1.225 kg/m^3, ν = 14.61 × 10^{-6} m^2/s, V = 40 m/s, skin-friction drag F_f = 1184 N.

SI

Eq. 7.6: $\mathbf{R} = (2.5)40/(14.61 \times 10^{-6}) = 6.84 \times 10^6$; Fig. 9.10 (prolate ellipsoid): $C_D = 0.08$

Eq. 9.1: $F_p = (0.08)1.225(40^2/2)(\pi/4)2.5^2 = 385$ N ◄

This is about 1/3 of the friction drag of 1184 N determined in Prob. 9.9. ◄

9.25 *A submerged 7.5-ft by 1.5-ft flat plate is dragged through 60°F water at 1.4 fps with the flat side normal to the direction of motion. (a) What is the approximate drag force? (b) How does this compare with the drag force of 0.1124 lb when pulled in the direction of the 7.5 ft length? [Hint: Assume that the drag coefficient for the plate of finite length is in the same ratio to the coefficient for the infinite plate as is the ratio of coefficients for the finite cylinder (L/D = 5) and the infinite cylinder of Fig. 9.13, for the same Reynolds number.]*

BG

(*a*) Table A.1 for 60°F water: ρ = 1.938 slugs/ft^3, ν = 1.217×10^{-5} ft^2/sec

R = 1.5(1.4)/(1.217 × 10^{-5}) = 172,600. From Fig. 9.13, for infinite plate, C_D = 1.76.

But (Fig. 9.13) for cylinders at **R** = 172,600, $\dfrac{C_D \text{ for } L/D = 5}{C_D \text{ for infinite cyl}} = \dfrac{0.70}{1.15}$

Thus, for the given finite plate (L/D = 5), approximately C_D = 1.76(0.70/1.15) = 1.071

Eq. 9.32: F_D = 1.071(1.938)(1.4^2/2)(7.5)(1.5) = 22.9 lb ◀

(*b*) This is 22.9/0.1124 = 204 times the friction drag when pulled in the direction of the 7.5 ft length (see Exer. 9.5.1). ◀

9.26 *A submerged 2.5-m by 0.5-m flat plate is dragged through 15°C water at 0.5 m/s with the flat side normal to the direction of motion. (a) What is the approximate drag force? (b) How does this compare with the drag force of 0.888 N when pulled in the direction of the 2.5 m length? [Hint: See Prob. 9.25.]*

Prob: 9.25: Hint: Assume that the drag coefficient for the plate of finite length is in the same ratio to the coefficient for the infinite plate as is the ratio of coefficients for the finite cylinder (L/D = 5) and the infinite cylinder of Fig. 9.13, for the same Reynolas number.

SI

(*a*) Table A.1 for 15°C water: ρ = 999.1 kg/m^3, ν = 1.139×10^{-6} m^2/s.

R = 0.5(0.5)/(1.139 × 10^{-6}) = 219 000. From Fig. 9.13, for infinite plate, C_D = 1.76.

But for cylinders at **R** = 439000, $\dfrac{C_D \text{ for } L/D = 5}{C_D \text{ for } L/D = \infty} = \dfrac{0.68}{1.1}$

Thus, for the given finite plate, approximately C_D = 1.76(0.68/1.1) = 1.088

Eq. 9.32: F_D = 1.088(999.1)(0.5^2/2)2.5(0.5) = 169.8 N ◀

(*b*) This is 169.8/0.888 = 191 times the friction drag when pulled in the direction of the 2.5 m length. ◀

9.27 *Compare the velocity of an 0.15-in-diameter drop of water falling through standard air at sea level, with that of a spherical bubble of air of the same size rising through water at the same temperature. Neglect the weight of the air.*

BG

Fig. 2.2 or Table A.3 for standard air at seal level: $T = 59°\text{F}$.

(*a*) For air bubble rising through water:

Table A.1 for water at 59°F (by interpolation): $\rho = 1.938$ slugs/ft^3, $\nu = 0.00001236$ ft^2/sec.

At terminal velocity, neglecting weight of air, $F_D = F_B$

$\therefore$ Using Eq. 9.32: $F_D = C_D(1.938)\dfrac{(\pi/4)(0.15/12)^2}{2}V^2 = F_B = 62.4\dfrac{\pi}{6}\left(\dfrac{0.15}{12}\right)^3 = 63.8 \times 10^{-6}$ lb.

From which $V = 0.733/C_D^{1/2}$

Eq. 7.6: $\mathbf{R} = \dfrac{DV}{\nu} = \dfrac{(0.15/12)V}{0.00001236} = 1011V$

By trial, with Fig. 9.10: $C_D = 0.47$, $V = 1.069$ ft/sec ◀ and $\mathbf{R} = 1104$

(*b*) For water drop falling through air:

Table A.2 for air at 59°F (by interpolation): $\rho = 0.002379$ slugs/ft^3, $\nu = 0.0001574$ ft^2/sec.

At terminal velocity, neglecting weight of air, $W = F_D$

$\therefore$ Using Eq. 9.32: $W = 63.8 \times 10^{-6}$ lb $= F_D = C_D(0.002379)(V^2/2)(\pi/4)(0.15/12)^2$

from which $V = 20.9/C_D^{1/2}$; also $\mathbf{R} = \dfrac{(0.15/12)V}{0.0001574} = 79.4V$

By trial, with Fig. 9.10: $C_D = 0.40$, $V = 33.1$ fps ◀ and $\mathbf{R} = 2618$

Velocity ratio $= V_{\text{drop}}/V_{\text{bubble}} = 33.1/1.1069 = 31.0$ ◀

9.28 *Compare the velocity of a 3-mm-diameter drop of water falling through standard air at sea level, with that of a spherical bubble of air of the same size rising through water at the same temperature. Neglect the weight of the air.*

SI

Figure 2.2 or Table A.3 for standard air at sea level: $T = 15°C$.

(*a*) For air bubble rising through water:

Table A.1 for water at 15°C: $\rho = 999.1$ kg/m^3, $\nu = 1.139 \times 10^{-6}$ m^2/s.

At terminal velocity, neglecting weight of air, $F_D = F_B$

Eq. 9.32: $F_D = C_D(999.1)(V^2/2)(\pi/4)(0.003)^2 = F_B = 9810(\pi/6)0.003^3 = 1.387 \times 10^{-4}$ N

from which $V = 0.1982/C_D^{1/2}$; Eq. 7.6: $\mathbf{R} = \dfrac{DV}{\nu} = \dfrac{0.003V}{1.139 \times 10^{-6}} = 2634V$

By trial, with Fig. 9.10: $C_D = 0.49$, $V = 0.283$ m/s ◄ and $\mathbf{R} = 746$

(*b*) For water drop falling through the air:

Table A.2 for air at 15°C, by interpolation: $\rho = 1.227$ kg/m^3, $\nu = 1.455 \times 10^{-5}$ m^2/s.

At terminal velocity, neglecting weight of air, $W = F_D$

∴ Using Eq. 9.32: $W = 1.387 \times 10^{-4}$ N $= F_D = C_D(1.227)(V^2/2)(\pi/4)(0.003)^2$

from which $V = 5.66/C_D^{1/2}$; also $\mathbf{R} = \dfrac{0.003V}{1.455 \times 10^{-5}} = 206V$

By trial, with Fig. 9.10: $C_D = 0.41$, $V = 8.83$ m/s ◄ and $\mathbf{R} = 1821$

Velocity ratio $= V_{drop}/V_{bubble} = 8.83/0.283 = 31.2$ ◄

9.29 *Find the rate of fall of a spherical particle of sand (s = 2.65) in 15°C water if the diameter is (a) 0.1 mm; (b) 1.0 mm; (c) 10 mm. Express answers in mm/s.*

SI

Table A.1 for water at 15°C: $\gamma = 9798$ N/m^3, $\rho = 999.1$ kg/m^3, $\nu = 1.139 \times 10^{-6}$ m^2/s.

At terminal velocity, net weight in water $= W - F_B = F_D$

So using Eq. 9.32: $(\pi/6)D^3(2.65 - 1)9798 = C_D(999.1)(V^2/2)\pi D^2/4$

which gives $V = 4.64(D/C_D)^{1/2}$. Also $\mathbf{R} = DV/\nu = DV/(1.139 \times 10^{-6}) = 878\,000DV$. From which:

	(*a*)	(*b*)	(*c*)	
D (m)	0.0001	0.001	0.01	
V	$0.0464/C_D^{1/2}$	$0.1469/C_D^{1/2}$	$0.464/C_D^{1/2}$	
$\mathbf{R}$	$87.8V$	$878V$	$8780V$	
C_D (by trial, using Fig. 9.10 sphere)	34.5*	0.82	0.37	
V (m/s)	0.00791	0.1622	0.764	◄◄
$\mathbf{R}$	0.694	142.4	6700	

*Stokes' Law, $C_D = 24/\mathbf{R}$, for $\mathbf{R} < 1$.

9.30 *A regulation football has a shape very similar to the prolate ellipsoid of Fig. 9.10, with a diameter of 6.78 in and a weight of 14.5 oz (Fig. P9.30). (a) Find the resistance when the ball is passed through still air (14.5 psia and 80°F) at a velocity of 45 fps. Neglect the effect of spin about the longitudinal axis. What is the deceleration at the beginning of the trajectory? (b) Find the percentage change in resistance if the air temperature is 30°F rather than 80°F, and the drag coefficient is unchanged.*

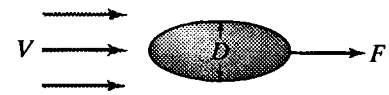


Figure P9.30

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Table A.5 for air: $R = 1715\ \text{ft}^2/(\text{sec}^2\cdot{}^\circ\text{R})$

(*a*) At 80°F, Table A.2 for air: $\nu = 0.000169\ \text{ft}^2/\text{sec}$

Eq. 2.5: $$\gamma = \frac{gp}{RT} = \frac{32.2(14.7 \times 144)}{1715(460 + 80)} = 0.0736\ \text{lb/ft}^3$$

$$\mathbf{R} = \frac{(6.78/12)45}{0.000\,169} = 1.504 \times 10^5; \text{ from Fig. 9.10: } C_D = 0.056$$

Eq. 9.32: $$F_D = (0.056)\frac{0.0736}{32.2}\left(\frac{45^2}{2}\right)\frac{\pi}{4}\left(\frac{6.78}{12}\right)^2 = 0.0325\ \text{lb} \quad ◀$$

At start of trajectory, $$a = \frac{F_D}{m} = \frac{0.0325(32.2)}{14.5/16} = 1.155\ \text{ft/sec}^2 \quad ◀$$

(*b*) At 30°F, Table A.2 for air: $\nu = 0.000142\ \text{ft}^2/\text{sec}$.

Eq. 2.5: $$\gamma = \frac{32.2(14.7 \times 144)}{1715(460 + 30)} = 0.0811\ \text{lb/ft}^3$$

$\mathbf{R} = (6.78/12)45/0.000\,142 = 1.790 \times 10^5$; from Fig. 9.10: $C_D = 0.055$

Eq. 9.32: $$F_D = (0.055)\frac{0.0811}{32.2}\left(\frac{45^2}{2}\right)\frac{\pi}{4}\left(\frac{6.78}{12}\right)^2 = 0.0352\ \text{lb or } 8.24\%\ \text{increase} \quad ◀$$

9.31 *An eight-oar racing shell is traveling through 60°F water at a mean velocity of 12 mph. Each oar is 9 ft long, with a length of 6 ft from the oarlock to the center of the "spoon", which has a projected area of 120 in². Assume that all drag is due to the spoon, and that this drag is equal to that of a disk of equal area. (a) If the "stroke" is 32 per min, and if each oarsmen sweeps a right angle in one-fourth of his rowing cycle, what is the maximum thrust of the oars? It must be assumed that the shell moves at something less (say, 20 percent) than its mean velocity when the oars are in the water. (b) The maximum velocity occurs during the backstroke when the oarsmen shift their weight toward the stern. Why? (c) The oarsman on his backstroke moves at half the angular velocity of his forward stroke, while the shell moves at perhaps 10% above its mean velocity. Find the drag in 60°F air resulting from a "feathered" oar (turbulent boundary layer, Fig 9.7) and an unfeathered one, in percentage of the forward thrust from part (a). What is the ratio of these two drag forces?*

BG

(*a*) Period of rowing cycle = 60/32 = 1.875 sec

Period of 1/4 cycle = 0.469 sec; angular sweep = $\pi/2$ radians

Velocity of oar relative to boat = $r\omega$ = $6(\pi/2)(1/0.469)$ = 20.1 fps

Forward velocity of boat = (1 − 0.2)12(44/30) = 14.08 fps

Velocity of oar relative to water = 20.1 − 14.08 = 6.02 fps

Table A.1 for water at 60°F: ρ = 1.938 slugs/ft³, ν = 0.000 012 17 ft²/sec.

$$\mathbf{R} = \frac{[(4/\pi)120]^{1/2}(6.02)}{12(0.000\,012\,17)} = 5.10 \times 10^5;$$ Fig. 9.10 for perpendicular disk: C_D = 1.1

Eq. 9.32: $F_D = (1.1)1.938(6.02^2/2)(120/144) = 32.3$ lb ; Total thrust = 8(32.3) = 258 lb ◄

(*b*) $V = V_{max}$ when oarsmen's weight is shifted toward the stern because of conservation of momentum. The reduced forward momentum of the oarsmen increases the forward momentum and velocity of the shell. ◄

(*c*) Backstroke: $r\omega$ = 20.1/2 = 10.05 fps

V_{boat} = (1.1)12(44/30) = 19.36 fps ; $V_{oar\ rel\ to\ air}$ = 10.05 + 19.36 = 29.4 fps

Table A.2 for air at 60°F: ρ = 0.002 37 slug/ft³, ν = 0.000 158 ft²/sec.

Unfeathered oar: $$\mathbf{R} = \frac{[(4/\pi)120]^{1/2}(29.4)}{12(0.000\,158)} = 1.917 \times 10^5$$

Fig. 9.10 for perpendicular disk: C_D = 1.1

Eq. 9.32: $F_D = 1.1(0.002\,37)(29.4^2/2)(120/144) = 0.940$ lb

or 0.940/32.3 = 2.91% of forward thrust ◄

Feathered oar (two surfaces): Fig. 9.7 (turbulent) for $\mathbf{R} = 1.917 \times 10^5$: C_f = 0.0063

Eq. 9.2 assuming $BL = A$: $F_f = 2(0.0063)0.002\,37(29.4^2/2)(120/144) = 0.010\,76$ lb

or 0.010 76/32.3 = 0.0334% of forward thrust. ◄

Drag force ratio (unfeathered/feathered) = 0.940/0.010 76 = 87.3 ◄

So an unfeathered oar causes about 87 times the resistance of a feathered oar.

Sec. 9.8: Drag on Two-Dimensional Bodies (Incompressible Flow) -- *Exercises (2)*

9.8.1 BG *What is the bending moment at the base of a vertical 0.3-in-diameter cylindrical radio antenna 6 ft tall on an automobile traveling at 80 mph through standard air at sea level?*

Table A.3 for air at sea level: $\mu = 3.737 \times 10^{-7}$ lb·sec/ft^2; $\rho = 0.002\,377$ slug/ft^3

$V = 80(44/30) = 117.3$ fps

Eq. 7.6: $\mathbf{R} = \dfrac{0.30(117.3)0.002\,377}{12(3.737 \times 10^{-7})} = 18{,}660$; from Fig. 9.13 (infinite cylinder), $C_D = 1.15$

Eq. 9.32: $F_D = 1.15(0.002\,377)(117.3^2/2)(0.3/12)6 = 2.82$ lb

$M = F_D(L/2) = 2.82(6/2) = 8.47$ ft·lb ◄

9.8.2 BG *A 70-mph wind blows across an 0.1-in-diameter wire. Find the frequency of oscillation which results in standard atmosphere (a) at sea level; (b) at 10,000 ft elevation.*

$V = 70$ mph $= 70(44/30) = 102.7$ fps;

(*a*) Table A.3 for air at sea level: $\rho = 0.002\,377$ slug/ft^3; $\nu = 0.000\,1572$ ft^2/sec

$$\mathbf{R} = \left(\frac{0.1}{12}\right)\frac{102.7}{0.000\,1572} = 5442\ ; \quad \text{Eq. 9.34: } f \approx 0.20\frac{102.7}{(0.1/12)}\left(1 - \frac{20}{5442}\right) = 2455 \text{ Hz} \quad ◄$$

(*b*) Table A.3 for air at 10,000 ft: $\rho = 0.001\,756$ slug/ft^3; $\nu = 0.000\,2013$ ft^2/sec

$$\mathbf{R} = \left(\frac{0.1}{12}\right)\frac{102.7}{0.000\,2013} = 4250\ ; \quad \text{Eq. 9.34: } f \approx 0.20\frac{102.7}{(0.1/12)}\left(1 - \frac{20}{4250}\right) = 2452 \text{ Hz} \quad ◄$$

Sec. 9.8: Drag on Two-Dimensional Bodies (Incompressible Flow) -- *Problems 9.32–9.33*

9.32 SI *(a) Find the bending moment at the base of a 300-mm-diameter cylindrical light post 12 m high when it is subject to a uniform wind velocity of 30 m/s at standard sea level. Neglect end effects. (b) Discuss the consequences of considering the atmospheric boundary layer above the surface of the earth.*

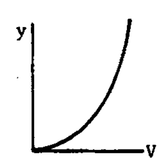
(*a*) Table A.3 for air at sea level: $\rho = 1.225$ kg/m^3, $\nu = 14.61 \times 10^{-6}$ m^2/s

Eq. 7.6: $\mathbf{R} \doteq \dfrac{0.30(30)}{14.61 \times 10^{-6}} = 6.16 \times 10^5$; from Fig. 9.13, $C_D = 0.31$

Eq. 9.32: $F_D = 0.31(1.225)(30^2/2)(12)0.30 = 615$ N

$M = F_D(L/2) = 615(15/2) = 3690$ N·m ◄

(*b*) In the atmospheric boundary layer wind velocities are reduced close to the surface of the earth. For velocities < 24.3 m/s ($\mathbf{R}$ < 500 000) Fig. 9.13 indicates that C_D increases suddenly, by up to four times. But the resulting increased drag forces have small moment arms about the base. So the net effect of the atmospheric boundary layer is probably a small increase of bending moment. ◄



9.33 *(a) Repeat Prob. 9.32(a) for the case where the pole has a uniform taper from 350 mm diameter at the base to 250 mm at its top. (b) Determine if the drag force would be larger or smaller if the pole had instead been tapered from 350 mm down to 150 mm. Why?*

Prob. 9.32 (a): Find the bending moment at the base of a 12-m-high light pole subject to a uniform wind velocity of 30 m/s at standard sea level (neglect end effects).

SI

Table A.3 for air at sea level: $\rho = 1.225\ \text{kg/m}^3$, $\nu = 14.61 \times 10^{-6}\ \text{m}^2/\text{s}$

(*a*) Given the wind velocity is uniform everywhere 30 m/s (actually it falls to zero at ground level).

Position on pole	Base	Top
D (m)	0.35	0.25
$\mathbf{R} = D(30)/(14.61 \times 10^{-6})$	7.19×10^5	5.13×10^5
C_D (Fig. 9.13)	0.32	0.30
Drag force per unit length (Eq. 9.32)		
$F_D/L = C_D \rho (V^2/2) D$ (N/m)	61.7	41.3

In this diameter **R** range C_D varies little (see Fig. 9.13). Thus the drag force depends entirely on D, and so its variation is almost linear.

Thus total drag force = ½(61.7 + 41.3)12 = 619 N ◄

$M = 41.3(12)6 + (61.7 - 41.3)(12/2)4 = 3470\ \text{N·m}$ ◄

(*b*) For $D = 0.15$ m, $\mathbf{R} = 3.08 \times 10^5$, $C_D = 0.90$, $F_D = 74.4$ N/m

Thus if the pole had been tapered from 350 mm at the base down to 150 mm (instead of 250 mm) at the top, the drag force per unit length on the upper portion of the pole would have been considerably larger (by about 80%) than for the 250 mm diameter, because of the abrupt rise in C_D as the Reynolds number is reduced (see Fig. 9.13). ◄

Sec. 9.9: Lift and Circulation – *Exercise (1)*

9.9.1 *Estimate the lift per unit length of span if the mean velocity along the top of a wing having a 10 ft chord is 100 mph and that along the bottom of the wing is 80 mph when the wing moves at 85 mph through still air ($\gamma = 0.072\ lb/ft^3$).*

BG

Velocities (100, 80, and 85 mph) are 146.7, 117.3, and 124.7 fps.

At a distance where the velocity is undisturbed and uniform (= U), let the pressure = p_u (as in Sec. 9.10).

Then $$\frac{U^2}{2g} + \frac{p_u}{\gamma} = \frac{V_{\text{top}}^2}{2g} + \frac{p_{\text{top}}}{\gamma}; \quad \frac{U^2}{2g} + \frac{p_u}{\gamma} = \frac{V_{\text{bot}}^2}{2g} + \frac{p_{\text{bot}}}{\gamma}$$

Subtracting: $$\frac{p_{\text{top}} - p_{\text{bot}}}{\gamma} = \frac{V_{\text{bot}}^2 - V_{\text{top}}^2}{2g} = \frac{117.3^2 - 146.7^2}{2(32.2)} = -120.2 \text{ ft of air}$$

$\therefore\ \Delta p = 120.2(0.072) = 8.66\ \text{lb/ft}^2$; lift per unit length of span $= \Delta p(c) = 8.66(10) = 86.6$ lb/ft ◄

Sec. 9.10: Ideal Flow About a Cylinder -- *Exercises (2)*

9.10.1 *Imagine that a circulation of 25 ft^2/sec is superimposed around a 1-ft-diameter cylinder immersed in 60°F water flowing at 10 fps perpendicular to the cylinder axis. Find the location of the stagnation points, and the lift on a 20-ft length of cylinder.*

BG

Table A.1 for water at 60°F: ρ = 1.938 slug/ft^3

Eq. 9.41: $\Gamma = 25 = -4\pi(0.5)10\sin\theta_0$; $\sin\theta_0 = -0.398$; $\theta_0 = -23.4°$ ◄

Eq. 9.40: $F_L = 1.938(20)10(25) = 9690$ lb ◄

9.10.2 *Imagine that a circulation of 2.4 m^2/s is superimposed around a 300-mm-diameter cylinder immersed in 15°C water flowing at 3.5 m/s perpendicular to the cylinder axis. Find the location of the stagnation points, and the lift on a 10-m length of cylinder.*

SI

Table A.1 for water at 15°C: ρ = 999.1 kg/m^3

Eq. 9.41: $\Gamma = 2.4 = -4\pi(0.3/2)3.5\sin\theta_0$; $\sin\theta_0 = -0.364$; $\theta_0 = -21.3°$ ◄

Eq. 9.40: $F_L = 999.1(10)3.5(2.4) = 83\,900$ N ◄

Sec. 9.10: Ideal Flow About a Cylinder -- *Problems 9.34–9.42*

9.34 *Beginning with the expression before Eq. (9.40), fill in the steps leading to Eq. (9.40). Take care to account for all changes in sign.*

N

Expression above Eq. 9.40: $F_L = -B\int_0^{2\pi}(p - p_u)R\sin\theta\, d\theta$; substituting for $p - p_u$ from Eq. 9.39:

$$F_L = -B\int_0^{2\pi}\rho/2[U^2 - (2U\sin\theta + \Gamma/2\pi R)^2]R\sin\theta\, d\theta$$

$$= -\rho/2BR\int_0^{2\pi}[U^2\sin\theta - 4U^2\sin^3\theta - (2\Gamma U/\pi R)\sin^2\theta + \Gamma^2\sin\theta/(4\pi^2R^2)]d\theta$$

Noting: $\int_0^{2\pi}\sin\theta\, d\theta = 0$, $\int_0^{2\pi}\sin^3\theta\, d\theta = [-⅓\cos\theta(\sin^2\theta + 2)]_0^{2\pi} = 0$

and $\int_0^{2\pi}\sin^2\theta\, d\theta = [½\theta - ¼\sin 2\theta]_0^{2\pi} = \pi$

Thus, $F_L = -(\rho/2)BR[0 - 0 - (2\Gamma U/\pi R)\pi - 0] = \rho BU\Gamma$ Q.E.D. ◄

9.35 *For the rotating cylinder in Sample Prob. 9.7, calculate the value of the lift coefficient. Assume the effective circulation is half the theoretical. Assuming the drag coefficient to be unchanged by the rotation of the cylinder, find the magnitude and direction of the total wind force on the rotor.*

Sample Prob. 9.7: U = 120 fps, D = 4 ft, γ of air = 0.0765 lb/ft^3, theoretical Γ = 237 ft^2/sec, theoretical F_L = 1688 lb.

BG

Given effective = ½(theoretical). ∴ effective Γ = 118.4 ft^2/sec, effective F_L = 844 lb.

From Eqs. 9.40 and 9.44: $F_L = C_L\rho(U^2/2)BD = \rho BU\Gamma$

∴ $C_L = 2\Gamma/UD = [2(118.4)]/(120 \times 4) = 0.493$ ◄

Table A.2 for air with γ = 0.0765 lb/ft^3 (given), by interpolation: ν = 0.000 1564 ft^2/sec

∴ **R** = 4(120)/0.000 1576 = 3.05 × 10^6; From Fig. 9.13: C_D = 0.35

∴ From Eqs. 9.32 and 9.44: $F_D = (C_D/C_L)F_L = (0.35/0.493)844 = 599$ lb

Total force = $(844^2 + 599^2)^{1/2}$ = 1035 lb ◄

Direction = $\tan^{-1}(844/599)$ = 54.7° from wind direction ◄

9.36 *Assume the rotor in Sample Problem 9.7 to be installed upright on a ship traveling due north at 20 knots (Fig. P9.36). The wind has an absolute velocity of 35 mph due east. If the drag coefficient of the cylinder is 1.0 and the "lines" of stagnation are separated by 120° on the rotor, find approximately the component of the total air force on the rotor in the direction of the ship's motion. Assume standard air.*

Sample Prob. 9.7: $D = 4$ ft, $B = 25$ ft

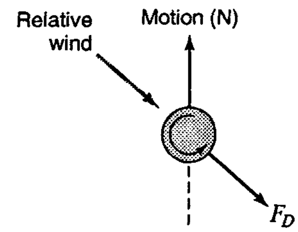


Figure P9.36

BG

Ship velocity = 20 knots = 20(1.688) fps = 33.8 fps

Wind velocity = 35 mph = 35(44/30) fps = 51.3 fps

Wind relative to ship: $V = \sqrt{51.2^2 + 33.8^2} = 61.4$ fps at

$\tan^{-1}(51.3/33.8) = 56.7°$ E of S.

Eq. 9.41: $\Gamma = -4\pi(2)61.4\sin(-30°) = 772$ ft²/sec

Table A.2 for standard air at 60°F: $\rho = 0.002\,37$ slug/ft³.

Eq. 9.40: $F_L = 0.002\,37(25)61.4(772) = 2810$ lb

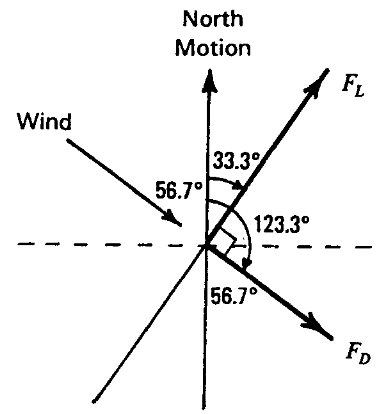
Given $C_D = 1.00$.

Eq. 9.32: $F_D = (1.0)0.002\,37(61.4^2/2)4(25) = 447$ lb

Angle of F_D to ship's forward axis is 180° − 56.7° = 123.3°

Angle of F_L to ship's forward axis is 90° - 56.7° = 33.3°

Net forward component, $F_{\text{north}} = 2810\cos 33.3° + 447\cos 123.3° = 2350 - 246 = 2100$ lb ◄



9.37 *Consider a cylinder of radius a in a stream of ideal fluid in which the undisturbed velocity and pressure are U and p_u and the density is ρ. (a) Using Eq. (9.36) and the Bernoulli theorem, evaluate the dimensionless pressure coefficient $(p - p_u)/(\rho U^2/2)$ for every 10° over the surface of one quadrant of the cylinder, measuring the angle from the forward stagnation point. Plot the pressure coefficient to scale, plotting it radially from the cylinder surface. (b) What is the actual pressure in pounds per square inch on the surface of a 1-ft-diameter cylinder, 70° from the forward stagnation point, if the cylinder is 15 ft below the free surface of a stream of water at 60°F, flowing at 12 fps?*

BG

(*a*) Bernoulli theorem from point distant from cylinder (U, p_u) to point on surface (v, p):

$$\frac{p_u}{\gamma} + \frac{U^2}{2g} = \frac{p}{\gamma} + \frac{v^2}{2g}; \quad \text{so} \quad \frac{p - p_u}{\rho U^2/2} = 1 - \frac{v^2}{U^2} \quad \text{and by substituting Eq. 9.36:}$$

$$= 1 - \frac{(2U\sin\theta)^2}{U^2} = 1 - 4\sin^2\theta$$

$\theta°$	0	10	20	30	40	50	60	70	80	90
$\dfrac{p - p_u}{\rho U^2/2}$	1	0.879	0.532	0	−0.653	−1.347	−2	−2.53	−2.88	−3 ◄

(*b*) Table A.1 for water at 60°F: $\rho = 1.938$ slug/ft³, $\gamma = 62.37$ lb/ft³

$$\text{At } \theta = 70°, \quad \frac{p - p_u}{\rho U^2/2} = 1 - (4 \times 0.940^2) = -2.53$$

$p = p_u - 2.53\rho U^2/2 = 15(62.37) - 2.53(1.938)12^2/2 = 936 - 353 = 582$ psf or 4.04 psi ◄

9.38 *Consider a cylinder of radius a in a stream of ideal fluid in which the undisturbed velocity and pressure are U and p_u and the density is ρ. (a) Using Eq. (9.36) and the Bernoulli theorem, evaluate the dimensionless pressure coefficient $(p - p_u)/(\rho U^2/2)$ for every 10° over the surface of one quadrant of the cylinder, measuring the angle from the forward stagnation point. Plot the pressure coefficient to scale, plotting it radially from the cylinder surface. (b) What is the actual pressure in newtons per square meter on the surface of a 300-mm-diameter cylinder, 70° from the forward stagnation point, if the cylinder is 5 m below the free surface of a stream of water at 15°C, flowing at 4 m/s?*

SI

(*a*) Solution is the same as for Prob. 9.37(*a*)

(*b*) Table A.1 for water at 15°C: $\rho = 999.1\ \text{kg/m}^3$, $\gamma = 9.798\ \text{kN/m}^3 = 9798\ \text{N/m}^3$

At $\theta = 70°$, $\dfrac{p - p_u}{\rho U^2/2} = 1 - (4 \times 0.940^2) = -2.53$

$p = p_u - 2.53\rho U^2/2 = 5(9798) - 2.53(999.1)4^2/2 = 49\ 000 - 20\ 200 = 28\ 800\ \text{N/m}^2$ ◄

9.39 *A double stagnation point is observed to occur on a 4-ft-diameter cylinder rotating in an air stream moving at 60 fps. Given standard atmospheric conditions at sea level, find (a) the lift force per foot length of the cylinder; (b) the lift coefficient.*

BG

(*a*) Eq. 9.41 with $\theta_0 = -90°$: $\Gamma = -4\pi(2)60\sin(-90°) = 1508\ \text{ft}^2/\text{sec}$

Table A.3 for standard air at sea level: $\rho = 0.00238\ \text{slug/ft}^3$

Eq. 9.40: $F_L = (0.002\,38)1(60)1508 = 215$ lb/ft of length ◄

(*b*) Eq. 9.44: $215 = C_L(0.002\,38)(60^2/2)4(1)$; $C_L = 12.57$ ◄

9.40 *A double stagnation point is observed to occur on a 1.2-m-diameter cylinder rotating in an air stream moving at 18 m/s. Given standard atmospheric conditions at sea level, find (a) the lift force per meter length of the cylinder; (b) the lift coefficient.*

SI

(*a*) Eq. 9.41 with $\theta_0 = -90°$: $\Gamma = -4\pi(1.2/2)18\sin(-90°) = 135.7\ \text{m}^2/\text{s}$

Table A.3 for standard air at sea level: $\rho = 1.225\ \text{kg/m}^3$

Eq. 9.40: $F_L = (1.225)1(18)135.7 = 2990$ N/m of length ◄

(*b*) Eq. 9.44: $2990 = C_L(1.225)(18^2/2)1.2(1)$; $C_L = 12.57$ ◄

9.41 *There have been many arguments over the validity and extent of the curve of a pitched baseball. According to tests (Life, July 27, 1953), a pitched baseball was found to rotate 1400 rpm while traveling at 43 mph. The horizontal projection of the trajectory revealed a smooth curve of about 800 ft radius. If the ball had a circumference of 9 in and weighed 5 oz, find the transverse force required to produce the observed curvature. Assume the shape of the ball to be roughly that of a cylinder having a diameter equal to the ball's diameter and a length of two-thirds its diameter. Find the circulation that would be required to produce the transverse force. Compare this with that obtained by assuming no slip at the equator of the ball. Assume standard air at sea level.*

BG

$U = 43(44/30) = 63.1$ fps ; $F_L = ma_n = m\dfrac{U^2}{r} = \left(\dfrac{5/16}{32.2}\right)\dfrac{63.1^2}{800} = 0.0483$ lb ◄

$D = (9/\pi)/12 = 0.239$ ft; given $B = (2/3)0.239 = 0.1592$ ft

Table A.3 for standard air at sea level: $\rho = 0.002\,38\ \text{slug/ft}^3$

Eq. 9.40: $0.0483 = 0.002\,38(0.1592)63.1\Gamma$; $\Gamma = 2.02\ \text{ft}^2/\text{sec}$ ◄

Comparison: From rotation at $\omega = 1400(2\pi/60) = 46.6$ rad/sec,

$v = r\omega = (0.239/2)146.6 = 17.50$ fps ; Eq. 9.37: $17.50 = \Gamma/[2\pi(0.239/2)]$; $\Gamma = 13.12\ \text{ft}^2/\text{sec}$ ◄

Note: Even with 50% slip, clearly only the middle 1/4 of the ball is effective in producing circulation.

9.42 *Solve Prob. 9.41 for a speed of 69 km/h and a trajectory radius of 245 m, with a ball weighing 1.4 N and having a circumference of 230 mm.*

Prob. 9.41: Find the transverse force which produces the observed curved path of the pitched baseball, rotating at 1400 rpm. Assume the shape of the ball to be roughly that of a cylinder having a diameter equal to the ball's diameter and a length of two-thirds its diameter. Find the circulation that would be required to produce the transverse force. Compare this with that obtained by assuming no slip at the equator of the ball. Assume standard air at sea level.

SI

$U = 69(1000/3600) = 19.17$ m/s ; $F_L = ma_n = m\dfrac{U^2}{r} = \left(\dfrac{1.4}{9.81}\right)\dfrac{19.17^2}{245} = 0.214$ N ◄

$D = 0.23/\pi = 0.0732$ m; $B = (2/3)0.0732 = 0.0488$ m

Eq. 9.40: $0.214 = 1.225(0.0488)19.17\Gamma$; $\Gamma = 0.1867$ m²/s ◄

Comparison: From rotation at $\omega = 1400(2\pi/60) = 146.6$ rad/s,

$v = r\omega = (0.0732/2)146.6 = 5.37$ m/s ; Eq. 9.37: $5.37 = \Gamma/[2\pi(0.0732/2)]$; $\Gamma = 1.234$ m²/s ◄

Note: Even with 50% slip, clearly only the middle 1/4 of the ball is effective in producing circulation.

Sec. 9.12: Induced Drag on Airfoil of Finite Length -- *Exercise (1)*

9.12.1 *A wing with a 15-m span and a 45-m² "planform" area moves horizontally through the standard atmosphere at 6000 m with a velocity of 750 km/hr. If the wing supports 180 kN, find (a) the required value of the lift coefficient; (b) the downwash velocity, assuming an elliptical distribution of lift over the span; (c) the induced drag.*

SI

Table A.3 for standard air at 6 km altitude: $\rho = 0.660$; $V = 750(1000/3600) = 208$ m/s

(*a*) Eq. 9.44: $F_L = 180\,000 = C_L(0.660)(208^2/2)45$; so $C_L = 0.279$ ◄

(*b*) $\dfrac{B^2}{A} = \dfrac{15^2}{45} = 5$; $\dfrac{B}{C} = \dfrac{15}{3} = 5$; Eq. 9.45: $\dfrac{V_i}{V} = \dfrac{C_L}{\pi(B^2/A)} = \dfrac{0.279}{\pi(5)} = 0.017\,78$

$\therefore V_i = 0.017\,78(208) = 3.70$ m/s ◄

(*c*) Eq. 9.46: $C_{Di} = (0.279)^2/[\pi(15^2/45)] = 0.004\,96$

From Eqs. 9.42 and 9.44: $F_{Di} = (0.004\,96/0.279)180\,000 = 3200$ N ◄

Sec. 9.12: Induced Drag on Airfoil of Finite Length -- *Problem 9.43*

9.43 *A wing of 44-ft span and a 350-ft² "planform" area moves horizontally through standard atmosphere at 20,000 ft with a velocity of 250 mph. If the wing supports 9000 lb, find (a) the required value of the lift coefficient; (b) the downwash velocity, assuming an elliptical distribution of lift over the span; (c) the induced drag.*

BG

Table A.3 for standard air at 20,000 ft altitude: $\rho = 0.001\,267$ slug/ft³

$V = 250(5280/3600) = 367$ fps

(*a*) Eq. 9.44: $F_L = 9000 = C_L(0.001\,267)(367^2/2)350$; so $C_L = 0.302$ ◄

(*b*) Eq. 9.45: $\dfrac{V_i}{V} = \dfrac{C_L}{\pi(B^2/A)} = \dfrac{0.302}{\pi(44^2/350)} = 0.017\,37$; $V_i = 0.017\,37(367) = 6.37$ fps ◄

(*c*) Eq. 9.46: $C_{Di} = (0.302)^2/[\pi(44^2/350)] = 0.005\,25$

From Eqs. 9.42 and 9.44: $F_{Di} = (0.005\,25/0.302)9000 = 156.4$ lb ◄

Sec. 9.13: Lift and Drag Diagrams -- *Exercises (3)*

9.13.1 *A sailplane including its load weighs 300 lb. It has a Clark Y section wing with a 3.5-ft chord by 21-ft span. Given that it has the same characteristics as the larger wing of the same aspect ratio shown in Fig. 9.24, find the angle of glide through standard air at 2000 ft which will produce the greatest horizontal distance range. Neglect air forces on the fuselage and the tail. (Note: The aspect ratio of 6 is here chosen to facilitate working the problem with the available data. Actually, the sailplane may be constructed with an aspect ratio of about twice this, in order to reduce the drag to a minimum.)*

BG

Fig. 9.24 for max C_l/C_D: $\alpha = -0.1°$, $C_L = 0.4$, $C_D = 0.018$

For max range, glide angle β is minimum. $\beta = \tan^{-1}(F_D/F_L) = \tan^{-1}(0.018/0.4) = 2.58°$ ◄

9.13.2 *Solve Exer. 9.13.1 for a sailplane with a 1-m chord by 6-m span wing, flying at 600 m altitude with a total weight of 1800 N.*

Exer. 9.13.1: A sailplane has a Clark Y section wing. Given that it has the same characteristics as the larger wing of the same aspect ratio shown in Fig. 9.24, find the angle of glide through standard air which will produce the greatest horizontal distance range. Neglect air forces on the fuselage and the tail. (Note: The aspect ratio of 6 is here chosen to facilitate working the problem with the available data. Actually, the sailplane may be constructed with an aspect ratio of about twice this, in order to reduce the drag to a minimum.)

SI

Fig. 9.24 for max C_l/C_D: $\alpha = -0.1°$, $C_L = 0.4$, $C_D = 0.018$

For max range, glide angle β is minimum. $\beta = \tan^{-1}(F_D/F_L) = \tan^{-1}(0.018/0.4) = 2.58°$ ◄

[Solution is identical to that for Exer. 9.13.1.]

9.13.3 *A 1400-lb airplane has a Clark Y airfoil wing of 6-ft chord by 36-ft span, with polar diagram given in Fig. 9.24. Find (a) the speed required to get the plane off the ground; (b) the horsepower required; (c) the circulation about the wing; (d) the strength of the starting vortex. Assume standard air at sea level and the angle of attack for maximum ratio of lift to drag. Neglect aerodynamic forces on the fuselage and tail.*

BG

(*a*) Fig. 9.24: For max ratio C_l/C_D, $\alpha = -0.1°$, $C_L = 0.4$, $C_D = 0.018$

Table A.3 for standard air at sea level: $\rho = 0.002\,38$ slug/ft^3

Eq. 9.44: $1400 = (0.4)0.002\,38(V^2/2)6(36)$; $V = 116.7$ fps ◄

(*b*) From Eqs. 9.32 and 9.44: $F_D = (C_D/C_L)F_L = (0.018/0.40)1400 = 63.0$ lb

Power $= (63.0)116.7/550 = 13.37$ hp ◄

(*c*) Eq. 9.40: $1400 = 0.002\,38(36)116.7\Gamma$; $\Gamma = 140.0$ ft^2/sec ◄

(*d*) Sec. 9.9, vortex strength: $C = \Gamma/(2\pi) = 140.0/(2\pi) = 22.3$ ft^2/sec ◄

Sec. 9.13: Lift and Drag Diagrams -- ***Problems 9.44–9.49***

9.44 *Determine the induced angle of attack and the inadced drag if the plan form of the wing in Exer. 9.12.1 were rectangular.*

Exer. 9.12.1: The wing with span B = 15 m and planform area A = 45 m² supports 180 kN when moving horizontally at 750 km/h through the standard atmosphere at an elevation of 6000 m.

SI

Table A.3 for standard air at 6 km altitude: ρ = 0.660 ; V = 750(1000/3600) = 208 m/s

Eq. 9.44: $F_L = 180\,000 = C_L(0.660)(208^2/2)45$; so $C_L = 0.279$ ◄

Fig. 9.25 for $B/c = B^2/A = 15^2/45 = 5.00$: $\tau = 0.154$, $\sigma = 0.041$

Eq. 9.49: $\alpha_i = \dfrac{0.279}{\pi(5)}(1 + 0.154) = 0.0205$ radian; $\therefore\ \alpha_i = 0.0205(180/\pi)^\circ = 1.175^\circ$ ◄

Eq. 9.50: $C_{Di} = \dfrac{0.279^2}{\pi(5)}(1 + 0.041) = 0.005\,17$

From Eqs. 9.42 and 9.44: $F_{Di} = (C_{Di}/C_L)F_L = (0.005\,17/0.279)180 = 3.33$ kN ◄

9.45 *Determine the induced angle of attack and the inadced drag if the plan form of the wing in Prob. 9.43 were rectangular.*

Prob. 9.43: The wing with span B = 44 ft and planform area A = 350 ft² supports 9000 lb when moving horizontally at 250 mph through the standard atmosphere at an elevation of 20,000 ft.

BG

Table A.3 for standard air at 20,000 ft altitude: ρ = 0.001 267 slug/ft³ ; V = 250(5280/3600) = 367 fps

Eq. 9.44: $F_L = 9000 = C_L(0.001\,267)(367^2/2)350$; so $C_L = 0.302$ ◄

Fig. 9.25 for $B/c = B^2/A = 44^2/350 = 5.53$: $\tau = 0.166$, $\sigma = 0.046$

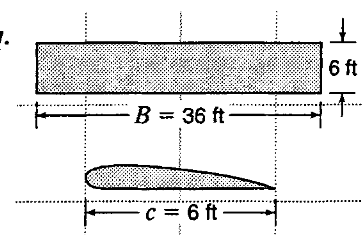
Eq. 9.49: $\alpha_i = \dfrac{0.302}{\pi(5.53)}(1 + 0.166) = 0.0203$ radian; $\therefore\ \alpha_i = 0.0203(180/\pi)^\circ = 1.161^\circ$ ◄

Eq. 9.50: $C_{Di} = \dfrac{0.302^2}{\pi(5.53)}(1 + 0.046) = 0.005\,49$

From Eqs. 9.42 and 9.44: $F_{Di} = (C_{Di}/C_L)F_L = (0.005\,49/0.302)9000 = 163.6$ lb ◄

9.46 *For the Clark Y airfoil of Fig. 9.24, evaluate the friction coefficient η of Eq. (9.48) for values of C_L of 0.6, 1.0, and 1.4.*

N



Part of Figure 9.24

Fig. 9.24: $B/c = 36/6 = 6$. Fig. 9.25 for $B/c = 6$: $\tau = 0.175$

At $C_L = 1.0$, Eq. 9.49: $\alpha_i = [1.0/\pi(6)](1 + 0.175) = 0.0623$ radian;

$\therefore\ \alpha_i = 0.0623(180/\pi)^\circ = 3.57^\circ$ ◄

Fig. 9.24: for $C_L = 1.0$, $\alpha = 8.0^\circ$; for $C_L = 0$, $\alpha = -5.6^\circ$

Fig. 9.22: $\alpha_0 = \alpha - \alpha_i = 8.0 - 3.57 = 4.43^\circ$

From zero angle of attack (Sec. 9.13), $\alpha_0' = 4.43 + 5.6 = 10.03^\circ = 0.1750$ radian

Eq. 9.48: $1.0 = 2\pi\eta(0.1750)$; $\eta = 0.909$ (at $C_L = 1.0$) ◄

Similarly				
	C_L	0.6	1.0	1.4
	α_i	2.14°	3.57°	5.00°
	α	2.6°	8.0°	14.3°
	α_0	0.457°	4.43°	9.30°
	α_0'	6.06°	10.03°	14.90°
	η	0.903	0.909	0.857 ◄◄

9.47 *A kite has the shape of a rectangular airfoil with a chord length of 2.5 ft and a span of 4.5 ft. When rigged and oriented as shown in the figure, the guideline exerts a tension T of 14 lb when the wind velocity V is 28 mph in standard air at 1500 ft altitude. Find C_L, C_{D0}, and the friction coefficient η.*

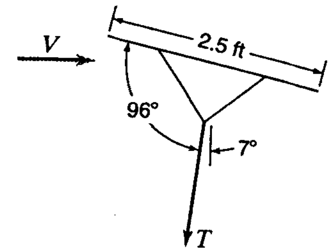


Figure P9.47

BG

From Fig. P9.47: $F_L = 14\cos 7° = 13.90$ lb; $F_D = 14\sin 7° = 1.706$ lb

Table A.3 for standard air at 1500 ft by interpolation:

$\rho = 0.002\,28$ slug/ft^3; $V = 28(44/30) = 41.1$ fps

Eq. 9.44: $13.90 = C_L(0.002\,28)(41.1^2/2)2.5(4.5)$; $\therefore\ C_L = 0.642$

Eqs. 9.32 and 9.44: $C_D = 0.642(1.706/13.90) = 0.0789$

Fig. 9.25 for $B/c = 4.5/2.5 = 1.80$: $\tau \approx 0.072$, $\sigma \approx 0.007$

Eq. 9.50: $C_{Di} = \dfrac{0.642^2}{\pi(1.80)}(1 + 0.007) = 0.0735$

From Eq. 9.43: $0.0789 = C_{D0} + 0.0735$; $C_{D0} = 0.005\,38$ ◄

Eq. 9.49: $\alpha_i = [0.642/\pi(1.80)](1 + 0.072) = 0.1218$ radian or $\alpha_i = 0.1218(180/\pi)° = 6.98°$ ◄

Fig. P9.47: $\alpha = 96° + 7° - 90° = 13°$; Fig. 9.22: $\alpha_0 = \alpha - \alpha_i = 13 - 6.98 = 6.02°$

Since the angle of zero lift is undoubtedly zero, per Sec. 9.13: $\alpha_0' = \alpha_0 + 0 = 6.02° = 0.1051$ radian

Eq. 9.48: $0.642 = 2\pi\eta(0.1051)$; $\therefore\ \eta = 0.973$ ◄

9.48 *A rectangular airfoil with a 2.5-m chord and 15-m span has a drag coefficient of 0.062 and a lift coefficient of 0.95 at an angle of attack of 6.7°. What would be the corresponding (a) lift coefficient, (b) drag coefficient, and (c) angle of attack for a wing having the same profile but with an aspect ratio of 7.6?*

SI

(*a*) $C_L = 0.95$ ◄ irrespective of aspect ratio B/c (Sec. 9.13).

(*b*) Fig. 9.25 for $B/c = 15/2.5 = 6$: $\tau = 0.175$, $\sigma = 0.0505$

Fig. 9.25 for $B/c = 7.6$, $\tau = 0.204$, $\sigma = 0.066$

Eq. 9.47 with $B^2/A = B/c = 6$: $0.062 = C_{D0} + 0.95^2/(\pi 6)](1 + 0.0505)$; $C_{D0} = 0.011\,70$

Eq. 9.47 with $B^2/A = B/c = 7.6$: $C_D = 0.011\,76 + [0.95^2/(\pi 7.6)](1 + 0.066) = 0.0520$ ◄

(*c*) Eq. 9.49 with $B/c = 6$: $\alpha_i = [0.95/(\pi 6)](1 + 0.175) = 0.0592$ radian $= 0.0592(180/\pi)° = 3.39°$

Thus (Fig. 9.22): $\alpha_0 = \alpha - \alpha_i = 6.7 - 3.39 = 3.31°$

and for $B/c = 7.6$: $\alpha = \alpha_0 + \alpha_i = 3.31° + [0.95/(\pi 7.6)](1 + 0.204) = 3.31° + 0.0479$ radian $= 6.05°$ ◄

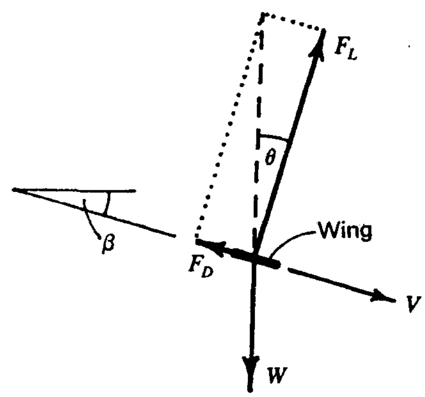
9.49 *Determine the angle of glide that will allow the sailplane in Exer. 9.13.1 to remain in the air for the longest time? (Note: A trial and error solution will be required here.)*

Exer. 9.13.1: The sailplane has a Clark Y section wing with c = 3.5 ft, B = 21 ft, and the same characteristics as the larger wing of the same aspect ratio in Fig. 9.24. Its total weight is 300 lb, as it glides through standard air at 2000 ft altitude.

BG

Table A.3 by linear interpolation for standard air at 2000 ft:

$$\rho = 0.002\,25 \text{ slug/ft}^3$$



For the maximum time in the air, we need a minimum velocity of descent, V_z. This will occur when the largest upward force on the wing, the resultant of F_L and F_D, is vertical (see figure), i.e., when the glide angle

$$\beta = \theta = \tan^{-1}(F_D/F_L) = \tan^{-1}(C_D/C_L)$$

Then $W\cos\beta = F_L$, and using Eq. 9.44

$$300\cos\beta = C_L(0.002\,25)(V^2/2)3.5(21)$$

from which $V = \sqrt{3628\cos\beta/C_L}$ and $V_z = V\sin\beta$

Trying various values of α given in Fig. 9.24:

$\alpha°$	C_L	C_D	$\beta°$	V (fps)	V_z (fps)	
2.6	0.60	0.0309	2.95	77.7	4.00	
3.9	0.70	0.0375	3.07	71.9	3.85	min
5.4	0.80	0.0473	3.38	67.3	3.97	

The minimum V_z occurs at a glide angle $\beta = 3.07°$ ◀

(Compare with Exer. 9.13.2 for maximum range, where $\alpha = -0.1°$, $C_L = 0.4$, $\beta = 2.58°$, $V = 95.2$ fps, $V_z = 4.28$ fps.)

Sec. 9.14: Effects of Compressibility on Drag and Lift -- *Exercises* (2)

9.14.1 *The blunt-nosed projectile (cylinder) of Fig. 9.27 has a diameter of 16 in and weighs 500 lb. Find its rate of deceleration when moving through standard sea level atmosphere (a) horizontally at 1200 mph; (b) upward at an angle of 40° with the horizontal at a velocity of 1200 mph.*

BG

1200 mph = 1200(5280/3600) fps = 1760 fps

Table A.3 for standard air at sea level: $\rho = 0.002\,38$ slug/ft^3, $c = 1116$ fps

$V/c = 1760/1117 = 1.577$, for which (Fig. 9.27 cylinder) $C_D = 1.36$

Eq. 9.32: $F_D = C_D\rho A(V^2/2) = 1.36(0.002\,38)[\pi(8/12)^2](1760^2/2) = 7000$ lb

(*a*) $F_D = 7000 = -(W/g)a = -(500/32.2)a$; $a = -451$ ft/sec^2 ◀

(*b*) $F_D + W\sin\theta = (W/g)a$; $7000 + 500\sin 40° = -(500/32.2)a$; $a = -471$ ft/sec^2 ◀

9.14.2 *The blunt-nosed projectile (cylinder) of Fig. 9.27 has a diameter of 0.45 m and weighs 2500 N. Find its rate of deceleration when moving through standard sea level atmosphere (a) horizontally at 450 m/s; (b) upward at an angle of 45° with the horizontal at a velocity of 450 m/s.*

SI

Table A.3 for standard air at sea level: $\rho = 1.225\ \text{kg/m}^3$, $c = 340.3\ \text{m/s}$

$V/c = 450/340.3 = 1.322$, for which (Fig. 9.27, cylinder) $C_D = 1.24$

Eq. 9.32: $F_D = 1.24(1.225)[\pi(0.45/2)^2]450^2/2 = 24\,500\ \text{N}$

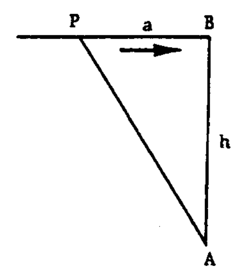
(*a*) $F_D = 24\ 500 = -(W/g)a = -(2500/9.81)a$; $a = -96.0\ \text{m/s}^2$ ◄

(*b*) $F_D + W\sin\theta = (W/g)a$; $24\,500 + 2500\sin 45° = -(2500/9.81)a$; $a = -102.9\ \text{m/s}^2$ ◄

Sec. 9.14: Effects of Compressibility on Drag and Lift -- *Problems 9.50–9.53*

9.50 *A supersonic jet plane is flying horizontally at 1500 mph. How soon after it passes overhead at an elevation of 5000 ft will the shock wave will be felt at sea level?*

BG



Let V = speed of aircraft, c = speed of shock wave. The first shock that arrives at A will have emanated from some point P. Let t = time of travel of the airplane from P to B. Thus $a = Vt$.

Time after plane passes B until shock is felt at A is

$$T = \frac{PA}{c} - t = \frac{\sqrt{h^2 + a^2}}{c} - t = \frac{(h^2 + V^2t^2)^{0.5}}{c} - t$$

T is minimum when $\dfrac{\partial T}{\partial t} = 0$, i.e., $\dfrac{\partial T}{\partial t} = \dfrac{0.5(h^2 + V^2t^2)^{-0.5}}{c}2V^2t - 1 = 0$

$$\therefore\ V^2t = c\sqrt{h^2 + V^2t^2};\quad V^4t^2 = c^2h^2 + c^2V^2t^2;\quad c^2h^2 = t^2V^2(V^2 - c^2);\quad t = ch/\left(V\sqrt{V^2 - c^2}\right)$$

$$\therefore\ T = \frac{1}{c}\sqrt{h^2 + V^2\frac{c^2h^2}{V^2(V^2 - c^2)}} - \frac{ch}{V\sqrt{V^2 - c^2}} = h\left(\frac{V}{c} - \frac{c}{V}\right)/\sqrt{V^2 - c^2} \qquad (1)$$

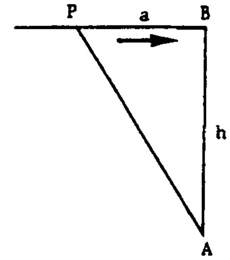
$V = 1500\ \text{mph} = 1500(5280/3600)\ \text{fps} = 2200\ \text{fps}$

Table A.3, at 5000 ft: $c = 1097$ fps; at sea level: $c = 1116$ fps. So average $c = 1107$ fps

Eq. (1): $T = 5000\left(\dfrac{2200}{1107} - \dfrac{1107}{2200}\right)/\sqrt{2200^2 - 1107^2} = 3.91\ \text{sec}$ ◄

9.51 *A supersonic jet plane is flying horizontally at 2800 km/h. How soon after it passes overhead at an elevation of 2 km will the shock wave will be felt at sea level?*

SI



Let V = speed of aircraft, c = speed of shock wave. The first shock that arrives at A will have emanated from some point P. Let t = time of travel of the airplane from P to B. Thus $a = Vt$.

Time after plane passes B until shock is felt at A is

$$T = \frac{PA}{c} - t = \frac{\sqrt{h^2 + a^2}}{c} - t = \frac{(h^2 + V^2t^2)^{0.5}}{c} - t$$

T is minimum when $\dfrac{\partial T}{\partial t} = 0$, i.e., $\dfrac{\partial T}{\partial t} = \dfrac{0.5(h^2) + V^2t^2)^{-0.5}}{c} 2V^2t - 1 = 0$

$$\therefore\ V^2t = c\sqrt{h^2 + V^2t^2};\quad V^4t^2 = c^2h^2 + c^2V^2t^2;\quad c^2h^2 = t^2V^2(V^2 - c^2);\quad t = ch/\left(V\sqrt{V^2 - c^2}\right)$$

$$\therefore\ T = \frac{1}{c}\sqrt{h^2 + V^2\frac{c^2h^2}{V^2(V^2 - c^2)}} - \frac{ch}{V\sqrt{V^2 - c^2}} = h\left(\frac{V}{c} - \frac{c}{V}\right)/\sqrt{V^2 - c^2} \qquad (1)$$

[The above developement of equation (1) for T is identical to that in the solution for Prob. 9.50.]

V = 2800 kph = 2800(1000/3600) = 778 m/s

Table A.3 for air at 2 km altitude: c = 333 m/s; at sea level: c = 340 m/s. So average c = 336 m/s

Eq. (1): $T = 2000\left(\dfrac{778}{336} - \dfrac{336}{778}\right)/\sqrt{778^2 - 336^2} = 5.37$ s ◄

9.52 *If the round-nosed and sharp-nosed projectiles of Fig. 9.27 each weigh 850 lb with a diameter of 18 in, and assuming they travel vertically downward nose first, find their terminal velocities in standard air at sea level.*

BG

At terminal velocity $W = F_D$. $\therefore$ using Eq. 9.32: 850 lb = $C_D\rho A(V^2/2)$; $A = \pi(9/12)^2 = 1.767$ ft^2

Table A.3 for standard air at sea level: ρ = 0.002 38 slug/ft^3, c = 1116 fps

$\therefore$ 850 = $C_D(0.002\,38)(1.767)(V^2/2)$; C_DV^2 = 404,000; $V = \sqrt{404{,}000/C_D}$ (1)

(*a*) For round nose: Solve by trial

Try C_D	V [Eq. (1)]	V/c	C_D (Fig. 9.27)
0.50	899 fps	0.806	0.24
0.24	1298	1.163	0.56
0.37	1045	0.937	0.29
0.29	1181	1.058	0.40
0.33	1107	0.992	0.35
0.34	1090	0.977	0.33

Thus $V_{\text{terminal}} \approx$ 1090 fps (round nose) ◄

(*b*) For sharp nose: Solve by trial

0.40	1005 fps	0.901	0.23
0.23	1326	1.188	0.35
0.30	1161	1.040	0.29
0.29	1181	1.058	0.29

Thus $V_{\text{terminal}} \approx$ 1181 fps (sharp nose) ◄

9.53 *If the round-nosed and sharp-nosed projectiles of Fig. 9.27 each weigh 3500 N with a diameter of 450 mm, and assuming they travel vertically downward nose first, find their terminal velocities in standard air at sea level.*

SI

At terminal velocity $W = F_D$. $\therefore$ using Eq. 9.32: $3500\text{ N} = C_D \rho A(V^2/2)$; $A = \pi(0.225)^2 = 0.1590\text{ m}^2$

Table A.3 for standard air at sea level: $\rho = 1.225\text{ kg/m}^3$, $c = 340\text{ m/s}$

$\therefore\ 3500 = C_D(1.225)(0.1590)(V^2/2)$; $C_D V^2 = 35\,900$; $V = \sqrt{35\,900/C_D}$ (1)

(*a*) For round nose: Solve by trial

Try C_D	V [Eq. (1)]	V/c	C_D (Fig. 9.27)
0.50	268 m/s	0.788	0.23
0.23	395	1.162	0.54
0.37	312	0.917	0.28
0.28	358	1.054	0.41
0.33	330	0.970	0.33

Thus $V_{\text{terminal}} \approx 330$ m/s (round nose) ◀

(*b*) For sharp nose: Solve by trial

0.40	300 m/s	0.881	0.21
0.21	414	1.217	0.37
0.30	346	1.018	0.28
0.28	358	1.054	0.29
0.29	352	1.035	0.29

Thus $V_{\text{terminal}} \approx 352$ m/s (sharp nose) ◀

Chapter 10

Steady Flow in Open Channels

PROBLEM SELECTION GUIDE

Sec.	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
10.1	*Open Channels*						
	P 10.1	BG	Medium	Medium	1		Uses Secs 8.6 & 8.13
10.3	*Solution of Uniform Flow Problems*						
	X[1] 10.3.1	BG	Easy	Medium	1		Plot; Manning
	10.3.2	BG	Easy	Short	1	10.3.3	Manning
	10.3.3	SI	Easy	Short	1	10.3.2	Manning
	10.3.4	SI	Easy	Medium	1		☐ T & E (Trial & Error), Manning
	10.3.5	BG	Easy	Short	1		Manning
	10.3.6	SI	Easy	Short	1		Manning
	10.3.7	BG	V Easy	V Short	1	10.3.8	Manning
	10.3.8	SI	V Easy	V Short	1	10.3.7	Manning
	10.3.9	BG	Easy	Short	1	10.3.10	Composite section, Manning
	10.3.10	SI	Easy	Short	1	10.3.9	Composite section, Manning
	P 10.2	BG	Medium	Medium	1		Uses Secs 8.6 & 8.12−16, (T & E?)
	10.3	BG	Medium	Medium	2	10.5	☐ T & E for (*b*), Manning
	10.4	BG	Medium	Medium	1	10.6	Uses Sec 8.14, Manning
	10.5	SI	Medium	Medium	2	10.3	☐ T & E for (*b*), Manning
	10.6	SI	Medium	Medium	1	10.4	Uses Secs 8.6, 8.12, 8.14, Manning
	10.7	BG	Medium	Medium	2	10.8	Uses Secs 8.5−6 & 8.13−14, Manning
	10.8	SI	Medium	Medium	2	10.7	Uses Secs 8.5−6 & 8.13−14, Manning
	10.9	BG	Medium	Medium	1		Manning
	10.10	BG	Medium	Medium	1		Composite section, Manning
10.4	*Velocity Distribution in Open Channels*						
	X 10.4.1	BG	Medium	Medium	1		Uses (next) Sec. 10.5
	10.4.2	SI	Easy	Medium	1		Plot
	P 10.11	BG	Medium	Medium	1		Uses (next) Sec. 10.5
10.5	*"Wide and Shallow" Flow*						
	X 10.5.1	BG	V Easy	V Short	1	10.5.2	
	10.5.2	SI	V Easy	V Short	1	10.5.1	
	10.5.3	BG	Easy	Short	1		

/cont...

[1] For all Exercises (identified by "X"), answers are given in Appendix F of the textbook.
† Answers are sensitive to values (y/D, Q/Q_{full}, V/V_{full}, y/E, A, P) that are or may be read from a figure.
X = Exercise, P = (end-of-chapter) Problem, S = Sample Problem. ☐ = could use computing aids.

Sec.	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
10.6	***Most Efficient Cross Section***						
	X 10.6.1	BG	Easy	Short	1		
	10.6.2	BG	Easy	Medium	3	10.6.3	
	10.6.3	SI	Easy	Medium	3	10.6.2	
	P 10.12	N	Medium	Medium	1		
	10.13	BG	Easy	Medium	1		Plot
	10.14	N	Hard	Medium	1		Differentiation
	10.15	N	Hard	Medium	1		Differentiation
	10.16	BG	Medium	Medium	2	10.17	☐ T & E
	10.17	SI	Medium	Medium	2	10.16	☐ T & E
	10.18	BG	Medium	Medium	1	10.19	
	10.19	SI	Medium	Medium	1	10.18	
10.7	***Circular Sections Not Flowing Full***						
	X 10.7.1	BG	Easy	Short	1		†
	10.7.2	SI	Easy	Short	2		†
	P 10.20	N	Hard	Long	1		☐ Differentiation, T & E (Alt: Plot)
10.8	***Laminar Flow in Open Channels***						
	X 10.8.1	BG	Easy	Short	2	10.8.2	Uses Sec 2.11
	10.8.2	SI	Easy	Short	2	10.8.1	Uses Sec 2.11
	10.8.3	BG	Medium	Medium	1	10.8.4	
	10.8.4	SI	Medium	Medium	1	10.8.3	
	P 10.21	N	Medium	Medium	1		Uses Secs 8.6 & 8.7
	10.22	N	Medium	Medium	1		Integration; uses Sec 5.1
10.9	***Specific Energy and Alternate Depths of Flow in Rectangular Channels***						
	X 10.9.1	BG	Easy	Short	1	10.9.2	Depth = critical
	10.9.2	SI	Easy	Short	1	10.9.1	Depth = critical
	10.9.3	SI	Medium	Short	1		Wide and shallow
	P 10.23	N	Medium	Short	1		Wide and shallow
	10.24	N	Medium	Short	1		Differentiation
	10.25	BG	Medium	Medium	1		
	10.26	BG	Easy	Short	1		
	10.27	BG	Medium	Medium	2		†
	10.28	BG	Medium	Medium	3	10.29	†
	10.29	SI	Medium	Medium	3	10.28	☐ T & E
10.10	***Subcritical and Supercritical Flow***						
	X 10.10.1	BG	Easy	Short	1		Critical flow, wide and shallow
	10.10.2	BG	Medium	Medium	1		Critical flow, wide and shallow

/cont...

Sec.	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
10.11	***Critical Depth in Nonrectangular Channels***						
	X 10.11.1	BG	Easy	Short	1	10.11.2	
	10.11.2	SI	Easy	Short	1	10.11.1	
	10.11.3	BG	Easy	Medium	2		
	10.11.4	SI	Medium	Medium	1		☐ T & E
	P 10.30	BG	Medium	Medium	2	10.31	☐ T & E
	10.31	SI	Medium	Medium	2	10.30	☐ T & E
	10.32	BG	Medium	Long	1		☐ Plot?
	10.33	BG	Medium	Long	2	10.34	† ☐ Plot?
	10.34	SI	Medium	Long	2	10.33	† ☐ Plot?
10.13	***Humps and Contractions***						
	X 10.13.1	SI	Easy	Short	1		
	10.13.2	BG	Easy	Medium	1	10.13.3	☐ T & E
	10.13.3	SI	Easy	Medium	1	10.13.2	☐ T & E
	10.13.4	BG	Medium	Medium	1	P10.40	☐ T & E
	P 10.35	BG	Hard	Long	4	S10.8	☐ Damming action; T & E
	10.36	SI	Medium	Medium	3		Damming action
	10.37	BG	Hard	Long	3	S10.8	☐ T & E
	10.38	BG	Medium	Medium	1	10.39	☐ T & E
	10.39	SI	Medium	Medium	1	10.38	☐ T & E
	10.40	BG	Medium	Medium	1	X10.13.4	☐ T & E
	10.41	BG	Medium	Medium	1	10.42	☐ T & E
	10.42	SI	Medium	Medium	1	10.41	☐ T & E
10.15	***Energy Equation for Gradually Varied Flow***						
	X 10.15.1	BG	Medium	Medium	1	10.15.2	Assume/confirm dir'n of depth decrease
	10.15.2	SI	Medium	Medium	1	10.15.1	Assume/confirm dir'n of depth decrease
	10.15.3	BG	Medium	Medium	1	10.15.4	
	10.15.4	SI	Medium	Medium	1	10.15.3	
	P 10.43	BG	Medium	Medium	1	10.44	Sketch
	10.44	SI	Medium	Medium	1	10.43	Sketch
	10.45	BG	Medium	Long	1	10.46	☐ T & E
	10.46	SI	Medium	Long	1	10.45	☐ T & E
	10.47	BG	Medium	Long	1	10.48	☐ T & E; uses (next) Sec. 10.16
	10.48	SI	Medium	Long	1	10.47	☐ T & E; uses (next) Sec. 10.16
	10.49	BG	Hard	Long	2	10.50	☐
	10.50	SI	Hard	Long	2	10.49	☐
	10.51	BG	Hard	V Long	2		☐ T & E; wide and shallow
	10.52	BG	Hard	Long	2		☐ Chézy, T & E
	10.53	BG	V Hard	Long	3		† GVF in circular section; sketch
	10.54	BG	Medium	Long	1		† Ass./confirm dir'n of depth decrease
	10.55	BG	Medium	Medium	1		Uses Chézy *C*

/cont...

Sec.	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
10.17	***Examples of Water Surface Profiles***						
	X 10.17.1	N	Easy	Short	1		
	10.17.2	BG	Medium	Medium	1	10.17.3	☐ T & E
	10.17.3	SI	Medium	Medium	1	10.17.2	☐ T & E
	10.17.4	BG	Easy	Short	1	10.17.5	
	10.17.5	SI	Easy	Short	1	10.17.4	
	10.17.6	BG	Easy	Medium	1		Sketch
	10.17.7	BG	Medium	Medium	1	10.17.8	☐ T & E; sketch
	10.17.8	SI	Medium	Medium	1	10.17.7	☐ T & E; sketch
	10.17.9	BG	Medium	Medium	1		☐ T & E; damming action
	P 10.56	BG	Medium	Medium	1	10.57	☐ T & E; sketch
	10.57	SI	Medium	Medium	1	10.56	☐ T & E; sketch
	10.58	BG	Medium	Long	1		☐
	10.59	BG	Hard	Long	2		☐ Damming action, T & E; sketch
	10.60	BG	Medium	Medium	1		
	10.61	BG	Medium	Medium	1	10.62	☐ Break in slopes, T & E; sketch
	10.62	BG	Medium	Medium	1	10.61	☐ Break in slopes, T & E; sketch
	10.63	BG	Hard	Long	1		☐ Break in slopes, T & E; sketch
	10.64	BG	Hard	Long	3		☐ Damming action, T & E
10.18	***The Hydraulic Jump***						
	X 10.18.1	BG	Medium	Medium	2	10.18.2	
	10.18.2	SI	Medium	Medium	2	10.18.1	
	10.18.3	BG	Medium	Medium	1		
	P 10.65	N	Medium	Medium	1		Derivation
	10.66	BG	Medium	Medium	2	10.67	
	10.67	SI	Medium	Medium	2	10.66	
	10.68	BG	Hard	Medium	2	10.69	☐ Jump on slope, T & E
	10.69	SI	Hard	Medium	2	10.68	☐ Jump on slope, T & E
	10.70	BG	Hard	Medium	1		Moving jump
	10.71	BG	Hard	Medium	1	10.72	☐ T & E
	10.72	SI	Hard	Medium	1	10.71	☐ T & E
10.19	***Location of Hydraulic Jump***						
	P 10.73	BG	Hard	Long	1		Wide and shallow
	10.74	BG	Hard	Long	1		☐ T & E
10.20	***Velocity of Gravity Waves***						
	X 10.20.1	BG	Easy	V Short	1		
	P 10.75	BG	Easy	Medium	1	10.76	
	10.76	SI	Easy	Medium	1	10.75	

/cont...

Sec.	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
10.21	***Flow Around Channel Bends***						
	X 10.21.1	SI	Easy	Short	1		Subcritical
	P 10.77	BG	Medium	Medium	2		Sub and supercritical
10.22	***Transitions***						
	X 10.22.1	BG	Medium	Medium	1		☐ T & E
	10.22.2	SI	Hard	V Long	1		☐ Assume/confirm flow condit's; T&E
	P 10.78	BG	Medium	Medium	2	S10.12	☐ Assume/confirm flow condit's; T&E
	10.79	SI	Medium	Long	3		☐ T & E
	10.80	BG	Medium	Long	2		☐ Assume/confirm flow condit's; T&E
10.23	***Hydraulics of Culverts***						
	X 10.23.1	SI	Medium	Long	1	S10.13	☐ Assume/confirm flow condit's; T&E
	P 10.81	BG	Hard	Long	3		☐ Assume/confirm flow condit's; T&E
	10.82	BG	Hard	Long	2	10.81*a*	☐ Assume/confirm flow condit's; T&E
	10.83	SI	Hard	Long	2		☐ Assume/confirm flow condit's; T&E
	10.84	BG	Hard	Long	1		

Chapter 10
STEADY FLOW IN OPEN CHANNELS

Sec. 10.1: Open Channels -- *Problem 10.1*

10.1 *For the channel of Sample Prob. 10.1 compute the "open-channel Reynolds number" assuming that water at 50°F is flowing. Refer to Fig. 8.11 to verify whether or not the flow is fully rough. Determine e from Fig. 8.11 and compare it with the value computed in the example.*

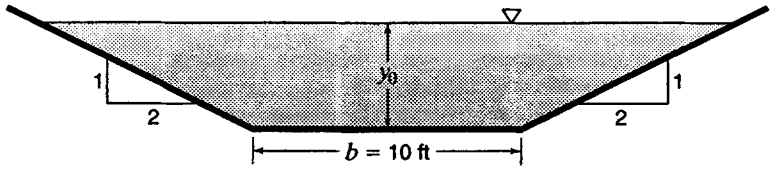


Figure S10.1

BG

Per Sample Prob. 10.1: R_h = 2.27 ft, A = 57.1 ft^2, Q = 225 cfs, f = 0.0226, e = 0.015 85

$\therefore$ $V = Q/A$ = 225/57.1 = 3.94 ft/s; Table A.1 at 50°F: ν = 1.410 × 10^{-5} ft^2/s

Open channel Sec. 10.1: $\mathbf{R} = \dfrac{R_h V}{\nu} = \dfrac{2.27(3.94)}{1.410 \times 10^{-5}} = 6.34 \times 10^5$ ◄

Equivalent pipe Eq. 8.23: $\mathbf{R} = 4(6.34 \times 10^5) = 2.53 \times 10^6$

Fig. 8.11 for $\mathbf{R} = 2.53 \times 10^6$ and f = 0.0226: e/D = 0.00172 and the flow is (just) fully rough ◄

$\therefore$ $e = 0.001\,72D = 0.001\,72(4R_h) = 0.001\,72(4)2.27 = 0.015\,43$ ft ◄

This e from Fig. 8.11 is very close to 0.015 85 ft as computed in Sample Prob. 10.1. ◄

Sec. 10.3: Solution of Uniform Flow Problems -- *Exercises (10)*

10.3.1 *For the channel of Sample Prob. 10.1, compute the flow rate for depths of 2, 4, 6, and 8 ft. Plot a curve of Q versus y.*
Sample Prob. 10.1: The channel of Fig. S10.1 has S_0 = 0.0006 and Manning's n = 0.016.

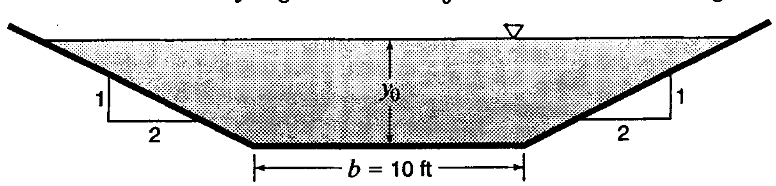


Figure S10.1

BG

Manning's Eq. 10.8*a* with n = 0.016 and S_0 = 0.0006: $Q = (1.486/0.016)AR^{2/3}(0.0006)^{1/2} = 2.28AR^{2/3}$

y ft	A ft^2	P ft	R_h ft	$R_h^{2/3}$	Q cfs
2	28	18.94	1.478	1.300	82.7
4	72	27.9	2.58	1.881	308
6	132	36.8	3.58	2.34	703
8	208	45.8	4.54	2.74	1298

▲

10.3.2 BG *Water flows uniformly in a trapezoidal channel with a 16-ft-wide bed and 1.9:1 (H:V) side slopes. If the bed slope is 1.2 ft/mile and n = 0.015, find the flow rate when the depth is 9 ft.*

$A = [16 + 1.9(9)]9 = 298\ \text{ft}^2$; $P = 16 + (2)9\sqrt{1 + 1.9^2} = 54.6$ ft; $R_h = A/P = 5.45$ ft.

Eq. 10.8*a*: $Q = (1.486/0.015)298(5.45)^{2/3}(1.2/5280)^{1/2} = 1378$ cfs ◄

10.3.3 SI *Water flows uniformly in a 2.5-m-wide rectangular channel at a depth of 300 mm. The channel slope is 0.0028 and n = 0.014. Find the flow rate in* m^3/s.

$A = 2.5(0.30) = 0.75\ \text{m}^2$, $P = 2(0.30) + 2.5 = 3.1$ m, $R_h = A/P = 0.242$ m

Eq. 10.8*b*: $Q = (1/0.014)0.75(0.242)^{2/3}(0.0028)^{1/2} = 1.101\ \text{m}^3/\text{s}$ ◄

10.3.4 SI *At what depth will water flow in a 4-m-wide rectangular channel if* $n = 0.018$, $S_0 = 0.0009$, *and* $Q = 7\ m^3/s$?

Depth = y, $A = 4y$, $P = 4 + 2y$, $R_h = A/P$

Eq. 10.8*b*: $7 = (1/0.018)4y[4y/(4 + 2y)]^{2/3}(0.0009)^{1/2}$

Solve by trial or by equation solver per Sample Prob. 10.1 to obtain $y = 1.251$ m ◄

10.3.5 BG *Figure X10.3.5 shows a tunnel section on the Colorado River Aqueduct. The area of the water cross section is 191* ft^2, *and the wetted perimeter is 39.1 ft. The flow is 1600 cfs. If n = 0.013 for its concrete lining, find the slope.*

Eq. 8.3: $R_h = A/P = 191/39.1 = 4.88$ ft

Eq. 10.8*a*: $1600 = (1.486/0.013)191(4.88)^{2/3}S_0^{1/2}$

$S_0 = 0.000\,648$ or 3.42 ft/mile ◄

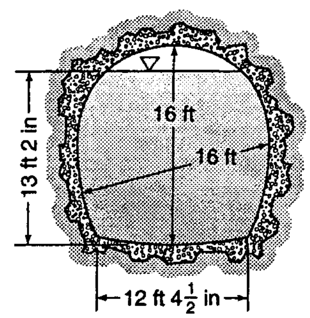


Figure X10.3.5

10.3.6 SI *For the channel cross section of Fig. X10.3.6, a = 1 m, b = 3 m, d = 2 m, w = 8 m, and n = 0.015. What bed slope is required so that the flow will be 16* m^3/s *when the depth of flow is 1.50 m?*

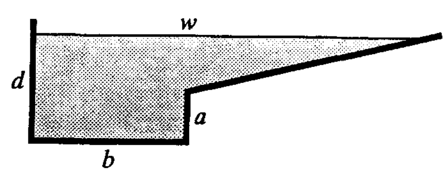


Figure X10.3.6

$A = 1.5(3) + 0.5(4)/2 = 5.50\ \text{m}^2$;

$P = 1.5 + 3.0 + 1.0 + \sqrt{0.5^2 + 4^2} = 9.53$ m;

Eq. 8.3: $R_h = A/P = 0.577$ m

Eq. 10.8*a*: $16 = (1/0.015)5.50(0.577)^{2/3}S_0^{1/2}$; $S_0 = 0.00396$ ◄

10.3.7 BG *Water flows in a rectangular flume 6 ft wide made of unplaned timber (n = 0.013). Find the necessary channel slope if the water flows uniformly at a depth of 3 ft and at 20 fps.*

Eq. 10.7*b*: $20 = (1.486/0.013)(3 \times 6/12)^{2/3}S_0^{1/2}$; $S_0 = 0.01783$ ◄

10.3.8 *Water flows in a rectangular flume 1.5 m wide made of unplaned timber (n = 0.013). Find the necessary channel slope if the water flows uniformly at a depth of 0.6 m and at 4.5 m/s.*

SI

Eq. 10.7*a*: $4.5 = (1/0.013)(0.6 \times 1.5/2.7)^{2/3}S_0^{1/2}$; $S_0 = 0.01481$ ◄

10.3.9 *Refer to Fig. 10.5. Suppose the widths of A_1, A_2, and A_3 are 120, 50, and 240 ft and the total depths are 3, 12, and 4 ft. Compute the flow rate if $S_0 = 0.0018$, $n_1 = n_3 = 0.06$, and $n_2 = 0.030$.*

BG

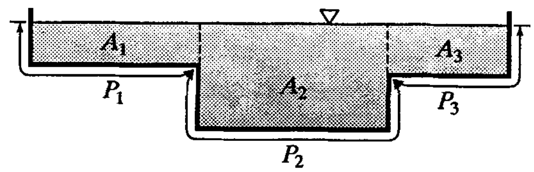


Figure 10.5

Eq. 10.11: $Q = Q_1 + Q_2 + Q_3$

$$= \frac{1.486}{0.06}(120 \times 3)\left(\frac{120 \times 3}{120 + 3}\right)^{2/3}(0.0018)^{1/2} +$$

$$+ \frac{1.486}{0.030}(50 \times 12)\left(\frac{50 \times 12}{9 + 50 + 8}\right)^{2/3}(0.0018)^{1/2} + \frac{1.486}{0.06}(240 \times 4)\left(\frac{240 \times 4}{240 + 4}\right)^{2/3}(0.0018)^{1/2}$$

$Q = 774 + 5438 + 2514 = 8730$ cfs ◄

10.3.10 *Refer to Fig. 10.5. Suppose the widths of A_1, A_2, and A_3 are 25, 8, and 45 m and the total depths are 0.4, 2.8, and 1.1 m. Compute the flow rate if $S_0 = 0.0018$, $n_1 = n_3 = 0.035$, and $n_2 = 0.022$.*

SI

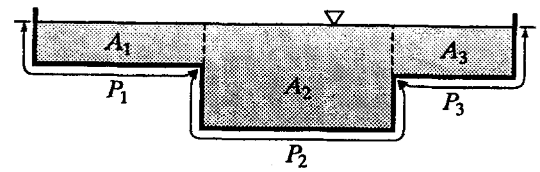


Figure 10.5

Applying Eq. 10.8*b* to each section:

$$Q = Q_1 + Q_2 + Q_3 = \frac{25 \times 0.4}{0.035}\left(\frac{25 \times 0.4}{25 + 0.4}\right)^{2/3}(0.0018)^{1/2} +$$

$$+ \frac{8 \times 2.8}{0.022}\left(\frac{8 \times 2.8}{2.4 + 8 + 1.7}\right)^{2/3}(0.0018)^{1/2} + \frac{45 \times 1.1}{0.035}\left(\frac{45 \times 1.1}{45 + 1.1}\right)^{2/3}(0.0018)^{1/2}$$

$Q = 6.51 + 65.1 + 62.9 = 134.6\ m^3/s$ ◄

Sec. 10.3: Solution of Uniform Flow Problems -- *Problems 10.2–10.10*

10.2 *Assuming the values of f versus* **R** *and e/D given for pipes in Fig. 8.11 also apply to open channels, find the rate of discharge of water at 60°F in a 100-in-diameter smooth concrete pipe flowing half full, if the pipe is laid on a grade of 2 ft/mi. Why must D be replaced by $4R_h$?*

BG

Flow is uniform. From Table 8.1 (concrete, smooth), choose $e = 0.001$ ft $= 0.012$ inches;

$e/D = 0.000\ 12$; From Fig. 8.11, try complete turbulent flow $f = 0.0125$

D must be replaced by $4R_h$ per Sec. 8.6 because it is open channel flow, not pipe flow. ◄

Eq. 8.20: $R_h = D/4 = (100''/12)/4 = 2.08$ ft; $S_0 = 2/5280 = 0.000\ 379$

Eq. 10.5: $V = [(8g/f)R_hS]^{1/2} = [(8 \times 32.2/0.0125)2.08 \times 0.000\ 379]^{1/2} = 4.03$ fps

$D''V = 4R''V = 4(2.08)12(4.03) = 403$ and $f = 0.0130$. Try this.

Eq. 10.5: $V = [(8 \times 32.2/0.0130)2.08 \times 0.000\ 379]^{1/2} = 3.95$ fps

$D''V = 4(2.08)12(3.95) = 395$ and $f = 0.0130$ which checks.

$\therefore\ Q = AV = (1/2)(\pi/4)(100/12)^2(3.95) = 107.8$ cfs ◄

Alternatively, we can use Eq. 8.52 instead of Fig. 8.11, to find f for trials.

Alternatively, we can avoid trials and f by using Eq. 8.56*b* (for full-pipe flow $= 2Q$). $Q = 108.0$ cfs ◄

10.3 *Figure P10.3 shows a cross section of a canal that is to carry 2000 cfs. The canal is lined with concrete, for which n is 0.014. (a) What is the slope of this canal, and what is the drop in elevation per mile? (b) If the flow in the canal were to decrease to 1000 cfs, all other data, including the slope and n, being the same, what would be the depth of the water?*

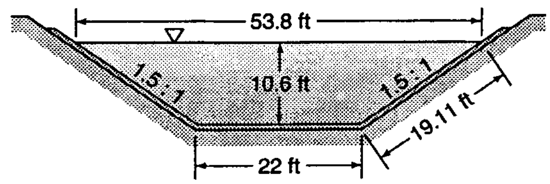


Figure P10.3

BG

(*a*) $A = [(53.8 + 22)/2]10.6 = 37.9(10.6) = 402\ \text{ft}^2$

$V = Q/A = 2000/402 = 4.98$ fps

$P = 22 + 2(19.11) = 60.2$ ft; $R_h = A/P = 402/60.2 = 6.67$ ft

Eq. 10.7*b*: $4.98 = (1.486/0.014)(6.67)^{2/3}S_0^{1/2}$; $S_0^{1/2} = 0.013\,24$

$S_0 = 0.000\,1752$ ◀ or 0.925 ft per mile ◀

Note: A very small drop per mile will produce a substantial velocity.

(*b*) Eq. 10.8*a*: $1000 = (1.486/0.014)AR_h^{2/3}(0.000\ 1752)^{1/2} = 1.405AR_h^{2/3}$

i.e. $712 = AR_h^{2/3} = A(A^{2/3}/P^{2/3}) = A^{5/3}/P^{2/3}$

where $A = 22y + 1.5y^2$ and $P = 22 + 2y(19.11/10.6) = 22 + 3.61y$

$\therefore\ (22y + 1.5y^2)^{5/3} = 712(22 + 3.61y)^{2/3}$.

Solve by trial or by equation solver per Sample Prob. 10.1 to obtain $y = 7.36$ ft ◀

10.4 *In Prob. 10.3(a) find the corresponding value of e and compare it with values previously given for concrete pipe. Does it fall in the range given?*

Prob. 10.3(a): Figure P10.3 shows a cross section of a canal that is to carry 2000 cfs. The canal is lined with concrete, for which n is 0.014. What is the slope of this canal, and what is the drop in elevation per mile?

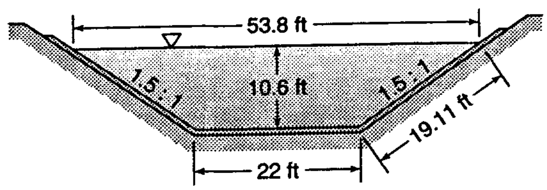


Figure P10.3

BG

$A = [(53.8 + 22)/2]10.6 = 37.9(10.6) = 402\ \text{ft}^2$

$P = 22 + 2(19.11) = 60.2$ ft; $R_h = A/P = 402/60.2 = 6.67$ ft

From Eqs. 10.9 and 10.10*a*:

$1/f^{1/2} = 2\log[14.8(6.67/e)] = 0.0926(6.67)^{1/6}/0.014 = 9.08$; $e = 0.00286$ ft ◀

Table 8.1 gives $e = 0.001$ to 0.01 ft for concrete ◀

So the computed value of *e* <u>does</u> fall in the range given ◀

10.5 *Figure P10.5 shows a cross section of a canal forming a portion of the Colorado River Aqueduct, which is to carry 44 m³/s. The canal is lined with concrete, for which n is 0.014. (a) What is the slope of this canal, and what is the drop in elevation per kilometer? (b) If the flow in the canal were to decrease to 22 m³/s, all other data, including the slope and n, being the same, what would be the depth of the water?*

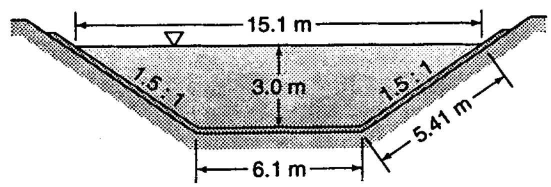


Figure P10.5

SI

(*a*) $A = [(15.1 + 6.1)/2]3 = 31.8\ \text{m}^2$; $V = Q/A = 44/31.8 = 1.384$ m/s

$P = 6.1 + 2(5.41) = 16.92$ m; $R_h = A/P = 31.8/16.92 = 1.879$ m

Eq. 10.7*a*: $1.384 = (1/0.014)(1.879)^{2/3}S_0^{1/2}$; $S_0^{1/2} = 0.012\,72$

$S_0 = 0.000\,1618$ ◀ or 0.1618 m/km ◀

(*b*) Eq. 10.8*b*: $22 = (A/0.014)R_h^{2/3}(0.000\ 1618)^{1/2} = 0.909AR_h^{2/3}$

i.e. $24.2 = AR_h^{2/3} = A(A^{2/3}/P^{2/3}) = A^{5/3}/P^{2/3}$

where $A = 6.1y + 1.5y^2$ and $P = 6.1 + 2y(5.41/3) = 6.1 + 3.61y$

$\therefore\ (6.1y + 1.5y^2)^{5/3} = 24.2(6.1 + 3.61y)^{2/3}$.

Solve by trial or by equation solver per Sample Prob. 10.1 to obtain $y = 2.08$ m ◀

10.6 *In Prob. 10.5 find the corresponding value of e and compare it with values previously given for concrete pipe. Does it fall in the range given?*

Prob. 10.5: Figure P10.5 shows a cross section of a canal forming a portion of the Colorado River Aqueduct, which is to carry 44 m³/s. The canal is lined with concrete, for which n is 0.014.

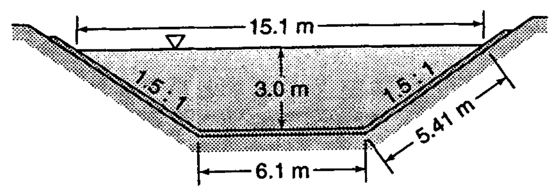


Figure P10.5

SI

$A = [(15.1 + 6.1)/2]3 = 31.8\ \text{m}^2$

$V = Q/A = 44/31.8 = 1.384$ m/s

$P = 6.1 + 2(5.41) = 16.92$ m ; $R_h = A/P = 31.8/16.92 = 1.879$ m

Per Sec. 10.2, substituting $D = 4R_h$ (Eq. 8.21) into fully-rough-pipe flow Eq. 8.50:

$$\frac{1}{f^{1/2}} = 2\log\left(\frac{3.7(4R_h)}{e}\right) = 2\log\left(\frac{14.8R_h}{e}\right).\quad \text{From Eq. 10.10}b\text{:}\ \frac{1}{f^{1/2}} = \frac{0.1129R_h^{1/6}}{n}$$

Equating, to eliminate f: $2\log\left(\dfrac{14.8R_h}{e}\right) = \dfrac{0.1129R_h^{1/6}}{n}$ or $2\log\left(\dfrac{14.8\times 1.879}{e}\right) = \dfrac{0.1129(1.879)^{1/6}}{0.014}$

from which $e = 0.000\ 914$ m $= 0.914$ mm ◀ Table 8.1 for concrete: $e = 0.3$ to 3.0 mm ◀

So the computed value of e does fall in the range given ◀

10.7 *A monolithic concrete inverted siphon is circular in cross section and is 20 ft in diameter. Obviously, it is completely filled with water. (a) If n = 0.013, find the slope of the hydraulic grade line for a flow of 2100 cfs. (b) Solve the same problem using the methods of Chap. 8 and assuming a mean value of e from Table 8.1 for concrete pipe. Compare the result with that of part (a).*

BG

(*a*) $A = (\pi/4)20^2 = 314\ \text{ft}^2$; Eq. 8.20: $R_h = D/4 = 20/4 = 5$ ft

For our constant cross section, the flow is uniform, so S_0 in Manning's Eq. = slope S of the HGL.

Eq. 10.8*a*: $2100 = (1.486/0.013)(314)5^{2/3}S^{1/2}$

$S = 0.000\,400$ or 2.11 ft per mile ◄

(*b*) Table 8.1: Mean value of *e* for concrete = (0.01 + 0.001)/2 = 0.0055 ft

Then $e/D = 0.0055/20 = 0.000\,275$; $V = Q/A = 2100/314 = 6.68$ fps

$D''V = 20(12)6.68 = 1604$; Fig. 8.11 with $e/D = 0.000\,275$, $D''V = 1604$: $f = 0.0147$

Eq. 8.14: $h_f/L = S = (0.0147/20)6.68^2/(2 \times 32.2) = 0.000\,510$ ◄

This is within 28% of the value found in Part (*a*) ◄

10.8 *A monolithic concrete inverted siphon on the Colorado River Aqueduct is circular in cross section and is 4.88 m in diameter. Obviously, it is completely filled with water. (a) If n = 0.013, find the slope of the hydraulic grade line for a flow of 45 m³/s. (b) Solve the same problem using the methods of Chap. 8 and assuming a mean value of e from Table 8.1 for concrete pipe. Compare the result with that of part (a).*

SI

(*a*) $A = (\pi/4)4.88^2 = 18.70\ \text{m}^2$; Eq. 8.20: $R_h = D/4 = 4.88/4 = 1.22$ m

For our constant cross section, the flow is uniform, so S_0 in Manning's Eq. = slope S of the HGL.

Eq. 10.8*b*: $45 = (18.70/0.013)(1.22)^{2/3}S^{1/2}$; $S = 0.000\,750$ or 0.750 m/km ◄

(*b*) Table 8.1: Mean value of *e* for concrete = (0.3 + 3.0)/2 = 1.65 mm

So $e/D = 1.65/4880 = 0.000\,338$; $V = Q/A = 45/18.70 = 2.41$ m/s

Table A.1 for water at standard 15°C: $\nu = 1.139 \times 10^{-6}\ \text{m}^2/\text{s}$

$\mathbf{R} = DV/\nu = 4.88(2.41)/(1.139 \times 10^{-6}) = 1.031 \times 10^7$

Fig. 8.11 (or Eq. 8.40) with $e/D = 0.000\,338$ and $\mathbf{R} = 1.031 \times 10^7$: $f = 0.0154$

Eq. 8.14: $h_f/L = S = (0.0154/4.88)2.41^2/(2 \times 9.81) = 0.000\,931$ ◄

This is within 24% of the value found in Part (*a*). ◄

10.9 *A 30-in-diameter pipe is known to have a Manning's n of 0.014. What is Manning's n for a 96-in-diameter pipe having exactly the same e as the smaller pipe?*

BG

30 inch dia pipe with $n = 0.014$: $R_h = A/P = D/4 = (1/4)30/12 = 0.625$ ft

Sec. 10.2: $$e = 14.8(0.625)\exp\left[\frac{1.486(\ln 10)0.625^{1/6}}{4\sqrt{2(32.2)}\ 0.014}\right] = 0.008\,10\ \text{ft}$$ ◄

96 inch dia pipe with same *e*: $R_h = (1/4)96/12 = 2.00$ ft

Sec. 10.2: $$n = \frac{1.486(2.00)^{1/6}}{4\sqrt{2(32.2)}\log\left[\dfrac{14.8(2.00)}{0.008\,10}\right]} = 0.014\,59\ \text{ft}$$ ◄

10.10 *Refer to Fig. X10.3.6. Find the flow rate at water depths of 1, 2.5, 4, 5.5, and 7 ft if $n = 0.025$ and $S = 0.0020$. The dimensions are as follows: $a = 4$ ft, $b = 5$ ft, $d = 7$ ft, and $w = 35$ ft.*

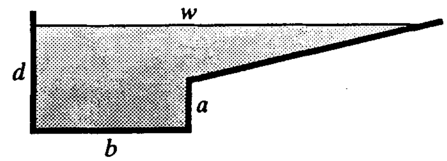


Figure X10.3.6

BG

Given $n = 0.025$, $S_0 = S = 0.0020$.

Eq. 10.8*a*: $Q = (1.486/0.025)AR_h^{2/3}0.0020^{1/2} = 2.66AR_h^{2/3}$

y ft	A ft^2	P ft	R_h ft	$R_h^{2/3}$	Q cfs
1	5	7	0.714	0.799	10.62
2.5	12.5	10	1.250	1.160	38.56
4	20	13	1.538	1.333	70.85

▲

For depths greater than 4 ft the flow should be computed in parts as shown below, i.e., using Eq. 10.11.

$y = 5.5$ ft: $Q_1 = (1.486/0.025)27.5(27.5/14.5)^{2/3}(0.0020)^{1/2} = 112.0$ cfs

$Q_2 = (1.486/0.025)11.25(11.25/15.07)^{2/3}(0.0020)^{1/2} = 24.6$ cfs; $Q = Q_1 + Q_2 = 136.6$ cfs ◄

$y = 7$ ft: $Q_1 = (1.486/0.025)35(35/16)^{2/3}(0.0020)^{1/2} = 156.8$ cfs

$Q_2 = (1.486/0.025)45(45/30.1)^{2/3}(0.0020)^{1/2} = 156.2$ cfs; $Q = Q_1 + Q_2 = 313$ cfs ◄

Sec. 10.4: Velocity Distribution in Open Channels – *Exercises (2)*

10.4.1 *In Fig. X10.4.1 with uniform flow in a very wide open channel ($R_h = y$), $a = 3.20$ ft. Find V and b if $n = 0.015$.*

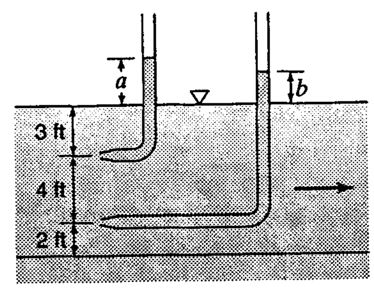


Figure X10.4.1

BG

$R_h = y_0 = 9$ ft (given).

Eq. 10.8*a*: $q = 9 \times V = (1.486/0.015)(9 \times 1)(9)^{2/3}S_0^{1/2}$

thus $S_0^{1/2} = 0.00233V$

Velocity head $u_a^2/2g = a = 3.2$ ft, ∴ $u_a = 14.36$ fps

Eq. 10.12: $u_a = V + \frac{1}{0.4}[32.2(9)]^{1/2}(0.002\,33V)\left(1 + 2.3\log_{10}\frac{6}{9}\right)$

$= 14.36$ fps

From which mean velocity, $V = 13.55$ fps ◄

Eq. 10.12: $u_b = 13.55 + (1/0.4)(32.2 \times 9)^{1/2}(0.002\,33 \times 13.55)(1 + 2.3\log_{10}2/9)$

$u_b = 12.88$ fps; $b = (12.88^2/2g) = 2.58$ ft ◄

10.4.2 *Water flows uniformly in a very wide rectangular channel ($R_h = y$) at a depth y_0 of 2.0 m. $S_0 = 0.005$ and $n = 0.018$. Calculate the velocities at y-values of 0.2, 0.4, 0.8, 1.2, and 2.0 m above the bed and plot the velocity profile. Note the value of the maximum velocity at the water surface.*

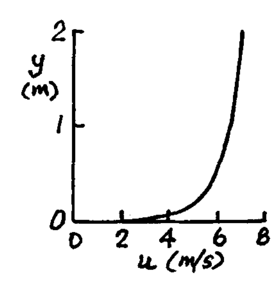
BG

For a very wide channel, $R_h = y_0 = 2.0$ m (given)

Eq. 10.7*a*: $V = (1/0.018)2.0^{2/3}(0.005)^{1/2} = 6.24$ m/s

Eq. 10.12: $u = 6.24 + (1/0.40)[9.81(2.0)0.005]^{1/2}[1 + 2.3\log(y/2.0)]$

y m:	0.20	0.40	0.80	1.20	2.00
u m/s:	5.22	5.76	6.30	6.62	7.02 = u_{max} ◄◄



Sec. 10.4: Velocity Distribution in Open Channels -- ***Problem*** *10.11*

10.11 *Using Eq. (10.12), determine the depth below the surface at which the velocity is equal to the mean velocity. Also find the average of the velocities at 0.2 and 0.8 depths. Let y = 6 ft, S = 0.0005, and n = 0.024.*

BG

Let $y = y'$ when $u = V$. Then Eq. 10.12: $u = V + (1/K)(gy_0S_0)^{1/2}(1 + 2.3\log y'/y_0)$

where $u = V$, so $V = V + (1/K)(gy_0S_0)^{1/2}(1 + 2.3\log y'/y_0)$

$\therefore\ 1 + 2.3\log y'/y_0 = 0;\ \log y'/y_0 = -(1/2.3) = 0.435$

$\log y_0/y' = 0.435,\ y_0/y' = 10^{0.435} = 2.72;\ y' = y_0/2.72 = 0.367y_0$

$\therefore$ The depth below the surface at which $u = V$ is equal to $y_0 - y'$ or $y_0 - 0.368y_0 = 0.633y_0$, where y_0 is the depth of water in the channel. ◄

Given to use Eq. 10.12, which is for "wide and shallow" flow, so assume $R_h = y_0$ (Sec. 10.5).

Eq. 10.7*b*: Mean velocity, $V = (1.486/0.024)6^{2/3}(0.0005)^{1/2} = 4.57$ fps

From Eq. 10.12 with $K = 0.4$: At 0.2 depth, $y/y_0 = 0.8$, $u = 5.18$ fps
At 0.8 depth, $y/y_0 = 0.2$, $u = 4.10$ fps

Average of these two u's = (5.18 + 4.10)/2 = 4.64 fps ◄

Therefore the average of the velocities at 0.2 depth and 0.8 depth is almost identical to V.

Sec. 10.5: "Wide and Shallow" Flow -- ***Exercises*** *(3)*

10.5.1 *Water flows 0.15 in deep in a 9-in-wide rectangular flume. What would be the percentage error in the hydraulic radius if this were assumed to be wide and shallow open channel flow?*

BG

$b/y = 9/0.15 = 60$

Error percent = 200/60 = 3.33% ◄

10.5.2 *Water flows 7 mm deep in a 0.5-m-wide rectangular flume. What would be the percentage error in the hydraulic radius if this were assumed to be wide and shallow open channel flow?*

SI

$b/y = 500/7 = 71.4$

Error percent = 200/71.4 = 2.80% ◄

10.5.3 *At a location where the cross section of a stream is reasonably rectangular, the width is 15 ft and the average depth is computed to be 20.6 in. What would be the percentage error in the hydraulic radius if this stream were assumed to be "wide and shallow"? Would it be reasonable to make this assumption?*

BG

$b/y = 15(12)/20.6 = 8.74$

Error percent = 200/8.74 = 22.9% ◄

This error is large, and $b/y < 10$, so the "wide and shallow" assumption would not be reasonable. ◄

Sec. 10.6: Most Efficient Cross Section -- *Exercises* (3)

10.6.1 *What diameter of semicircular channel will have the same capacity as a 3-ft-deep rectangular channel 8 ft wide. Assume S_0 and n are the same for both channels. Compare the lengths of the wetted perimeters.*

BG

$Q_1 = Q_2$; ∴ From Eq. 10.8*a* with $(1.486/n)S_0^{1/2}$ = constant, $A_1R_1^{2/3} = A_2R_2^{2/3}$

Substituting (from Eq. 8.3) for $R_h = A/P$: $A_1^{5/3}/P_1^{2/3} = A_2^{5/3}/P_2^{2/3}$

$$\therefore\ \frac{(\pi D_1^2/8)^{5/3}}{(\pi D_1/2)^{2/3}} = \frac{(8\times 3)^{5/3}}{14^{2/3}}\ ;\quad \left[\frac{\pi}{8}\right]\frac{D_1^{8/3}}{4^{2/3}} = 34.3\ ;\quad D_1 = 7.57\text{ ft} \quad ◄$$

$P_1 = \pi D_1/2 = 11.88$ ft, compared with $P_2 = 14$ ft ◄

10.6.2 *A rectangular flume of planed timber (n = 0.012) slopes 0.5 ft per 1000 ft. (a) Compute the rate of discharge if the width is 7 ft and the depth of water is 3.5 ft (Fig. X10.6.2). (b) What would be the rate of discharge if the width were 3.5 ft and the depth of water 7 ft? (c) Which of the two forms would have the greater capacity and which would require less lumber?*

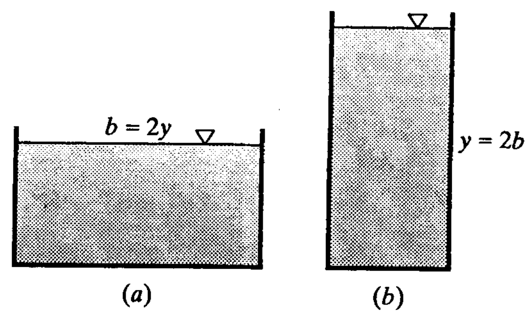


Figure X10.6.2

BG

(*a*) $b = 7,\ y = 3.5,\ \therefore A = 24.5,\ P = 14$

$R_h = 24.5/14 = 1.75$ ft; $n = 0.012,\ S = 0.0005$.

Eq. 10.7*b*: $V = \frac{1.486}{0.012}(1.75)^{2/3}(0.0005)^{1/2} = 4.02$ fps

$\therefore Q = 24.5(4.02) = 98.5$ cfs ◄

(*b*) $b = 3.5,\ y = 7,\ \therefore A = 24.5,\ P = 17.5,\ R_h = 1.4$ ft; $n = 0.012,\ S_0 = 0.0005$.

$V = \frac{1.486}{0.012}(1.4)^{2/3}(0.0005)^{1/2} = 3.47$ fps, $\therefore Q = 24.5(3.47) = 84.9$ cfs ◄

(*c*) Lumber ratio = 14/17.5 = 0.80; flow ratio = 98.5/84.9 = 1.160
The first design provides 16% more flow capacity while requiring only 80% as much lumber. ◄

10.6.3 *A rectangular flume of planed timber (n = 0.012) slopes 1 m per km. (a) Compute the rate of discharge if the width is 2 m and the depth of water is 1 m (Fig. X10.6.2). (b) What would be the rate of discharge if the width were 1 m and the depth of water 2 m? (c) Which of the two forms would have the greater capacity and which would require less lumber?*

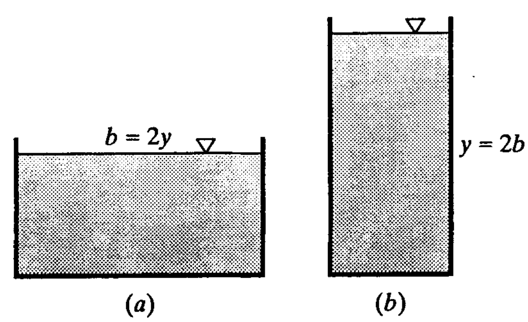


Figure X10.6.2

SI

(*a*) $b = 2,\ y = 1,\ \therefore A = 2,\ P = 4,\ R_h = 2/4 = 0.5$ m

$n = 0.012,\ S_0 = 0.001$.

Eq. 10.7*a*: $V = \frac{1}{0.012}(0.5)^{2/3}(0.001)^{1/2} = 1.660$ m/s

$\therefore Q = 2(1.660) = 3.32\ \text{m}^3/\text{s}$ ◄

(*b*) $b = 1,\ y = 2,\ \therefore A = 2,\ P = 5,\ R_h = 2/5 = 0.4$ m; $n = 0.012,\ S = 0.001$.

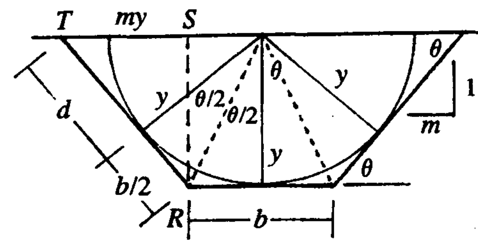
Eq. 10.7*a*: $V = (1/0.012)(0.4)^{2/3}(0.001)^{1/2} = 1.431$ m/s $\therefore Q = 2(1.431) = 2.86\ \text{m}^3/\text{s}$ ◄

(*c*) Lumber ratio = 4/5 = 0.8; flow ratio = 3.32/2.86 = 1.160
The first design provides 16% more flow capacity while requiring only 80% as much lumber. ◄

Sec. 10.6: Most Efficient Cross Section -- *Problems 10.12–10.19*

10.12 *Prove that the most efficient hydraulic trapezoidal cross section discussed in Sec. 10.6 envelopes a semi-circle with its center in the water surface.*

N



For such an enveloping trapezoid $\tan\theta = 1/m = y/d$,

so $d = my$

ΔRST, Pythagoras: $(d + b/2)^2 = y^2 + m^2y^2 = y^2(1 + m^2)$

from which $b = 2y\left(\sqrt{1+m^2} - m\right)$. Also

$P = 2(b + d) = 2\left[2y\left(\sqrt{1+m^2} - m\right) + my\right] = 2y\left(2\sqrt{1+m^2} - m\right)$

and $A = 2[(1/2)dy] + 4[(1/2)y(b/2)] = y(b + d)$,

so $R_h = A/P = [y(b + d)]/[2(b + d)] = y/2$

These results for P, b, and R_h are the same as those obtained in Sec. 10.6 for the optimum trapezoid by differentiation. Therefore the hydraulically optimal trapezoid must envelope such a semicircle. Q.E.D.

10.13 *Consider a variety of rectangular sections all of which have a cross-sectional area of 20 ft². Plot the hydraulic radii versus channel widths for a range of channel widths from 2 to 20 ft and note the depth:width ratio when R_h is maximum.*

BG

$A = bh = 20$ ft², $P = b + 2h$. Sample calculations are:

b ft	$h = A/b$ ft	P ft	h/b	$R_h = A/P$ ft
2	10.0	22	5.0	0.909
4	5.0	14	1.250	1.429
6	3.33	12.67	0.556	1.579
6.324	3.163	12.649	0.500	1.581 ◄ (max)
8	2.5	13	0.313	1.538
10	2	14	0.20	1.429
20	1	22	0.05	0.909

▲

10.14 *Set up a general expression for the wetted perimeter P of a trapezoidal channel in terms of the cross-sectional area A, depth y, and angle of side slope ϕ (Fig. P10.14). Then differentiate P with respect to y with A and ϕ held constant. From this prove that $R_h = y/2$ for the section of greatest hydraulic efficiency (i.e., smallest P for a given A).*

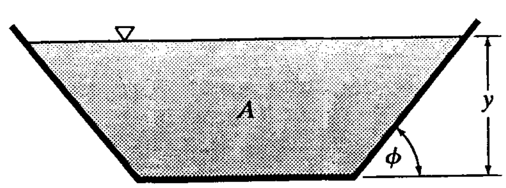


Figure P10.14

N

Let b = bottom width, y = depth, ϕ = side slope angle

$A = by + y(y\tan\phi) = by + y^2\tan\phi$; $b = (A/y) - y\tan\phi$

$P = b + 2y\sec\phi = (A/y) - y\tan\phi + 2y\sec\phi$. Differentiate with respect to y:

$dP/dy = -A/y^2 - \tan\phi - 2\sec\phi$. Substituting for A: $dP/dy = -(by + y^2\tan\phi)/y^2 - \tan\phi + 2\sec\phi$

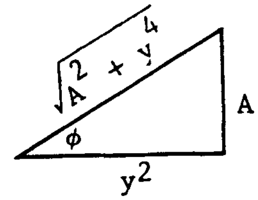
Equating to 0: $(by + y^2\tan\phi)/y^2 = 2\sec\phi - \tan\phi$; $b = 2y\sec\phi - 2y\tan\phi = 2y(\sec\phi - \tan\phi)$ (1)

Substituting for b from (1) into $R_h = A/P = (by + y^2\tan\phi)/(b + 2y\sec\phi)$ gives $R_h = y/2$ Q.E.D. ◄

10.15 *Using the results of Prob. 10.14, prove that the most efficient triangular section is the one with a 90° vertex angle.*

N

Prob. 10.14 with $b = 0$: $A = y^2 \tan\phi$, $P = 2y\sec\phi = 2y/\cos\phi$



$\tan\phi = A/y^2$, $\therefore$ $\cos\phi = \dfrac{y^2}{\sqrt{A^2 + y^4}}$; $P = \dfrac{2y\sqrt{A^2 + y^4}}{y^2} = 2(A^2 + y^4)^{1/2} y^{-1}$

For a given A, P is minimum when

$dP/dy = 2y^{-1}(1/2)(A^2 + y^4)^{-1/2}(4y^3) + 2(A^2 + y^4)^{1/2}(-y^{-2}) = 0$

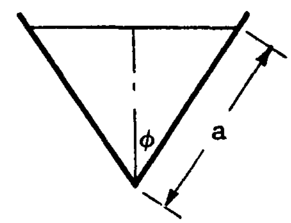
i.e., $4y^2(A^2 + y^4)^{-1/2} - 2(A^2 + y^4)^{1/2} y^{-2} = 0$; i.e., $4y^4 - 2(A^2 + y^4) = 0$

i.e., $2y^4 = 2A^2$, or $y = A^{1/2}$ when P is min

Then $\tan\phi = A/y^2 = y^2/y^2 = 1$, $\phi = 45° =$ side slope, and vertex angle $= 90°$ Q.E.D. ◄

Alternate solution: $A = a^2 \sin\phi\cos\phi$; $P = 2a$; $R_h = (a/2)\sin\phi\cos\phi$

For a given A, R_h is minimum when



$dR_h/d\phi = (a/2)(\cos^2\phi - \sin^2\phi) = 0$; i.e., $\cos\phi = \sin\phi$

i.e., $\phi = 45°$, i.e., vertex angle $= 2\phi = 90°$ Q.E.D. ◄

10.16 *The amount of water to be carried by a canal excavated in smooth earth (n = 0.030) is 370 cfs. It has side slopes of 2:1 (see Fig. P10.16), and a bed slope of 2.5 ft/mile. (a) If the depth of water y is to be 5 ft, what must be the bottom width b? (b) How does this compare with the depth and bottom width for the most efficient trapezoidal section for the given conditions?*

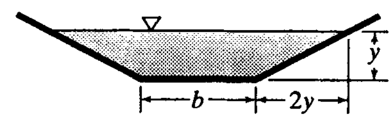


Figure P10.16

BG

(*a*) $A = 5(b + 10)$; $P = b + 2(5^2 + 10^2)^{1/2} = b + 22.36$

$R_h = A/P = 5(b + 10)/(b + 22.36)$; $S_0 = 2.5/5280 = 0.000473$

Eq. 10.8*a*: $370 = \dfrac{1.486}{0.03}\dfrac{[5(b + 10)]^{5/3}}{(b + 22.36)^{2/3}}\sqrt{0.000\,473}$

After several trials, or by equation solver per Sample Prob. 10.1*d*, we find $b = 19.63$ ft. ◄

For this b: $R_h = 3.53$ ft $\neq y/2 = 2.5$ ft.

(*b*) Given Q, n, m, and S_0: This is Case 2, Sec. 10.6:

$y_{opt} = 2^{1/4}\left[\dfrac{370(0.030)}{1.486(2\sqrt{5} - 2)\sqrt{2.5/5280}}\right]^{3/8} = 7.56$ ft ◄ $b_{opt} = 2(\sqrt{5}-2)7.56 = 3.57$ ft ◄

For this b: $A = 141.4$ ft^2, $P = 37.4$ ft, and $R_h = 3.78$ ft $= y/2$.

10.17 *The amount of water to be carried by a canal excavated in smooth earth (n = 0.030) is 12 m³/s. It has side slopes of 2:1 (see Fig. P10.16), and a bed slope of 600 mm/km. (a) If the depth of water y is to be 2.0 m, what must be the bottom width b? (b) How does this compare with the depth and bottom width for the most efficient trapezoidal section for the given conditions?*

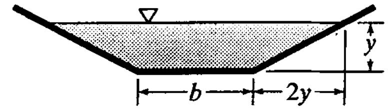


Figure P10.16

SI

(*a*) $A = 2.0(b + 4)$; $P = b + 2(2.0^2 + 4^2)^{1/2} = b + 8.944$

$R_h = A/P = 2.0(b + 4)/(b + 8.944)$; $S_0 = 0.60/1000 = 0.00060$

Eq. 10.8*b*: $$12 = \frac{1}{0.03}\frac{[2.0(b+4)]^{5/3}}{(b+8.944)^{2/3}}\sqrt{0.000\,60}$$

After several trials, or by equation solver per Sample Prob. 10.1*d*, we find $b = 2.69$ m ◄

For this *b*: $R_h = 1.150 \text{ m} \neq y/2 = 1.0$ m.

(*b*) Given Q, n, m, and S_0: This is Case 2, Sec. 10.6:

$$y_{opt} = 2^{1/4}\left[\frac{12(0.030)}{(2\sqrt{5}-2)\sqrt{0.6/1000}}\right]^{3/8} = 2.32 \text{ m} \quad ◄ \qquad b_{opt} = 2(\sqrt{5}-2)2.32 = 1.096 \text{ m} \quad ◄$$

For this *b*: $A = 13.31$ m², $P = 11.47$ m, and $R_h = 1.160 \text{ m} = y/2$.

10.18 *Refer to Fig. P10.16. (a) If the discharge in the canal (n = 0.030) is to be 200 cfs while the depth y is 5 ft and the velocity is not to exceed 150 ft/min, what must be the minimum bottom width b and the maximum drop in elevation per mile? (b) Compare this with the depth and bottom width for maximum efficiency when using the same side slopes, roughness, bed slope, and discharge.*

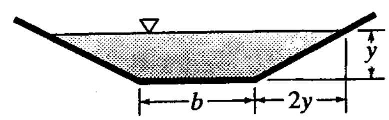


Figure P10.16

BG

(*a*) $V_{max} = 150/60 = 2.5$ fps; $A_{min} = Q/V_{max} = 200/2.5 = 80$ ft²

$A_{min} = 5(b_{min} + 10) = 80$, $\therefore b_{min} = 6$ ft ◄

$P_{min} = 6 + 2(5^2 + 10^2)^{1/2} = 28.36$ ft; min $R_h = 80/28.36 = 2.82$ ft; min $R_h^{2/3} = 1.996$

$\therefore$ from Eq. 10.7*b*: max $S_0^{1/2} = 2.5(0.030)/[1.486(1.996)] = 0.0253$; max $S_0 = 0.000639$

$\therefore$ max drop $= 0.000\,639(5280) = 3.37$ ft per mile ◄

(*b*) Given Q, n, m, and S_0: This is Case 2, Sec. 10.6:

$$y_{opt} = 2^{1/4}\left[\frac{200(0.030)}{1.486(2\sqrt{5}-2)\sqrt{0.000639}}\right]^{3/8} = 5.68 \text{ ft} \quad ◄ \qquad b_{opt} = 2(\sqrt{5}-2)5.68 = 2.68 \text{ ft} \quad ◄$$

For this *b*: $A = 79.7$ ft², $P = 28.1$ ft, and $R_h = 2.84 \text{ ft} = y/2$.

10.19 *Refer to Fig. P10.16. (a) If the discharge in the canal (n = 0.028) is to be 6 m³/s while the depth y is 1.8 m and the velocity is not to exceed 50 m/min, what must be the minimum bottom width b and the maximum drop in elevation per km? (b) Compare this with the depth and bottom width for maximum efficiency when using the same side slopes, roughness, bed slope, and discharge.*

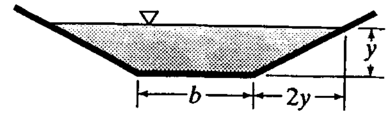


Figure P10.16

SI

(*a*) $V_{max} = 50/60 = 0.833$ m/s; $A_{min} = Q/V_{max} = 6/0.833 = 7.20\ \text{m}^2$

$A_{min} = 1.8(b_{min} + 3.6) = 7.20, \quad \therefore\ b_{min} = 0.400$ m ◄

$P_{min} = 0.400 + 2(1.8^2 + 3.6^2)^{1/2} = 8.450$ m; min $R_h = 7.20/8.450 = 0.852$ m; min $R_h^{2/3} = 0.899$

∴ from Eq. 10.7*a*: max $S_0^{1/2} = 0.833(0.028)/0.899 = 0.0260$; max $S_0 = 0.000674$

∴ max drop = 0.000 674(1000) = 0.674 m/km ◄

(*b*) Given Q, n, m, and S_0: This is Case 2, Sec. 10.6:

$$y_{opt} = 2^{1/4}\left[\frac{6(0.028)}{(2\sqrt{5}-2)\sqrt{0.000674}}\right]^{3/8} = 1.706 \text{ m} \quad ◄ \quad b_{opt} = 2(\sqrt{5}-2)1.706 = 0.805 \text{ m} \quad ◄$$

For this b: $A = 7.19\ \text{m}^2$, $P = 8.43$ m, and $R_h = 0.853\ \text{m} = y/2$.

Sec. 10.7: Circular Sections Not Flowing Full -- *Exercises (2)*

10.7.1 *Water flows uniformly in a 14-ft-diameter concrete pipe (n = 0.014, S_0 = 0.0004) at a depth of 6 ft. Using Fig. 10.10, determine the flow rate and the average flow velocity.*

BG

Eq. 10.8*a*: $Q_{full} = (1.486/0.014)\pi 7^2(\pi 7^2/14\pi)^{2/3}(0.0004)^{1/2} = 753$ cfs; $V_{full} = 753/(\pi 7^2) = 4.89$ fps

Fig. 10.10 for $y/D = 6/14 = 0.429$: $Q/Q_{full} \approx 0.315$, $V/V_{full} \approx 0.725$

∴ $Q \approx 0.315(753) = 237$ cfs ◄ $V \approx 0.725(4.89) = 3.55$ fps ◄

10.7.2 *At what depth will water flow at 0.30 m³/s in a 1.20-m-diameter concrete pipe on a slope of 0.003? (a) Assume n = 0.013. (b) Repeat with n = 0.015 and compare results.*

SI

(*a*) Eq. 10.8*b*: $Q_{full} = (1/0.013)(\pi\times 0.6^2)[(\pi\times 0.6^2)/(\pi\times 1.2)]^{2/3}(0.003)^{1/2} = 2.14\ \text{m}^3/\text{s}$

$Q/Q_{full} = (0.30/2.14) = 0.1405$; ∴ from Fig. 10.10, $y/D \approx 0.275$

Thus depth ≈ 0.275(1.20) = 0.330 m = 330 mm ◄

(*b*) With n changed to 0.015, $Q_{full} = 2.14(0.013/0.015) = 1.851\ \text{m}^3/\text{s}$

$Q/Q_{full} = 0.30/1.851 = 0.1621$; ∴ from Fig. 10.10, $y/D \approx 0.295$

Thus depth ≈ 0.295(1.20) = 0.354 m = 354 mm ◄ Larger n increased the depth ◄

Sec. 10.7: Circular Sections Not Flowing Full -- *Problem 10.20*

10.20 *Prove that the value of θ given in Sec. 10.7 for the point of maximum discharge (assuming constant n) is correct. After differentiating, a trial-and-error type of solution will be found most practical here.*

N

Eq. 10.8: $Q \propto AR_h^{2/3}$ for variable θ

Sec. 10.7: $A = (D^2/4)(\theta - 0.5\sin 2\theta)$ and $R_h = (D/4)[1 - (\sin 2\theta)/2\theta]$

∴ Write an expression for $AR_h^{2/3}$, differentiate it with respect to θ, and equate to zero.

By trial and error or equation solver solve the resulting expression for θ; $\theta = 151.2°$ ◄

<u>Alternate solution</u>: Plot values of $AR_h^{2/3}$ vs θ to find the approx. value of θ for which $AR_h^{2/3}$ is a max.

Sec. 10.8: Laminar Flow in Open Channels -- *Exercises (4)*

10.8.1 *Eastern lubricating oil (SAE 30) at 100°F flows down a flat plate 12 ft wide. (a) What is the maximum rate of discharge at which laminar flow may be ensured, assuming that the critical Reynolds number is 500? (b) What should be the slope of the plate to secure a depth of 8 in at this flow rate?*

BG

$b/y = 12/(8/12) = 18$. Sec. 10.5: As $b/y > 10$, flow is "wide and shallow."

$\mathbf{R} = R_h V/\nu = y_0(q/y_0)/\nu = q/\nu = 500$

Fig. A.2 for SAE 30 Eastern lubricating oil at 100°F: $\nu = 0.0011$ ft²/sec

$\therefore\ q = 500\nu = 500(0.0011) = 0.55$ cfs per ft

(*a*) For $B = 12$ ft: $Q = Bq = 12(0.55) = 6.60$ cfs ◄

(*b*) From Eq. 10.15: $S_0 = \dfrac{3q\nu}{gy_0^3} = \dfrac{3(0.55)0.0011}{32.2(8/12)^3} = 0.0001902$ ◄

10.8.2 *Eastern lubricating oil (SAE 30) at 30°C flows down a flat plate 3 m wide. (a) What is the maximum rate of discharge at which laminar flow may be ensured, assuming that the critical Reynolds number is 500? (b) What should be the slope of the plate to secure a depth of 150 mm at this flow rate?*

SI

$b/y = 3/0.15 = 20$. Sec. 10.5: As $b/y > 10$, flow is "wide and shallow."

$\mathbf{R} = R_h V/\nu = y_0(q/y_0)/\nu = q/\nu = 500$

Fig. A.2 for SAE 30 Eastern lubricating oil at 30°C: $\nu = 0.000\,15$ m²/s

$\therefore\ q = 500\nu = 500(0.000\,15) = 0.075$ m³/s per m

(*a*) For $B = 3$ m: $Q = Ba = 3(0.075) = 0.225$ m³/s ◄

(*b*) From Eq. 10.15: $S_0 = \dfrac{3q\nu}{gy_0^3} = \dfrac{3(0.075)0.000\,15}{9.81(0.15)^3} = 0.001019$ ◄

10.8.3 *At what rate (cfs/ft of width) will 70°F water flow in a wide and shallow, smooth channel on a slope of 0.000 10, if the depth is 0.02 ft? Assume laminar flow and justify this assumption by computing the Reynolds number.*

BG

Table A.1 for water at 70°F: $\nu = 1.059 \times 10^{-5}$ ft²/sec

Assuming the flow is laminar, Eq. 10.15: $q = \dfrac{32.2(0.02)^3 \times 0.0001}{3(1.059 \times 10^{-5})} = 8.11 \times 10^{-4}$ cfs/ft ◄

Sec. 10.1: $\mathbf{R} = \dfrac{R_h V}{\nu} = \dfrac{y_0 V}{\nu} = \dfrac{y_0(q/y_0)}{\nu} = \dfrac{q}{\nu} = \dfrac{8.11 \times 10^{-4}}{1.059 \times 10^{-5}} = 76.6$ ◄

Thus **R** is < 500, and the assumption that the flow is laminar is justified.

10.8.4 *At what rate (L/s per meter of width) will water at 20°C flow in a wide and shallow, smooth channel on a slope of 0.0004, if the depth is 6.0 mm? Assume laminar flow and justify this assumption by computing the Reynolds number.*

SI

Table A.1 for water at 20°C: $\nu = 1.003 \times 10^{-6}$ m²/s

Assuming the flow is laminar, Eq. 10.15: $q = \dfrac{(9.81\ \text{m/s}^2)(0.006\ \text{m})^3 0.0004}{3(1.003 \times 10^{-6}\ \text{m}^2/\text{s})}$

$q = 0.000\,282$ m²/s $= 0.282$ L/s per m of width ◄

Sec. 10.1: $\mathbf{R} = R_h V/\nu = y_0 V/\nu = q/\nu = 0.000\,282/(1.003 \times 10^{-6}) = 281$ ◄

Thus **R** is < 500, therefore the flow is laminar and the answer is valid.

Sec. 10.8: Laminar Flow in Open Channels -- *Problems 10.21 – 10.22*

10.21 *Evaluate the friction factor f for laminar flow in a wide and shallow open channel in terms of the Reynolds number, and compare it with f for pipe flow using Eq. (8.29). (Note: Recall that for a wide and shallow channel the hydraulic radius is approximately equal to the depth.)*

N

Sec. 10.1: $\mathbf{R}_{oc} = R_h V/\nu$; Eq. 8.23: $\mathbf{R}_{pipe} = 4R_h V/\nu$. $\therefore\ \mathbf{R}_{oc} \equiv \mathbf{R}_{pipe}/4$

For laminar open channel flow, Eq. 10.15: $V = Q/A = q/y_0 = gS_0 y_0^2/(3\nu)$; Eq. 10.5: $V = \sqrt{(8g/f)R_h S_0}$

$\therefore\ V^2 = (8g/f)R_h S_0 = VgS_0 y_0^2/(3\nu)$, i.e., $f_{oc} = 24\nu R_h/(Vy_0^2)$

But $R_h = y_0$ for a wide and shallow channel, so: $f_{oc} = 24\nu/(VR_h) = 24/\mathbf{R}_{oc}$ ◄

For laminar pipe flow; Eq. 8.29: $f_{pipe} = 64/\mathbf{R}_{pipe}$

$\therefore\ f_{oc} = 24/\mathbf{R}_{oc} \equiv 24(4)/\mathbf{R}_{pipe} = 96/\mathbf{R}_{pipe} = 96(f_{pipe}/64) = 1.5f_{pipe}$ ◄

10.22 *In Sec. 10.8 the velocity distribution in laminar sheet flow was found to be given by $u = (gS/2\nu)y_0^2[1 - (y/y_0)^2]$, where y in this case is the variable distance downward from the surface. Evaluate α, and compare it with the result of Sample Prob. 5.1.*

N

Eq. 5.4: $\alpha = \dfrac{1}{V^3 y_0}\displaystyle\int_0^{y_0}\left\{\frac{gS_0}{2\nu}y_0^2\left[1 - \left(\frac{y}{y_0}\right)^2\right]\right\}^3 dy = \frac{16}{35V^3}\left\{\frac{gS_0}{2\nu}y_0^2\right\}^3$

With $V = (2/3)u_{max} = (2/3)(gS_0/2\nu)y_0^2(1 - 0/y_0)$; $\alpha = 1.542$ ◄

In comparison, $\alpha = 2$ for flow in a circular pipe (Sample Prob. 5.1) ◄

Sec. 10.9: Specific Energy and Alternate Depths of Flow in Rectangular Channels -- *Exercises (3)*

10.9.1 *A long straight rectangular channel 12 ft wide is observed to have a wavy water surface at a depth of about 5 ft (Fig. X10.9.1). Estimate the rate of discharge.*

BG

Sec. 10.9: Wavy surface means the depth is near critical.

If $y_c = 5$ ft, by Eq. 10.29: $q = \sqrt{32.2(5)^3} = 63.4$ cfs/ft

So $Q = bq = 12(63.4) = 761$ cfs ◄

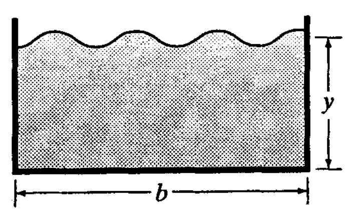


Figure X10.9.1

10.9.2 *A long straight rectangular channel 4 m wide is observed to have a wavy water surface at a depth of about 2 m (Fig. X10.9.1). Estimate the rate of discharge.*

SI

Sec. 10.9: A wavy surface means the depth is near critical.

If $y_c = 2$ m, by Eq. 10.29: $q = \sqrt{9.81(2)^3} = 8.86$ m³/s per m

$Q = Bq = 4(8.86) = 35.4$ m³/s ◄

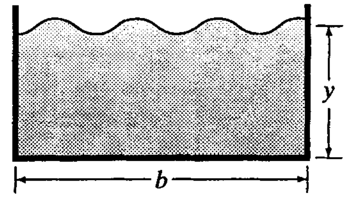


Figure X10.9.1

10.9.3 *Water flows down a wide and shallow rectangular channel of concrete ($n = 0.015$) laid on a slope of 0.004. Find the depth and rate of flow in SI units for critical conditions in this channel.*

SI

For wide and shallow rectangular channel (Sec. 10.5), $R_h = y$ and $V = q/y$.

Equating V's from Eq. 10.22 and from Eq. 10.7*a* (with $R_h = y$):

$(9.81y_c)^{1/2} = (1/0.015)y_c^{2/3}(0.004)^{1/2}$; $y_c^{1/6} = 0.743$; $y_c = 0.168$ m ◀

Eq. 10.22: $V_c = (gy_c)^{1/2} = (9.81 \times 0.168)^{1/2} = 1.284$ m/s

$\therefore\ q_c = V_c y_c = 1.284(0.168) = 0.216$ m^3/s per m ◀

Sec. 10.9: Specific Energy and Alternate Depths of Flow in Rectangular Channels -- *Problems 10.23–10.29*

10.23 *Consider a wide and shallow rectangular channel on a given slope. With what power of the discharge does the depth vary? With what power of the discharge does the critical depth vary? As the flow increases, does the Froude number increase or decrease? Assume Manning's equation, with a constant value of n.*

N

Assuming Manning's Eq. with constant "n" and $R_h = y$ (wide and shallow):

From Eq. 10.7*b*: $q = yV = y(1.486/n)y^{2/3}S_0^{1/2}$ or $q \propto y^{5/3}$, or $y \propto q^{3/5}$ ◀

Eq. 10.23: $y_c = (q^2/g)^{1/3}$, or $y_c \propto q^{2/3}$ ◀

Eq. 7.8: $\mathbf{F} = \dfrac{V}{\sqrt{gL}} = \dfrac{V}{\sqrt{gy}} = \dfrac{q}{y\sqrt{gy}} = \dfrac{q}{g^{1/2}y^{3/2}} \propto \dfrac{q}{(q^{3/5})^{3/2}} = q^{1/10}$

So as the flow, q, increases, the Froude number <u>in</u>creases. ◀

10.24 *Differentiate Eq. (10.17) to obtain the expression for y_c given in Eq. (10.23).*

N

Eq. 10.17: $E = y + (q^2/2gy^2) = y + (q^2/2g)y^{-2}$; $y = y_c$ when $dE/dy = 0$

i.e., $1 + (q^2/2g)(-2y^{-3}) = 0$, i.e., $y_c = (q^2/g)^{1/3}$ Q.E.D. ◀

10.25 *A flow of 90 cfs is carried in a rectangular channel 10 ft wide at a depth of 1.6 ft. If the channel is made of rough concrete ($n = 0.016$), find the slope necessary to sustain uniform flow at this depth. What roughness coefficient would be required to produce uniform critical flow for the given rate of discharge on this slope?*

BG

$V = 90/(10 \times 1.6) = 5.63$ fps; $R_h = A/P = 1.6(10)/[(10 + 2(1.6)] = 1.212$ ft

Eq. 10.7*b*: $5.63 = (1.486/0.016)1.212^{2/3}S_0^{1/2}$; $S_0^{1/2} = 0.0533$; $S_0 = 0.002\,84$ ◀

For $q = 90/10 = 9.00$ cfs/ft, Eq. 10.23: $y_c = (9.00^2/32.2)^{1/3} = 1.360$ ft

To find n required, $V_c = 9.00/1.360 = 6.62$ fps; $R_{h_c} = 10(1.360)/[10 + 2(1.360)] = 1.069$ ft

Eq. 10.7*b*: $6.62 = (1.486/n)(1.069)^{2/3}(0.0533)$; $n = 0.012\,51$ ◀

10.26 *A rectangular channel 15 ft wide carries a flow of 320 cfs. Find the critical depth and the critical velocity for this flow. Find also the critical slope if n = 0.015.*

BG

$q = Q/b = 320/15 = 21.33$ cfs/ft; Eq. 10.23: $y_c = (21.33^2/32.2)^{1/3} = 2.42$ ft ◄

Eq. 10.22: $V_c = (32.2 \times 2.42)^{1/2} = 8.82$ fps ◄ Sec. 10.10: When $y = y_c$, then $S_0 = S_c$.

Eq. 10.8*a*: $320 = \dfrac{1.486}{0.015}(15 \times 2.42)\left(\dfrac{15(2.42)}{15 + 2(2.42)}\right)^{2/3} S_c^{1/2}$; $S_c = 0.00355$ ◄

Note that Eq. 10.30 (which yields $S_c = 0.00244$) is not accurate because the channel is not sufficiently wide and shallow.

10.27 *Water flows with a velocity of 6 fps and at a depth of 3 ft in a wide rectangular channel. Is the flow subcritical or supercritical? Find the alternate depth for the same discharge and specific energy by two methods; (a) by direct solution of Eq. (10.17); (b) by use of Fig. 10.13.*

BG

$V^2/2g = 6^2/(2 \times 32.2) = 0.559$ ft and $y/2 = 1.5$ ft; Sec. 10.10: $0.559 < 1.5$ so flow is subcritical ◄

To find alternate (supercritical) depth:

(*a*) Eq. 10.17: $E = 3 + 0.559 = 3.559$ ft; $q = 6(3) = 18$ cfs/ft

Eq. 10.17: $3.559 = y + 18^2/[2(32.2)y^2]$ or $y^3 - 3.559y^2 + 5.031 = 0$

Since we know $y = 3$ is one solution (root), we can find the other two from Eq. B.9,

yielding $y = -1.045$ ft (meaningless), and $y = 1.604$ ft, the alternate depth ◄

(*b*) $y/E = 3/3.559 = 0.843$ ($> 2/3$, ∴ subcritical)

So in Fig. 10.13: $q/q_{max} \approx 0.865$, alternate $y/E \approx 0.45$ (supercritical)

∴ $y \approx 3.559(0.45) = 1.602$ ft ◄

(*b*) <u>Alt</u>: Eq. 10.28: $y_c = (2/3)3.559 = 2.37$ ft ; Eq. 10.29: $q_{max} = (32.2 \times 2.37^3)^{1/2} = 20.70$ cfs/ft

Thus, abscissa of Fig. 10.13 is $q/q_{max} = 18/20.70 = 0.869$

For supercritical limb, $y/E \approx 0.44$, or $y \approx 0.44(3.559) = 1.566$ ft ◄

10.28 *Water is released from a sluice gate in a rectangular channel 6 ft wide such that the depth is 3 ft and the velocity is 18 fps (Fig. P10.28). Find (a) the critical depth for this specific energy; (b) the critical depth for this rate of discharge; (c) the type of flow and the alternate depth by either direct solution or the discharge curve.*

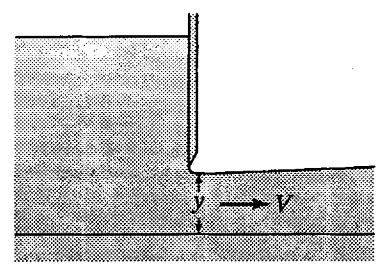


Figure P10.28

BG

(*a*) Eq. 10.17: $E = 3 + 18^2/[2(32.2)] = 8.03$ ft

For this given (constant) E (Fig. 10.13*b*, q may vary),

Eq. 10.28: $y_c = (2/3)8.03 = 5.35$ ft ◄

(*b*) $q = yV = 3(18) = 54$ cfs/ft.

For this given (constant) q (Fig. 10.12, E may vary), Eq. 10.23: $y_c = (54^2/32.2)^{1/3} = 4.49$ ft ◄

(*c*) $y < y_c$, ∴ the flow is <u>supercritical</u> ◄

By direct solution of Eq. 10.17: $8.03 = y + 54^2/[2(32.2)y^2]$ or $y^3 - 8.03y^2 + 45.28 = 0$

By trial, the alternate depth $y = 7.14$ ft ◄

<u>Alternatively</u>: As we know $y = 3$ ft is one solution (root), we can find the other two from Eq. B.9,

yielding $y = -2.11$ ft (meaningless), and $y = 7.14$ ft, the alternate depth ◄

10.29 *Water is released from a sluice gate in a rectangular channel 1.5 m wide such that the depth is 0.6 m and the velocity is 4.5 m/s (Fig. P10.28). Find (a) the critical depth for this specific energy; (b) the critical depth for this rate of discharge; (c) the type of flow and the alternate depth by either direct solution or the discharge curve.*

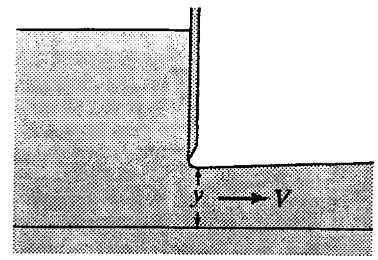


Figure P10.28

SI

(*a*) Eq. 10.17: $E = 0.6 + 4.5^2/[2(9.81)] = 1.632$ m

For this given (constant) E (Fig. 10.13*b*, q may vary),

Eq. 10.28: $y_c = (2/3)1.632 = 1.088$ m ◄

(*b*) $q = yV = 0.6(4.5) = 2.70$ m³/s per m

For this given (constant) q (Fig. 10.12, E may vary), Eq. 10.23: $y_c = \left(\frac{2.70^2}{9.81}\right)^{1/3} = 0.906$ m ◄

(*c*) $y < y_c$, ∴ the flow is <u>supercritical</u> ◄

By direct solution of Eq. 10.17: $E = 1.632 = y + 2.70^2/[2(9.81)y^2]$ or $y^3 - 1.632y^2 + 0.372 = 0$

By trial, the alternate depth $y = 1.457$ m ◄

<u>Alternatively</u>: As we know $y = 0.6$ m is one solution (root), we can find the other two from Eq. B.9, yielding $y = -0.425$ m (meaningless), and $y = 1.457$ m, the alternate depth ◄

Sec. 10.10: Subcritical and Supercritical Flow -- ***Exercises*** *(2)*

10.10.1 *A rectangular channel 50 ft wide (n = 0.015) carries a flow of 375 cfs. For this flow, find the critical depth, the critical velocity, and the critical bed slope.*

BG

$q = Q/b = 375/50 = 7.5$ cfs/ft; Eq. 10.23: $y_c = (7.5^2/32.2)^{1/3} = 1.204$ ft ◄

Eq. 10.22: $V_c = \sqrt{32.2(1.204)} = 6.23$ fps ◄

$b/y_c = 50/1.204 = 41.5$, ∴ (Sec. 10.5) critical flow is "wide and shallow," $R_h \approx y_c$.

Eq. 10.30: $S_c = \left(\frac{0.015}{1.486}\right)^2 \frac{32.2}{1.204^{1/3}} = 0.003\,08$ ◄

10.10.2 *A 10-ft-wide rectangular channel (n = 0.020) carries 400 cfs. Find the critical bed slope from Eq. (10.30) and by using the Manning formula. Which of these answers is correct, and why? What is the percentage error in the incorrect answer?*

BG

$q = Q/b = 400/10 = 40$ cfs/ft; Eq. 10.23: $y_c = (q^2/g)^{1/3} = (40^2/32.2)^{1/3} = 3.68$ ft

Eq. 10.22: $V_c = q/y_c = 40/3.68 = 10.88$ fps

Eq. 10.30: $S_c = \left[\frac{n}{1.486}\right]^2 \frac{g}{y_c^{1/3}} = \left[\frac{0.020}{1.486}\right]^2 \frac{32.2}{3.68^{1/3}} = 0.003\ 78$ ◄

Manning formula for critical flow conditions: $V_c = \frac{1.486}{n} R_{h_c}^{2/3} S_c^{1/2}$

so $S_c = \left[\frac{nV_c}{1.486 R_{h_c}^{2/3}}\right]^2 = \left[\frac{0.020(10.88)}{1.486\left[\frac{3.68 \times 10}{2(3.68) + 10}\right]^{2/3}}\right]^2 = 0.007\ 88$ ◄

Eq. 10.30 assumes "wide and shallow" flow (Sec. 10.5), i.e., $R_h \approx y$.

(*a*) $b/y = 10/3.676 = 2.72$. This is far from being > 10,

∴ (Sec. 10.5) it is not reasonable to assume that $y \approx R_h$.

(*b*) Sec. 10.5: Percentage error in using y for R_h is $200(y/b) = 200(3.67/10) = 125\%$

This error is very large. Therefore the use of Eq. 10.30 is incorrect.

The correct answer is $S_c = 0.007\ 88$ ◄

The other answer is incorrect because the flow is not "wide and shallow."

Error $= \frac{0.00788 - 0.00378}{0.00788}(100\%) = 52.0\%$ small ◄

Sec. 10.11: Critical Depth in Nonrectangular Channels -- *Exercises (4)*

10.11.1 *A flow of 12 cfs of water is carried in a 90° triangular flume built of planed timber (n = 0.011). Find the critical depth and the critical slope.*

BG

$A = y^2$, $B = 2y$; Eq. 10.33: $12^2/32.2 = y_c^6/2y_c$; $y_c^5 = 8.94$; $y_c = 1.550$ ft ◄

Then: $A_c = (1.550)^2 = 2.40$ ft²; $V_c = Q/A_c = 12/2.40 = 5.00$ fps

$P = 2(2^{1/2})1.550 = 4.38$ ft; $R_h = A_c/P = 2.40/4.38 = 0.548$ ft

Eq. 10.7*b*: $5.00 = (1.486/0.011)(0.548)^{2/3} S_c^{1/2}$; $S_c^{1/2} = 0.0552$; $S_c = 0.003\ 05$ ◄

10.11.2 *A flow of 0.28 m³/s of water is carried in a 90° triangular flume built of planed timber (n = 0.011). Find the critical depth and the critical slope.*

SI

$A = y^2$, $B = 2y$; Eq. 10.33: $0.28^2/9.81 = y_c^6/2y_c$; $y_c^5 = 0.015\ 98$; $y_c = 0.437$ m ◄

Then: $A_c = (0.437)^2 = 0.1912$ m²; $V_c = Q/A_c = 0.28/0.1912 = 1.464$ m/s

$P = 2(2^{1/2})0.437 = 1.237$ m; $R_h = A_c/P = 0.1912/1.237 = 0.1546$ ft

Eq. 10.8*b*: $0.28 = \frac{1}{0.011} 0.1912\,(0.1546)^{2/3} S_c^{1/2}$; $S_c^{1/2} = 0.0559$, $S_c = 0.003\ 13$ ◄

10.11.3 BG *A circular conduit of well-laid (smooth) brickwork when flowing half full is to carry 340 cfs at a velocity of 8.2 fps. (a) What will be the necessary fall per mile? (b) Will the flow be subcritical or supercritical?*

(*a*) Table 10.1: For well-laid brickwork, minimum $n = 0.012$

$A = 340/8.2 = 41.46\ \text{ft}^2 = 0.5(\pi/4)D^2$; Eq. 8.20: $D = 10.28$ ft ; $R_h = 10.28/4 = 2.57$ ft

Eq. 10.7*b*: $8.2 = (1.486/0.012)(2.57)^{2/3}S_0^{1/2}$; $S_0 = 0.001\,246 = 6.58$ ft/mile ◄

(*b*) For flow half full, $B = D$.

Sec. 10.11: $\mathbf{F} = V/\sqrt{gy_h} = V\sqrt{B/gA} = 8.2\sqrt{10.28/(32.2 \times 41.46)} = 0.719$

Sec. 10.10: $\mathbf{F} < 1$, so the flow is subcritical. ◄

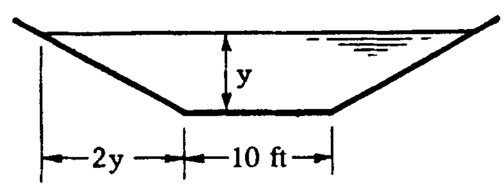
10.11.4 SI *A trapezoidal channel has a 4-m-wide bed and m = 2 (i.e., 2H:1V). What is the critical flow depth when it is carrying 85 m³/s?*

Equation preceding Sample Prob. 10.7: $\dfrac{Q^2}{g} = \dfrac{85^2}{9.81} = \dfrac{(4y_c + 2y_c^2)^3}{4 + 4y_c} = \dfrac{8(2y_c + y_c^2)^3}{4(1 + y_c)}$

i.e., $368 = (2y_c + y_c^2)^3/(1 + y_c)$; by T & E, or by equation solver: $y_c = 2.44$ m. ◄

Sec. 10.11: Critical Depth in Nonrectangular Channels -- ***Problems*** *10.30–10.34*

10.30 BG *A trapezoidal canal with side slopes of 2:1 (H:V) has a bottom width of 10 ft and carries a flow of 600 cfs. (a) Find the critical depth and critical velocity. (b) If the canal is lined with brick (n = 0.015), find the critical slope for the same rate of discharge.*



(*a*) $A = 10y + 2y^2 = 2y(5 + y)$

$B = 10 + 4y = 4(2.5 + y)$

When $y = y_c$, from Eq. 10.33: $\dfrac{600^2}{32.2} = \dfrac{8y_c^3(5 + y_c)^3}{4(2.5 + y_c)}$

i.e., $11{,}180 = \dfrac{2y_c^3(5 + y_c)^3}{2.5 + y_c}$

By trial and error, or by polynomial or equation solver per Sample Prob. 10.1: $y_c = 3.74$ ft ◄

From above: $A_c = 65.3\ \text{ft}^2$; $B_c = 25.0$ ft; $V_c = Q/A_c = 600/65.3 = 9.18$ fps ◄

(*b*) $P_c = 10 + 2(3.74)5^{1/2} = 26.7$ ft; $R_h = A_c/P_c = 65.3/26.7 = 2.45$ ft

Eq. 10.7*b*: $9.18 = (1.486/0.015)2.45^{2/3}S_c^{1/2}$; $S_c = 0.002\,61$ ◄

Note: Eq. 10.30 (yielding $S_c = 0.002\,11$) is not valid (as channel is not "wide and shallow").

10.31 SI *A trapezoidal canal with side slopes of 1:1 has a bottom width of 4 m and carries a flow of 25 m³/s. (a) Find the critical depth and critical velocity. (b) If the canal is lined with brick (n = 0.015), find the critical slope for the same rate of discharge.*

(*a*) $A = y(4 + y)$; $B = 4 + 2y$

When $y = y_c$, Eq. 10.33: $25^2/9.81 = 63.71 = y_c^3(4 + y_c)^3/(4 + 2y_c)$

By trial and error, or by equation solver per Sample Prob. 10.1: $y_c = 1.401$ m ◄

From above, $A_c = 7.57\ \text{m}^2$; $B_c = 6.80$ m; $V_c = Q/A_c = 25/7.57 = 3.30$ m/s ◄

(*b*) $P_c = 4 + 2(1.401)2^{1/2} = 7.96$ m; $R_h = A_c/P_c = 7.57/7.96 = 0.950$ m

Eq. 10.7*a*: $3.30 = (1/0.015)(0.950)^{2/3}S_c^{1/2}$; $S_c = 0.002\,63$ ◄

Note: Eq. 10.30 (yielding $S_c = 0.001\,973$) is not valid (as channel is not "wide and shallow").

10.32 *For a circular conduit with a diameter of 12 ft, compute the specific energy for a flow of 120 cfs at depths of 2, 4, 6, and 10 ft assuming $\alpha = 1.0$. At what depth is E the least? Check to see if Eq. (10.33) is satisfied at this depth.*

BG

Given: $D = 12$ ft, $Q = 120$ cfs, $\alpha = 1$. Sec. 10.7: $A = (D^2/4)(\theta - 0.5\sin 2\theta)$

Compute $A(y)$, or use Tables such as Table 7.4 in Brater et al's "Handbook of Hydraulics," 7th ed, McGraw-Hill, 1996. A spreadsheet could be helpful with this problem.

y ft	θ deg	A ft²	V fps (= Q/A)	$V^2/2g$ ft	E ft (Eq. 10.16)
2	48.2	12.4	9.69	1.46	3.46
2.5	54.3	17.1	7.03	0.77	3.27
3	60.0	22.1	5.43	0.46	3.46
4	70.5	33.0	3.64	0.21	4.21
6	90.0	56.6	2.12	0.07	6.07
10	131.8	100.7	1.19	0.02	10.02

▲

Plot the above and more data and note that $E_{min} \approx 3.26$ ft at $y \approx 2.42$ feet ◄

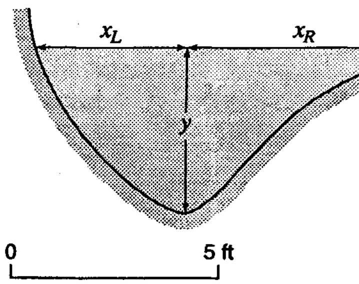
Check: When $y = 2.42$ ft, $A = 16.29$ ft², $B = 9.63$ ft, and so in Eq. 10.33:

$Q^2/g = 120^2/32.2 = 447$ and $A^3/B = 16.29^3/9.63 = 449$

∴ Eq. 10.33 is satisfied well at this depth. ◄

10.33 *Figure P10.33 describes the cross section of an open channel for which S_0 = 0.03 and n = 0.020. The sketch is drawn to the scale shown. When the flow rate is 120 cfs, find (a) the depth for uniform flow and (b) the critical depth. Hint: The cross-sectional area may be found by planimetry or counting squares and the wetted perimeter by use of dividers.*

BG



y, ft	x_L, ft	x_R, ft
0	0	0
1	1.5	1.1
2	2.6	2.0
3	3.25	3.3
4	3.6	5.5
5	3.85	8.2

Figure P10.33

Given: Q = 120 cfs, S_0 = 0.03, n = 0.020

(*a*)

y_0 ft	A^* ft^2	P^* ft	$R_h = A/P$ ft	$R_h^{2/3}$	Q cfs Eq. 10.8a
1	1.30	3.29	0.395	0.538	9.01
2	4.90	6.12	0.800	0.862	54.4
3	10.48	8.96	1.170	1.110	149.7
4	18.30	12.43	1.472	1.294	304.8
5	28.88	16.34	1.767	1.462	543.1

* From Fig. P10.33 (measurements may vary)

Plot Q vs y_0; from the plot, for Q = 120 cfs, find that y_0 = 2.70 ft ◄

(*b*)

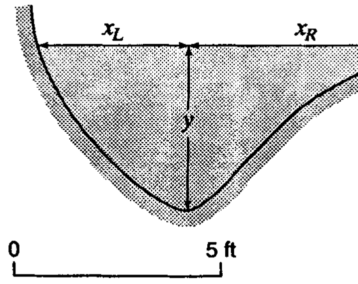
y_c ft	A^* ft^2	$B = x_L + x_R$ ft	gA^3/B	$Q = \sqrt{gA^3/B}$ cfs Eq. 10.33
1	1.30	2.60	27.2	5.22
2	4.90	4.60	823	28.7
3	10.48	6.55	5,650	75.2
4	18.30	9.10	21,700	147.3
5	28.88	12.05	64,000	254

* From Fig. P10.33 (measurements may vary)

Plot Q vs y_c; from the plot, for Q = 120 cfs, find that y_c = 3.75 ft ◄

10.34 *Refer to the figure, table, and hint for Prob. 10.33, replacing feet dimensions with meters. Let the slope be 0.009 with n = 0.020. When the flow rate is 60 m³/s, find (a) depth for uniform flow; (b) the critical depth.*

Prob. 10.33: Figure P10.33 describes the cross section of an open channel; the sketch is drawn to the scale shown. Hint: The cross-sectional area may be found by planimetry or counting squares and the wetted perimeter by use of dividers.



y, ft	x_L, ft	x_R, ft
0	0	0
1	1.5	1.1
2	2.6	2.0
3	3.25	3.3
4	3.6	5.5
5	3.85	8.2

Figure P10.33

SI

Given: $Q = 60$ m³/s, $S_0 = 0.009$, $n = 0.020$

(*a*)

y_0 m	A^* m²	P^* m	$R_h = A/P$ m	$R_h^{2/3}$	Q m³/s Eq. 10.8b
1	1.30	3.29	0.395	0.538	3.32
2	4.90	6.12	0.800	0.862	20.0
3	10.48	8.96	1.170	1.110	55.2
4	18.30	12.43	1.472	1.294	112.3
5	28.88	16.34	1.767	1.462	200.2

* From Fig. P10.33 (measurements may vary)

Plot Q vs y_0; from the plot, for $Q = 60$ m³/s, find that $y_0 = 3.10$ m ◄

(*b*)

y_c m	A^* m²	$B = x_L + x_R$ m	gA^3/B	$Q = \sqrt{gA^3/B}$ m³/s Eq. 10.33
1	1.30	2.60	8.29	2.88
2	4.90	4.60	250	15.84
3	10.48	6.55	1 721	41.5
4	18.30	9.10	6 610	81.3
5	28.88	12.05	19 600	140.0

Plot Q vs y_c; from the plot, for $Q = 60$ m³/s, find that $y_c = 3.50$ m ◄

Sec. 10.13: Humps and Contractions – *Exercises (4)*

10.13.1 *A rectangular channel 3 m wide carries 4 m³/s of water in subcritical uniform flow at a depth of 1.2 m. A frictionless hump is to be installed across the bed. Find the critical hump height (i.e., the minimum hump which causes y_c on it).*

SI

$V_0 = Q/A_0 = 4/(3 \times 1.2) = 1.111$ m/s

Eq. 10.17: $E_0 = 1.2 + 1.111^2/[2(9.81)] = 1.263$ m; $q = Q/b = 4/3 = 1.333$ m³/s per m

Eq. 10.23: $y_c = (1.333^2/9.81)^{1/3} = 0.566$ m; Eq. 10.25: $E_{min} = (3/2)0.566 = 0.849$ m

Sec 10.13: $(\Delta z)_{crit} = E_0 - E_{min} = 1.263 - 0.849 = 0.414$ m ◄

10.13.2 *Suppose that the depth of uniform flow in a 4-ft-wide rectangular channel is 1.10 ft. Find the change in water-surface elevation caused by a 1-ft-wide bridge pier placed in the middle of the channel. The flow rate is 50 cfs.*

BG

$V_0 = Q/(by_0) = 50/(4 \times 1.10) = 11.36$ fps ; Eq. 10.17: $E_0 = 1.10 + 11.36^2/(2 \times 32.2) = 3.11$ ft

Eq. 10.23: $(y_c)_0 = [(50/4)^2/32.2]^{1/3} = 1.693$ ft, so $y_0 = 1.10$ ft is supercritical.

With a critical constriction, $E_{min} = 3.11 \text{ ft} = (3/2)y_c$, $y_c = 2.07$ ft

and $q = \sqrt{gy_c^3} = \sqrt{32.2(2.07)^3} = 16.90$ cfs/ft, $b_{crit} = Q/q = 50/16.90 = 2.96$ ft.

At the given constriction, $b = 4 - 1 = 3 \text{ ft} > b_{crit}$, so $V = (50/3)/y = 16.67/y$

and $3.11 = y + (16.67/y)^2/(2 \times 32.2)$

By trial and error or by polynomial or equation solver, $y = 1.865$ ft (supercritical) or 2.262 ft (subcritical) or -1.022 ft (meaningless).

The flow will remain supercritical, so $y = 1.865$ ft

Water surface rise $= y_2 - y_0 = 1.869 - 1.100 = 0.769$ ft ◄

10.13.3 *Suppose that the depth of uniform flow in a 1.2-m-wide rectangular channel is 270 mm. Find the change in water-surface elevation caused by a 300-mm-wide bridge pier placed in the middle of the channel. The flow rate is 1.1 m³/s.*

SI

$V_0 = \dfrac{Q}{by_0} = \dfrac{1.1}{1.2 \times 0.27} = 3.40$ m/s ; Eq. 10.17: $E_0 = 0.27 + \dfrac{3.40^2}{2(9.81)} = 0.857$ m

Eq. 10.23: $(y_c)_0 = [(1.1/1.2)^2/9.81] = 0.0857$ m, so $y_0 = 0.270$ is subcritical.

With a critical constriction, $E_{min} = 0.857 \text{ m} = (3/2)y_c$, $y_c = 0.572$ m

and $q = \sqrt{gy_c^3} = \sqrt{9.81(0.572)^3} = 1.354$ m³/s per m, $b_{crit} = Q/q = 1.1/1.354 = 0.813$ m.

At the given constriction, $b = 1.2 - 0.3 = 0.9 \text{ m} > b_{crit}$, so $V = 1.1/(0.9y) = 1.222/y$

and $0.857 = y + (1.222/y)^2/(2 \times 9.81)$

By trial and error or by polynomial or equation solver, $y = 0.4146$ m (supercritical) or 0.704 m (subcritical) or -0.261 m (meaningless).

The flow will remain supercritical, so $y = 0.4146$ m

Water surface rise $= y_2 - y_0 = 0.4146 - 0.2700 = 0.1446$ m ◄

10.13.4 *A rectangular channel 12 ft wide carries 24 cfs in uniform flow at a depth of 0.80 ft. Find the local change in water-surface elevation caused by a frictionless hump 0.12 ft high across the floor of the channel.*

BG

$q = Q/b = 24/12 = 2$ cfs/ft. Eq. 10.23: $y_c = (2^2/32.2)^{1/3} = 0.499$ ft.

Eq. 10.25: $E_{min} = 1.5y_c = 0.748$ ft

Given $y_1 = 0.8$: $y_1 > y_c$ so flow is subcritical; $V_1 = q/y_1 = 2/0.8 = 2.50$ fps; $V_1^2/2g = 0.0970$ ft

Eq. 10.17: $E_1 = 0.8 + 0.0970 = 0.897$ ft; $\Delta z_{crit} = E_1 - E_{min} = 0.897 - 0.748 = 0.1486$ ft

$\therefore\ \Delta z < \Delta z_{crit}$. At 0.12-ft rise in bed, $E_2 = E_1 - \Delta z = 0.897 - 0.12 = 0.777$ ft

Eq. 10.17: $E_2 = y_2 + 2^2/[2(32.3)y_2^2] = 0.777$ ft. By T & E or by equation solver, $y_2 = 0.610$ ft.

Then change in water surface $= y_1 - (\Delta z + y_2) = 0.800 - (0.120 + 0.610) = 0.070$ ft drop ◄

Sec. 10.13: Humps and Contractions -- *Problems* 10.35–10.42

10.35 *Work Sample Prob. 10.8 ($y_0 = 2.0$ ft) for the case where the flow rate is 275 cfs.*

Sample Prob. 10.8: The flow is uniform in a 20-ft-wide rectangular channel. (a) Is it subcritical or supercritical? (b) If a hump of height $\Delta z = 0.30$ ft is placed in the bottom of the channel, calculate the water depth on the hump, and the change in the water surface level at the hump. (c) If the hump height is raised to $\Delta z = 0.60$ ft, what then are the water depths upstream and downstream of the hump? (d) If the 0.30-ft hump is accompanied by a local contraction to 18 ft, find the water depth on the hump. In all cases neglect head losses over the hump and through the contraction.

BG

Given: $y_0 = 2.00$ ft, $Q = 275$ cfs. *Note* the needs for extra significant figures.

(*a*) $q = Q/b = 275/20 = 13.75$ cfs/ft; Eq. 10.23: $y_c = (13.75^2/32.2)^{1/3} = 1.804$ ft

$y_0 > y_c$, so flow is subcritical (and slope is mild) ◀

(*b*) Instal hump, height $\Delta z = 0.30$ ft. First find critical hump height.

Eq. 10.17: $E_0 = 2.00 + [1/(2 \times 32.2)](13.75/2.00)^2 = 2.7339$ ft

Eq. 10.25: $E_{min} = 1.5(1.804) = 2.7061$ ft

Sec 10.13: $E_0 = \Delta z_{crit} + E_{min}$, so $\Delta z_{crit} = 2.7339 - 2.7061 = 0.0279$ ft

$\Delta z > \Delta z_{crit}$, so damming action occurs and y_c occurs on the hump. On hump $y = 1.804$ ft ◀

(*c*) Increase hump height Δz to 0.60 ft. Still $\Delta z > \Delta z_{crit}$, so y_c occurs on the hump.

Let y = upstream or downstream depth, with specific energy E. Then, Sec. 10.13: $E = \Delta z + E_{min}$

Eq. 10.17: $y + [1/(2 \times 32.2)](13.75/y)^2 = 0.60 + 2.706$ i.e., $y + (2.94/y^2) = 3.306$

By trial and error or by equation solver: $y = 2.97$, 1.17, or a negative root which has no physical meaning.

This hump causes damming action, and the depths just upstream and downstream of the hump are 2.97 ft (subcritical) and 1.17 ft (supercritical) respectively. ◀

See Sample Prob. 10.8(*c*) for an explanation of which depth occurs where.

(*d*) At section 2 with 0.30 ft hump and contraction to 18 ft:

$q_2 = 275/18 = 15.28$ cfs/ft; $y_{c2} = (15.28^2/32.2)^{1/3} = 1.935$ ft

$(E_{min})_2 = 1.5(1.935) = 2.903$ ft ; $\Delta z_{crit} = E_1 - (E_{min})_2 = 2.734 - 2.903 = -0.169$ ft

So $\Delta z > \Delta z_{crit}$, damming action will occur with critical depth on the hump.

$\therefore\ y_2 = y_{c2} = 1.935$ ft ◀

10.36 ***A flow of 2.0 m^3/s is carried in a rectangular channel 1.8 m wide at a depth of 1.0 m. Will critical depth occur at a section where (a) a frictionless hump 180 mm high is installed across the bed, (b) a frictionless sidewall constriction (with no hump) reduces the channel width to 1.4 m, and (c) the hump and the sidewall constriction are installed together? Show calculations.***

SI

Given: $Q = 2.0$ m³/s, channel $b = 1.8$ m, $y_0 = 1.0$ m

(*a*) With hump 0.18 m high: $q = Q/b = 2.0/1.8 = 1.111$ m³/s per m

Eq. 10.17: $E_0 = 1.0 + [1/(2 \times 9.81)](1.111/1.0)^2 = 1.063$ m

Eq. 10.23: $y_c = (1.111^2/9.81)^{1/3} = 0.501$ m

Eq. 10.25: $E_{min} = 1.5(0.501) = 0.752$ m; Sec. 10.13: $\Delta z_{crit} = E_0 - E_{min} = 0.311$ m

$\therefore\ \Delta z = 0.18$ m $< \Delta z_{crit}$, so y_c does <u>not</u> occur at the hump. ◀

<u>Alternative solution (*a*):</u>

$E_h = E_0 - \Delta z = 1.063 - 0.180 = 0.883$ m

$E_h > E_{min} = 0.752$, so y_c does <u>not</u> occur at the hump. ◀

(*b*) With constricted $b = 1.4$ m, and no hump: $q = 2.0/1.4 = 1.429$ m³/s per m

Eq. 10.23: $y_c = (1.429^2/9.81)^{1/3} = 0.593$ m; Eq. 10.25: $E_{min} = 1.5(0.593) = 0.889$ m

$\therefore\ E_{min} < E_0 = 1.063$, so y_c does <u>not</u> occur at constriction. ◀

(*c*) With both 0.18-m hump and constricted $b = 1.4$ m:

From part (*b*): $q_{constriction} = 1.429$ m³/s per m, $y_c = 0.593$ m, $E_{min} = 0.889$ m

$\Delta z_{crit} = E_0 - E_{min} = 1.063 - 0.889 = 0.1741$ m

So hump height $= 0.18$ m $> \Delta z_{crit}$, so y_c <u>does</u> occur at hump + constriction ◀

Also therefore, the flow must back up, and so increase the y and E just upstream of the hump.

<u>Alternative solution (*c*):</u>

$E_h = E_0 - \Delta z = 0.883$; $E_{min} = 0.889$ m

$E_h < E_{min}$ is impossible, so y_c <u>does</u> occur and flow must back up, etc. ◀

10.37 *Work Sample Prob. 10.8 (y_0 = 2.0 ft) for the case where the flow rate is 90 cfs.*

Sample Prob. 10.8: The flow is uniform in a 20-ft-wide rectangular channel. (a) Is it subcritical or supercritical? (b) If a hump of height Δz = 0.30 ft is placed in the bottom of the channel, calculate the water depth on the hump, and the change in the water surface level at the hump. (c) If the hump height is raised to Δz = 0.60 ft, what then are the water depths upstream and downstream of the hump? (d) If the 0.30-ft hump is accompanied by a local contraction to 18 ft, find the water depth on the hump. In all cases neglect head losses over the hump and through the contraction.

BG

Given: $y_0 = 2.00$ ft, $Q = 90$ cfs

(*a*) $q = Q/b = 90/20 = 4.5$ cfs/ft; Eq. 10.23: $y_c = (4.5^2/32.2)^{1/3} = 0.857$ ft

$y_0 > y_c$, so flow is <u>sub</u>critical (and slope is mild) ◄

(*b*) Instal hump, height $\Delta z = 0.30$ ft. First find critical hump height.

Eq. 10.17: $E_0 = 2.00 + [1/(2 \times 32.2)](4.5/2.00)^2 = 2.079$ ft

Eq. 10.25: $E_{\min} = 1.5(0.857) = 1.286$ ft

Sec 10.13: $E_0 = \Delta z_{\text{crit}} + E_{\min}$, so $\Delta z_{\text{crit}} = 2.079 - 1.286 = 0.793$ ft

$\Delta z < \Delta z_{\text{crit}}$, so y_c does not occur on the hump, and damming action does not result.

Sec. 10.13: $E_h = E_0 - \Delta z$

$\therefore$ Eq. 10.17: $y_h + [1/(2 \times 32.2)](4.5/y_h)^2 = 2.079 - 0.30$ i.e., $y_h + 0.314/y_h^2 = 1.779$

By trial and error or by equation solver: $y_h = 1.6652$ (subcritical) or 0.495 (supercritical) or a negative root which has no physical meaning. The flow does not pass through y_c so it cannot become supercritical.

$\therefore$ Water depth on 0.30-ft hump = 1.665 ft ◄

Change in water surface level at hump = 2.00 − (1.6652 + 0.30) = 0.0348 ft drop ◄

(*c*) Increase hump height Δz to 0.60 ft

Still $\Delta z < \Delta z_{\text{crit}}$, so y_c does not occur on the hump and damming action does not result.

Sec 10.13: $E_h = E_0 - \Delta z$

$\therefore$ using Eq. 10.17: $y_h + [1/(2 \times 32.2)](4.5/y_h)^2 = 2.079 - 0.60$ i.e., $y_h + 0.314/y_h^2 = 1.479$

By trial and error or by polynomial or equation solver: y = 1.290 ft (subcritical) or 0.597 ft (supercritical) or a negative root which has no physical meaning. The flow does not pass through y_c so it cannot become supercritical. So water depth on the 0.60-ft hump is 1.290 ft.

With no damming action, water depths upstream and downstream of the hump = y_0 = 2.00 ft ◄

(*d*) At section 2 with 0.3-ft hump and contraction to 18 ft:

$q_2 = 90/18 = 5$ cfs/ft; $y_{c2} = (5^2/32.2^2)^{1/3} = 0.919$ ft.

$(E_{\min})_2 = 1.5(0.919) = 1.379$ ft

$\Delta z_{\text{crit}} = E_1 - (E_{\min})_2 = 2.079 - 1.379 = 0.700$ ft.

So still $\Delta z < \Delta z_{\text{crit}}$, and damming action does not occur, y_c does not occur on the hump.

$E_2 = E_1 - \Delta z = 2.079 - 0.3 = 1.779$ ft.

Eq. 10.17: $1.779 = y_2 + (5/y_2)^2/(2 \times 32.2)$

By trial and error, or by polynomial or equation solver, y_2 = 1.633 ft (subcritical) or 0.556 ft (supercritical) or −0.420 ft (meaningless).

Per (*a*), approaching flow is subcritical. $\therefore$ it will remain so, and y_2 = 1.633 ft ◄

10.38 *A rectangular channel 5 ft wide carries 50 cfs of water in uniform flow at a depth of 3.20 ft. (a) If a bridge pier 1.5 ft wide is placed in the middle of this channel, find the local change in the water-surface elevation. (b) What is the minimum width of constricted channel which will not cause a rise in water surface upstream?*

BG

(*a*) Given $y_0 = 3.20$ ft, from Sec. 10.9: $V_0 = q/y_0 = (Q/b)/y_0 = (50/5)/3.2 = 3.125$ fps

At constriction, $V = (50/3.5)/y = 14.29/y$

Eq. 10.17: $E_0 = 3.20 + 3.125^2/(2 \times 32.2) = 3.35 \text{ ft} = y + (14.29/y)^2/(2 \times 32.2)$

By trial and error or by equation solver: $y = 2.999$ ft or 1.218 ft. Supercritical conditions are impossible at the pier. $\therefore$ $y = 3.00$ ft.

Change in water depth = 3.20 − 2.999 = 0.201 ft drop ◄

(*b*) The minimum width of constricted channel which would produce critical flow is found by getting q_{max} for $E = 3.35$ ft.

From Eqs. 10.28 and 10.29: $q_{max} = g^{1/2}(2E/3)^{3/2} = 32.2^{1/2}(2/3 \times 3.35)^{3/2} = 18.95$ cfs/ft

Minimum channel width = 50/18.95 = 2.64 ft ◄

(A width less than this would produce backwater/damming action.)

10.39 *A rectangular channel 1.2 m wide carries 1.1 m³/s of water in uniform flow at a depth of 0.85 m. If a bridge pier 0.3 m wide is placed in the middle of this channel, find the local change in the water-surface elevation. What is the minimum width of constricted channel which will not cause a rise in water surface upstream?*

SI

Given $y_0 = 0.85$ m, $V_0 = q/y_0 = (Q/b)/y_0 = (1.1/1.2)/0.85 = 1.078$ m/s

At constriction, $V = (1.1/0.9)/y = 1.222/y$

Eq. 10.17: $E_0 = 0.85 + 1.078^2/(2 \times 9.81) = 0.909 \text{ m} = y + (1.222/y)^2/(2 \times 9.81)$

By trial and error or be equation solver: $y = 0.7861$ m or 0.379 m. Supercritical conditions are impossible at the pier so $y = 0.786$ ft.

Change in water depth = 0.850 − 0.7861 = 0.0639 m drop ◄

From Eqs. 10.28 and 10.29: $q_{max} = g^{1/2}(2E/3)^{3/2} = 9.81^{1/2}(2 \times 0.909/3)^{3/2} = 1.478$ m³/s per m

Minimum channel width = 1.1/1.478 = 0.744 m ◄

(A width less than this would produce backwater/damming action.)

10.40 *A rectangular channel 12 ft wide carries 24 cfs in uniform flow at a depth of 0.32 ft. Find the local change in water-surface elevation caused by a 0.15-ft-high obstruction across the channel bed.*

BG

$q = Q/b = 24/12 = 2$ cfs/ft. Eq. 10.23: $y_c = (2^2/32.2)^{1/3} = 0.499$ ft. Eq. 10.25: $E_{min} = 1.5y_c = 0.748$ ft

Given $y_1 = 0.32$ ft: $\therefore$ $y_1 < y_c$, flow is supercritical, and $V_1 = 2/0.32 = 6.25$ fps ; $V_1^2/2g = 0.607$ ft

$E_1 = 0.320 + 0.607 = 0.927$ ft ; $\Delta z_{crit} = E_1 - E_{min} = 0.927 - 0.748 = 0.1781$ ft

So $\Delta z < \Delta z_{crit}$, $\therefore$ at 0.15-ft rise in bed, $E_2 = E_1 - \Delta z = 0.927 - 0.150 = 0.777$ ft

Also, Eq. 10.17: $E_2 = y_2 + (2/y_2)^2/(2 \times 32.2) = 0.777$ ft

or $y_2^3 - 0.777y_2^2 + 0.0621 = 0$; By trial or by polynomial or equation equation solver:

$y_2 = 0.414$ ft (supercritical) or 0.609 ft (subcritical) or −0.246 ft (meaningless).

$y609$ ft is $> y_c = 0.499$ ft, so is impossible; $y_2 = 0.414$ ft.

So water surface change = (0.150 + 0.414) − 0.320 = 0.244 ft rise ◄

10.41 *Fifty cubic feet per second of water flows uniformly in a 6-ft-wide rectangular channel at a depth of 2.5 ft (Fig. P10.41). What is the change in water-surface elevation at a section contracted to a 4-ft width with an 0.2-ft depression in the bottom?*

BG

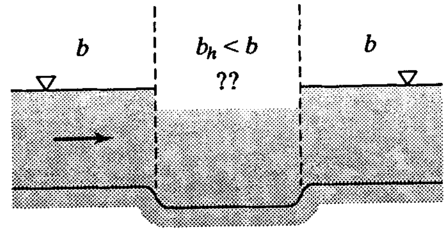
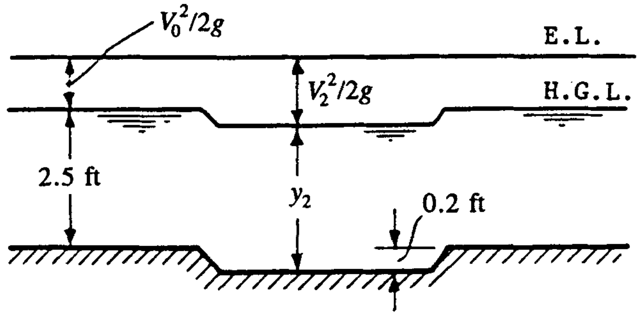


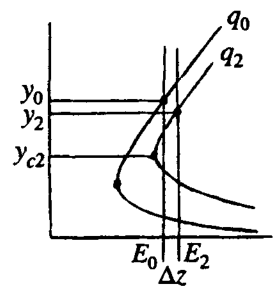
Figure P10.41

$q_2 = Q/b = 50/4 = 12.5$ cfs/ft

Eq. 10.23: $y_{c2} = (12.5^2/32.2)^{1/3} = 1.693$ ft

$V_0 = 50/(6 \times 2.5) = 3.33$ fps, $V_2 = 50/(4 \times y_2)$





Sec. 10.13 and Eq. 10.17: $E_0 = E_2 - \Delta z$, or $2.5 + V_0^2/2g = y_2 + V_2^2/2g - 0.2$

$\therefore\ 2.5 + 3.33^2/(2 \times 32.2) = 2.673 \text{ ft} = y + (12.5/y)^2/(2 \times 32.2) - 0.2$

By trial or by polynomial or equation solver: $y = 2.477$ (subcritical) or 1.207 ft (supercritical).

y_2 cannot be less than y_{c2}, therefore $y_2 = 2.48$ ft.

Water-surface change $= y_0 - (y_2 - \Delta z) = 2.50 - (2.477 - 0.20) = 0.223$ ft drop ◄

10.42 *Water flows uniformly in a 2.0-m-wide rectangular channel at a rate of 1.6 m³/s and a depth of 0.75 m (Fig. P10.41). What is the change in water-surface elevation at a section contracted to a 1.4 m width with an 80-mm depression in the bottom?*

SI

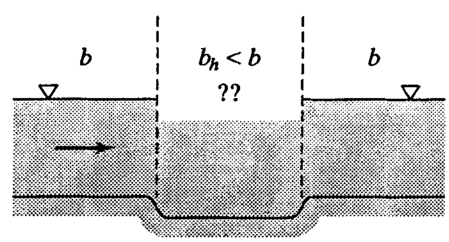
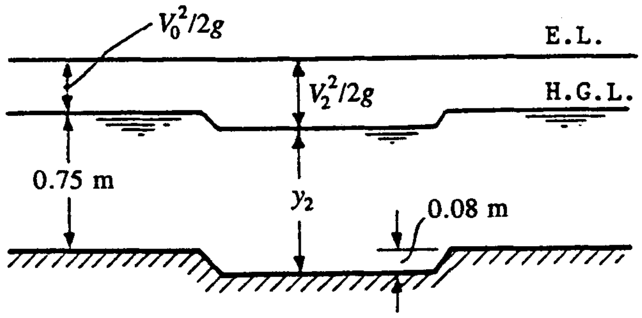


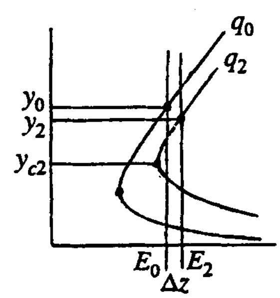
Figure P10.41

$q_2 = Q/b = 1.6/1.4 = 1.143$ m³/s per m

Eq. 10.23: $y_{c2} = (1.143^2/9.81)^{1/3} = 0.511$ m

$V_0 = 1.6/(2.0 \times 0.75) = 1.067$ m/s, $V_2 = 1.6/(1.4 y_2) = 1.143/y_2$





Sec. 10.13 and Eq. 10.17: $E_0 = E_2 - \Delta z$, $\therefore\ 0.75 + V_0^2/2g = y_2 + V_2^2/2g - 0.08$

$\therefore\ 0.75 + 1.067^2/(2 \times 9.81) = 0.808 \text{ m} = y_2 + (1.143/y_2)^2/(2 \times 9.81) - 0.080$

By trial or by polynomial or equation solver, $y_2 = 0.7780$ m (subcritical) or 0.353 m (supercritical).

y_2 cannot be less than $y_{c2} = 0.511$ m, therefore $y_2 = 0.778$ m.

Water-surface change $= y_0 - (y_2 - \Delta z) = 0.750 - (0.7780 - 0.080) = 0.0520$ m drop ◄

Sec. 10.15: Energy Equation for Gradually Varied Flow -- *Exercises (4)*

10.15.1 *A rectangular flume of planed timber (n = 0.012) is 5 ft wide and carries 60 cfs of water. The bed slope is 0.0006, and at a certain section the depth is 3 ft. Find the distance (in one reach) to the section where the depth is 2.5 ft. Is the distance upstream or downstream?*

BG

Eq. 10.8*a*: $60 = (1.486/0.012)(5y_0)[5y_0/(5 + 2y_0)]^{2/3}(0.0006)^{1/2}$;

By trial and error or by equation solver per Sample Prob. 10.1, $y_0 = 3.17$ ft

Eq. 10.23: Critical depth $y_c = (q^2/g)^{1/3} = [(60/5)^2/32.2]^{1/3} = 1.648$ ft

$3.17 \text{ ft} = y_0 > 2.5$ ft to 3.0 ft $> y_c = 1.648$ ft, so depth range does not cross y_0 or y_c.

Assume the 2.5 ft depth is downstream. Then (for a single reach):

$A = by = 5y;\ \ P = b + 2y = 5 + 2y;\ \ R_h = A/P;\ \ V = Q/A = 60/A;\ \ E = y + V^2/2g$

y ft	V fps	$V^2/2g$ ft	E ft	A ft^2	P ft	R_h ft	$\bar{R}$ ft	$\bar{V}$ fps
3.0	4.0	0.248	3.248	15.0	11	1.364	1.3068	4.4
2.5	4.8	0.358	2.858	12.5	10	1.250		

Eq. 10.38a: $S = [0.012 \times 4.40/(1.486 \times 1.3068^{2/3})]^2 = 0.000\,884$

Eq. 10.37: $3 + 0.248 = 2.5 + 0.358 + (0.000\,884 - 0.0006)\Delta x;\ \ \Delta x = +1377$ ft ◀

Since Δx is positive, the 2.5-ft depth is downstream, as assumed. ◀

10.15.2 *A rectangular flume of planed timber (n = 0.013) is 1.6 m wide and carries 1.7 m³/s of water. The bed slope is 0.0005, and at a certain section the depth is 1.0 m. Find the distance (in one reach) to the section where the depth is 0.85 m. Is the distance upstream or downstream?*

SI

Eq. 10.8*b*: $1.7 = (1/0.013)(1.6y_0)[1.6y_0/(1.6 + 2y_0)]^{2/3}(0.0005)^{1/2}$

By trial and error or by equation solver per Sample Prob. 10.1, $y_0 = 1.047$ m

Eq. 10.23: Critical depth $y_c = [(1.7/1.6)^2/9.81]^{1/3} = 0.486$ m

$1.047 \text{ m} = y_0 > 0.85$ m to 1.00 m $> y_c = 0.486$ m, so depth range does not cross y_0 or y_c.

Assume the 0.85 m depth is downstream. Then (for a single reach):

$A = by = 1.6y;\ \ P = b + 2y = 1.6 + 2y;\ \ R_h = A/P;\ \ V = Q/A = 1.9/A;\ \ E = y + V^2/2g$

y m	V m/s	$V^2/2g$ m	E m	A m^2	P m	R_h m	$\bar{R}$ m	$\bar{V}$ m/s
1.00	1.063	0.0575	1.0575	1.60	3.6	0.444	0.428	1.156
0.85	1.250	0.0796	0.9296	1.36	3.3	0.412		

Eq. 10.38b: $S = [0.013 \times 1.156/0.428^{2/3}]^2 = 0.000\,700$

Eq. 10.37: $1.0 + 0.0575 = 0.85 + 0.0796 + (0.000\,700 - 0.0005)\Delta x;\ \ \Delta x = +640$ m ◀

Since Δx is positive, the 0.85-m depth is downstream, as assumed. ◀

10.15.3 *A test on a rectangular glass flume 9 in wide yielded the following data on a reach of 40 ft length: with still water, $z_1 - z_2 = 0.009$ ft; with a measured flow of 0.1446 cfs, $y_1 = 0.381$ ft, $y_2 = 0.386$ ft. Find the value of Mnning's roughness coefficient n using only one reach.*

BG

$S_0 = 0.009/40 = 0.000\,225$; $B_0 = 9$ in $= 0.750$ ft

$A_1 = 0.286$ ft^2; $P_1 = 1.512$ ft; $R_1 = A_1/P_1 = 0.1890$ ft; $V_1 = Q/A_1 = 0.506$ fps; $V_1^2/2g = 0.003\,98$ ft

$A_2 = 0.290$ ft^2; $P_2 = 1.522$ ft; $R_2 = 0.1902$ ft; $V_2 = Q/A_2 = 0.499$ fps; $V_2^2/2g = 0.003\,87$ ft;

$\therefore\ \bar{R} = 0.1896$ ft; $\bar{V} = 0.503$ fps

Eq. 10.37: $0.381 + 0.003\,98 = 0.386 + 0.003\,87 + (S - 0.000\,225)40$; $S = 0.000\,1026$

With these values of S, $\bar{R}$, and $\bar{V}$ in Eq. 10.38*a*: $n = 0.009\,88$ ◀

10.15.4 *A test on a rectangular glass flume 250 mm wide yielded the following data on a reach of 9-m length: with still water, $z_1 - z_2 = 2.7$ mm; with a measured flow of 4.3 L/s, $y_1 = 110.2$ mm, $y_2 = 111.7$ mm. Find the value of Manning's roughness coefficient n using only one reach.*

SI

$S_0 = 0.0027/9 = 0.0003$; $B_0 = 0.25$ m

$A_1 = 0.0276$ m^2; $P_1 = 0.470$ m; $R_1 = A_1/P_1 = 0.0586$ m; $V_1 = Q/A_1 = 0.1561$ m/s;

$V_1^2/2g = 0.001\,242$ m; $A_2 = 0.0279$ m^2; $P_2 = 0.473$ m; $R_2 = 0.0590$ m;

$V_2 = Q/A_2 = 0.1540$ m/s; $V_2^2/2g = 0.001\,209$;

$\therefore\ \bar{R} = 0.0588$; $\bar{V} = 0.1550$ m/s

Eq. 10.37: $0.1102 + 0.001\,242 = 0.1117 + 0.001\,209 + (S - 0.0003)9$; $S = 0.000\,1370$

With these values of S, $\bar{R}$, and $\bar{V}$ in Eq. 10.38*b*: $n = 0.011\,41$ ◀

Sec. 10.15: Energy Equation for Gradually Varied Flow – ***Problems 10.43–10.55***

10.43 *A 5-ft-wide rectangular flume ($n = 0.012$) carries 60 cfs of water. At one point the water depth is found to be 4 ft; 1000 ft downstream is is measured at 3 ft. Calculate the bed slope of the flume, using one reach. Sketch the flume bed, water surface, and energy line, to check that the answer is reasonable.*

BG

$y_1 = 4$ ft; $A_1 = 20$ ft^2; $P_1 = 13$ ft; $R_1 = A_1/P_1 = 1.538$ ft; $V_1 = Q/A_1 = 3$ fps; $V_1^2/2g = 0.1398$ ft;

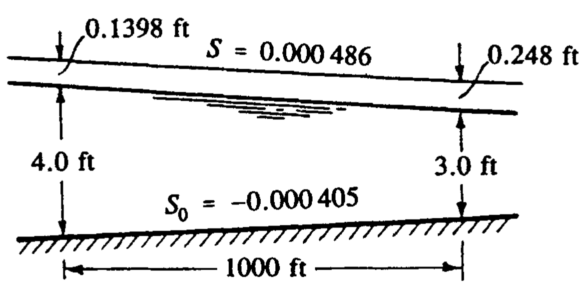
$y_2 = 3$ ft; $A_2 = 15$ ft^2; $P_2 = 11$ ft; $R_2 = 1.364$ ft; $V_2 = 4$ fps; $V_2^2/2g = 0.248$ ft;

$\bar{R} = (R_1 + R_2)/2 = 1.451$ ft and $\bar{V} = (V_1 + V_2)/2 = 3.5$ fps

Eq. 10.38*a*: $S = [0.012 \times 3.5/(1.486 \times 1.451^{2/3})]^2 = 0.000\,486$

Eq. 10.37: $4 + 0.1398 = 3 + 0.248 + (0.000\,486 - S_0)1000$; $S_0 = -0.000\,405$ ◀

The minus sign indicates the slope is <u>adverse</u>. Sketch:



◀

10.44 *A 1.6-m-wide rectangular flume (n = 0.013) carries 1.9 m³/s of water. At one point the water depth is found to be 1.3 m; 320 m downstream is is measured at 1.0 m. Calculate the bed slope of the flume, using one reach. Sketch the flume bed, water surface, and energy line, to check that the answer is reasonable.*

SI

$y_1 = 1.3$ m; $A_1 = 1.3 \times 1.6 = 2.08$ m²; $P_1 = 4.2$ m; $R_1 = 0.495$ m

$V_1 = 1.9/2.08 = 0.913$ m/s; $V_1^2/2g = 0.0425$ m;

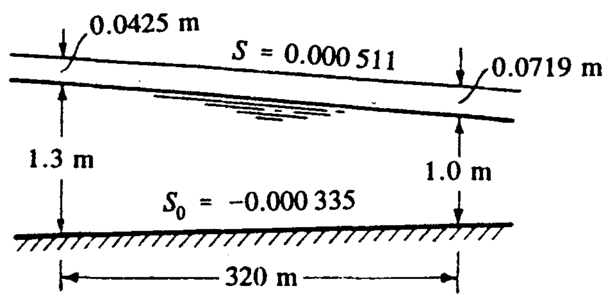
$y_2 = 1.0$ m; $A_2 = 1.0 \times 1.6 = 1.60$ m²; $P_2 = 3.6$ m; $R_2 = 0.444$ m

$V_2 = 1.9/1.60 = 1.1875$ m/s; $V_2^2/2g = 0.0719$ m; $\bar{R} = 0.470$ m; $\bar{V} = 1.050$ m/s

Eq. 10.38*b*: $S = [0.013 \times 1.050/0.470^{2/3}]^2 = 0.000\,511$

Eq. 10.37: $1.3 + 0.0425 = 1.0 + 0.0719 + (0.000\,511 - S_0)320$; $S_0 = -000\,335$ ◀

The minus sign indicates the slope is <u>adverse</u>. Sketch:



◀

10.45 *A 5-ft-wide rectangular flume ($S_0 = 0.01$, n = 0.012) carries 60 cfs of water. Find the depth 50 ft downstream from a section where the flow is 1.5 ft deep. Use only one reach. Is the flow subcritical or supercritical? [Note: In this case it will not be possible to make a direct solution from Eq. (10.37). A trial-and-error solution may best be set in the form of a table with the headings y_2, V_2, $\bar{V}$, A_2, P_2, R_2, $\bar{R}$, $\bar{R}^{2/3}$, S, etc.]*

BG

$q = Q/b = 60/5 = 12$ cfs/ft; Eq. 10.23: $y_c = (12^2/32.2)^{1/3} = 1.648$ ft

As $y_1 = 1.5 < y_c$, the flow is <u>supercritical</u> ◀

$A_1 = 7.5$ ft²; $P_1 = 5 + 2(1.5) = 8$ ft; $R_1 = A_1/P_1 = 0.938$ ft;

$V_1 = Q/A_1 = 8$ fps; $V_1^2/2g = 0.994$ ft

Eq. 10.37: $1.5 + 0.994 = y_2 + (12/y_2)^2/2g + (S - 0.01)50$

where, by Eq. 10.38*a*: $S = (0.012\bar{V}/1.486\bar{R}^{2/3})^2$

This expression for S may be substituted into the equation above it and by successive trials of y_2 or by equation solver yields the result: $y_2 = 1.241$ ft. ◀

10.46 *A 1.6-m-wide rectangular flume (S_0 = 0.008, n = 0.013) carries 1.9 m³/s of water. Find the depth 17 m downstream from a section where the flow is 0.5 m deep. Use only one reach. Is the flow subcritical or supercritical? [Note: In this case it will not be possible to make a direct solution from Eq. (10.37). A trial-and-error solution may best be set in the form of a table with the headings y_2, V_2, $\bar{V}$, A_2, P_2, R_2, $\bar{R}$, $\bar{R}^{2/3}$, S, etc.]*

SI

$q = Q/b = 1.9/1.6 = 1.1875\ \text{m}^3/\text{s per m}$

Eq. 10.23: $y_c = (1.1875^2/9.81)^{1/3} = 0.524$ m

As $y_1 = 0.50$ m $< y_c$, the flow is <u>supercritical</u> ◄

$A_1 = 0.800\ \text{m}^2$; $P_1 = 1.6 + 2(0.50) = 2.6$ m; $R_1 = A_1/P_1 = 0.308$ m;

$V_1 = Q/A_1 = 2.375$ m/s; $V_1^2/2g = 0.2875$ m

Eq. 10.37: $0.50 + 0.2875 = y_2 + (1.1875/y_2)^2/2g + (S - 0.008)17$

where, by Eq. 10.38*b*: $S = (0.013\bar{V}/\bar{R}^{2/3})^2$

This expression for S may be substituted into the equation above it and by successive trials of y_2 or by equation solver yields the result: $y_2 = 0.423$ m. ◄

10.47 *Repeat Prob. 10.45, but increase the 50-ft distance to 500 ft.*

Prob. 10.45: Rectangular flume, b = 5 ft, S_0 = 0.01, n = 0.012, Q = 60 cfs, y_1 = 1.5 ft. Find the downstream y_2 using only one reach. Is the flow subcritical or supercritical?

BG

$q = Q/b = 12$ cfs/ft; Eq. 10.23: $y_c = (12^2/32.2)^{1/3} = 1.648$ ft

As $y_1 = 1.5 < y_c$, the flow is <u>supercritical</u> ◄

To find y_2 at $\Delta x = 500$ ft downstream of y_1. First find y_0:

From Eq. 10.8*a*: $60 = \dfrac{1.486}{0.012}\dfrac{(5y_0)^{5/3}}{(5 + 2y_0)^{2/3}}0.01^{1/2}$;

By trial and error or by equation solver, per Sample Prob. 10.1, $y_0 = 1.140$ ft

Find the distance to $y = y_0$. Conditions at Section 1 are as for Prob. 10.45. At the section where $y = y_0$:

$A_0 = 5.70\ \text{ft}^2$; $P_0 = 7.28$ ft; $R_0 = A_0/P_0 = 0.783$ ft

$V_0 = Q/A_0 = 10.53$ fps; $V_0^2/2g = 1.721$ ft; $\bar{R} = 0.860$ ft; $\bar{V} = 9.26$ fps

Eq. 10.38*a*: $S = [0.012 \times 9.26/(1.486 \times 0.860^{2/3})]^2 = 0.006\,84$

Eq. 10.39: $\Delta x = [(1.5 + 0.994) - (1.140 + 1.721)]/(0.006\,84 - 0.01) = +116.0$ ft

Downstream, where $\Delta x > 116$ ft, y remains at y_0

$\therefore$ At $\Delta x = 500$ ft, $y_2 = y_0 = 1.140$ ft ◄

<u>Note</u>: Calculation of y_2 using Eq. 10.37 with $\Delta x = 500$ ft would give an erroneous (impossible) result with $y_2 < y_0$.

10.48 *Repeat Prob. 10.46, but increase the 17-m distance to 170 m.*

Prob. 10.46: Rectangular flume, $b = 1.6$ m, $S_0 = 0.008$, $n = 0.013$, $Q = 1.9$ m³/s, $y_1 = 0.50$ m. Find the downstream y_2 using only one reach. Is the flow subcritical or supercritical?

SI

$q = Q/b = 1.9/1.6 = 1.1875$ m³/s per m; Eq. 10.23: $y_c = (1.1875^2/9.81)^{1/3} = 0.524$ m

As $y_1 = 0.50$ m $< y_c$, the flow is <u>supercritical</u> ◄

To find y_2 at $\Delta x = 150$ m downstream of y_1. First find y_0:

From Eq. 10.8*b*: $$1.9 = \frac{1}{0.013}\frac{(1.6y_0)^{5/3}}{(1.6 + 2y_0)^{2/3}}(0.008)^{1/2}$$

By trial and error or by equation solver per Sample Prob. 10.1, $y_0 = 0.411$ m

Find the distance to $y = y_0$. Conditions at Section 1 are as for Prob. 10.46. At the section where $y = y_0$:

$A_0 = 0.658$ m²; $P_0 = 2.42$ m; $R_0 = A_0/P_0 = 0.272$ m;

$V_0 = Q/A_0 = 2.89$ m/s; $V_0^2/2g = 0.425$ m; $\bar{R} = 0.290$ m; $\bar{V} = 2.63$ m/s

Eq. 10.38*b*: $S = [0.013 \times 2.63/0.290^{2/3}]^2 = 0.006\,11$

Eq. 10.39: $\Delta x = [(0.50 + 0.287) - (0.411 + 0.425)]/(0.006\,11 - 0.008) = +25.9$ m

Downstream, where $\Delta x > 26$ m, y remains at y_0

$\therefore$ At $\Delta x = 170$ m, $y_2 = y_0 = 0.411$ m ◄

<u>Note</u>: Calculation of y_2 using Eq. 10.37 with $\Delta x = 170$ ft would give an erroneous (impossible) result with $y_2 < y_0$.

10.49 *A rectangular flume 9 ft wide is built of planed timber (n = 0.012) on a bed slope of 0.3 ft per 1000 ft, ending in a free overfall (Fig. P10.49). If the measured depth at the fall is 1.67 ft, find (a) the rate of flow; (b) the distance upstream from the fall to where the depth is 3.6 ft. [Note: Assume that critical depth occurs at a distance of $4y_c$ upstream from the fall, and use reaches with end depths of 2.5, 2.8, 3.2, and 3.6 ft].*

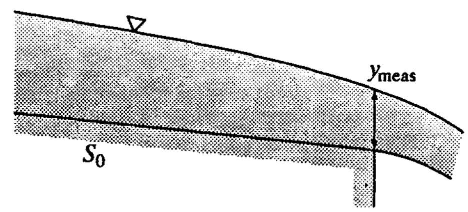


Figure P10.49

BG

(*a*) Given: $b = 9$ ft, $n = 0.012$, $S_0 = 0.3/1000 = 0.0003$

Sec. 10.12: $y_b \approx 0.72 y_c = 1.67$ ft; $\therefore\ y_c \approx 1.67/0.72 = 2.32$ ft

At $y = y_c$, by Eq. 10.29: $q = (32.2 \times 2.32^3)^{1/2} = 20.04$ cfs/ft

$Q = bq = 9(20.05) = 180.40$ cfs ◀

Note: Between y_b and y_c the flow is rapidly varying. So Manning's equation is not valid there, and cannot be used there to determine Q.

(*b*)

y ft	V fps	$V^2/2g$ ft	E ft	A ft²	P ft	R ft	$\bar{R}$ ft	$\bar{V}$ fps	S Eq. 10.38a	$\Delta x = \dfrac{E_1 - E_2}{S - S_0}$ ft
3.6	5.57	0.481	4.081	32.4	16.2	2.00				
							1.94	5.92	0.000 941	+424
3.2	6.26	0.609	3.809	28.8	15.4	1.87				
							1.798	6.71	0.001 336	+206
2.8	7.16	0.796	3.596	25.2	14.6	1.726				
							1.667	7.59	0.001 890	+61
2.5	8.02	0.998	3.498	22.5	14.0	1.607				
							1.569	8.33	0.002 47	+9
2.32	8.64	1.159	3.479	20.88	13.64	1.531				
										$4y_c$ = +9
y_b									$\Sigma(\Delta x)$	= +710 ◀

Note: A spreadsheet could be helpful on this problem.

10.50 *A rectangular flume 3 m wide is built of planed timber (n = 0.012) on a bed slope of 200 mm per km, ending in a free overfall (Fig. P10.49). If the measured depth at the fall is 0.55 m, find (a) the rate of flow; (b) the distance upstream from the fall to where the depth is 1.2 m. [Note: Assume that critical depth occurs at a distance of $4y_c$ upstream from the fall, and use reaches with end depths of 0.82, 0.9, 1.0, and 1.2 m.]*

SI

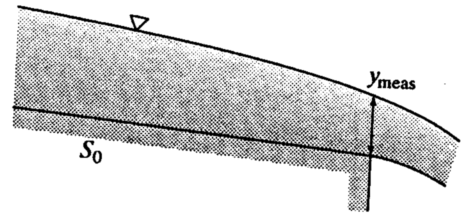


Figure P10.49

(a) Given: b = 3 m, n = 0.012, S_0 = 0.2/1000 = 0.0002

Sec. 10.12: $y_b \approx 0.72 y_c = 0.55$ m; $y_c \approx 0.55/0.72 = 0.764$ m

At $y = y_c$, by Eq. 10.29: $q = \sqrt{9.81(0.764)^3} = 2.09$ m³/s per m

$Q = bq = 3(2.09) = 6.27$ m³/s ◀

Note: Between y_b and y_c the flow is rapidly varying. So Manning's equation is not valid there, and cannot be used there to determine Q.

(b)

y m	V m/s	$V^2/2g$ m	E m	A m²	P m	R m	$\bar{R}$ m	$\bar{V}$ m/s	S Eq. 10.38b	$\Delta x = \frac{E_1 - E_2}{S - S_0}$ m
1.2	1.743	0.1548	1.355	3.6	5.40	0.667				
							0.633	1.917	0.000 973	+171
1.0	2.09	0.223	1.223	3.0	5.00	0.600				
							0.581	2.21	0.001 446	+38
0.9	2.32	0.275	1.175	2.7	4.80	0.563				
							0.546	2.44	0.001 915	+14
0.82	2.55	0.331	1.151	2.46	4.64	0.530				
							0.518	2.64	0.002 42	+3
0.764	2.74	0.382	1.146	2.29	4.53	0.506				
										$4y_c$ = +3
y_b									$\Sigma(\Delta x)$	= +228 ◀

Note: A spreadsheet could be helpful on this problem.

10.51 *A wide and shallow rectangular channel dredged in earth (n = 0.035) is laid on a slope of 9 ft/mi and carries a flow of 90 cfs/ft of width. (a) Find the water depth 2 miles upstream from a section where the depth is 26.1 ft, using a single reach. (b) Compare the result with that obtained using three reaches of equal length.*

BG

Given: q = 90 cfs/ft, channel is "wide and shallow," ∴ (Sec. 10.5) $R_h = y$.

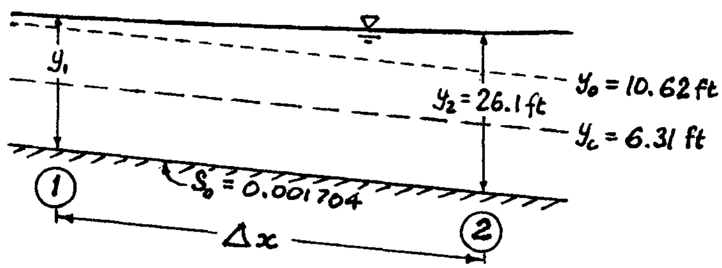
$n = 0.035$, $S_0 = 9/5280 = 0.001\,704$, $y_2 = 26.1$ ft; find y_1 at $\Sigma(\Delta x) = 2$ mi = 10,560 ft upstream.

From Manning Eq. 10.8*a*: $V_0 = \dfrac{q}{y_0} = \dfrac{1.486}{n} y_0^{2/3} S_0^{1/2}$; $q = \dfrac{1.486}{n} y_0^{5/3} S_0^{1/2}$

$90 = \dfrac{1.486}{0.035} y_0^{5/3} (0.001\,704)^{1/2}$; $y_0 = 10.62$ ft

Eq. 10.23: $y_c = (q^2/g)^{1/3} = (90^2/32.2)^{1/3} = 63.1$ ft

∴ $y = 26.1$ ft $> y_0 > y_c$, so per Fig. 10.20 S_0 = Mild, the water surface profile = M_1, and the depth increases downstream.



(*a*) Using one reach, of $\Delta x = 2(5280) = 10{,}560$ ft:

$$\text{Eq. 10.30:}\quad 10{,}560 = \frac{\left[y_1 + \dfrac{1}{2g}\left(\dfrac{90}{y_1}\right)^2\right] - \left[26.1 + \dfrac{1}{2g}\left(\dfrac{90}{26.1}\right)^2\right]}{S - 0.001\,704}$$

where, by Eq. 10.38*a*: $S = \left[0.035\bar{V}/(1.486\bar{R}^{2/3})\right]^2$ and $\bar{V} = (90/26.1 + 90/y_1)/2$ and

$\bar{R} = \bar{y} = (26.1 + y_1)/2$

Solving the above equations by trial and error or by equation solver, $y_1 = 11.191$ ft ◀

where $S = 0.000\,3703$, $\bar{V} = 5.745$ fps, $\bar{R} = 18.646$ ft.

(*b*) Using three equal reaches of $\Delta x = 10{,}560/3 = 3520$ ft each:

Reach	$y = R$ ft	$V = 90/y$ fps	$\frac{V^2}{2g}$ ft	E ft	$E_1 - E_2$ ft	$\bar{V}$ fps	$\bar{R}$ ft	S Eq. 10.38*a*	$\Delta x = \frac{E_1 - E_2}{S - S_0}$ Want 3520 ft
	26.1	3.45	0.185	26.28					
3					−5.54	3.93	23.27	0.000 1287	+3518
	20.44*	4.40	0.301	20.74					
2					−4.89	5.14	17.88	0.000 3136	+3519
	15.31*	5.88	0.537	15.85					
1					−3.21	6.78	13.52	0.000 7919	+3518
	11.72*	7.68	0.916	12.64					

*By equation solver or by trial and error, (4 - 6 trials) until Δx closely = 3520 ft.

Thus, using 3 reaches, $y_1 = 11.72$ ft. ◀ Note: A spreadsheet could be helpful on this problem.

10.52 *The slope of a stream of rectangular cross section is S_0 = 0.0003, the width is 170 ft, and the value of the Chézy C is 78.3 $ft^{1/2}/s$. (a) Find the depth for a uniform flow of 101.54 cfs per foot of width of the stream. (b) If a dam raises the water level so that at a certain distance upstream the increase is 5 ft, how far from this latter section will the increase by only 1 ft? Use reaches with 1-ft depth increments.*

BG

Given: q = 101.54 cfs/ft, b = 170 ft, Chézy C = 78.3, S_0 = 0.0003

(*a*) Uniform flow: $V_0 = q/y_0$ and Eq. 10.6: $V_0 = C(RS)^{1/2}$, where $R = A/P = 170y_0/(170 + 2y_0)$

Equating V_0's: $101.54/y_0 = 78.3\sqrt{170y_0 \times 0.0003/(170 + 2y_0)}$

Solve by trial and error or by equation solver, to obtain y_0 = 19.00 ft ◀

(*b*) GVF with 4 reaches, from y_1 = 20 ft to y_2 = 24 ft

y	A	P	R	V	$\frac{V^2}{2g}$	E	$E_1 - E_2$	$\bar{V}$	$\bar{R}$	S	$\Delta x = \frac{E_1 - E}{S - S_0}$
	$170y$		A/P	q/y						Eq. 10.38a	
ft	ft²	ft	ft	fps	ft	ft	ft	fps	ft		ft
20	3400	210	16.19	5.08	0.400	20.400					
							−0.963	4.96	16.52	0.000 2426	+16,772
21	3570	212	16.84	4.84	0.363	21.363					
							−0.968	4.73	17.16	0.000 2123	+11,030
22	3740	214	17.48	4.62	0.331	22.331					
							−0.972	4.52	17.79	0.000 1869	+8,595
23	3910	216	18.10	4.41	0.303	23.303					
							−0.975	4.32	18.41	0.000 1656	+7,255
24	4080	218	18.72	4.23	0.278	24.278					
										$\Sigma(\Delta x)$	= +43,651

Note: A spreadsheet could be helpful on this problem. ▲

10.53 *A portion of an outfall sewer is approximately a circular conduit 5 ft in diameter and with a slope of 1 ft in 1100 ft. It is of brick, for which n = 0.013 (neglect variation with depth). (a) What would be its maximum capacity for uniform flow? (b) If it discharges 120 cfs with a depth at the end of 2.90 ft (Fig. P10.53), how far back from the end must it become a pressure conduit? Proceeding from the mouth upstream, find by tabular solution the length of sewer that is not flowing full. Use three reaches with equal depth increments. (c) Find the pressure at the top and bottom of the pipe at the section that is 600 ft upstream from the point at which the sewer no longer flows full. Sketch the energy line and hydraulic grade line from this point to the end of the sewer.*

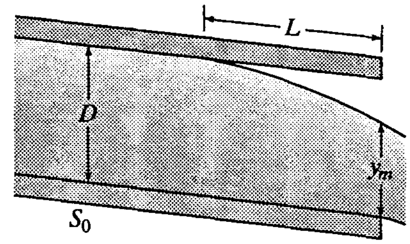


Figure P10.53

BG

Given: $D = 5$ ft, $S_0 = 1/1100 = 0.000\,909$, $n = 0.013$

(*a*) For uniform flow: Eq. 10.8*a*: $Q_{full} = (1.486/0.013)(\pi/4)5^2(5/4)^{2/3}0.000\,909^{1/2} = 78.5$ cfs

Fig. 10.10: $Q_{max}/Q_{full} \approx 1.025$; $\therefore$ $Q_{max} \approx 1.025(78.5) = 80.5$ cfs ◀

(*b*) GVF profile for part-full flow with $Q = 120$ cfs:

y ft	θ deg	A ft² (Sec. 10.7)	P ft	R A/P ft	V Q/A fps	$\frac{V^2}{2g}$ ft	E ft	E_1-E_2 ft	$\bar{V}$ fps	$\bar{R}$ ft	S Eq 10.38*a*	$\Delta x = \frac{E_1-E_2}{S-S_0}$ ft
5.0	180.0	19.63	15.71	1.250	6.11	0.580	5.580					
								0.587	6.40	1.382	0.002 03	+522
4.3	136.1	17.96	11.87	1.513	6.68	0.693	4.993					
								0.417	7.30	1.503	0.002 37	+285
3.6	116.1	15.13	10.13	1.494	7.93	0.976	4.576					
								0.073	9.05	1.429	0.003 89	+24
2.9	99.2	11.81	8.66	1.364	10.16	1.603	4.503					
											$\Sigma(\Delta x)$ =	+831

The sewer must become a pressure conduit 831 ft back from the end. ◀

Note: A spreadsheet could be helpful with the above.

(*c*) First 600 ft of pipe flowing full: Eq. 10.8*a*: $120 = (1.486/0.013)(\pi/4)5^2(5/4)^{2/3}S^{1/2}$; $S = 0.002\,12$

For 600 ft length AB: $h_L = SL = 0.002\,12(600) = 1.274$ ft water

Eq. 5.14: $(p/\gamma + z + V^2/2g)_A = (p/\gamma + z + V^2/2g)_B + h_L$; $V_A = V_B$

$\therefore$ $(p_A - p_B)/\gamma = h_L - (z_A - z_B) = 1.274 - 600(1/1100) = 0.728$ ft water

$\therefore$ $p_A - p_B = 62.4(0.728) = 45.4$ psf $= 0.316$ psi

At the point B where the sewer no longer flows full, $(p_B)_{top} = p_{atmos} = 0$ psig

At the point A 600 ft upstream of B, $(p_A)_{top} = (p_B)_{top} + 0.316 = 0 + 0.316 = 0.316$ psig ◀

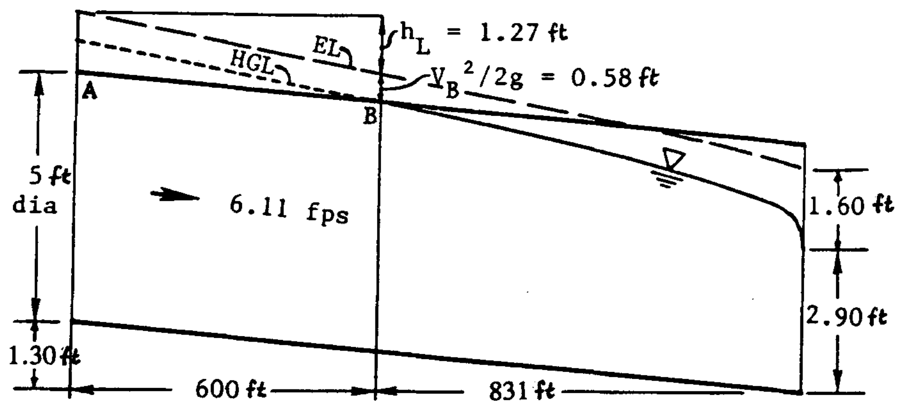
The vertical pressure distribution is hydrostatic for parallel streamlines.

$\therefore$ $(p_{bottom} - p_{top})/\gamma = 5$ ft water; $(p_A)_{bottom} - (p_A)_{top} = 5(62.4) = 312$ psf $= 2.17$ psi

$(p_A)_{bottom} = (p_A)_{top} + 2.17 = 0.316 + 2.17 = 2.48$ psi ◀

Sketch:

/cont...



10.54 *For the channel of Prob. 10.33 find the distance between a section where the depth is 3.5 ft to another where the depth is 3.0 ft. Which section is upstream? Use only one reach. See the hint of Prob. 10.33.*

Prob. 10.33: $S_0 = 0.03$, $n = 0.020$, and $Q = 120$ cfs. The sketch is drawn to the scale shown. Hint: The cross-sectional area may be found by planimetry or counting squares and the wetted perimeter by use of dividers.

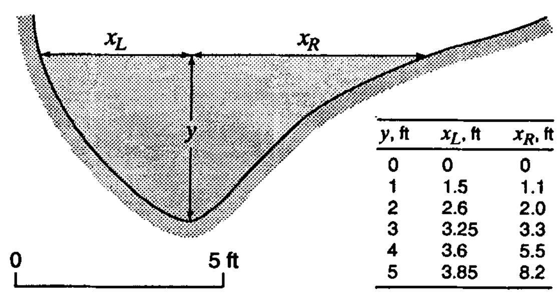


Figure P10.33

y, ft	x_L, ft	x_R, ft
0	0	0
1	1.5	1.1
2	2.6	2.0
3	3.25	3.3
4	3.6	5.5
5	3.85	8.2

BG

Assume the 3.0-ft depth is upstream:

$y_1 = 3.0$ ft, $y_2 = 3.5$ ft.

y ft	A^* ft^2	P^* ft	R ft	V fps	$V^2/2g$ ft	E ft	$\bar{V}$ fps	$\bar{R}$ ft
3.0	10.48	8.96	1.170	11.45	2.04	5.036		
							9.99	1.243
3.5	14.07	10.695	1.316	8.53	1.130	4.630		

*From Fig. P10.33 (measurements may vary).

Eq. 10.38*a*: $S = [0.020 \times 9.99/(1.486 \times 1.243^{2/3})]^2 = 0.013\,53$

Eq. 10.39: $\Delta x = (E_1 - E_2)/(S - S_0) = (5.036 - 4.630)/(0.013\,53 - 0.03) = -24.7$ ft

Distance Δx between sections = 24.7 ft ◄

Negative Δx indicates the assumption was incorrect; the 3.5-ft depth is upstream ◄

10.55 *When the flow in a certain natural stream is 7600 cfs, it is required to find the elevation of the water surface at different sections upstream from a certain initial point. A survey of the channel shows that conditions are fairly similar for a length of 1500 ft upstream from the initial point, and then beyond that there is another stretch of 2200 ft, and so on. Assuming a rise in the water surface in the distance of 1500 ft to be 0.20 ft, a study of the stream bed shows the average values of the area and wetted perimeter to be as given in the table below. Using Eq. (10.6), the computed head loss, based on average velocity and hydraulic radius, is seen to be 0.283 ft, which is greater than that assumed. Hence assume a larger value and repeat. Note that average areas and wetted perimeters have also been determined for alternative water-surface slopes. Complete the following table, and find the probable rise in elevation in the first 1500 ft. [In a similar manner, the rises in other lengths may be computed, and the sum of all of them up to the desired point will give the elevation at that point above the initial.] Assume n = 0.036.*

Assumed rise SL, ft	$\bar{A}$ ft²	$\bar{P}$ ft	$\bar{R}$ ft	$\bar{V}$ fps	$SL = LV^2/C^2R$, ft
0.20	3100	350	8.86	2.45	0.283
0.25	3180	359	8.86		
0.26	3190	360	8.86		
0.27	3220	363	8.86		
0.28	3230	364	8.86		

BG

Given: $Q = 7600$ cfs, $n = 0.036$

Comparing Eqs. 10.6 and 10.7*b*: Chézy $C = (1.486/n)R_h^{1/6}$

Here R_h = constant, $\therefore$ $C = (1.486/0.036)8.86^{1/6} = 59.4$

Completing the indicated spaces in the given table, the best agreement for the first 1500 ft is found with an assumed rise of $SL = 0.27$ ft.

Then $V = Q/A = 7600/3220 = 2.36$ fps and $LV^2/C^2R = 1500(2.36)^2/(59.4^2 \times 8.86) = 0.268$ ft

Thus the probable rise in the water surface over this 1500-ft reach is 0.27 ft ◄

Sec. 10.17: Examples of Water Surface Profiles -- *Exercises (9)*

10.17.1 *Under what different conditions can supercritical flow leaving a hump ($\Delta z > \Delta z_{crit}$) return to normal depth without passing through a hydraulic jump? Explain your answers.*

N

Supercritical flow can return to y_0 without a jump only if $y_0 \le y_c$ (it must jump if $y_0 > y_c$).

From Fig. 10.20: Thus return without a jump is possible only by S_2, S_3, or C_3 profiles. ◄

10.17.2 *Classify the water-surface profile of Exer. 10.15.1 as one of the forms shown in Fig. 10.20. Show all necessary calculations.*

Exer. 10.15.1: $n = 0.012$, $b = 5$ ft, $S_0 = 0.0006$, $Q = 60$ cfs, y varies from 2.5 to 3.0 ft.

BG

Eq. 10.8*a*: $60 = (1.486/0.012)(5y_0)[5y_0/(5 + 2y_0)]^{2/3}(0.0006)^{1/2}$;

By trial and error or by equation solver per Sample Prob. 10.1, $y_0 = 3.17$ ft

Eq. 10.23: Critical depth $y_c = (q^2/g)^{1/3} = [(60/5)^2/32.2]^{1/3} = 1.648$ ft

Fig. 10.20: $y_0 = 3.17$ ft $> y_c = 1.648$ ft, so the slope is Mild.

Given y varies from 2.5 to 3.0 ft. Fig. 10.20: Since $y_c < y < y_0$ this is an M_2 profile. ◄

10.17.3 *Classify the water-surface profile of Exer. 10.15.2 as one of the forms shown in Fig. 10.20. Show all necessary calculations.*

Exer. 10.15.2: Rectangular flume, $n = 0.013$, $b = 1.6$ m, $S_0 = 0.0005$, $Q = 1.7$ m³/s, y varies from 0.85 to 1.00 m.

SI

Eq. 10.8*b*: $1.7 = (1/0.013)(1.6y_0)[1.6y_0/(1.6 + 2y_0)]^{2/3}(0.0005)^{1/2}$

By trial and error or by equation solver per Sample Prob. 10.1, $y_0 = 1.047$ m

Eq. 10.23: Critical depth $y_c = [(1.7/1.6)^2/9.81]^{1/3} = 0.486$ m

Fig. 10.20: $y_0 = 1.047$ m $> y_c = 0.486$ m, so the slope is Mild.

Given y varies from 0.85 m to 1.00 m. Fig. 10.20: since $y_c < y < y_0$ this is an M_2 profile. ◄

10.17.4 *Repeat Exer. 10.17.2 when the channel slope is −0.000 40.*

Exer. 10.17.2: $n = 0.012$, $b = 5$ ft, $Q = 60$ cfs, y varies from 2.5 to 3.0 ft. Classify the water-surface profile as one of the forms shown in Fig. 10.20. Show all necessary calculations.

BG

Given: $S_0 = -0.000\,40$. Since $S_0 < 0$, the slope is Adverse, $y_0 = \infty$. y_c is independent of S_0.

Eq. 10.23: $y_c = (q^2/g)^{1/3} = [(60/5)^2/32.2]^{1/3} = 1.648$ ft.

Given y varies from 2.5 to 3.0 ft. Fig. 10.20: Since $y_c < y < y_0$, this is an A_2 profile ◄

10.17.5 *Repeat Exer. 10.17.3 when the channel slope is −0.000 40.*

Exer. 10.17.3: $n = 0.013$, $b = 1.6$ m, $Q = 1.7$ m³/s, y varies from 0.85 to 1.00 m. Classify the water-surface profile as one of the forms shown in Fig. 10.20. Show all necessary calculations.

SI

Since $S_0 < 0$, the slope is Adverse, $y_0 = \infty$. y_c is independent of S_0.

Eq. 10.23: $y_c = (q^2/g)^{1/3} = [(1.7/1.6)^2/9.81]^{1/3} = 0.486$ m.

Given y varies from 0.85 m to 1.00 m. Fig. 10.20: Since $y_c < y < y_0$, this is an A_2 profile. ◄

10.17.6 *The flow in a 15-ft-wide rectangular channel that has a constant bottom slope is 1400 cfs. A computation using Manning's equation indicates that the normal depth is 6.0 ft. At a certain section the depth of flow in the channel is 2.8 ft. Does the depth increase, decrease, or remain the same as one proceeds downstream from this section? Sketch a physical situation where this type of flow will occur.*

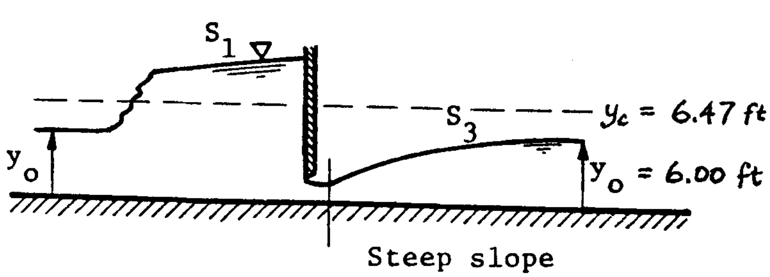
BG

Eq. 10.23: $y_c = [(1400/15)^2/32.2]^{1/3} = 6.47$ ft; $y_0 = 6.0$ ft (given).

Fig. 10.20: $y_0 = 6.0$ ft $< y_c = 6.47$ ft, so the slope of the channel is Steep.

Fig. 10.20: Since the given $y = 2.8$ ft $< y_0 < y_c$, this is an S_3 curve, and the depth will increase (asymptotically to y_0) as one proceeds downstream. ◄

Sketch of a physical situation (from Fig. 10.20):



◄

10.17.7 *A laboratory flume (n = 0.012) is 10 inches wide and set on a slope of 0.0003. With a measured flow of 0.1516 cfs the depth is observed to vary between 0.361 and 0.366 ft. Classify the water-surface profile as one of the forms of Fig. 10.20. Show all necessary calculations, and sketch the profile.*

BG

$b = 10$ in $= 0.833$ ft, $Q = 0.1516$ cfs, $S_0 = 0.0003$

Eq. 10.8*a*: $0.1516 = (1.486/0.012)(0.833y_0)[0.833y_0/(0.833 + 2y_0)]^{2/3}(0.0003)^{1/2}$

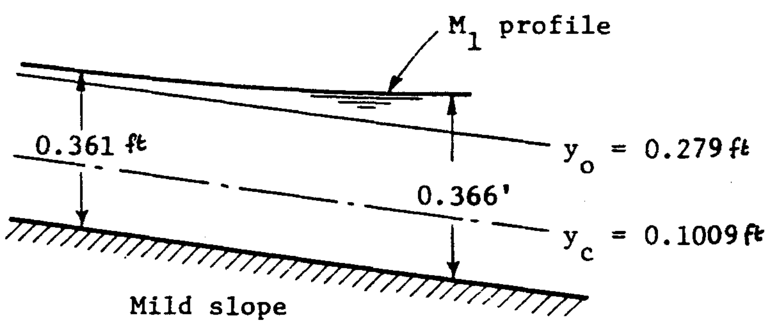
By trial and error or by equation solver per Sample Prob. 10.1: $y_0 = 0.279$ ft

Eq. 10.23: $y_c = [(0.1516/0.833)^2/32.2]^{1/3} = 0.1009$ ft

Fig. 10.20 for $y_0 = 0.279$ ft $> y_c = 0.1009$ ft; the slope is Mild.

Given y varies in the range 0.361 to 0.366 ft

Fig. 10.20: Since given $y > y_0 > y_c$, this is an M_1 profile. ◄



◄

10.17.8 *A laboratory flume (n = 0.012) is 250 mm wide and set on a slope of 0.0003. With a measured flow of 4.3 L/s, the depth is observed to vary between 110.2 and 111.7 mm. Classify the water-surface profile as one of the forms of Fig. 10.20. Show all necessary calculations, and sketch the profile.*

SI

$b = 0.25$ m, $Q = 0.0043$ m³/s, $S_0 = 0.0003$

Eq. 10.8*b*: $0.0043 = (1/0.012)(0.25y_0)[0.25y_0/(0.25 + 2y_0)]^{2/3}(0.0003)^{1/2}$

By trial and error or by equation solver per Sample Prob. 10.1: $y_0 = 0.0865$ m;

Eq. 10.23: $y_c = [(0.0043/0.25)^2/9.81]^{1/3} = 0.0311$ m

Fig. 10.20: $y_0 = 0.0865$ m $> y_c = 0.0311$ m; the slope is Mild.

Given 0.1102 m $< y <$ 0.1117 m; Fig. 10.20: Since given $y > y_0 > y_c$, this is an M_1 profile. ◄

The sketch is the same as above (Exer. 10.17.7), but with the SI dimensions substituted. ◄

10.17.9 *In an 8-ft-wide rectangular channel ($S_0 = 0.003$, n = 0.015) water flows at 300 cfs. A low dam (broad-crested weir) placed in the channel raises the water to a depth of 8.0 ft. Analyze the water surface profile upstream from the dam.*

BG

Eq. 10.8*a*: $300 = (1.486/0.015)(8y_0)[8y_0/(8 + 2y_0)]^{2/3}(0.003)^{1/2}$

By trial and error or by equation solver per Sample Prob. 10.1: $y_0 = 4.26$ ft

Eq. 10.23: $y_c = [(300/8)^2/32.2]^{1/3} = 3.52$ ft

Fig. 10.20: $y_0 > y_c$ so the channel slope is Mild. $y = 8.0$ ft caused by the dam is $> y_0 > y_c$.

Therefore, per Fig. 10.20: Upstream of the dam is an M_1 profile of great length with the depth gradually decreasing upstream to $y_0 = 4.26$ ft. ◄

Note: As damming action has occurred (Sec. 10.13), critical depth $y_c = 3.52$ ft occurs on the dam.

Sec. 10.17: Examples of Water Surface Profiles -- *Problems* 10.56–10.64

10.56 *Classify the water-surface profile of Prob. 10.45 as one of the forms shown in Fig. 10.20. Show all necessary calculations. Sketch the profile.*

Prob. 10.45: Rectangular flume, b = 5 ft, n = 0.012, S_0 = 0.01, Q = 60 cfs, y_1 = 1.5 ft.

BG

Eq. 10.8*a*: $60 = (1.486/0.012)(5y_0)[5y_0/(5 + 2y_0)]^{2/3}(0.01)^{1/2}$

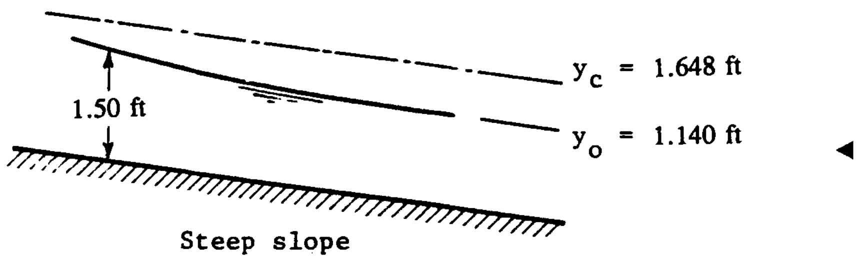
By trial and error or by equation solver per Sample Prob. 10.1: y_0 = 1.140 ft

Eq. 10.23: $y_c = (q^2/g)^{1/3} = [(60/5)^2/32.2]^{1/3} = 1.648$ ft.

Fig. 10.20: y_0 = 1.140 ft < y_c = 1.648 ft, so the slope is Steep.

Fig. 10.20: Since y_0 < given y_1 = 1.5 ft < y_c, this is an S_2 profile. ◄

Sketch:



10.57 *Classify the water-surface profile of Prob. 10.46 as one of the forms shown in Fig. 10.20. Show all necessary calculations. Sketch the profile.*

Prob. 10.46: Rectangular flume, b = 1.6 m, n = 0.013, S_0 = 0.008, Q = 1.9 m³/s, y_1 = 0.50 m.

SI

Eq. 10.8*b*: $1.9 = (1/0.013)(1.6y_0)[1.6y_0/(1.6 + 2y_0)]^{2/3}(0.008)^{1/2}$

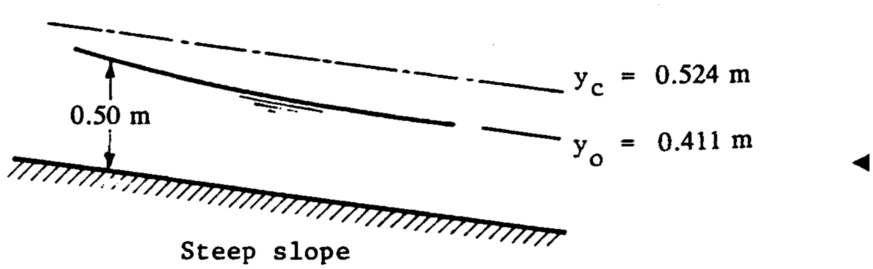
By trial and error or by equation solver per Sample Prob. 10.1, y_0 = 0.411 m

Eq. 10.23: $y_c = (q^2/g)^{1/3} = [(1.9/1.6)^2/9.81]^{1/3} = 0.524$ m.

Fig. 10.20: y_0 = 0.411 m < y_c = 0.524 m, so slope is Steep.

Fig. 10.20: Since y_0 < given y_1 = 0.50 m < y_c, this is an S_2 profile. ◄

Sketch:



10.58 *A trapezoidal canal dredged in smooth earth ($n = 0.030$) has a bottom width of 15 ft, side slopes of 1:1, and a bed slope $S_0 = 0.0003$. With a flow of 800 cfs, $y_c = 4.05$ ft, and $y_0 = 10.8$ ft. Find the distance (along an M_2 curve) from a free overfall back to where the depth is 10 ft. Use reaches with end depths of 6, 8, and 10 ft.*

BG

Given: $Q = 800$ cfs in trapezoidal channel with $b = 15$ ft and 1:1 side slopes, $n = 0.030$, $S_0 = 0.0003$, $y_0 = 10.8$ ft, $y_c = 4.05$ ft.

y ft	V fps	$V^2/2g$ ft	E ft	A ft²	P ft	$R = A/P$ ft	$\bar{R}$ ft	$\bar{V}$ fps	S Eq. 10.38a	$\Delta x = \dfrac{E_1 - E_2}{S - S_0}$ ft
10	3.20	0.159	10.159	250	43.3	5.78				
							5.33	3.77	0.000 623	+5775
8	4.35	0.294	8.294	184	37.6	4.89				
							4.42	5.35	0.001 609	+1273
6	6.35	0.626	6.626	126	32.0	3.94				
							3.43	8.36	0.005 51	+174
4.05	10.37	1.670	5.720	77.2	26.5	2.92				
									Assume $4y_c$	= +16
y_b										
										+7239

▲

10.59 *A rectangular channel 10 ft wide carries 120 cfs in uniform flow at a depth of 2.00 ft. Suppose that an obstruction such as a submerged weir is placed across the channel, rising to a height of 8 in above the bottom. (a) Does this obstruction cause a hydraulic jump to form upstream? Why or why not? (b) Find the water depth over the obstruction, and just upstream of it. Classify the surface profile, if possible, to be found upstream from the weir. Sketch the resulting water surface profile and energy line, showing y_c and y_0.*

BG

(*a*) $q = Q/b = 120/10 = 12$ cfs/ft; Eq. 10.23: $y_c = (12^2/32.2)^{1/3} = 1.648$ ft

Fig. 10.20: The given $y_0 = 2.00$ ft $> y_c = 1.648$ ft, so the slope is Mild.

For $y = y_0$, the flow is subcritical, so no jump can occur ◄

(*b*) Eq. 10.17: $E_0 = 2.00 + (12/2.00)^2/(2 \times 32.2) = 2.56$ ft

Eq. 10.25: $E_{min} = 1.5(1.648) = 2.47$ ft; Sec. 10.13: $\Delta z_{crit} = E_0 - E_{min} = 2.56 - 2.47 = 0.0877$ ft

$\Delta z = 0.667$ ft $> \Delta z_{crit}$, so there is damming action, y_c occurs over the obstruction.

Water depth over obstruction = 1.648 ft ◄

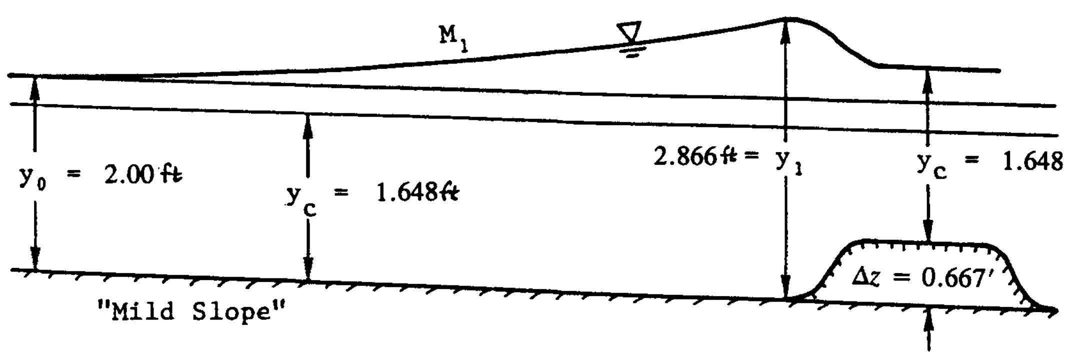
Let y_1 = water depth just upstream of obstruction. Assuming no energy loss over obstruction,

$E_1 = E_{min} + \Delta z$, i.e., $y_1 + (12/y_1)^2/(2 \times 32.2) = 2.47 + 0.667 = 3.14$

By trial and error or by equation solver per Sample Prob. 10.1, $y_1 = 2.866$ ft, 1.030 ft, or a negative root which has no physical meaning.

Fig. 10.20: There is no *M* profile which enables normal flow to become supercritical further downstream, i.e., $y_1 = 1.030$ ft is not possible here. Thus $y_1 = 2.866$ ft. ◄

Fig. 10.20: Upstream of y_1, the water surface tends asymptotically to $y_0 = 2.00$ along an M_1 profile. ◄



◄

10.60 *Suppose that the slope and roughness of the channel in Prob. 10.59 are such that uniform flow of 120 cfs occurs at a depth of 1.20 ft. Consider an obstruction rising 3 in above the bottom of the channel. Will a hydraulic jump form upstream? Classify the surface profile found just upstream from the obstruction.*
Prob. 10.59: Rectangular channel, b = 10 ft.

BG

$q = Q/b = 120/10 = 12$ cfs/ft; Eq. 10.23: $y_c = (12^2/32.2)^{1/3} = 1.648$ ft

Fig. 10.20: The given $y_0 = 1.20$ ft $< y_c = 1.648$ ft, so the slope is Steep.

For $y = y_0$, the flow is supercritical. Eq. 10.17: $E_0 = 1.20 + (12/1.20)^2/(2 \times 32.2) = 2.75$ ft

Eq. 10.25: $E_{min} = 1.5(1.648) = 2.47$ ft. Sec. 10.13: $\Delta z_{crit} = E_0 - E_{min} = 2.75 - 2.47 = 0.281$ ft

$\Delta z = 3$ in $= 0.250$ ft $< \Delta z_{crit}$, so the obstruction is not sufficiently high to produce critical flow.

Upstream the flow is straight supercritical uniform flow, and a hydraulic jump cannot form. ◄

10.61 *Analyze the water-surface profile in a long rectangular channel (n = 0.014). The channel is 12 ft wide, the flow rate is 480 cfs, and there is an abrupt change in slope from 0.0019 to 0.018. Make a sketch showing normal depths, critical depths, and water surface profile types.*

BG

Eq. 10.23: $y_c = (q^2/g)^{1/3} = [(480/12)^2/32.2]^{1/3} = 3.68$ ft

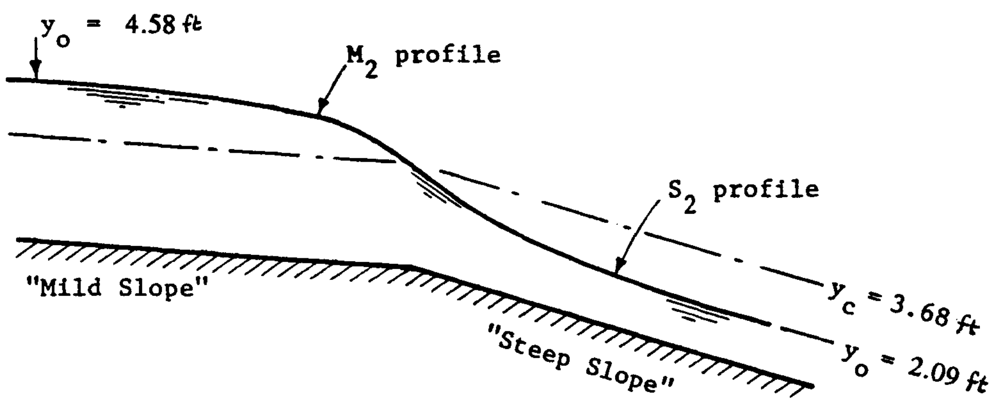
On the upper slope ($S_{01} = 0.0019$), Eq. 10.8*a*: $480 = \dfrac{1.486}{0.014}(12y_{01})\left(\dfrac{12y_{01}}{12 + 2y_{01}}\right)^{2/3}(0.0019)^{1/2}$

By trial and error or by equation solver per Sample Prob. 10.1, $y_{01} = 4.58$ ft.

Using the same equation for the lower slope ($S_{02} = 0.018$), we find $y_{02} = 2.09$ ft.

Thus the flow is subcritical ($y_0 > y_c$) before the break and supercritical ($y_0 < y_c$) after the break.

Sketch (using Fig. 10.20):



Critical depth occurs in the vicinity of the break in slope. We can find depths at various distances upstream and downstream of the break by using Eq. 10.39.

10.62 *Repeat Prob. 10.61 for the case where the flow rate is 180 cfs.*

Prob. 10.61: Analyze the water-surface profile in a long rectangular channel (n = 0.014). The channel is 12 ft wide, and there is an abrupt change in slope from 0.0019 to 0.018. Make a sketch showing normal depths, critical depths, and water surface profile types.

BG

Eq. 10.23: $y_c = [(180/12)^2/32.2]^{1/3} = 1.912$ ft

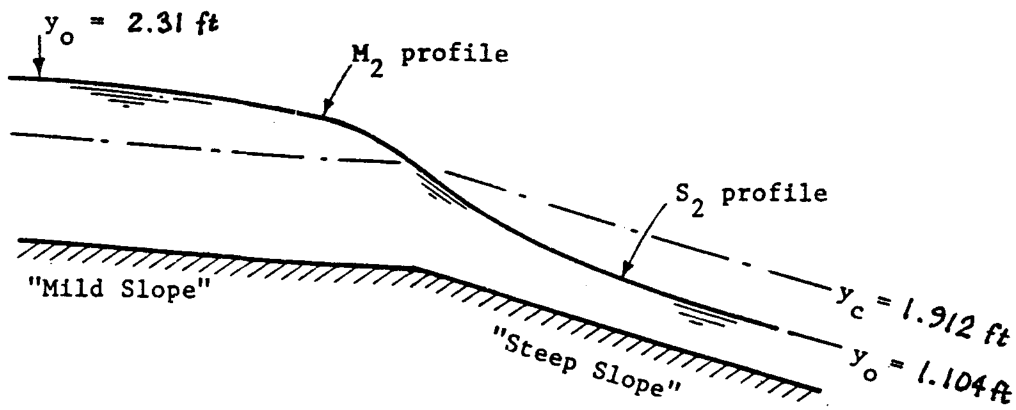
On the upper slope ($S_{01} = 0.0019$), Eq. 10.8*a*: $180 = (1.486/0.014)12y_0[12y_0/(12 + 2y_0)]^{2/3}0.0019^{1/2}$

By trial and error or by equation solver per Sample Prob. 10.1: $y_{01} = 2.31$ ft

Similarly, on the lower slope with $S_{02} = 0.0180$: $y_{02} = 1.104$ ft. $y_{01} > y_c > y_{02}$.

Thus the flow is subcritical ($y_0 > y_c$) before the break and supercritical ($y_0 < y_c$) after the break.

Sketch (using Fig. 10.20):



Critical depth occurs in the vicinity of the break in slope. We can find depths at various distances upstream and downstream of the break by using Eq. 10.39.

10.63 *Repeat Prob. 10.61 for the case where the slope change is from 0.0019 to 0.0007, and the flow rate is 180 cfs. Compute the approximate distance upstream from the break to the point where normal depth occurs, using one reach.*

Prob. 10.61: Analyze the water-surface profile in a long rectangular channel (n = 0.014). The channel is 12 ft wide, and there is an abrupt change in slope. Make a sketch showing normal depths, critical depths, and water surface profile types.

BG

Eq. 10.23: $y_c = (q^2/g)^{1/3} = [(180/12)^2/32.2]^{1/3} = 1.912$ ft

on the upper slope ($S_{01} = 0.0019$), Eq. 10.8*a*: $180 = \frac{1.486}{0.014}(12y_{01})\left(\frac{12y_{01}}{12+2y_{01}}\right)^{2/3}(0.0019)^{1/2}$

By trial and error or by equation solver per Sample Prob. 10.1, $y_{01} = 2.31$ ft.

Similarly, on the lower slope ($S_{02} = 0.0007$) we find: $y_{02} = 3.25$ ft

The flow is subcritical ($y_0 > y_c$) before the break in slope and is also subcritical ($y_0 > y_c$) after the break.

Sketch (using Fig. 10.20):

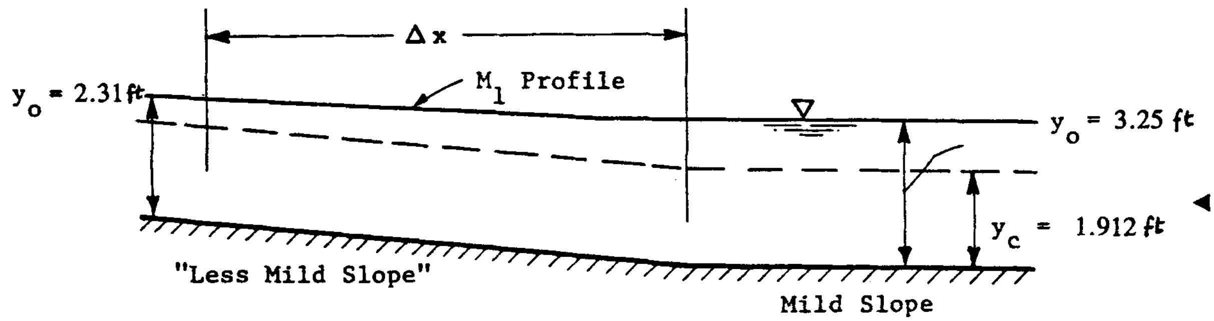


Fig. 10.20: A water surface on a mild slope cannot diverge from y_0 in the upstream direction. Therefore the depth at the break in slope is 3.24 ft.

For the length of the M_1 profile (given Q = 180 cfs, b = 12 ft):

Sta	y ft	A ft^2 10y	P ft 12 + 2y	R_h ft A/P	V fps 180/A	$V^2/2g$ ft	E ft $y + V^2/2g$
1	2.31	27.7	16.61	1.668	6.49	0.655	2.965
2	3.25	39.0	18.50	2.108	4.62	0.331	3.581

$\bar{R} = 1.888$ ft, $\bar{V} = 5.55$ fps

Eq. 10.38*a*: $S = [0.014 \times 5.55/(1.486 \times 1.888^{2/3})]^2 = 0.001\,174$

Eq. 10.39: $\Delta x = (E_1 - E_2)/(S - S_0) = (2.965 - 3.581)/(0.001\,174 - 0.0019) = +848$ ft ◄

Greater accuracy is possible by taking more steps.

10.64 *In an 8-ft-wide rectangular channel (n = 0.015) water flows at 300 cfs. A low dam (broad-crested weir) placed in the channel raises the water to a depth of 8.0 ft. Analyze the water-surface profile upstream from the dam back to uniform depth if the channel slope is (a) 0.0006, (b) 0.0009, and (c) 0.006.*

BG

y_c is independent of slope. Eq. 10.23: $y_c = (q^2/g)^{1/3} = [(300/8)^2/32.2]^{1/3} = 3.52$ ft

(*a*) With $S_0 = 0.0006$: Eq. 10.8*a*: $300 = (1.486/0.015)8y_0[8y_0/(8+2y_0)]^{2/3}0.0006^{1/2}$

By trial and error or by equation solver per Sample Prob. 10.1, $y_0 = 8.03$ ft

Fig. 10.20: $y_0 = 8.03 > y_c = 3.52$, so the slope is Mild.

Insertion of a dam cannot lower the water surface below y_0, to $y = 8.0$ ft, so this situation is impossible. ◄

(*b*) With $S_0 = 0.0009$: Repeating the procedure of Part (*a*), by trial or by equation solver, $y_0 = 6.81$ ft

Fig. 10.20: $y_0 = 6.81 > y_c = 3.52$, so the slope is Mild.

$y = 8.0$ ft caused by the dam is $> y_0 > y_c$. Therefore, per Fig. 10.20: Upstream of the dam is a long M_1 profile with the depth gradually decreasing upstream to $y_0 = 6.81$ ft. As damming action has occurred (Sec. 10.13), depth $y_c = 3.52$ ft occurs on the dam. ◄

(*c*) With $S_0 = 0.006$: Repeating the procedure of Part (*a*), by trial or by equation solver, $y_0 = 3.29$ ft

Fig. 10.20: $y_0 = 3.29 < y_c = 3.52$, so the slope is Steep.

$y = 8.0$ ft caused by the dam is $> y_c > y_0$. Therefore, per Fig. 10.20: Upstream of the dam is an S_1 profile, preceded by a hydraulic jump. Upstream of the jump is straight supercritical uniform flow with depth $y_0 = 3.29$ ft. ◄

Sec. 10.18: The Hydraulic Jump -- *Exercises (3)*

10.18.1 *A hydraulic jump occurs in a 20-ft-wide rectangular channel carrying 250 cfs on a slope of 0.005. The depth after the jump is 4.0 ft. (a) What must be the depth before the jump? (b) What are the losses of energy and power in the jump?*

BG

(*a*) Neglecting the frictional effect and weight component (small slope),

$q = 250/20 = 12.50$ cfs/ft; Eq. 10.46*b* with $y_2 = 4.0$ ft:

$y_1 = (4.0/2)\left(-1 + \sqrt{1 + (8 \times 12.50^2)/(32.2 \times 4.0^3)}\right) = 0.535$ ft ◄

(*b*) Eq. 10.47: $h_L = \left[0.535 + (1/2g)(12.50/0.535)^2\right] - \left[4.0 + (1/2g)(12.50/4.0)^2\right] = 4.86$ ft ◄

Eq. 5.40: Power loss $= 62.4(250)4.86/550 = 137.8$ hp ◄

10.18.2 *A hydraulic jump occurs in a 5-m-wide rectangular channel carrying 8 m³/s on a slope of 0.006. The depth after the jump is 1.75 m. (a) What must be the depth before the jump? (b) What are the losses of energy and power in the jump?*

SI

(*a*) $q = 8/5 = 1.600$ m³/s per m; Eq. 10.46*b* with $y_2 = 1.75$ m:

$y_1 = (1.75/2)\left(-1 + \sqrt{1 + (8 \times 1.600^2)/(9.81 \times 1.75^3)}\right) = 0.1564$ m ◄

(*b*) $V = Q/A = q/y$;

Eq. 10.47: $h_L = \left[0.1564 + (1/2g)(1.600/0.1564)^2\right] - \left[1.75 + (1/2g)(1.600/1.75)^2\right] = 3.70$ m ◄

Eq. 5.41: Power loss $= 9810(8)3.70/1000 = 290$ kW ◄

10.18.3 *The hydraulic jump may be used as a crude flowmeter. Suppose that in a horizontal rectangular channel 6 ft wide the observed depths before and after a hydraulic jump are 0.80 and 3.60 ft, respectively. Find the rate of flow and the head loss.*

BG

From Eq. 10.45: $q = \sqrt{32.2(0.80)3.60(0.80 + 3.60)/2} = 14.28$ cfs/ft; $Q = bq = 6(14.28) = 85.7$ cfs ◄

(An alternative, but less convenient, solution for Q uses Eq. 10.42.)

$V_1 = 14.28/0.80 = 17.85$ fps, $V_1^2/2g = 4.95$ ft; $V_2 = 14.28/3.60 = 3.97$ fps, $V_2^2/2g = 0.244$ ft

$h_L = E_1 - E_2 = (0.80 + 4.95) - (3.60 + 0.244) = 1.906$ ft ◄

Sec. 10.18: The Hydraulic Jump – *Problems 10.65–10.72*

10.65 *Derive Eq. (10.48) in the manner suggested in Sec. 10.18.*

N

Eq. 10.47: $h_{Lj} = E_1 - E_2 = y_1 - y_2 + (V_1^2 - V_2^2)/2g$; writing $V = q/y$ this becomes:

$$h_{Lj} = (y_1 - y_2) + \frac{q^2}{2g}\left(\frac{1}{y_1^2} - \frac{1}{y_2^2}\right) = (y_1 - y_2) + \frac{q^2}{2g}\left(\frac{y_2^2 - y_1^2}{y_1^2 y_2^2}\right) = \frac{q^2}{2g}\frac{(y_2 - y_1)(y_2 + y_1)}{y_1^2 y_2^2} - (y_2 - y_1)$$

Also, Eq. 10.45 states that: $q^2/g = y_1 y_2 (y_1 + y_2)/2$; Substituting to eliminate q^2/g:

$$h_{Lj} = \frac{y_1 y_2 (y_2 + y_1)}{4}\frac{(y_2 - y_1)(y_2 + y_1)}{y_1^2 y_2^2} - (y_2 - y_1) = \left[\frac{(y_2 + y_1)^2}{4y_1 y_2} - 1\right](y_2 - y_1)$$

$$= \left[\frac{y_2^2 + 2y_1 y_2 + y_1^2 - 4y_1 y_2}{4y_1 y_2}\right](y_2 - y_1) = \left[\frac{(y_2 - y_1)^2}{4y_1 y_2}\right](y_2 - y_1) = \frac{(y_2 - y_1)^3}{4y_1 y_2} = \text{Eq. 10.48}$$ ◄

10.66 *In a rectangular channel 10 ft wide with a flow of 200 cfs the depth is 1 ft. If a hydraulic jump is produced, (a) what will be the depth immediately after it? (b) What will be the loss of energy?*

BG

(*a*) Given: $Q = 200$ cfs, $b = 10$ ft, $y_1 = 1.00$ ft; ∴ $q = 200/10 = 20$ cfs/ft

Eq. 10.46*a*: $y_2 = (1.00/2)(-1 + [1 + (8 \times 20^2)/(32.2 \times 1.00^3)]^{1/2}) = 4.51$ ft ◄

(*b*) $V_1 = 20/1.00 = 20.0$ fps, $V_1^2/2g = 20^2/(2 \times 32.2) = 6.21$ ft;

$V_2 = 20/4.51 = 4.44$ fps, $V_2^2/2g = 0.305$ ft

Eq. 10.47: $h_L = E_1 - E_2 = (1.00 + 6.21) - (4.51 + 0.305) = 2.40$ ft·lb/lb ◄

(*b*) Alt: Eq. 10.48: $h_L = (4.51 - 1.00)^3/[4(1.00)4.51] = 2.40$ ft·lb/lb (or ft) ◄

10.67 *In a rectangular channel 4 m wide with a flow of 7.65 m³/s the depth is 0.4 m. If a hydraulic jump is produced, (a) what will be the depth immediately after it? (b) What will be the loss of energy?*

SI

(*a*) Given: $Q = 7.65$ m³/s, $b = 4$ m, $y_1 = 0.4$ m; ∴ $q = 7.65/4 = 1.9125$ m³/s per m

Eq. 10.46*a*: $y_2 = (0.4/2)(-1 + [1 + (8 \times 1.9125^2)/(9.81 \times 0.4^3)]^{1/2}) = 1.180$ m ◄

(*b*) $V_1 = 1.9125/0.4 = 4.78$ m/s, $V_1^2/2g = 4.78^2/(2 \times 9.81) = 1.165$ m;

$V_2 = 1.9125/1.180 = 1.621$ m/s, $V_2^2/2g = 0.1339$ m

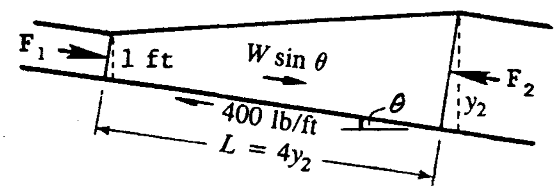
Eq. 10.47: $h_L = E_1 - E_2 = (0.4 + 1.165) - (1.180 + 0.1339) = 0.251$ N·m/N ◄

(*b*) Alt: Eq. 10.48: $h_L = (1.180 - 0.4)^3/[4(0.4)1.180] = 0.251$ N·m/N (or J/N or m) ◄

10.68 *Repeat Prob. 10.66 for the case where the channel bed slopes at 10°. For this slope, jump length ≈ $4y_2$. Assume friction force = 400 lb/ft of width. Also find the horsepower loss.*

Prob. 10.66: Rectangular channel, b = 10 ft, Q = 200 cfs, y = 1 ft. If a hydraulic jump is produced, (a) what will be the depth after it? (b) What will be the loss of energy?

BG



Note (per Secs. 10.1 and 10.2) that L is measured along the channel bed and depth is measured vertically.

$q = Q/b = 200/10 = 20$ cfs/ft. $V = q/y$

(*a*) Impulse-momentum equation 6.7*a* parallel to the bed, per foot of channel width:

$F_1 - F_2 + W\sin\theta - 400 = \rho q(V_2 - V_1)$ where $F_1 = (\gamma y_1/2)(y_1\cos 10°) = 30.7$ lb/ft,

$F_2 = \gamma y_2^2 \cos 10°/2 = 30.7y_2^2$ lb/ft, and $W = 0.5(1 + y_2)\cos 10°(4y_2)62.4 = 122.9y_2(1 + y_2)$

Thus $30.7(1 - y_2^2) + 122.9y_2(1 + y_2)\sin 10° - 400 = 1.938(20)[(20/y_2) - 20]/\cos 10°$

This is a cubic equation in y_2. By trial and error or by equation solver, $y_2 = 6.95$ ft ◄

Note to instructor: In Chow's *Open Channel Hydraulics*, McGraw-Hill, 1959, a procedure for jumps in sloping channels is given in Sec. 15-16. By that method $y_2 = 6.3$ ft.

(*b*) $V_2 = Q/(by_2\cos 10°) = 200/(10 \times 6.95\cos 10°) = 2.92$ fps; similarly, $V_1 = 20.3$ fps

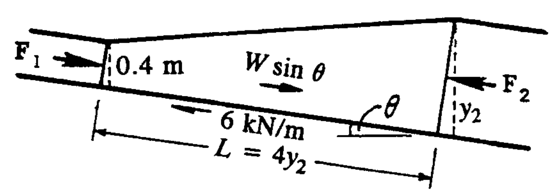
Eq. 10.47: $h_L = [1 + (20.3^2/2g)] - [6.95 + (2.92^2/2g)] = 7.40 - 7.08 = 0.325$ ft ◄

Eq. 5.40: Power loss $= \gamma Q h_L/550 = 62.4(200)0.325/550 = 7.38$ hp ◄

10.69 *Repeat Prob. 10.67 for the case where the channel bed slopes at 10°. For this slope, jump length ≈ $4y_2$. Assume friction force = 6 kN/m of width. Also find the power (kW) loss.*

Prob. 10.67: Rectangular channel, b = 4 m, Q = 7.65 m³/s, y = 0.4 m. If a hydraulic jump is produced, (a) what will be the depth after it? (b) what will be the loss of energy?

SI



Note (per Secs. 10.1 and 10.2) that L is measured along the channel bed and depth is measured vertically.

$q = Q/b = 7.65/4 = 1.913$ m³/s per m. $V = q/y$

(*a*) Impulse-momentum equation 6.7*a* parallel to the bed, per meter of channel width:

$F_1 - F_2 + W\sin\theta - 6000 = \rho q(V_2 - V_1)$ where $F_1 = (\gamma y_1/2)(y_1\cos 10°) = 773$ N/m,

$F_2 = \gamma y_2^2\cos 10°/2 = 4830y_2^2$ N/m, and $W = 0.5(0.4 + y_2)\cos 10°(4y_2)9810 = 19\,320y_2(0.4 + y_2)$

Thus $773 - 4830y_2^2 + 19\,320y_2(0.4 + y_2)\sin 10° - 6000 = 1000(1.913)^2[(1/y_2) - 1/0.4]/\cos 10°$

This is a cubic equation in y_2. By trial and error or by equation solver, $y_2 = 0.918$ m ◄

Note to instructor: In Chow's *Open Channel Hydraulics*, McGraw-Hill, 1959, a procedure for jumps in sloping channels is given in Sec. 15-16. By that method $y_2 = 6.3$ ft.

(*b*) Eq. 10.47: $h_L = [0.4 + (1.913/0.4\cos 10°)^2/2g] - [0.918 + (1.913/0.918\cos 10°)^2/2g] = 0.455$ m ◄

Eq. 5.41: Power loss $= \gamma Q h_L/1000 = 9810(7.65)0.455/1000 = 34.2$ kW ◄

10.70 *The tidal bore, which carries the tide into the estuary of a large river, is an example of an abrupt translatory wave, or moving hydraulic jump. Suppose such a bore rises to a height of 13 ft above the normal low-tide river depth of 10 ft. The speed of travel of the bore upstream is 13 mph. Find the velocity of the undisturbed river. Does this represent subcritical or supercritical flow? [Note: The theory developed in Sec. 10.18 is based on the hydraulic jump in a fixed position. In the case of a moving jump, all kinematic terms must be considered relative to the moving wave as a frame of reference.]*

BG

y_1 = 10 ft; y_2 = 10 + 13 = 23 ft; Eq. 10.45: $q^2/32.2 = (23)10(23+10)/2 = 3795$;

from which q = 349.6 cfs/ft, and $V_1 = 349.6/10 = 34.96$ fps (relative to the jump)

Velocity of jump = 13(44/30) = 19.07 fps

The jump is moving upstream at a velocity of 19.07 fps while the river is moving into the jump at 34.96 fps. Therefore, the river is moving downstream at (34.96 − 19.07) = 15.89 fps. ◄

$V_2/2g = 15.89^2/(2 \times 32.2) = 3.92$ ft $< y/2 = 5$ ft; From Sec. 10.10: The flow is subcritical. ◄

10.71 *A hydraulic jump occurs in a triangular flume having side slopes at 1:1. The flow rate is 18 cfs and the depth before the jump is 1.2 ft. Find the depth after the jump and the power loss in the jump.*

BG

Divide Eq. 10.42 by γ and substitute $V_i = Q/A_i$: $Q^2/gA_1 + h_{c1}A_1 = Q^2/gA_2 + h_{c2}A_2$

For triangular section $h_c = y/3$, $A = y^2$; y_1 = 1.2 ft, Q = 18 cfs (given)

Thus $18^2/[32.2(1.2)^2] + (1.2/3)(1.2)^2 = 18^2/(32.2y_2^2) + (y_2/3)(y_2)^2$

or $22.7 = 30.2/y_2^2 + y_2^3$; by trial and error or by equation solver, y_2 = 2.64 ft ◄

Eq. 10.47: $h_L = [1.2 + (1/2g)(18/1.2^2)^2] - [2.64 + (1/2g)(18/2.64^2)^2] = 0.883$ ft

Eq. 5.40: Power loss = 62.4(18)0.883/550 = 1.803 hp ◄

10.72 *A hydraulic jump occurs in a triangular flume having side slopes at 1:1. The flow rate is 0.45 m³/s and the depth before the jump is 0.30 m. Find the depth after the jump and the power loss in the jump.*

SI

Divide Eq. 10.42 by γ and substitute $V_i = Q/A_i$: $Q^2/gA_1 + h_{c1}A_1 = Q^2/gA_2 + h_{c2}A_2$

For triangular section $h_c = y/3$, $A = y^2$; y_1 = 0.3 m, Q = 0.45 m³/s (given)

Thus $0.45^2/[9.81(0.3)^2] + (0.3/3)(0.3)^2 = 0.45^2/(9.81y_2^2) + (y_2/3)(y_2)^2$

or $0.715 = 0.0619/y_2^2 + y_2^3$; by trial and error or by equation solver, y_2 = 0.858 m ◄

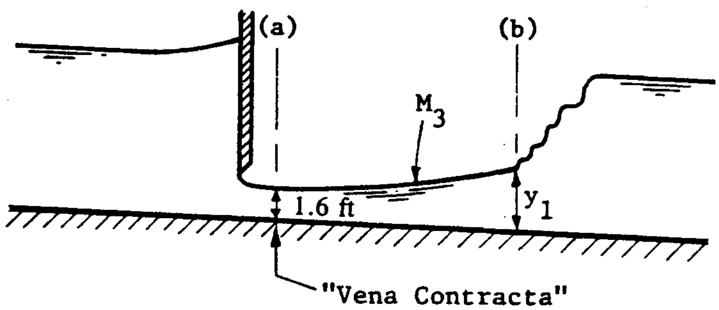
Eq. 10.47: $h_L = [0.3 + (1/2g)(0.45/0.3^2)^2] - [0.858 + (1/2g)(0.45/0.858^2)^2] = 0.697$ m

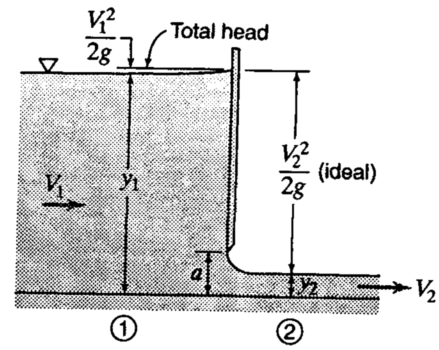
Eq. 5.41: Power loss = 9810(0.45)0.697/1000 = 3.08 kW ◄

Sec. 10.19: Location of Hydraulic Jump -- ***Problems*** *10.73–10.74*

10.73 *A wide and shallow rectangular channel with bed slope S_0 = 0.0004 and roughness n = 0.022 carries a steady flow of 65 cfs/ft of width. If a sluice gate (Fig. 11.34) is adjusted as to produce a minimum depth of 1.6 ft in the channel, determine whether a hydraulic jump will form downstream, and if so, find (using one reach) the distance from the gate to the jump.*

BG





(*a*) Free flow ($y_2 = C_c a$)

Figure 11.34*a*

Given: q = 65 cfs/ft, n = 0.022, S_0 = 0.0004, and the channel is "wide and shallow" so that $R_h = y$

Eq. 10.8*a* for unit width: $65 = (1.486/0.022)(1y_0)(y_0)^{2/3}(0.0004)^{1/2}$; y_0 = 10.22 ft

Eq. 10.23: $y_c = (65^2/32.2)^{1/3}$ = 5.08 ft. $y_0 > y_c$, so (per Fig. 10.20) the channel slope is Mild.

Alternatively, from Eq. 10.30 or 31: S_c = 0.004 11. ∴ $S_0 < S_c$ and (Sec. 10.10) the bed slope is Mild.

y = 1.6 ft caused by the gate is $< y_c < y_0$. Therefore, per Fig. 10.20:

Downstream of the sluice gate is an M_3 profile, which must be followed by a hydraulic jump to enable the flow to return to normal depth. ◀

Sec. 10.18: The channel slope (0.0004) is much less than 3° (or S = 0.0524), so the weight component in the jump may be neglected.

Eq. 10.46*b* with $y_2 = y_0$ = 10.20: y_1 = 2.093 ft

For the distance along the M_3 profile from y = 1.6 ft to y_1:

Sta	$y = R_h$ ft	$V = q/y$ fps	$V^2/2g$ ft	$E = y + V^2/2g$ ft	$\bar{R}$ ft	$\bar{V}$ fps
(*a*)	1.6	40.6	25.63	27.227	1.846	35.84
(*b*)	2.093	31.1	14.98	17.071		

Eq. 10.38*a*: $S = [(0.022 \times 35.84)/(1.486 \times 1.846^{2/3})]^2 = 0.1243$

Eq. 10.39: $\Delta x = (E_1 - E_2)/(S - S_0) = (27.227 - 17.071)/(0.1243 - 0.0004) = +82.0$ ft ◀

10.74 *Solve for and sketch the water surface profile in a very long 10-ft-wide rectangular open channel of rough brickwork (n = 0.017) when the flow rate is 400 cfs. The bed slope is 0.020, and a 5-ft-high streamlined hump is at the downstream end. Locate all hydraulic jumps, treating any gradually varied flow as a single reach. How does the water surface* <u>*elevation*</u> *vary between key points?*

BG

Eq. 10.8*a*: $400 = \dfrac{1.486}{0.017}(10y_0)\left[\dfrac{10y_0}{10+2y_0}\right]^{2/3} 0.020^{1/2}$

By trial and error or equation solver: $y_0 = 2.36$ ft ◀ $q = Q/b = 400/10 = 40$ cfs/ft

Eq. 10.23: $y_c = \left[\dfrac{40^2}{32.2}\right]^{1/3} = 3.68$ ft ◀

Because $y_0 < y_c$, the flow is supercritical.

$E_0 = 2.36 + \dfrac{1}{2(32.2)}\left[\dfrac{40}{2.36}\right]^2 = 6.82$ ft

Table 10.3: At the hump, $\Delta z_{crit} = E_u - \frac{3}{2}y_c = E_0 - \frac{3}{2}y_c = 6.82 - \frac{3}{2}(3.68) = 1.302$ ft

$\Delta z = 5$ ft $> \Delta z_{crit}$, so depth on the hump $= y_c = 3.68$ ft. ◀

$E_H = E_{min} = \frac{3}{2}(3.68) = 5.51$ ft

Immediately upstream (Station 1), $E_1 = E_{min} + \Delta z = 5.51 + 5.00 = 10.51$ ft

Eq. 10.17: $10.51 = y_1 + \dfrac{1}{2(32.2)}\left[\dfrac{40}{y_1}\right]^2$; by T & E, $y_1 = 10.28$ ft

Eq. B.9: The other two roots are $y_1 = 1.677$ ft and -1.442 ft

Fig. 10.20: There is no way for $y_0 = 2.36$ ft far upstream to drop to 1.677 ft. $\therefore$ $y_1 = 10.28$ ft ◀

Fig. 10.20: $y_1 = 10.28 > y_0 = 2.36$ ft. So we have an S_1 profile immediately u/s of the hump ◀

Immed. d/s of hump: Water surface change $= y_1 - (\Delta z + y_c) = 10.279 - (5 + 3.676) = 1.603$ ft drop

U/s (upstream), the surface rises from y_0 to S_1 via a hydraulic jump.

At jump entrance (Station J1), $y_{J1} = y_0 = 2.36$ ft. ◀

At jump exit (Station J2), Eq. 10.46*a*: $y_{J2} = \dfrac{2.36}{2}\left[-1 + \sqrt{1 + \dfrac{8(40)^2}{32.2(2.36)}}\right] = 5.41$ ft ◀

Sec. 10.18: Jump length $\approx 5(5.41) = 27.1$ ft

Water surface rise across jump $= y_{J2} - LS_0 - y_{J1} = 5.41 - 27.1(0.020) - 2.36 = 2.51$ ft ◀

For GVF S_1 profile between J2 and Station (1):

Sta	y	P	A	$R_h = A/P$	$V = Q/A$	$V^2/2g$	E
J2	5.4130	20.8260	54.130	2.5992	7.3896	0.8479	6.2609
(1)	10.2794	30.5589	102.794	3.3638	3.8913	0.2351	10.5146
Mean				2.9815	5.64045		

Eq. 10.38: $S = \left[\dfrac{0.017(5.640\,45)}{1.486(2.9815)^{2/3}}\right]^2 = 0.000\,970$

Eq. 10.39: $\Delta x = \dfrac{6.26 - 10.51}{0.000970 - 0.020} = +223.53$ ft ◀

/cont...

Summary:

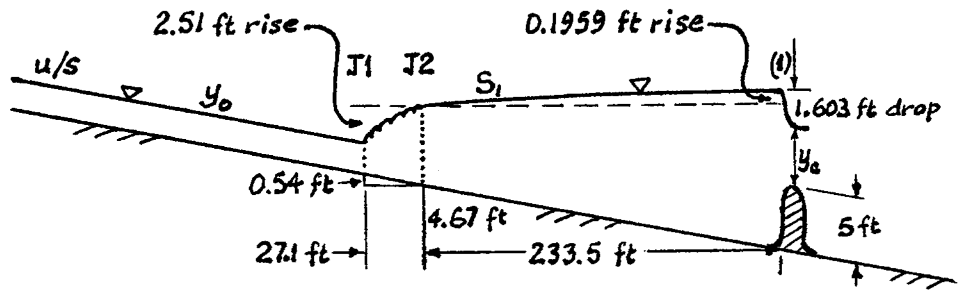
	Far u/s	J1	J2	(1)	Hump
E, ft	6.82	6.82	6.26	10.51	5.51
y, ft	2.36	2.36	5.41	10.28	3.68
Bed elev, ft		5.21	4.67	0.0	0.00
w.s. elev, ft		7.57	10.08	10.28	8.68 ◄

S_1 depth increase (J2 to 1) = 10.2795 − 5.4130 = 4.8665

Bed drop (J2 to 1) = LS_0 = 233.53(0.020) = 4.6706. Difference = 0.1959 ft

The water surface rises in elevation by 0.1959 ft from J2 to (1). ◄

Sketch:



Sec. 10.20: Velocity of Gravity Waves -- *Exercise (1)*

10.20.1 *A thin rod is placed vertically in a stream which is 4 ft deep, and the resulting small disturbance wave makes an angle of about 60° with the axis of the stream. Find the approximate velocity of the stream.*

BG

Eq. 10.49: $c \approx (32.2\times 4)^{1/2} = 11.35$ fps; Eq. 10.51: $\sin\beta = \sin 60° \approx 11.35/V$

$\therefore\ V \approx 11.35/\sin 60° = 13.10$ fps ◄

Sec. 10.20: Velocity of Gravity Waves -- *Problems 10.75–10.76*

10.75 *At a point in a shallow lake, the wave from a passing boat is observed to rise 0.5 ft above the undisturbed water surface. The observed speed of the wave is 8 mph. Find the approximate depth of the lake at this point. Compute it three ways and comment on them.*

BG

c = 8 mph = 8(44/30) fps = 11.73 fps; Δy = 0.5 ft. Using Eq. 10.49 three ways:

(*i*) $c^2 = 137.7 = g(y + 0.5)(y + 0.25)/y$ (quadratic); y = 3.49 ft or 0.0358 ft

y = 0.0358 cannot cause Δy = 0.5 ft. So y = 3.49 ft. ◄

(*ii*) $137.7 \approx g(y + 0.5)$; $y \approx 3.78$ ft ◄ (*iii*) $137.7 \approx gy$; $y \approx 4.28$ ft ◄

y = 3.49 ft is accurate; the other two values are approximations with errors of 8% and 23% respectively, on the high side. ◄

10.76 *At a point in a shallow lake, the wave from a passing boat is observed to rise 300 mm above the undisturbed water surface. The observed speed of the wave is 16 km/h. Find the approximate depth of the lake at this point. Compute it three ways and comment on them.*

SI

c = 16 kph = 16(1000/3600) m/s = 4.44 m/s; Δy = 0.30 m. Using Eq. 10.49 three ways:

(*i*) $c^2 = 19.75 = g(y + 0.3)(y + 0.15)/y$ (quadratic); y = 1.534 m or 0.0293 m.

y = 0.0293 m cannot cause Δy = 0.30 m. So y = 1.534 m ◀

(*ii*) $19.75 \approx g(y + 0.3)$; $y \approx 1.714$ m ◀ (*iii*) $19.75 \approx gy$; $y \approx 2.01$ m ◀

y = 1.534 m is accurate; the other two values are approximations with errors of 12% and 31% respectively, on the high side. ◀

Sec. 10.21: Flow Around Channel Bends -- *Exercise (1)*

10.21.1 *A rectangular channel 5 m wide carries 8 m³/s in uniform flow at a depth of 2 m. What will be the maximum difference in water-surface elevations between the inside and outside of a circular bend in this channel if the radius of the bend at the centerline is 20 m?*

SI

$V = Q/A = 8/(5\times 2) = 0.80$ m/s; Sec. 10.10: $\mathbf{F} = 0.8/\sqrt{9.81(2.0)} = 0.181$, so flow is subcritical

Eq. 10.52: $\Delta y = 0.8^2(5)/(9.81 \times 20) = 0.016\,31$ m = 16.31 mm ◀

Sec. 10.21: Flow Around Channel Bends -- *Problem 10.77*

10.77 *A rectangular channel 12 ft wide carries 360 cfs under uniform flow conditions. By how much should the outside wall be elevated above the inside wall for a bend of 45-ft radius to the center line of the channel, if (a) the normal depth is 4.4 ft and (b) the normal depth is 2.2 ft.*

BG

(*a*) $V = Q/A = 360/(12 \times 4.4) = 6.82$ fps

Sec. 10.10: $\mathbf{F} = 6.82/\sqrt{32.2(4.4)} = 0.573 < 1$, so flow is subcritical.

Eq. 10.52: Difference in wall elevations = $\Delta y = 6.82^2(12)/(32.2 \times 45) = 0.385$ ft ◀

(*b*) $V = Q/A = 360/(12 \times 2.2) = 13.64$ fps

Sec. 10.10: $\mathbf{F} = 13.64/(32.2 \times 2.2)^{1/2} = 1.620 > 1$, so flow is supercritical.

Eq. 10.52: $\Delta y = 13.64^2(12)/(32.2 \times 45) = 1.540$ ft

Because of (supercritical) wave action, maximum water depth at inside wall = y_0 and maximum water depth at outside wall = $(y_0 + \Delta y)$.

∴ Required difference in wall elevations = Δy = 1.540 ft ◀

Sec. 10.22: Transitions – *Exercises (2)*

10.22.1 *Water flows at 180 cfs from a trapezoidal channel (b = 10 ft, side slopes = 2.5 H:1V) into a rectangular channel with the same bed width. If the upstream depth is 3.5 ft and the transition is abrupt (k_t = 0.4), find the downstream depth.*

BG

Using Eq. 10.33 (for trapezoidal channel): $\dfrac{180^2}{32.2} = \dfrac{(10y_c + 2.5y_c^2)^3}{10 + 5y_c}$

By trials or equation solver, y_{c1} = 1.838 ft. $\therefore$ y_1 = 3.50 ft is subcritical.

Eq. 10.23 (for rectangular channel): $y_{c2} = (18^2/32.2)^{1/3} = 2.16$ ft

Trapezoidal: $A_1 = [10 + (2.5)3.5]3.5 = 65.6$ ft²; rectangular: $A_2 = 10y_2$

$V_1 = Q/A_1 = 180/65.6 = 2.74$ fps; $V_2 = Q/A_2 = 180/(10y_2) = 18/y_2$

Using Eq. 10.53 and assuming contracting flow ($V_2 > V_1$):

$$E_1 = E_2 + h_{Lt}, \quad \text{i.e.,} \quad 3.5 + \frac{2.74^2}{2(32.2)} = y_2 + \frac{V_2^2}{2(32.2)} + 0.4\left(\frac{V_2^2}{2(32.2)} - \frac{2.74^2}{2(32.2)}\right)$$

from which $3.66 = y_2 + 0.217V_2^2$ and substituting for $V_2 = 18/y_2$: $3.66 = y_2 + 7.04/y_2^2$;

By trial and error or by polynomial or equation solver, y_2 = 2.18 ft (subcritical) or 2.68 ft (subcritical) or −1.203 ft (meaningless).

As the water depth drops from y_1 = 3.50 ft, it will reach y_2 = 2.68 ft before reaching y_2 = 2.18 ft.

$\therefore$ The downstream water depth will remain at y_2 = 2.68 ft ◀

10.22.2 *A rectangular, 3-m-wide channel is connected to a trapezoidal channel (b = 1 m, side slopes = 1.5H:1V) by a warped transition. When 7.5 m^3/s of water is flowing, the depth in the upstream channel is 1.5 m. (a) Find y_2. (b) Is the flow through the transition expanding or contracting?*

SI

Upstream rectangular: $A_1 = 3(1.5) = 4.5\ m^2$; downstream trapezoidal: $A_2 = (1 + 1.5y_2)y_2$

$V_1 = Q/A_1 = 7.5/4.5 = 1.667$ m/s; $y_{c1} = [(7.5/3)^2/9.81]^{1/3} = 0.860$ m; $V_2 = Q/A_2 = 7.5/[(1 + 1.5y_2)y_2]$

From Eq. 10.33 with $A = y_c + 1.5y_c^2$ and $B = 1 + 3y_c$, by trials or equation solver: $y_{c2} = 1.099$ m

(*i*) Assume flow through transition is contracting ($A_2 < A_1$, $V_2 > V_1$).

We note that $A_2 = A_1$, i.e., $(1 + 1.5y_2)y_2 = 4.5$ when $1.5y_2^2 + y_2 - 4.5 = 0$,

i.e. $y_2 = 1.431$ m or -2.10 m (impossible),

$\therefore A_2 < A_1$ when $0 < y_2 < 1.431$ m and when $V_2 > V_1 = 1.667$ m/s.

Table 10.4 for contracting flow through warped transition: $k_t = 0.1$

Using Eq. 10.53: $E_1 = E_2 + h_{Lt}$, i.e., $1.5 + \dfrac{1.667^2}{2(9.81)} = y_2 + \dfrac{V_2^2}{2(9.81)} + 0.1\left(\dfrac{V_2^2}{2(9.81)} - \dfrac{1.667^2}{2(9.81)}\right)$

and substituting for V_2 this yields: $1.656 = y_2 + \dfrac{3.15}{[(1 + 1.5y_2)y_2]^2}$ (provided $0 < y_2 < 1.431$ m)

By trial and error or by equation solver, $y_2 = 0.869$ m (subcrit) [1.532 m and -1.232 m are invalid]

$\therefore A_2 = (1 + 1.5 \times 0.869)0.869 = 2.00\ m^2$

$A_2 = 2.00 < A_1 = 4.5$, so the flow is contracting, our assumption is valid.

Subcritical $y_2 = 0.869$ m can occur.

(*ii*) Assume flow is expanding ($A_2 > A_1$, $V_2 < V_1$). Table 10.4: $k_t = 0.3$

$A_2 > A_1$ when $y_2 > 1.431$ m and when $V_2 < V_1 = 1.667$ m/s.

Using Eq. 10.53: $E_1 = E_2 + h_{Lt}$, i.e., $1.5 + \dfrac{1.667^2}{2(9.81)} = y_2 + \dfrac{V_2^2}{2(9.81)} + 0.3\left(\dfrac{1.667^2}{2(9.81)} - \dfrac{V_2^2}{2(9.81)}\right)$

and substituting for V_2 this yields: $1.599 = y_2 + \dfrac{2.01}{[(1 + 1.5y_2)y_2]^2}$ (provided $y_2 > 1.431$ m)

By trial and error or by equation solver, $y_2 = 1.518$ m (subcrit) [0.726 m and -1.158 m are invalid]

$\therefore A_2 = (1 + 1.5 \times 1.518)1.518 = 4.97\ m^2$

$A_2 = 4.97 > A_1 = 4.5$, so the flow is expanding, our assumption is valid

Results: (*i*) Flow can contract, with subcritical $y_2 = 0.869$ m; ◄

(*ii*) Flow can expand, with subcritical $y_2 = 1.518$ m ◄

Which of these two occurs depends on $(y_0)_2$ which is unknown.

Sec. 10.22: Transitions -- *Problems 10.78–10.80*

10.78 *Refer to Fig. 10.32. A rectangular channel changes in width from 5 to 7.5 ft. Measurements indicate that y_1 = 3.00 ft and Q = 62 cfs. Determine the depth y_2 by (a) neglecting head loss; (b) considering the head loss to be given as shown on the figure.*

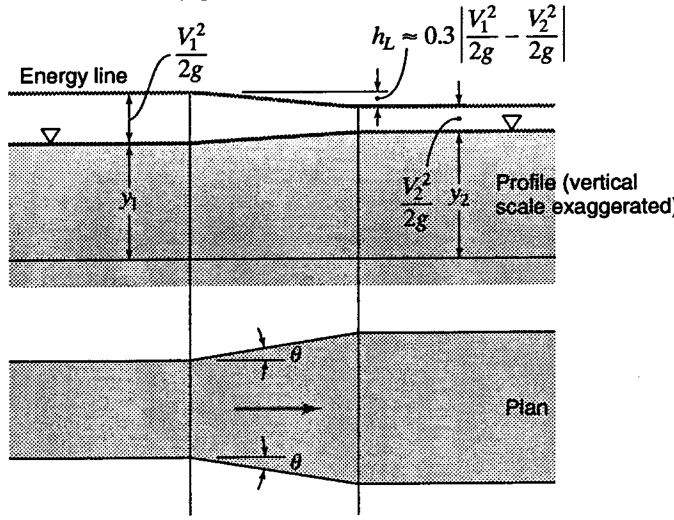


Figure 10.32

BG

Eq. 10.23: $y_{c1} = [(62/5)^2/32.2]^{1/3} = 1.684$ ft, $y_{c2} = 1.285$ ft

$y_1 = 3.00$ ft is subcritical. $V_1 = Q/A_1 = 62/(3 \times 5) = 4.13$ fps ; $V_2 = 62/7.5y_2 = 8.27/y_2$ fps

(*a*) $h_L = 0$, $\therefore E_1 = E_2$ i.e., $3.0 + 4.13^2/(2 \times 32.2) = y_2 + (8.27/y_2)^2/(2 \times 32.2)$

By trial and error or by polynomial or equation solver,

$y_2 = 3.16$ ft (subcrit) or 0.635 ft (supercrit) or −0.529 ft (impossible).

Assuming the flow does not change from subcrit to supercrit in the transition, $y_2 = 3.16$ ft ◄

(*b*) Fig. 10.32 and assuming expanding flow: $E_1 = E_2 + h_L = E_2 + 0.3(V_1^2/2g - V_2^2/2g)$

$y_1 + V_1^2/2g = y_2 + V_2^2/2g + 0.3(V_1^2/2g - V_2^2/2g)$ i.e., $y_1 + 0.7V_1^2/2g = y_2 + 0.7V_2^2/2g$

i.e., $3.0 + 0.186 = 3.186 = y_2 + 0.743/y_2^2$; by trial and error or by polynomial or equation solver,

$y_2 = 3.11$ ft (subcrit) or 0.529 ft (supercrit) or −0.452 ft (impossible).

Check: $A_1 = 5(3) = 15$ ft²; $A_2 = 7.5(3.11) = 23.3$ ft² or $7.5(0.529) = 3.92$ ft² (not expanding).

So flow is expanding, with $y_2 = 3.11$ ft ◄

10.79 *A rectangular channel changes in width from 1.35 m to 2.0 m with the wedge-shaped transition shown in Fig. 10.32. Given that y_1 = 0.75 m and Q = 1.5 m^3/s, find the depth y_2 by (a) neglecting head loss and (b) using a loss coefficient k_t of 0.4. (c) Does the head loss increase or decrease the flow expansion?*

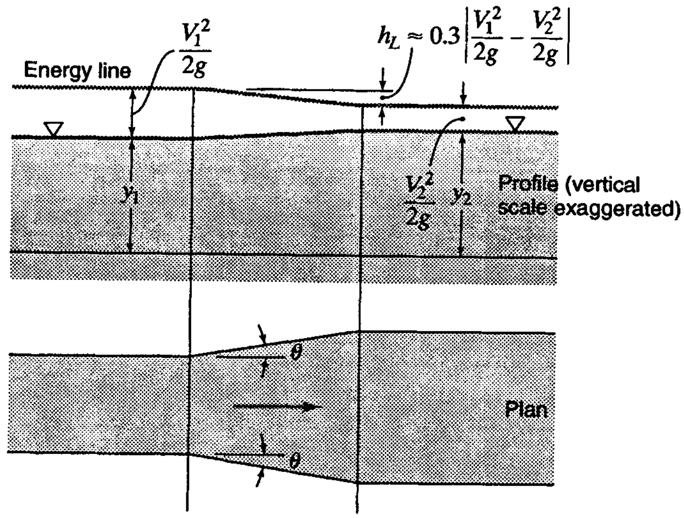


Figure 10.32

SI

Eq. 10.23: $y_{c1} = [(1.5/1.35)^2/9.81]^{1/3} = 0.501$ m, $y_{c2} = 0.386$ m

$y_1 = 0.75$ m is subcritical. $V_1 = Q/A_1 = 1.5/(1.35 \times 0.75) = 1.481$ m/s; $V_2 = 1.5/2y_2 = 0.75/y_2$ m/s

(*a*) $h_L = 0$, $\therefore$ $E_1 = E_2$, i.e., $0.75 + 1.481^2/(2 \times 9.81) = y_2 + (0.75/y_2)^2/(2 \times 9.81)$

By T & E or by polynomial or equation solver,

$y_2 = 0.819$ m (subcritical) or 0.210 m (supercritical) or -0.1669 m (impossible).

As we know of nothing to cause supercritical flow, $y_2 = 0.819$ m ◄

(*b*) Table 10.4 for wedge transition with $k_t = 0.4$: the flow expands, so $V_1 > V_2$

Per Fig. 10.32: $E_1 = E_2 + 0.4(V_1^2/2g - V_2^2/2g)$; $y_1 + V_1^2/2g = y_2 + V_2^2/2g + 0.4(V_1^2 - V_2^2)/2g$

$y_1 + 0.6V_1^2/2g = y_2 + 0.6V_2^2/2g$; $0.75 + 0.0671 = 0.817 = y_2 + 0.01720/y_2^2$

By trial and error or by polynomial or equation solver,

$y_2 = 0.790$ m (subcritical) or 0.1621 m (supercritical) or -0.1345 m (impossible).

Check: $A_1 = 1.35(0.75) = 1.013\ m^2$; $A_2 = 2(0.790) = 1.579\ m^2$ (expanded) or

$2(0.1621) = 0.324\ m^2$ (contracting, invalid). $\therefore$ $y_2 = 0.790$ m ◄

(*c*) Expansion (*a*) without head loss = $A_2/A_1 = 2(0.819)/(1.35 \times 0.75) = 1.618$

Expansion (*b*) with head loss = $2(0.790)/(1.35 \times 0.75) = 1.560$, so head loss reduces expansion ◄

10.80 *Flow from a trapezoidal channel (20-ft bed width, 1:1 side slopes) enters a 20-ft-wide rectangular channel via a cylinder-quadrant transition (Fig. P10.80). (a) If the upstream depth is 4 ft when the flow rate is 530 cfs, find the resulting downstream depth. (b) If the flow increases to 550 cfs, causing the upstream depth to increase to 4.1 ft, does the downstream depth increase or decrease? By how much?*

BG

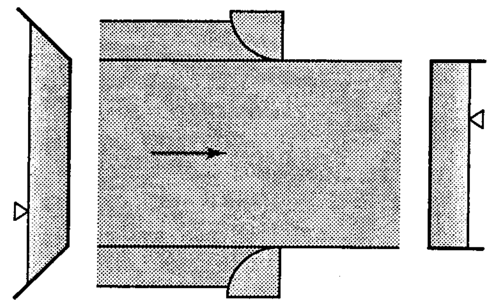


Figure P10.80

(*a*) $A_1 = (20 + 1y_1)y_1 = (20 + 4)4 = 96 \text{ ft}^2$; $A_2 = 20y_2$

$$V_1 = \frac{Q}{A_1} = \frac{530}{96} = 5.52 \text{ fps}; \quad V_2 = \frac{530}{20y_2} = \frac{26.5}{y_2}$$

The downstream rectangular channel is "narrower," so try contracting flow ($A_2 < A_1$, $V_2 > V_1$).

Table 10.4 for contracting flow through cylinder-quadrant transition: $k_t = 0.2$

Using Eq. 10.53: $E_1 = E_2 + h_L$, i.e., $4.0 + 5.52^2/2g = y_2 + V_2^2/2g + 0.2(V_2^2/2g - 5.52^2/2g)$

$4.57 = y_2 + 0.018\,63V_2^2$. Substituting for V_2: $4.57 = y_2 + 13.09/y_2^2$

By trial and error or by polynomial or equation solver, $y_2 = 3.4994$ ft or 2.54 ft or -1.472 ft

$y_1 = 4.00$ ft, so both y_2 alternates are contractions.

As the water depth drops from 4.00 ft, it will reach $y_2 = 3.50$ ft before reaching $y_2 = 2.54$ ft.

$\therefore$ The downstream water depth will remain at $y_2 = 3.50$ ft ◀

(*b*) $Q = 550$ cfs, $y_1 = 4.1$ ft. Repeating the procedure of part (*a*), $4.68 = y_2 + 14.09/y_2^2$

From which $y_2 = 3.5743$ ft or 2.61 ft or -1.509 ft (impossible).

Again, both y_2 alternatives are contractions.

The downstream water depth will remain at the higher, $y_2 = 3.5743$ ft

Downstream depth change $= 3.5743 - 3.4994 = 0.0750$ ft increase ◀

Sec. 10.23: Hydraulics of Culverts – *Exercise (1)*

10.23.1 *Repeat Sample Prob. 10.13 for the situation where the culvert must be 100 m long.*

Sample Prob. 10.13: $S_0 = 0.003$, $Q = 4.3\ m^3/s$, maximum headwater = 3.6 m above invert. Neglect velocity of approach, assume square-edged inlet with $k_e = 0.5$, $C_d = 0.65$, and free discharge at outlet. What size of corrugated pipe ($n = 0.025$) is needed?

SI

Sec. 10.23: Assume headwater depth/D > 1.2 so that entrance is submerged, per Fig. 10.33.

∴ assume that $D <$ headwater depth/1.2 = 3.6 m/1.2 = 3.0 m

Given discharge is free (not submerged), so flow is per Fig. 10.33*b* or 10.33*c*.

Assume the full flow of Fig. 10.33*b*: $V = Q/A = 4.3/[\pi D^2/4] = 5.47/D^2$; $R_h = D/4$

Fig. 10.33*b*: $\Delta h = (y_1 - y_2) + (z_1 - z_2) = y_1 - y_2 + S_0 L = 3.6 - D + 0.003(100) = 3.9 - D$

Eq. 10.56: (in SI units) $$\Delta h = \left(0.5 + \frac{(2)9.81(0.025)^2 100}{(D/4)^{4/3}} + 1\right)\frac{5.47^2}{2(9.81)D^4} = \left(1.5 + \frac{7.79}{D^{4/3}}\right)\frac{1.528}{D^4}$$

Equating these two expressions for Δh and simplifying: $3.9 = D + \left(1.5 + \frac{7.79}{D^{4/3}}\right)\frac{1.528}{D^4}$

By trial and error or by equation solver: $D = 1.409$ m

Thus the first assumption ($D < 3.0$ m, i.e. submerged entrance) is OK.

To check assumption 2, find the d_0 which just flows full with uniform flow:

Eq. 10.8*b*: $4.3 = (1/0.025)(\pi d_0^2/4)(d_0/4)^{2/3}0.003^{1/2}$; $d_0 = 1.994$ m

$D < d_0$, so the culvert flows full, we <u>do</u> have the assumed flow of Fig. 10.33*b*.

The assumptions and results are valid, $D = 1.409$ m.

Standard sizes are 1.35 and 1.50 m. ∴ Use $D = 1.50$ m ◄

Sec. 10.23: Hydraulics of Culverts -- *Problems 10.81–10.84*

10.81 *What is the capacity of a 5-ft by 5-ft concrete box culvert (n = 0.013) with a rounded entrance (k_e = 0.05, C_d = 0.95) if the culvert slope is 0.004, the length is 180 ft, and the headwater level is 7 ft above the culvert invert? Assume (a) free outlet conditions, (b) tailwater elevation 1 ft above top of box at outlet, and (c) tailwater elevation 2 ft above top of box at outlet. Neglect headwater and tailwater velocity heads.*

BG

(*a*) Find capacity with free outlet.

Sec. 10.23: Headwater/D = 7/5 = 1.4 > 1.2, so conditions are those of Fig. 10.33*b* or 10.33*c*.

Assume Case (*b*) of Fig. 10.33. $R = A/P = 5^2/(4 \times 5) = 1.25$ ft

$\Delta h = (y_1 - y_2) + (z_1 - z_2) = y_1 - y_2 + S_0 L$. Equating this to Δh in Eq. 10.56:

$7 - 5 + 0.004(180) = [0.05 + 29.2(0.013^2)180/1.25^{4/3} + 1]V^2/(2 \times 32.2) = 0.0265V^2$

from which V = 10.12 fps, $Q = AV = 5^2(10.12) = 253$ cfs

Now find the depth y_0 which occurs with normal/uniform flow at this Q:

Eq. 10.8*a*: $253 = (1.486/0.013)5y_0[5y_0/(5 + 2y_0)]^{2/3}0.004^{1/2}$

By trial and error or by equation solver per Sample Prob. 10.1, y_0 = 4.983 ft.

$y_0 < D$ = 5 ft, so the culvert does <u>not</u> flow full; free discharge at the outlet is given; therefore the preceeding assumption and computations are not valid, and the flow conditions must be Case (*c*).

Case (*c*) of Fig. 10.33, Eq. 10.57: $Q = C_d A\sqrt{2gh} = 0.95(5^2)\sqrt{2(32.2)(7 - 2.5)} = 404$ cfs ◄

(*b*) Find capacity when tailwater elevation is 1 ft above top of box at outlet.

Sec. 10.23: Headwater/D = 7/5 = 1.4 > 1.2, so conditions are per Fig. 10.33*a*, with y_2 = 6 ft.

Using the same equations as Part (*a*) (neglecting headwater and tailwater velocity heads):

$7 - 6 + 0.004(180) = 0.0265V^2$ from which V = 8.05 fps, $Q = 5^2(8.05) = 201$ cfs ◄

(*c*) TW elevation 2 ft above top of box at outlet. Conditions are again per Fig. 10.33*a*, but with y_2 = 7 ft.

Using the same equations as Part (*a*): $7 - 7 + 0.004(180) = 0.0265V^2$

from which V = 5.21 fps, so $Q = 5^2(5.21) = 130.2$ cfs ◄

10.82 *Repeat Prob. 10.81(a) for the cases where the culvert slope is (a) 0.03 and (b) 0.07.*

Prob. 10.81(a): Find the capacity of a 5-ft by 5-ft box culvert (n = 0.013) 180 ft long with a rounded entrance (k_e = 0.05, C_d = 0.95). The headwater is 7 ft above invert, and the outlet discharge is free; neglect headwater and tailwater velocity heads.

BG

(*a*) Find capacity when $S_0 = 0.03$

Sec. 10.23: Headwater/D = 7/5 = 1.4 > 1.2, so conditions are per Fig. 10.33*b* or 10.33*c*.

Assume Case (*b*) of Fig. 10.33. $R = A/P = 5^2/(4 \times 5) = 1.24$ ft.

$\Delta h = (y_1 - y_2) + (z_1 - z_2) = y_1 - y_2 + S_0 L$. Equating this to Δh in Eq. 10.56:

$7 - 5 + 0.03(180) = [0.05 + 29.2(0.013^2)180/1.25^{4/3} + 1]V^2/(2 \times 32.2) = 0.0265V^2$

from which $V = 16.70$ fps, $Q = AV = 5^2(16.70) = 417$ cfs

Now find the depth y_0 which occurs with normal/uniform flow at this Q:

Eq. 10.8*a*: $417 = (1.486/0.013)5y_0[5y_0/(5 + 2y_0)]^{2/3}0.03^{1/2}$

By trial and error or by equation solver per Sample Prob. 10.1, $y_0 = 3.33$ ft.

$y_0 < D = 5$ ft, so the culvert does <u>not</u> flow full and Case (*b*) cannot occur. Case (*c*) must occur, i.e., with entrance control:

Eq. 10.57: $Q = 0.95(5^2)\sqrt{2(32.2)(7 - 2.5)} = 404$ cfs ◄

(*b*) Find capacity when $S_0 = 0.07$. Conditions are again those of Fig. 10.33*b* or 10.33*c*.

Following the same procedure as in Part (*a*):

$7 - 5 + 0.07(180) = 0.0265V^2$ from which $V = 23.5$ fps, $Q = 586$ cfs, $y_0 = 3.12$ ft

$y_0 < D = 5$ ft, so the culvert does <u>not</u> flow full and Case (*b*) cannot occur. Case (*c*) must occur, i.e., with entrance control: As for Part (*a*): $Q = 404$ cfs ◄

10.83 *A culvert under a road must carry 4.5 m³/s. (a) If the culvert length is 32 m, the slope is 0.004, and the maximum permissible headwater level above the culvert invert is 3.8 m, what size of corrugated-pipe culvert (n = 0.025) would you select? The outlet will discharge freely. Neglect velocity of approach. Assume square-edged entrance with k_e = 0.5, C_d = 0.65. (b) Repeat for a culvert length of 110 m.*

SI

(*a*) Find diameter D when $L = 32$ m.

Sec. 10.23: Assume $D < 3.0$ m, so that headwater/$D > 3.8/3 = 1.267$ and the conditions are those of Fig. 10.33*b* or 10.33*c*.

Assume Case (*b*), Fig. 10.33. $V = Q/A = 4.5/(\pi D^2/4) = 5.73/D^2$; $R = D/4$

$\Delta h = (y_1 - y_2) + (z_1 - z_2) = y_1 - y_2 + S_0 L$; Equating this to Δh in Eq. 10.56:

$$3.8 - D + 0.004(32) = \left[0.5 + \frac{2(9.81)0.025^2(32)}{(D/4)^{4/3}} + 1\right]\frac{5.73^2}{2(9.81)D^4}$$

i.e., $3.93 - D = [1.5 + (2.49/D^{4/3})]1.673/D^4$

By trial and error or by equation solver per Sample Prob. 10.1, $D = 1.206$ m

Now find the diameter d_0 which just flows full with normal/uniform flow:

Eq. 10.8*b*: $4.5 = (1/0.025)(\pi d_0^2/4)(d_0/4)^{2/3}0.004^{1/2}$ from which, by trial and error, $d_0 = 1.922$ m.

$d_0 > D$, so the culvert <u>does</u> flow full; free discharge at outlet is given;

Therefore the above assumptions and results are valid; use standard $D = 1.35$ m ◀

(*b*) Find diameter D when $L = 110$ m

Using the same procedures as Part (*a*):

$$3.8 - D + 0.004(110) = \left[0.5 + \frac{(2)9.81(0.025)^2 110}{(D/4)^{4/3}} + 1\right]\frac{5.73^2}{2(9.81)D^4}$$

i.e., $4.24 - D = [1.5 + (8.56/D^{4/3})]1.673/D^4$

By trial and error or by equation solver per Sample Prob. 10.1, $D = 1.421$ m

As in Part (*a*), the diameter which just flows full with normal/uniform flow is $d_0 = 1.922$ m.

$d_0 > D$, so the culvert <u>does</u> flow full; free discharge at outlet is given; therefore the above assumptions and analysis are valid.

Standard sizes are 1.35 and 1.50 m. Use $D = 1.50$ m ◀

10.84 *A 120-ft-long corrugated metal pipe ($n = 0.022$) of 30-in diameter is tested in a laboratory. The headwater is maintained at a level which is 5 ft above the pipe invert at entrance, and the outlet cannot be submerged. Assume a square-edged entrance with $k_e = 0.5$, $C_d = 0.68$, and neglect headwater and tailwater velocity heads. Compute values of Q for S_0 values of 0.0, 0.01, 0.03, and 0.08. As the slope is increased, at what slope does the flow change to condition (c) of Fig. 10.33?*

BG

Given: conditions of Fig. 10.33*b* prevail when $y_0 > D$.

Sec. 10.23: Headwater/D = 5/2.5 = 2 > 1.2, so conditions are those of Fig. 10.33*b* or 10.33*c*.

For Case (*b*) of Fig. 10.33: $R = D/4 = 2.5/4 = 0.625$ ft

$\Delta h = (y_1 - y_2) + (z_1 - z_2) = y_1 - y_2 + S_0 L$

Equating this to Δh in Eq. 10.56: $5 - 2.5 + 120S_0 = [0.5 + 29.2(0.022)^2 120/0.625^{4/3} + 1]V^2/(2 \times 32.2)$

i.e., $2.5 + 120S_0 = 0.0726V^2$ (1)

The change in conditions, from Case (*b*) to Case (*c*), occurs when $y_0 = D = 2.5$ ft.

Then, using Eq. 10.7*b*: $V_{full} = (1.486/0.022)0.625^{2/3}S_0^{1/2}$

i.e., $V_{full} = 49.4S_0^{1/2}$; $(V_{full})^2 = 2438S_0$

Substituting this into Eq. (1): $2.5 + 120S_0 = 0.0726(2438S_0)$; $S_0 = 0.0439$ ◀

So when $S_0 > 0.0439$ orifice flow (Condition 10.33*c*) occurs.

For the required slopes:

Slope S_0	Condition	Eq.	V fps	Q cfs
0.0	Fig. 10.33*b*	(1)	5.87	28.8
0.01	Fig. 10.33*b*	(1)	7.14	35.0
0.03	Fig. 10.33*b*	(1)	9.17	45.0
(0.0439)	Fig. 10.33*b*	(1)	(10.35)	(50.8)
0.08	Fig. 10.33*c*	10.57*	10.57	51.9

*With $C_d = 0.68$, $h = 5 - D/2 = 3.75$ ft ▲

Chapter 11

Fluid Measurements

PROBLEM SELECTION GUIDE

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
11.1	***Measurement of Fluid Properties***						
	X[1] 11.1.1	BG	Easy	Short	1	11.1.2	Uses Sec. 2.3
	11.1.2	SI	Easy	Short	1	11.1.1	Uses Sec. 2.3
	11.1.3	BG	Easy	V Short	1	11.1.4	Uses Sec. 3.2
	11.1.4	SI	Easy	V Short	1	11.1.3	Uses Sec. 3.2
	11.1.5	BG	Medium	Short	1	11.1.6	Uses Sec. 2.11
	11.1.6	SI	Medium	Short	1	11.1.5	Uses Sec. 2.11
	11.1.7	B	Easy	V Short	1		
	11.1.8	BG	Medium	Short	1	11.1.9	Assume laminar flow
	11.1.9	SI	Medium	Short	1	11.1.8	Assume laminar flow
	11.1.10	BG	Easy	Short	1		Uses Sec. 7.4
	P 11.1	BG	Medium	Medium	1	11.2	
	11.2	SI	Medium	Medium	1	11.1	
	11.3	BG	Medium	Short	1		Assume laminar flow, uses Sec 2.11
	11.4	SI	Easy	Medium	1		Uses Secs 2.11, 7.4, & 8.7
	11.5	SI	Easy	Medium	1		Uses Secs 2.11 & 7.4
	11.6	BG	Medium	Short	1	11.7	Uses Sec 9.7. T & E (Trial and Error).
	11.7	SI	Medium	Short	1	11.6	Uses Sec 9.7. T & E
	11.8	BG	Easy	Medium	1		Plot
11.3	***Measurement of Velocity with Pitot Tubes***						
	X 11.3.1	SI	Easy	Short	1		
	11.3.2	BG	Easy	V Short	1	11.3.3	Uses Sec. 3.5
	11.3.3	SI	Easy	V Short	1	11.3.2	Uses Sec. 3.5
	11.3.4	BG	Easy	V Short	1	11.3.5	Uses Sec. 3.5
	11.3.5	SI	Easy	V Short	1	11.3.4	Uses Sec. 3.5
	11.3.6	BG	V Easy	V Short	1	11.3.7	Uses Sec. 3.5
	11.3.7	SI	V Easy	V Short	1	11.3.6	Uses Sec. 3.5
	P 11.9	BG	Medium	Short	1		Uses Sec. 2.7
	11.10	BG	Easy	Short	1		Uses Sec. 3.5
	11.11	BG	Easy	Short	1	11.12	Uses Sec. 3.5
	11.12	SI	Easy	Short	1	11.11	Uses Sec. 3.5
	11.13	BG	Easy	Medium	1		Plot
	11.14	BG	Medium	Medium	1		Uses Secs. 3.5, 8.11, & 8.13.
11.5	***Measurement of Discharge***						
	P 11.15	BG	Medium	Medium	1	11.16	Integration; uses Sec. 4.5
	11.16	SI	Medium	Medium	1	11.15	Integration; uses Sec. 4.5

/cont...

[1] For all Exercises (identified by "X"), answers are given in Appendix F of the textbook.
† Answers are sensitive to values (C, C_d, K, k', Y) that are read from graphs.
X = Exercise, P = (end-of-chapter) Problem, S = Sample Problem. ☐ = could use computing aids.

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
11.6	***Orifices, Nozzles, and Tubes***						
	X 11.6.1	BG	Easy	Short	1		
	11.6.2	BG	Easy	Short	1	11.6.3	Uses Sec 5.15
	11.6.3	SI	Easy	Short	1	11.6.2	Uses Sec 5.15
	11.6.4	BG	Easy	Short	1		
	11.6.5	BG	Easy	Short	2		
	11.6.6	BG	Easy	Short	1		
	11.6.7	BG	Easy	Medium	1		Uses Sec 5.6
	11.6.8	SI	Easy	Short	1		
	11.6.9	SI	Easy	Short	1		Interpolation
	11.6.10	SI	Easy	Short	1		
	P 11.17	BG	Medium	Medium	4	11.18	Uses Sec 5.6
	11.18	SI	Medium	Medium	4	11.17	Uses Sec 5.6
	11.19	BG	Medium	Medium	4	11.20	Uses Secs 5.6 & 5.8
	11.20	SI	Medium	Medium	4	11.19	Uses Secs 5.6 & 5.8
	11.21	BG	Medium	Medium	2	11.22	
	11.22	SI	Medium	Medium	2	11.21	
	11.23	BG	Medium	Medium	1	11.24	Uses Secs 3.6 & 5.6
	11.24	SI	Medium	Medium	1	11.23	Uses Secs 3.6 & 5.6
	11.25	BG	Medium	Medium	1		
11.7	***Venturi Meter***						
	X 11.7.1	BG	Medium	Medium	1	11.7.2	† Assume/verify *C* value.
	11.7.2	SI	Medium	Medium	1	11.7.1	† Assume/verify *C* value.
	11.7.3	SI	Medium	Medium	1		† Assume/verify *C*; uses Sec 7.4
	11.7.4	BG	Medium	Medium	1		† Assume/verify *C*; uses Sec 7.4
	P 11.26	BG	Medium	Long	3	11.27	† Assume/verify *C*; uses Sec 8.24
	11.27	SI	Medium	Long	3	11.26	† Assume/verify *C*; uses Sec 8.24
	11.28	BG	Medium	Long	3	11.29	† Assume/verify *C*; uses Sec 8.24
	11.29	SI	Medium	Long	3	11.28	† Assume/verify *C*; uses Sec 8.24
11.8	***Flow Nozzle***						
	X 11.8.1	SI	Medium	Medium	3	P11.30	†
	P 11.30	SI	Medium	Medium	3	X11.8.1	†
	11.31	BG	Medium	Medium	1		† Assume/verify *K* value; uses Sec 7.4
11.9	***Orifice Meter***						
	X 11.9.1	SI	Medium	Medium	3	P11.32	†
	P 11.32	SI	Medium	Medium	4	X11.9.1	†
	11.33	BG	Medium	Medium	1		† Assume/verify *K* value; uses Sec 7.4
	11.34	BG	Medium	Medium	3		† Interpolation

/cont…

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
11.10	***Flow Measurement of Compressible Fluids***						
	X 11.10.1	BG	Easy	Short	1		† Uses $pv = RT$ (Sec. 2.7)
	11.10.2	BG	Easy	Short	1		Uses $pv = RT$ (Sec. 2.7)
	11.10.3	BG	Easy	Short	1	11.10.4	† Uses $pv = RT$ (Sec. 2.7)
	11.10.4	SI	Easy	Short	1	11.10.3	† Uses $pv = RT$ (Sec. 2.7)
	11.10.5	BG	Easy	Short	1	11.10.6	† Uses $pv = RT$ (Sec. 2.7). Extrapol'n.
	11.10.6	SI	Easy	Short	1	11.10.5	† Uses $pv = RT$ (Sec. 2.7). Extrapol'n.
	11.10.7	BG	Medium	Short	2	11.10.8	Uses Secs. 13.3 & 13.5-6.
	11.10.8	SI	Medium	Short	2	11.10.7	Uses Secs. 13.3 & 13.5-6.
	P 11.35	BG	Medium	Medium	3	11.36	† Uses $pv = RT$ (Sec. 2.7)
	11.36	SI	Medium	Medium	3	11.35	† Uses $pv = RT$ (Sec. 2.7)
	11.37	BG	Easy	Short	1	11.38	†
	11.38	SI	Easy	Short	1	11.37	†
	11.39	SI	Medium	Long	1		† Uses Secs 2.7, 2.11, 4.7, interpolation
	11.40	SI	Hard	Long	4		† Uses Sec 13.8
11.11	***Thin-Plate Weirs***						
	X 11.11.1	BG	V Easy	V Short	1	11.11.2	
	11.11.2	SI	V Easy	V Short	1	11.11.1	
	11.11.3	BG	V Easy	V Short	2	11.11.4	
	11.11.4	SI	V Easy	V Short	2	11.11.3	
	11.11.5	BG	Easy	Medium	3		† Extrapolation.
	P 11.41	BG	Easy	Medium	1		Plots
	11.42	N	Medium	Medium	1		
	11.43	N	Medium	Medium	1		
	11.44	BG	Medium	Medium	1	11.45	† Differentiation.
	11.45	SI	Medium	Medium	1	11.44	† Differentiation.
11.12	***Streamlined Weirs and Free Outfall***						
	X 11.12.1	BG	Easy	Medium	1	11.12.2	T & E
	11.12.2	SI	Easy	Medium	1	11.12.1	T & E
	11.12.3	BG	Medium	Medium	1	11.12.4	T & E
	11.12.4	SI	Medium	Medium	1	11.12.3	T & E
	11.12.5	BG	Medium	Short	1	11.12.6	Uses Sec. 10.9
	11.12.6	SI	Medium	Short	1	11.12.5	Uses Sec. 10.9
	P 11.46	BG	Hard	Long	1		☐ Successive T & E; plots
	11.47	N	Hard	Medium	1		Integration.
11.14	***Sluice Gate***						
	X 11.14.1	BG	Medium	Short	1	11.14.2	
	11.14.2	SI	Medium	Short	1	11.14.1	
	P 11.48	BG	Medium	Medium	1	11.49	
	11.49	SI	Medium	Medium	1	11.48	
	11.50	SI	Medium	Medium	3		T & E for V

Chapter 11
FLUID MEASUREMENTS

Sec. 11.1: Measurement of Fluid Properties – *Exercises (10)*

11.1.1 *A small object has a volume of 0.0070 ft³ and weighs 1.50 lb in air and 1.18 lb in a liquid. What is the density of the liquid?*

BG

Buoyant force = 1.50 − 1.18 = 0.32 lb

γ = 0.32/0.007 = 45.7 lb/ft³ ; Eq. 2.1: $\rho = \gamma/g$ = 45.7/32.2 = 1.420 slugs/ft³ ◄

11.1.2 *A small object has a volume of 300 mL and weighs 15.50 N in air and 10.50 N in a liquid. What is the density of the liquid?*

SI

Buoyant force = 15.50 − 10.50 = 5.00 N

$\gamma = (5.00\text{ N})/(300 \times 10^{-6}\text{ m}^3) = 16\,670\text{ N/m}^3 = 16\,670\text{ kg}\cdot\text{m}/(\text{s}^2\cdot\text{m}^3)$

Eq. 2.1: $\rho = \gamma/g = [16670\text{ kg}\cdot\text{m}/(\text{s}^2\cdot\text{m}^3)]/(9.81\text{ m/s}^2) = 1699\text{ kg/m}^3$ ◄

11.1.3 *Carbon tetrachloride (s = 1.59) is placed in an open U tube. A liquid is poured into one of the legs of the tube. A column of this liquid 19.4 in high balances a carbon tetrachloride column 15.0 in high. What is the specific weight of the liquid? In Sec. 11.1 it is mentioned that this method will give only approximate values. Why is this so?*

BG

Eq. 3.4: $p = \gamma_1 h_1 = \gamma_2 h_2$; $19.4\gamma = 15(1.59\times 62.4)$; γ = 76.7 lb/ft³ ◄

Results are only approximate because of difficulty with menisci. ◄

11.1.4 *Carbon tetrachloride (s = 1.59) is placed in an open U tube. A liquid is poured into one of the legs of the tube. A column of this liquid 392 mm high balances a carbon tetrachloride column 300 mm high. What is the specific weight of the liquid? In Sec. 11.1 it is mentioned that this method will give only approximate values. Why is this so?*

SI

Eq. 3.4: $p = \gamma_1 h_1 = \gamma_2 h_2$; $0.392\gamma = 0.30(1.59\times 9810)$; γ = 11 940 N/m³ ◄

Results are only approximate because of difficulty with menisci. ◄

11.1.5 *A rotational viscometer is constructed of two concentric cylinders of height 15.0 in. The OD of the inner cylinder is 4.00 in, and the ID of the outer cylinder is 4.10 in. When a torque of 6.0 ft·lb is applied to the outer cylinder, it was found to rotate at 1 revolution per 3.8 sec. Find the (absolute) viscosity of the fluid. Neglect mechanical friction.*

BG

Gap distance = (4.10 − 4.00)/2 = 0.05 inches = 0.004 17 ft

Peripheral velocity = ωr = $[1(2\pi)/3.8]4.10/(2 \times 12)$ = 0.282 fps ; du/dy = 0.282/0.004 17 = 67.8 fps/ft

Eq. 2.9: $\tau = \mu\, du/dy$; $F = \tau A = (\mu\, du/dy)\pi DL$; $T = FD/2 = (\mu\, dy/dy)\pi D^2 L/2$

∴ $6.0 = \mu(67.8)\pi(4.10/12)^2(15/12)/2 = 15.54\mu$; μ = 0.386 lb·sec/ft² ◄

11.1.6 *A rotational viscometer is constructed of two concentric cylinders of height 300 mm. The OD of the inner cylinder is 100.0 mm, and the ID of the outer cylinder is 102.0 mm. When a torque of 8.0 N·m is applied to the outer cylinder, it was found to rotate at 1 revolution per 4.0 s. Find the (absolute) viscosity of the fluid. Neglect mechanical friction.*

SI

Gap distance = (10.20 - 10.00)/2 = 0.10 cm = 0.001 00 m

Peripheral velocity = ωr = $[1(2\pi)/4.0]10.2/(2 \times 100)$ = 0.0801 m/s

du/dy = 0.0801/0.00100 = 80.1 m/s per m

Eq. 2.9: $\tau = \mu\, du/dy$; $F = \tau A = (\mu\, du/dy)\pi DL$; $T = FD/2 = (\mu\, dy/dy)\pi D^2L/2$

$\therefore$ $8.0 = \mu(80.1)\pi(0.102)^2 0.30/2$; $\mu = 20.4\ \text{N·s/m}^2$ ◄

11.1.7 *A tube viscometer similar to the one of Fig. 11.1 has a diameter of 0.0420 in and a length of 3.10 in. A volume of 60 mL flowed through the tube in 128.7 sec, causing the vertical distance from the liquid surface in the reservoir to the tube outlet to change from 10.00 to 9.50 in. Find the kinematic viscosity of the liquid.*

B

Eq. 11.2: $\nu = \dfrac{\pi(0.0420/12)^4(9.75/12)}{128[60/(2.54 \times 12)^3](3.10/12)}(32.2)128.7 = 0.000\,0227\ \text{ft}^2/\text{sec}$ ◄

Check to confirm that flow is laminar: V = 1.711 fps, **R** = 265, $\therefore$ OK.

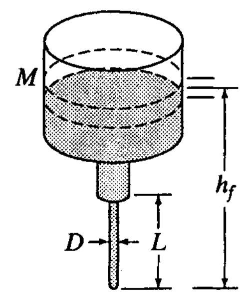


Figure 11.1

11.1.8 *If 60°F water flows through a tube-type viscometer in 120 sec, how long will it take the same volume of 100°F water to pass though the same viscometer?*

BG

Table A.1 for water: $\nu = 1.217 \times 10^{-5}\ \text{ft}^2/\text{sec}$ at 60°F, $0.739 \times 10^{-5}\ \text{ft}^2/\text{sec}$ at 100°F

Assume laminar flow (**R** < 2000) in both cases.

From Eq. 11.2, $\nu \propto t$: $t_{100°} = t_{60°}(\nu_{100°}/\nu_{60°}) = 120(0.739/1.217) = 72.9$ sec ◄

11.1.9 *If 15°C water flows through a tube-type viscometer in 85 s, how long will it take the same volume of 40°C water to pass though the same viscometer?*

SI

Table A.1 for water: $\nu = 1.139 \times 10^{-6}\ \text{m}^2/\text{s}$ at 15°C, $0.658 \times 10^{-6}\ \text{m}^2/\text{s}$ at 40°C

Assume laminar flow (**R** < 2000) in both cases.

From Eq. 11.2, $\nu \propto t$: $t_{40} = t_{15}(\nu_{40}/\nu_{15}) = 85(0.658/1.139) = 49.1$ s ◄

11.1.10 *A lead sphere (s = 11.4) with a diameter of 0.25 in falls through oil (s = 0.86) in a 2.25-in-diameter cylinder. If the sphere moves with a constant velocity of 0.150 fps, what is the absolute viscosity of the oil? Check* **R** *to see if it is less than 1.0.*

BG

Eq. 11.4: $\dfrac{V}{V_t} = 1 + \dfrac{9(0.25)}{4(2.25)} + \left(\dfrac{1}{4}\right)^2 = 1.313$; $V = 0.150(1.313) = 0.1969$ fps

Eq. 11.3: $\mu = \dfrac{(0.25/12)^2(11.4 - 0.86)62.4}{18(0.1969)} = 0.0806\ \text{lb·sec/ft}^2$ ◄

Check: Eq. 7.6: $\mathbf{R} = \dfrac{(0.25/12)(0.150)(1.940 \times 0.86)}{0.0806} = 0.0647$ ◄

Sec. 11.1: Measurement of Fluid Properties – *Problems 11.1–11.8*

11.1 *A hydrometer (Figs. 3.21 and 3.22a) is made in the form of an 8-in-long cylinder with a diameter of 3/8 in. Attached to the end of the cylinder is a 1.2-in-diameter sphere. The entire device weighs 0.9 oz. What range of specific gravities can be measured with this device? When floating in a liquid with a density of 1.80 slugs/ft³, how much of the cylinder will project above the surface?*

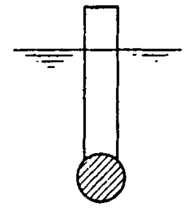


Figure 3.22a

BG

$$V_{sphere} + V_{rod} = \frac{\pi D^3}{6} + \frac{\pi}{4}d^2 h = \frac{\pi(1.2/12)^3}{6} + \frac{\pi}{4}\left(\frac{3/8}{12}\right)^2\left(\frac{8}{12}\right)$$

$$= 0.000\,524 + 0.000\,511 = 0.001\,035\ \text{ft}^3$$

$$\gamma_{min} = \frac{W}{V_{tot}} = \frac{0.9/16\ \text{lb}}{0.001\,035\ \text{ft}^3} = 54.4\ \text{lb/ft}^3\ ; \quad s_{min} = \frac{\gamma_{min}}{\gamma_{water}} = \frac{54.4}{62.4} = 0.871 \quad ◄$$

$$\gamma_{max} = \frac{W}{V_{sphere}} = \frac{0.9/16}{0.000\,524} = 107.4\ ; \quad s_{max} = \frac{\gamma_{max}}{\gamma_{water}} = \frac{107.4}{62.4} = 1.722 \quad ◄$$

$s = 1.80/1.940 = 0.928$; so $(s_{min} = 0.871) < (s = 0.928) < (s_{max} = 1.722)$

$\gamma = \rho g = 1.80(32.2) = 58.0$ pcf

Displaced $V = W/\gamma = (0.9/16)/58.0 = 0.000\,970\ \text{ft}^3 \approx 0.000\,524 + (L/8)0.000\,511$

Immersed $L = 6.99$ in ; ∴ Length projecting $= 8 - 6.99 = 1.01$ in ◄

11.2 *A hydrometer (Figs. 3.21 and 3.22a) is made in the form of a 200-mm-long cylinder with a diameter of 6 mm. Attached to the end of the cylinder is a 22-mm-diameter sphere. The mass of the entire device is 8 g. What range of specific gravities can be measured with this device? When floating in a liquid having a density of 900 kg/m³, how much of the cylinder will project above the surface?*

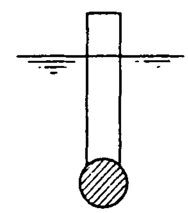


Figure 3.22a

SI

sphere: $V_s = \frac{\pi}{6}D^3 = \frac{\pi}{6}(22/1000)^3 = 5.58\times10^{-6}\ \text{m}^3$

cylinder: $V_c = \frac{\pi}{4}d^2 h = \frac{\pi}{4}(6/1000)^2(20/100) = 5.65\times10^{-6}\ \text{m}^3$

$V_{total} = 11.23\times10^{-6}\ \text{m}^3$

Device mass $= m = 8$ g $= 0.008$ kg

$$\rho_{min} = \frac{m}{V_{total}} = \frac{0.008\ \text{kg}}{11.23\times10^{-6}\ \text{m}^3} = 712\ \text{kg/m}^3\ ; \quad s_{min} = \frac{\rho_{min}}{\rho_{water}} = \frac{712}{1000} = 0.712 \quad ◄$$

$$\rho_{max} = \frac{m}{V_s} \approx \frac{0.008\ \text{kg}}{5.58\times10^{-6}\ \text{m}^3} = 1435\ \text{kg/m}^3\ ; \quad s_{max} = \frac{\rho_{max}}{\rho_{water}} = \frac{1435}{1000} = 1.435 \quad ◄$$

For $\gamma = 900\ \text{kg/m}^3$, displaced $V = W/\gamma = 0.008/900 = 8.89\times10^{-6}\ \text{m}^3$

$= V_s + (L/h)V_c = 5.58\times10^{-6} + (L/200)5.65\times10^{-6}$; immersed $L = 117.2$ mm

Length projecting above surface $= h - L = 200 - 117.2 = 82.8$ mm ◄

11.3 *Fifty milliliters of 70°F water flows through a certain tube-type viscometer in 53.7 sec, whereas the same volume of 50°F oil (s = 0.90) flows through it in 850 sec. Find the absolute viscosity of the oil.*

BG

Table A.1 for water at 70° F: $\nu = 1.059\times10^{-5}$ ft²/sec

Assume laminar flow (**R** < 2000) in both cases. From Eq. 11.2, $\nu \propto t$ or $\nu = kt$

Oil at 50°F: $\nu_{oil} = (k)850$; water at 70°F: $1.059 \times 10^{-5} = (k)53.7$

Dividing to eliminate k: $\nu_{oil} = 1.059 \times 10^{-5}(850/53.7) = 1.676 \times 10^{-4}$ ft²/sec

Eq. 2.11 for the oil: $\mu = \rho\nu = (1.940\times0.90)1.676\times10^{-4} = 2.93\times10^{-4}$ lb·sec/ft² ◄

11.4 *A liquid (ρ = 889 kg/m³) under a head of 500 mm flows at a steady rate of 30 mL/min through a glass tube of diameter 2.0 mm and length 4.5 m. What is the absolute viscosity and the kinematic viscosity of the liquid? Express the answers in stokes and poises. Confirm by calculation that the flow is laminar.*

SI

Q = 30 mL/min = 30 × 10³ mm³/(60 s) = 500 mm³/s

$V = Q/A = Q/(\pi r^2) = (500 \text{ mm}^3/\text{s})/(\pi 1^2 \text{ mm}^2) = 159.2 \text{ mm/s} = 0.1592 \text{ m/s}$

Sec 11.1: For such a small diameter tube, assume laminar flow.

Eq. 8.28: $0.5 = 32\nu\dfrac{4.5}{9.81(0.002)^2}0.1592$ from which $\nu = 0.856 \times 10^{-6}$ m²/s

Inside cover, Sec. 2.11: $\nu = (0.856 \times 10^{-6} \text{ m}^2/\text{s})(10^4 \text{ St per m}^2/\text{s}) = 0.008\,56 \text{ St} = 0.856 \text{ cSt}$ ◄

Eq. 2.11: $\mu = \rho\nu = (889 \text{ kg/m}^3)(0.856\times10^{-6} \text{ m}^2/\text{s}) = 761\times10^{-6}$ kg/(m·s) (or N·s/m²)

Using inside cover: $\mu = (761 \times 10^{-6} \text{ N·s/m}^2)(10 \text{ P per N·s/m}^2) = 7.61 \times 10^{-3}$ P ◄

Check assumption: $\mathbf{R} = \dfrac{DV}{\nu} = \dfrac{0.002 \text{ m}(0.1592 \text{ m/s})}{0.856\times10^{-6} \text{ m}^2/\text{s}} = 372 < 2000$ so flow is laminar. ◄

11.5 *A glass bead (s = 2.60) with a diameter of 16 mm falls through a liquid (s = 1.59) in a 100-mm-diameter tube. If the bead moves with a constant velocity of 145 mm/min, what is the absolute viscosity and the kinematic viscosity of the liquid?*

SI

Eq. 11.4: $\dfrac{V}{V_t} = 1 + \dfrac{9\times16}{4\times100} + \left(\dfrac{9\times16}{4\times100}\right)^2 = 1.490$; $V = 1.490(0.145/60) = 0.003\,60$ m/s

Eq. 11.3: $\mu = \dfrac{(0.016)^2 9810(2.60 - 1.59)}{18(0.003\,60)} = 39.1$ N·s/m² ◄

Eq. 2.11: $\nu = \mu/\rho = 39.1/(1.59\times1000) = 0.0247$ m²/s ◄

Check solution validity: Eq. 7.6: $\mathbf{R} = \dfrac{0.016(0.003\,60)}{0.0247} = 0.002\,34$

R < 1, so these answers are O.K. [If **R** > 1, we must use data from Fig. 9.10.]

11.6 *A 3.6-in-diameter tube contains oil (s = 0.93) with an absolute viscosity of 0.0065 lb·sec/ft². What is the maximum size of steel sphere (s = 7.84) that will satisfy Stokes' law when released into this oil. What will be the fall velocity of this sphere?*

BG

Sec 9.7: When D is max for Stokes' Law validity: $\mathbf{R} = \dfrac{DV_t(0.93\times 1.94)}{0.0065} = 1$; $V_t = \dfrac{1}{278D}$

Eq. 11.3: $\mu = 0.0065 = \dfrac{D^2(7.84-0.93)62.4}{18V}$; $V = 3685D^2$

Eq. 11.4: $\dfrac{V}{V_t} = 1 + \dfrac{9D}{4(3.6/12)} + \left[\dfrac{9D}{4(3.6/12)}\right]^2 = 1 + 7.5D + (7.5D)^2$

Solution of these three equations (by eliminating V and V_t) yields $1.023\times 10^6 D^3 = 1 + 7.5D + (7.5D)^2$

By trials: $D = 0.01019$ ft $= 0.1223$ inches ◄ ; $V = 0.383$ fps ◄

11.7 *A 100-mm-diameter tube contains oil (s = 0.9) with an absolute viscosity of 0.25 N·s/m². What is the maximum size of steel sphere (s = 7.8) that will satisfy Stokes' law when released into this oil. What will be the fall velocity of this sphere?*

SI

Sec 9.7: When D is max for Stokes' Law validity: $\mathbf{R} = \dfrac{DV_t(0.9\times 1000)}{0.25} = 1$; $V_t = \dfrac{1}{3600D}$

Eq. 11.3: $\mu = 0.25 = \dfrac{D^2(7.8-0.9)9810}{18V}$; $V = 15\,040D^2$

Eq. 11.4: $\dfrac{V}{V_t} = 1 + \dfrac{9D}{4(0.1)} + \left[\dfrac{9D}{4(0.1)}\right]^2 = 1 + 22.5D + (22.5D)^2$

Solution of the above three equations (by eliminating V and V_t) yields

$54.2\times 10^6 D^3 = 1 + 22.5D + (22.5D)^2$

By trials: $D = 0.00270$ m $= 2.70$ mm ◄ ; $V = 0.1097$ m/s ◄

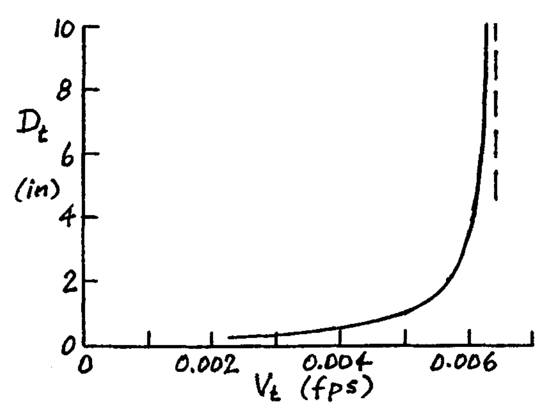
11.8 *An 0.10-in-diameter sphere has a velocity of 0.005 fps when falling through liquid in a 1.0-in-diameter tube. What will be its fall velocities through the same liquid in tubes of diameter 0.50, 2.0, 4.0, and 10.0 in. Plot fall velocity vs tube diameter.*

BG

Eq. 11.4: $\dfrac{V}{0.005} = 1 + \dfrac{9(0.1)}{4(1)} + \left[\dfrac{9(0.1)}{4(1)}\right]^2$; $V = 0.005(1 + 0.225 + 0.0506) = 0.00638$ fps

Eq. 11.4: $\dfrac{0.00638}{V_t} = 1 + \dfrac{9(0.1)}{4D_t} + \left[\dfrac{9(0.1)}{4D_t}\right]^2$

D_t (inches)	V_t (fps)	
0.5	0.003 87	◄
1.0	0.005 00	
2.0	0.005 66	◄
4.0	0.006 04	◄
10.0	0.006 25	◄
∞	0.006 38	



Sec. 11.3: Measurement of Velocity with Pitot Tubes – *Exercises (7)*

11.3.1 *In Fig. S11.1 kerosene (s = 0.81) is flowing. The pressure gages at A and B read 70 and 150 N/m². Find the velocity u assuming C_I = 1.0.*

SI

γ = 0.81(9.81) = 7.95 kN/m³

From Eq. 11.5: $\gamma(V^2/2g)$ = 150 – 70 = 80 N/m²

V^2 = 2(9.81)(80/7950) = 0.1974 ; $V = \sqrt{0.1974}$ = 0.444 m/s ◄

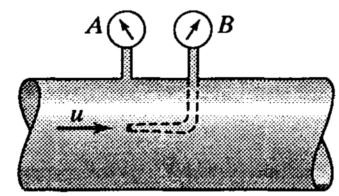


Figure S11.1

11.3.2 *A pitot tube and a wall piezometer tube are installed in a pipe carrying 60°F water. The tubes are connected to a water-mercury manometer which registers a differential of 3.4 in. Given that C_I = 0.98, find the flow velocity approaching the tube.*

BG

Eqs. 11.7 and 3.12: $u = C_I\sqrt{2g[(s_M/s_F) - 1]MR} = 0.98\sqrt{2(32.2)[(13.56/1.00) - 1]3.4/12}$ = 14.84 fps ◄

11.3.3 *A pitot tube and a wall piezometer tube are installed in a pipe carrying 15°C water. The tubes are connected to a water-mercury manometer which registers a differential of 78 mm. Given that C_I = 0.97, find the flow velocity approaching the tube.*

SI

Eqs. 11.7 and 3.12: $u = C_I\sqrt{2g[(s_M/s_F) - 1]MR} = 0.97\sqrt{2(9.81)[(13.56/1.00) - 1]0.078}$ = 4.25 m/s ◄

11.3.4 *The fluids of Exer. 11.3.2 are reversed so that mercury (s = 13.56) is flowing in the pipe and water is the gage fluid (with the manometer now inverted). With the same gage differential, find the flow velocity of the mercury.*

Exer. 11.3.2: A pitot tube and a wall piezometer tube are installed in a pipe carrying 60°F water. The tubes are connected to a water-mercury manometer which registers a differential of 3.4 in. Given that C_I = 0.98, find the flow velocity approaching the tube.

BG

Eqs. 11.7 and 3.13: $u = C_I\sqrt{2g[1 - (s_M/s_F)]MR} = 0.98\sqrt{2(32.2)[1 - (1.00/13.56)]3.4/12}$ = 4.03 fps ◄

11.3.5 *The fluids of Exer. 11.3.3 are reversed so that mercury (s = 13.56) is flowing in the pipe and water is the gage fluid (with the manometer now inverted). With the same gage differential, find the flow velocity of the mercury.*

Exer. 11.3.2: A pitot tube and a wall piezometer tube are installed in a pipe carrying 60°F water. The tubes are connected to a water-mercury manometer which registers a differential of 3.4 in. Given that C_I = 0.98, find the flow velocity approaching the tube.

SI

Eqs. 11.7 and 3.13: $u = C_I\sqrt{2g[1 - (s_M/s_F)]MR} = 0.97\sqrt{2(9.81)[1 - (1.00/13.56)]0.078}$ = 1.156 m/s ◄

11.3.6 *A pitot-static tube (C_I = 0.985) is connected to an inverted U tube containing oil (s = 0.875). Find the velocity of flowing water if the manometer reading is 3.6 in.*

BG

From Eq. 3.13: $\Delta p/\gamma$ = [1 – (0.875/1.00)]3.6/12 = 0.0375 ft

Eq. 11.7: $u = 0.985\sqrt{2(32.2)0.0375}$ = 1.531 fps ◄

11.3.7 *A pitot-static tube (C_I = 0.992) is connected to an inverted U tube containing oil (s = 0.91). Find the velocity of flowing water if the manometer reading is 127 mm.*

SI

From Eq. 3.13: $\Delta p/\gamma$ = [1 – (0.91/1.00)]0.127 = 0.011 43 m

Eq. 11.7: $u = 0.992\sqrt{2(9.81)0.011\,43)}$ = 0.470 m/s ◄

Sec. 11.3: Measurement of Velocity with Pitot Tubes -- *Problems 11.9–11.14*

11.9 *In Fig. S11.1, pressure gages A and B read 11.0 psi and 12.0 psi, respectively. If 60°F air is flowing, what is the velocity u? Atmospheric pressure is 27.0 inHg. Assume C_I = 1.0 and neglect compressibility effects.*

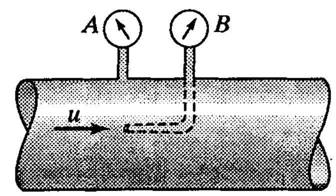


Figure S11.1

BG

$p_{abs} = p_{atmos} + p_{gage} = 14.7(27.0/29.92) + 11.0 = 24.3$ psia

Eq. 2.5: $\gamma = gp/RT = 32.2(24.3\times144)/[1715(460+60)] = 0.1262$ lb/ft³

$\Delta p = p_0 - p = (12.0 - 11.0)144 = 144$ psf

From Eq. 11.5: $u^2/2g = \Delta p/\gamma = 144/0.1262 = 1141$ ft ; $u = \sqrt{2(32.2)1141} = 271$ fps ◄

11.10 *Suppose the two pressure gages of Prob. 11.9 are replaced by a differential manometer containing water. What is the reading on the manometer?*

Prob. 11.9: The two gages are at A and B (Fig. S11.1). Air is flowing, γ = 0.1262 pcf; $\Delta p/\gamma$ = 1142 ft of air.

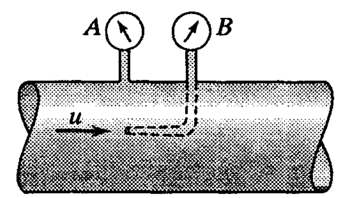


Figure S11.1

BG

From Eq. 3.12: $\dfrac{\Delta p}{\gamma} = \left(\dfrac{s_M}{s_F} - 1\right)\dfrac{R_m(\text{in})}{12}$; $1141 = \left(\dfrac{62.4}{0.1262} - 1\right)\dfrac{R_m}{12}$

$R_m = 27.7$ inches ◄

11.11 *The pitometers in Fig. P11.11 are connected to a mercury manometer, which reads 5.0 in. The velocity of flowing carbon tetrachloride (s = 1.59) is known to be 9.37 fps. What is C_I for this instrument?*

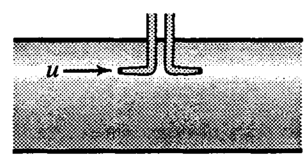


Figure P11.11

BG

Eq. 3.12: $\dfrac{\Delta p}{\gamma} = \left(\dfrac{13.56}{1.59} - 1\right)\dfrac{5.0}{12} = 3.14$ ft

Eq. 11.7: $9.37 = C_I\sqrt{2(32.2)3.14}$; $C_I = 0.659$ ◄

11.12 *The pitometers in Fig. P11.11 are connected to a mercury manometer, which reads 120 mm. The velocity of flowing carbon tetrachloride (s = 1.59) is known to be 2.76 m/s. What is C_I for this instrument?*

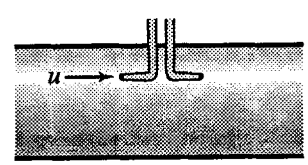


Figure P11.11

SI

Eq. 3.12: $\dfrac{\Delta p}{\gamma} = \left(\dfrac{13.56}{1.59} - 1\right)\dfrac{12}{100} = 0.903$ m

Eq. 11.7: $2.76 = C_I\sqrt{2(9.81)0.903}$; $C_I = 0.656$ ◄

11.13 *Given air at 60°F and standard atmospheric pressure is flowing in Fig. P11.11. The pitometers are attached to a manometer containing a liquid with s = 0.90. Plot the velocity u versus the manometer reading assuming C_I = 0.93.*

BG

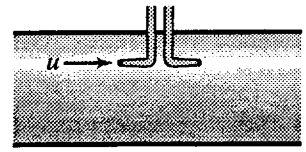


Figure P11.11

Table A.2: Density of air = 2.37×10^{-3} slug/ft^3.

Ratio of densities $= \dfrac{0.90(1.940)}{2.37 \times 10^{-3}} = 737$

Thus 1 foot of Manometer Reading (R_m) is equivalent to 737 ft of air, and $\Delta p/\gamma = 737(R_m)$

Eq. 11.7: $u = C_I\sqrt{2g(\Delta p/\gamma)} = 0.93\sqrt{2(32.2)737(R_m)} = 203\sqrt{R_m}$

R_m (in)	R_m (ft)	u (fps)
1.0	0.083	58.5
5.0	0.416	130.8
10.0	0.833	184.9
20.0	1.667	262

At larger manometer readings the effect of compressibility must be considered, see Eq. 13.31.

11.14 *Water at 90°F flows through a smooth, 12-in-diameter pipe. A Prandtl tube is placed on the center line of the pipe. The reading on a differential manometer attached to this Prandtl tube is 8 in of carbon tetrachloride (s = 1.59). What is the flow rate?*

BG

Table A.1 for water at 90°F: $\nu = 0.826 \times 10^{-5}$ ft^2/sec. Sec. 11.3 for Prandtl tube: $C_I = 1$

Eq. 3.12: $\Delta p/\gamma = [(1.59/1.00) - 1]8/12 = 0.393$ ft

Eq. 11.7 on centerline: $u_{max} = C_I\sqrt{2g(\Delta p/\gamma)} = 1\sqrt{2(32.2)0.393} = 5.03$ fps

Assume V = 5 fps, then $\mathbf{R} = 1(5)/(0.826 \times 10^{-5}) = 6.05 \times 10^5$

Fig. 8.11 for e = 0: f = 0.0127 ; Eq. 8.43: $\dfrac{V}{u_{max}} = \dfrac{1}{1 + 1.326\sqrt{f}} = \dfrac{1}{1.149}$

$V = u_{max}/1.149 = 5.03/1.149 = 4.38$ fps (**R** = 5.30 × 10^5, f is now 0.0130, little change)

$Q = (\pi/4)(1^2)4.38 = 3.44$ cfs ◄

Sec. 11.5: Measurement of Discharge – ***Problems*** *11.15–11.16*

11.15 *Assume the pipe diameter in Fig. 11.10 is 16 in, and the flow is laminar, i.e., $u = u_{max} - kr^2$, when u_{max} = 0.1 fps. Divide the circle into concentric rings with radii of 2, 4, 6, and 8 in, and compute the flow rate by the method of Fig. 11.10 by taking the velocities at radii of 1, 3, 5, and 7 in as representative of the rings. Compare the result with that obtained by integration.*

BG

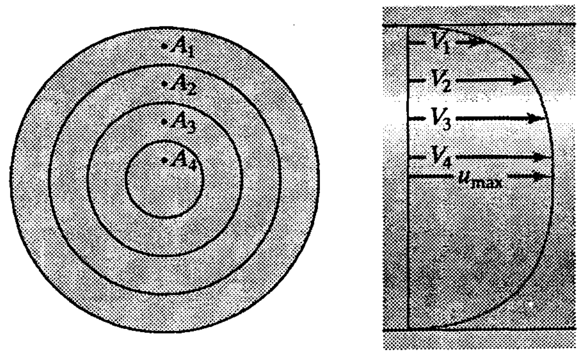


Figure 11.10

$u = 0.1 - kr^2$; when r = 8 inches, u = 0 fps

$\therefore \quad 0 = 0.1 - k(8)^2, \quad k = 1/640$

Hence $u = 0.1 - r^2/640$

$\bar{r}$ (in)	A (in²)	u (fps)	$\Delta Q = Au$
1	$\pi 2^2 = 4\pi$	0.0984	1.237
3	$\pi(4^2 - 2^2) = 12\pi$	0.0859	3.24
5	$\pi(6^2 - 4^2) = 20\pi$	0.0609	3.83
7	$\pi(8^2 - 6^2) = 28\pi$	0.0234	2.06
			Σ = 10.37 ft·in²/sec = 0.0720 cfs ◄

By integration: Eq. 4.3: $Q = \int_0^8 u 2\pi r dr = 2\pi \int_0^8 (0.1r - r^3/640) dr$

$Q = 2\pi[0.1r^2/2 - r^4/2560]_0^8 = 3.2\pi$ ft·in²/sec = 0.0698 cfs ◄

11.16 *Assume the pipe diameter in Fig. 11.10 is 480 mm, and the flow is laminar, i.e., $u = u_{max} - kr^2$, when u_{max} = 30 mm/s. Divide the circle into concentric rings with radii of 60, 120, 180, and 240 mm, and compute the flow rate by the method of Fig. 11.10 by taking the velocities at radii of 30, 90, 150, and 210 mm as representative of the rings. Compare the result with that obtained by integration.*

SI

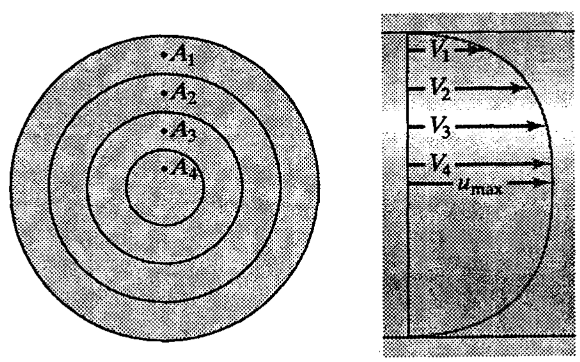


Figure 11.10

$u = 0.03 - kr^2$; when r = 0.24 m, u = 0 m/s

$\therefore \quad 0 = 0.03 - k(0.24)^2, \quad k = 0.521$

Hence $u = 0.03 - 0.521r^2$

$\bar{r}$ (m)	A (m²)	u (m/s)	$\Delta Q = Au$
0.03	$\pi(0.06)^2 = 0.003\,60\,\pi$	0.0295	0.000 334
0.09	$\pi(0.12^2 - 0.06^2) = 0.010\,80\pi$	0.0258	0.000 825
0.15	$\pi(0.18^2 - 0.12^2) = 0.018\,00\pi$	0.0183	0.001 034
0.21	$\pi(0.24^2 - 0.18^2) = 0.0252\pi$	0.007 03	0.000 557
			Σ = 0.002 80 m³/s ◄

By integration: Eq. 4.3: $Q = \int_0^{0.24} u 2\pi r dr = 2\pi \int_0^{0.24} (0.03r - 0.521r^3) dr$

$Q = 2\pi[0.03r^2/2 - 0.521r^4]_0^{0.24} = 0.002\,71$ m³/s ◄

Sec. 11.6: Orifices, Nozzles, and Tubes -- *Exercises (10)*

11.6.1 *Water issues from a circular 5-in-diameter orifice under a head of 45 ft. If 534 ft³ are discharged in 2 min, what is the coefficient of discharge? If the diameter at the vena contracta is measured to be 3.93 in, what is the coefficient of contraction and what is the coefficient of velocity?*

BG

Eqs. 11.9 and 11.11: $Q = C_d A_o \sqrt{2gh}$; $534/(2\times 60) = C_d \dfrac{\pi}{4}\left(\dfrac{5}{12}\right)^2 \sqrt{2(32.2)45}$; $C_d = 0.606$ ◄

Sec 11.6: $C_c = A/A_o = (3.93/5.00)^2 = 0.618$ ◄

Eq. 11.11: $C_v = C_d/C_c = 0.606/0.618 = 0.981$ ◄

11.6.2 *A jet discharges 0.131 cfs from a 1.25-in-diameter orifice in a vertical plane under a head of 10 ft. The jet center line passes through the point 11.47 ft horizontally from the vena contracta and 3.5 ft below the center of the orifice. Find the coefficients of (a) discharge, (b) velocity, and (c) contraction.*

BG

Eq. 5.45: $V = x\sqrt{g/2z} = 11.47\sqrt{32.2/(2\times 3.5)} = 24.6$ fps $= C_v\sqrt{2gh} = C_v\sqrt{2(32.2)10}$; $C_v = 0.969$ ◄

$A_o = (\pi/4)(1.25/12)^2 = 0.008\,52$ ft² ; Eq. 11.9: $0.131 = (0.008\,52)\sqrt{2(32.2)10}$; $C_d = 0.606$ ◄

From Eq. 11.11: $C_c = C_d/C_v = 0.606/0.969 = 0.625$ ◄

Alternatively: $A = Q/V = 0.131/24.6 = 0.005\,33$; $C_c = A/A_o = 0.005\,33/0.008\,52 = 0.625$ ◄

11.6.3 *A jet discharges 5.19 L/s from a 35-mm-diameter orifice in a vertical plane under a head of 4 m. The jet center line passes through the point 4.28 m horizontally from the vena contracta and 1.2 m below the center of the orifice. Find the coefficients of (a) discharge, (b) velocity, and (c) contraction.*

SI

Eq. 5.45: $V = x\sqrt{g/2z} = 4.28\sqrt{9.81/(2\times 1.2)} = 8.65$ m/s $= C_v\sqrt{2gh} = C_v\sqrt{2(9.81)4}$; $C_v = 0.977$ ◄

$A_o = (\pi/4)(0.035)^2 = 0.000\,962$ m² ; Eq. 11.9: $0.005\,19 = C_d(0.000\,962)\sqrt{2(9.81)4}$; $C_d = 0.609$ ◄

From Eq. 11.11: $C_c = C_d/C_v = 0.609/0.977 = 0.623$ ◄

Alternatively: $A = Q/V = 0.005\,19/8.65 = 0.000\,600$; $C_c = A/A_o = 0.000\,600/0.000\,962 = 0.623$ ◄

11.6.4 *A 4-in-diameter water jet discharges from a nozzle whose velocity coefficient is 0.96. The pressure in the 9-in-diameter pipe is 11 psi. Assuming the jet does not contract, what is the velocity at the tip of the nozzle? What is the flow rate?*

BG

Eq. 11.12: $V_{2i} = \dfrac{1}{\sqrt{1-(4/9)^4}}\sqrt{2(32.2)\dfrac{11(144)}{62.4}} = 41.2$ fps

$V_2 = C_v(V_{2i}) = 0.96(41.2) = 39.6$ fps ◄ $Q = A_2V_2 = (\pi/4)(4/12)^2 39.6 = 3.46$ cfs ◄

11.6.5 *In Fig X11.6.5 the area A is twice area A_o. If the diverging tube discharges water when h = 6 ft, find (a) the velocity at the throat; (b) the pressure head at the throat. Neglect all friction losses.*

BG

$V = \sqrt{2gh} = \sqrt{2(32.2)6} = 19.66$ fps

(*a*) Continuity: $V_o = V(A/A_o) = 19.66(2) = 39.3$ fps ◄

(*b*) Given $h_L = 0$. So Eq. 5.29 from surface to throat:

$$0 + 6.00 + 0 = \frac{p_o}{\gamma} + 0 + \frac{V_o^2}{2g}$$

$$\text{so}\quad \frac{p_o}{\gamma} = 6.00 - \frac{V_o^2}{2g} = 6.00 - \frac{39.3^2}{2(32.2)} = -18.00 \text{ ft}$$ ◄

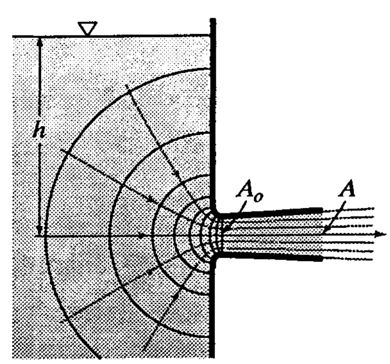


Figure X11.6.5

11.6.6 *For the same data as Exer. 11.6.5, with a barometric pressure of 14.8 psia and a water temperature of 90°F, what is the maximum value of h at which the tube will flow full? What will happen if h is made greater than this?*

Exer. 11.6.5: $A = 2A_o$, neglect friction.

BG

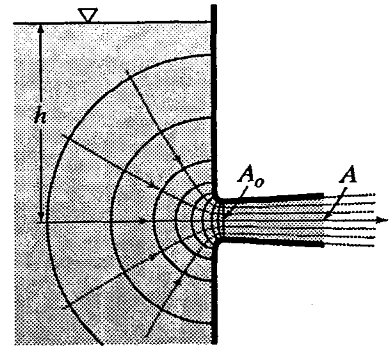


Figure X11.6.5

Table A.1 for water at 90°F: γ = 62.11 pcf, p_v = 0.70 psia

Given h_L - 0. Eq. 5.29 from throat to exit:

$$\frac{p_o}{\gamma} + 0 + \frac{V_o^2}{2g} = \frac{p_a}{\gamma} + 0 + \frac{V_x^2}{2g} ;$$

but $A_x = 2A_o$ (given), so from continuity, $V_o = 2V_x$.

When $h = h_{max}$ for full flow, $p_o = (p_o)_{min} = p_v$ (abs), so

$$\therefore \quad \frac{p_v}{\gamma} + \frac{4V_x^2}{2g} = \frac{p_a}{\gamma} + \frac{V_x^2}{2g} ; \quad \frac{3V_x^2}{2g} = \frac{p_a - p_v}{\gamma} = \frac{(14.8 - 0.70)144}{62.11} = 32.7 \text{ ft}$$

Eq. 5.29 from water surface to exit: $\frac{p_a}{\gamma} + h_{max} + 0 = \frac{p_a}{\gamma} + 0 + \frac{V_x^2}{2g}$

$\therefore\ h_{max} = V_x^2/2g = 32.7/3 = 10.90$ ft ◄

If $h > 10.90$ ft, the jet will spring free from the diverging tube, so that $p_o = p_a$, and the discharge will decrease. ◄

11.6.7 *For the rounded entrance and tube flowing full as in Fig. X11.6.5, C_c = 1.0 for both the throat and the exit, and thus $C_v = C_d$ for both sections. For the throat, assume the value of C_v as given for Fig. 11.16a, and assume that for the tube as a whole the discharge coefficient applied to the exit end is 0.72. Find the velocity at the throat and the pressure head at the throat if h = 6 ft. Compare your answer with Exer. 11.6.5, which neglected friction.*

Fig. 11.16a: C_v = 0.98. Exer. 11.6.5 neglecting friction: Ans = V_o = 39.3 fps, p_o/γ = −18.00 ft.

BG

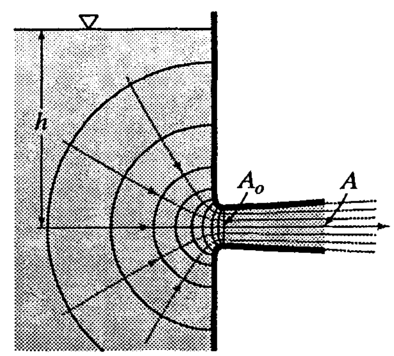


Figure X11.6.5

From Eq. 11.9: $V = Q/A = 0.72A\sqrt{2g(6)}/A = 14.15$ fps.

Given: $C_d = C_v = 0.98$

Continuity: $V_o = VA/A_o = 14.15(2) = 28.3$ fps ◄

From Eq. 11.15 (surface to throat): $h_L = \left(\frac{1 - C_v^2}{C_v^2}\right)\frac{(V_{2i} \times C_v)^2}{2g} = (1 - C_v^2)h = (1 - 0.98^2)6 = 0.238$ ft

Eq. 5.28, from surface to throat:

$0 + 6 + 0 - 0.238 = p_o/\gamma + 0 + 28.3^2/[2(32.2)]$; $p_o/\gamma = -6.68$ ft ◄

At throat:	V_o	p_o/γ
Included friction (this Exer.)	28.3 fps	−6.68 ft
Neglecting friction (Exer. 11.6.5)	39.3 fps	−18.00 ft

Friction decreases velocity and increases pressure at the throat, considerably. ◄

11.6.8 *Given that the diverging tube shown in Fig. X11.6.5 has $A = 1.75A_o$. Find the throat velocity and pressure head when h = 3 m of water. Neglect all friction losses.*

SI

$V = \sqrt{2gh} = \sqrt{2(9.81)3.0} = 7.67$ m/s

$V_o = V(A/A_0) = 7.67(1.75) = 13.43$ m/s ◄

Given $h_L = 0$. So Eq. 5.29, from surface to throat:

$$0 + 3.0 + 0 = \frac{p_o}{\gamma} + 0 + \frac{V_o^2}{2g};$$

$$\frac{p_o}{\gamma} = 3.0 - \frac{V_o^2}{2g} = 3.0 - \frac{13.43^2}{2(9.81)} = -6.19 \text{ m} \quad ◄$$

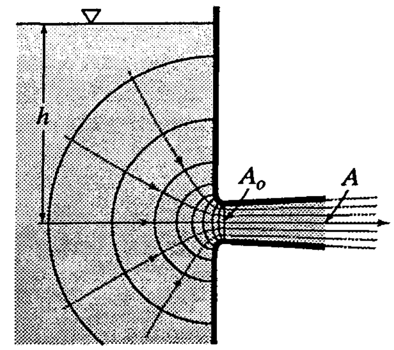


Figure X11.6.5

11.6.9 *Given that the diverging tube shown in Fig. X11.6.5 has $A = 1.75A_o$. If it is operating at 1000 m elevation under standard atmospheric conditions, what will be the maximum value of h for which the tube will flow full?*

SI

Table A.3 at 1000 m elevation:

p_{atm} = 89.9 kPa abs and T = 8.501°C

Table A.1 at 8.50°C by interpolation: $p_v/\gamma = 0.111$ m

$$\frac{p_o}{\gamma} + \frac{V_o^2}{2g} = \frac{p_a}{\gamma} + \frac{V^2}{2g}; \quad 0.111 + \frac{(1.75V)^2}{2g} = \frac{89.9}{9.81} + \frac{V^2}{2g}$$

$0.111 + 3.06V^2/2g = 9.16 + V^2/2g$; $2.06(V^2/2g) = 9.05$ m ; $V^2/2g = 4.39$ m

The tube will not flow full if $h > 4.39$ m. ◄

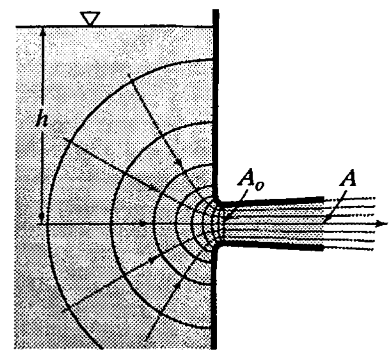


Figure X11.6.5

11.6.10 *Water flows through a 60-mm-diameter sharp-edged orifice which connects two adjacent tanks. The head on one side of the orifice is 2.5 m and 0.5 m on the other. Given $C_c = 0.62$ and $C_v = 0.95$, calculate the flow rate.*

SI

D = 60 mm = 0.06 m ; $A = \pi D^2/4 = \pi(0.06)^2/4 = 0.002\,83$ m²

Sec. 11.6 for submerged discharge: $Q = C_cC_vA\sqrt{2gh} = 0.62(0.95)0.002\,83\sqrt{(2)9.81(2.5-0.5)}$

$Q = 0.01043$ m³/s = 10.43 L/s ◄

Sec. 11.6: Orifices, Nozzles, and Tubes -- *Problems* *11.17–11.25*

11.17 *Water flows at 12 fps through a horizontal 3-in-diameter pipe. A nozzle at the end of the pipe has a velocity coefficient of 0.98. If the pressure in the pipe is *7.5 psi, find (a) the velocity in the jet; (b) the rate of discharge; (c) the diameter of the jet; and (d) the head loss through the nozzle. [*Note this correction to the textbook.]*

BG

(*a*) Eq. 5.29: $V_1^2/2g + p_1/\gamma = (V_j)_i^2/2g$ since $p_j = p_{atm} = 0$. $\therefore$ $(V_j)_i = \sqrt{2g(p_1/\gamma + V_1^2/2g)}$

Sec 11.6: $V_j = C_v(V_j)_i = 0.98\sqrt{2(32.2)\left(\dfrac{7.5(144)}{62.4} + \dfrac{12^2}{2(32.2)}\right)} = 34.8 \text{ fps}$ ◄

(*b*) $Q = A_1V_1 = (\pi/4)(3/12)^2 12 = 0.589 \text{ cfs}$ ◄

(*c*) $Q = A_jV_j = (\pi/4)D_j^2V_j$; $0.589 = (\pi/4)D_j^2(34.8)$; $D_j = 0.1469 \text{ ft} = 1.762 \text{ in}$ ◄

(*d*) $$h_L = \frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 - \left(\frac{V_j^2}{2g} + \frac{p_j}{\gamma} + z_j\right) = \frac{12^2}{2(32.2)} + \frac{7.5(144)}{62.4} + z_1 - \left(\frac{34.8^2}{2(32.2)} + 0 + z_1\right)$$

$= 2.24 + 17.31 - 18.77 = 0.774 \text{ ft}$ ◄

11.18 *Water flows at 3.5 m/s through a horizontal 120-mm-diameter pipe. A nozzle at the end of the pipe has a velocity coefficient of 0.98. If the pressure in the pipe is 45 kPa, find (a) the initial velocity of the jet; (b) the rate of discharge; (c) the diameter of the jet; and (d) the head loss through the nozzle.*

SI

(*a*) Eq. 5.29: $p_1/\gamma + 0 + V_1^2/2g = 0 + 0 + (V_j)_i^2/2g$ since $p_j = p_{atm} = 0$. $\therefore$ $(V_j)_i = \sqrt{2g(p_1/\gamma + V_1^2/2g)}$

Sec 11.6: $V_j = C_v(V_j)_i = 0.98\sqrt{2(9.81)\left(\dfrac{45}{9.81} + \dfrac{3.5^2}{2(9.81)}\right)} = 9.91 \text{ m/s}$ ◄

(*b*) $Q = \pi(0.12^2/4)3.5 = 0.0396 \text{ m}^3/\text{s}$ ◄

(*c*) $Q = A_jV_j = (\pi/4)D_j^2V_j$; $0.0396 = (\pi/4)D_j^2(9.91)$; $D_j = 0.005\,09 \text{ m} = 5.09 \text{ mm}$ ◄

(*d*) $$h_L = \frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 - \left(\frac{V_j^2}{2g} + \frac{p_j}{\gamma} + z_j\right) = \frac{3.5^2}{2(9.81)} + \frac{45}{9.81} + z_1 - \left(\frac{9.91^2}{2(9.81)} + 0 + z_j\right)$$

$= 0.206 \text{ m}$ ◄

11.19 *In Fig. P11.19 the pressure at the base of the nozzle, point 1, is 21.0 psi, and the power available in the jet at point 2 is 3.42 hp. Find (a) the theoretical height to which the jet will rise; (b) the coefficient of velocity for this nozzle; (c) the head loss between points 1 and 2; and (d) the theoretical diameter of the jet at a point 20 ft above point 2.*

BG

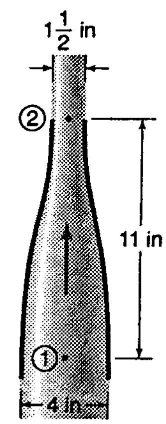


Figure P11.19

Eq. 5.39: $P = h\gamma Q = Q\gamma V_2^2/2g = A_2\gamma V_2^3/2g$ where $A_2 = \pi(1.5/12)^2/4 = 0.012\,27$ ft^2

$3.42(550) = 0.021\,27(62.4)V_2^3/(2 \times 32.2)$; $V_2^3 = 158{,}200$; $V_2 = 54.1$ fps

(*a*) Theoretical height $= V_2^2/2g = 54.1^2/(2 \times 32.2) = 45.4$ ft ◄

(*b*) Then, from continuity, $V_1^2/2g = 45.4(1.5/4)^4 = 0.898$ ft

Ideal Eq. 5.29 from point 1 to 2, writing (from Sec. 11.6) $V_i = V/C_v$:

$$\frac{21.0(144)}{62.4} + 0 + \frac{0.898}{C_v^2} = 0 + \frac{11}{12} + \frac{45.4}{C_v^2}\,; \quad C_v = 0.968 \quad ◄$$

(*c*) Eq. 11.14: $h_L = \left(\frac{1}{0.968^2} - 1\right)\left[1 - \left(\frac{1.5}{4}\right)^4\right]45.4 = 3.02$ ft ◄

The above answer is very sensitive to the value of C_v that is used.

(*d*) $Q = AV = (\pi/4)D^2V =$ constant, $\therefore$ $D_2^2V_2 = D_3^2V_3$ or $(D_3/D_2)^2 = V_2/V_3$

or $(D_3/D_2)^4 = (V_2/V_3)^2 = (V_2^2/2g)/(V_3^2/2g)$ from which

$$D_3 = D_2\left(\frac{V_2^2/2g}{V_3^2/2g}\right)^{1/4} = 1.5\left(\frac{45.4}{45.4 - 20}\right)^{1/4} = 1.734 \text{ in} \quad ◄$$

11.20 *In Fig. P11.19 the pressure at the base of the nozzle, point 1, is 145 kPa, and the power available in the jet at point 2 is 2.55 kW. Find (a) the theoretical height to which the jet will rise; (b) the coefficient of velocity for this nozzle; (c) the head loss between points 1 and 2; and (d) the theoretical diameter of the jet at a point 6 m above point 2.*

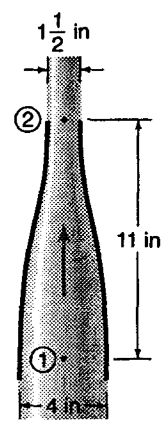


Figure P11.19

SI

Eq. 5.39: $P = h\gamma Q = Q\gamma V_2^2/2g = A_2\gamma V_2^3/2g$

2.55 kW $= 2.55$ kN·m/s $= (\pi/4)(1.5\times0.0254)^2(9.81)V_2^3/(2\times9.81) = 0.000\,570\,V_2^3$

so $V_2 = 16.48$ m/s

(*a*) Theoretical height $= V_2^2/2g = 16.48^2/(2\times9.81) = 13.84$ m ◄

(*b*) Then $V_1^2/2g = 13.84(1.5/4)^4 = 0.274$ m

Ideal Eq. 5.29 from point 1 to 2, writing (from Sec. 11.6) $V_i = V/C_v$:

$$\frac{145}{9.81} + 0 + \frac{0.274}{C_v^2} = 0 + 11(0.0254) + \frac{13.84}{C_v^2}; \quad C_v = 0.967 \quad ◄$$

(*c*) Eq. 11.14: $h_L = \left(\frac{1}{0.967^2} - 1\right)\left[1 - \left(\frac{1.5}{4}\right)^4\right]13.84 = 0.938$ m ◄

The above answer is very sensitive to the value of C_v that is used.

(*d*) $D_3 = D_2\left(\frac{V_2^2/2g}{V_3^2/2g}\right)^{1/4} = 1.5\left(\frac{13.84}{13.84 - 6}\right)^{1/4} = 1.729$ in $= 43.9$ mm ◄

11.21 *We can express the loss of head due to friction in a nozzle, tube, or orifice as $h_L = kV^2/2g$, where V is the actual velocity of the jet. (a) What is k for the three tubes in Fig. 11.16? (b) If these tubes discharge water under a head of 6 ft, what is the loss of head for each?*

Fig. 11.16: The three tubes have $C_v = 0.98$, 0.82, and 0.74.

BG

(*a*) Using Eq. 11.15: $h_L = k\frac{V^2}{2g} = \left[\frac{1}{C_v^2} - 1\right]\frac{V^2}{2g} \quad \therefore \quad k = \left[\frac{1}{C_v^2} - 1\right]$

Fig 11.16: $C_v = 0.98$, ∴ $k = 0.0412$ ◄

$C_v = 0.82$, $k = 0.487$ ◄

$C_v = 0.74$, $k = 0.826$ ◄

(*b*) $V = C_v\sqrt{2gH}$ so $V^2 = C_v^2(2gH)$ and $V^2/2g = C_v^2H$

So using Eq. 11.15: $h_L = \left[\frac{1}{C_v^2} - 1\right]\frac{V^2}{2g} = \left[\frac{1}{C_v^2} - 1\right]C_v^2H = (1 - C_v^2)H$

The respective head losses are: 0.238 ft, 1.966 ft, and 2.71 ft ◄◄

11.22 *We can express the loss of head due to friction in a nozzle, tube, or orifice as $h_L = kV^2/2g$, where V is the actual velocity of the jet. (a) What is k for the three tubes in Fig. 11.16? (b) If these tubes discharge water under a head of 1.8 m, what is the loss of head for each?*

Fig. 11.16: The three tubes have C_v = 0.98 , 0.82, and 0.74.

SI

(*a*) Using Eq. 11.15: $h_L = k\dfrac{V^2}{2g} = \left(\dfrac{1}{C_v^2} - 1\right)\dfrac{V^2}{2g}$ $\quad\therefore\quad$ $k = \dfrac{1}{C_v^2} - 1$

Fig 11.16: $C_v = 0.98, \therefore\ k = 0.0412$ ◄
$C_v = 0.82, \quad k = 0.487$ ◄
$C_v = 0.74, \quad k = 0.826$ ◄

(*b*) $V = C_v\sqrt{2gH}$ so $V^2 = C_v^{\,2}(2gH)$ and $V^2/2g = C_v^{\,2}H$

So using Eq. 11.15: $h_L = \left(\dfrac{1}{C_v^{\,2}} - 1\right)\dfrac{V^2}{2g} = \left(\dfrac{1}{C_v^2} - 1\right)C_v^{\,2}H = (1 - C_v^{\,2})H$

The respective head losses are: 0.713 m, 0.590 m, and 0.814 m ◄◄

11.23 *Find the maximum theoretical head h at which the Borda tube of Fig. 11.17 will flow full if the liquid is 90°F water and the barometer reads 29.2 inHg. Assume C_d = 0.72 when the tube is flowing full.*

BG

Flowing full: $V = 0.72\sqrt{2gh}$, thus $h = \dfrac{1}{(0.72)^2}\dfrac{V^2}{2g}$ or $\dfrac{V^2}{2g} = (0.72)^2h$

If, while flowing full, $C_c = 0.52$ and $C_v = 0.98$ for the contracted throat section (per Sec. 11.6),

$V_o^2/(2g) = (1/0.52)^2(V^2/2g) = (0.72)^2h/(0.52)^2 = 1.917h$

Per Eq. 5.28 from surface to throat: $p_a/\gamma + h + 0 = p_o/\gamma + 0 + V_o^2/2g + h_L$

Table A.1 for water at 90°F: $p_v/\gamma = 1.61$ ft

When $h = h_{max}$ for full flow, $p_o = (p_o)_{min} = p_v$, $\therefore$ $(p_o/\gamma)_{min} = 1.61$ ft

Thus $(29.2/12)13.56 + h + 0 = 1.61 + 0 + 1.917h + [1 - (0.98)^2](h + 33.9)$

where (per Sec 3.6) the quantity (h + 33.9) ft is the total absolute pressure head acting on the orifice.

$\therefore$ $h_{max} = (33.0 - 1.61 - 1.342)/0.957 = 31.4$ ft ◄

11.24 *Find the maximum theoretical head h at which the Borda tube of Fig. 11.17 will flow full if the liquid is 30°C water and the barometer reads 735 mmHg. Assume C_d = 0.72 when the tube is flowing full.*

SI

Flowing full: $V = 0.72\sqrt{2gh}$, thus $h = (1/0.72)^2(V^2/2g)$ or $V^2/(2g) = (0.72)^2h$

If, while flowing full, $C_c = 0.52$ and $C_v = 0.98$ for the contracted throat section (per Sec. 11.6),

$V_o^2/(2g) = (1/0.52)^2(V^2/2g) = (0.72)^2h/(0.52)^2 = 1.917h$

Per Eq. 5.28 from surface to throat: $p_a/\gamma + h + 0 = p_o/\gamma + 0 + V_o^2/2g + h_L$

Table A.1 at 30°C: $p_v/\gamma = 0.44$ m

When $h = h_{max}$ for full flow, $p_o = (p_o)_{min} = p_v$, $\therefore$ $(p_o/\gamma)_{min} = 0.44$ m

Thus $0.735(13.56) + h + 0 = 0.44 + 0 + 1.917h + [1 - (0.98)^2](h + 10.3)$

where (per Sec 3.6) the quantity (h + 10.3) m is the total absolute pressure head acting on the orifice.

$\therefore$ $h_{max} = (9.97 - 0.44 - 0.408)/0.957 = 9.53$ m ◄

11.25 *In Fig. P11.25 the orifice at the bottom of the large open tank has a diameter of 1.27 in. If the flow rate is 0.19 cfs, and the pitot tube registers a pressure of 16.8 psi, find the C_c and the C_v of the orifice. Neglect air resistance.*

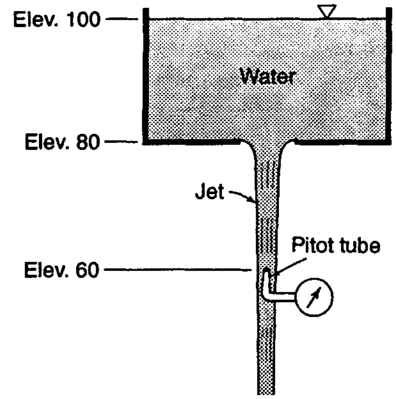


Figure P11.25

BG

Given: $Q = 0.19$ cfs

Let subscript 3 be at pitot tube, and j be at orifice.

$V_3^2/2g = 16.8(144)/62.4 = 38.8$ ft, $V_3 = 50.0$ fps

$V_j^2/2g + 80 = 38.8 + 60$; $V_j = 34.8$ fps

$V_{ideal} = \sqrt{2g(20)} = 35.9$ fps ; $C_v = 34.8/35.9 = 0.969$ ◀

$Q = A_jV_j = C_cAV_j$; $0.19 = C_c\left(\frac{\pi}{4}\right)\left(\frac{1.27}{12}\right)^2 34.8$; $C_c = 0.621$ ◀

Sec. 11.7: Venturi Meter -- *Exercises (4)*

11.7.1 *The venturi meter of Fig. S11.4 has $D_1 = 8$ in, $D_2 = 4$ in, and $\Delta z = 1.5$ ft. Assume the discharge coefficients of Fig. 11.19 are applicable. Find the flow rate of 72°F water when the manometer containing carbon tetrachloride ($s = 1.59$) reads $R_m = 4$ in.*

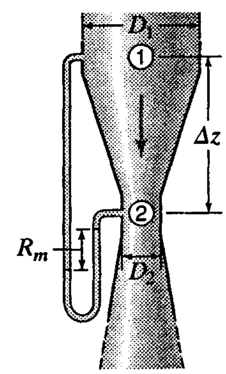


Figure S11.4

BG

Eq. 11.17: $\Delta(z + p/\gamma) = (4/12)[(1.59/1.00) - 1.0] = 0.1967$ ft

$A_2 = (\pi/4)(4/12)^2 = 0.0873$ ft²

From Fig. 11.19: As a first trial assume $C = 0.95$

Eq. 11.16: $Q = \dfrac{0.95(0.0873)}{\sqrt{1 - (4/8)^4}}\sqrt{2(32.2)0.1967} = 0.305$ cfs

$V_2 = Q/A_2 = 0.305/0.0873 = 3.49$ fps

$D_2''V_2 = 13.88$, and $T = 72°$F (given); Fig. 11.19: $C = 0.985$; so $Q = \dfrac{0.985}{0.95}0.305 = 0.316$ cfs ◀

Note: The given $\Delta z = 1.5$ ft is not used in the solution.

11.7.2 *The venturi meter of Fig. S11.4 has $D_1 = 200$ mm, $D_2 = 100$ mm, and $\Delta z = 450$ mm. Assume the discharge coefficients of Fig. 11.19 are applicable. Find the flow rate of 20°C water when the manometer containing carbon tetrachloride ($s = 1.59$) reads $R_m = 100$ mm.*

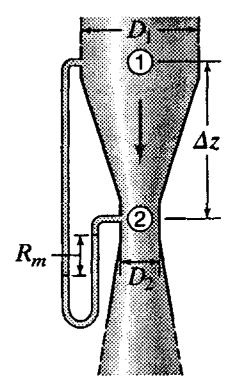


Figure S11.4

SI

Eq. 11.17: $\Delta(z + p/\gamma) = 0.1[(1.59/1.00) - 1.0] = 0.0590$ m.

$A_2 = (\pi/4)(0.1)^2 = 0.007\,85$ m²

$T = 20°$C $= 68°$F ; this is close enough to 72°F to use Fig. 11.22

Venturi meter: $D_1 \times D_2 = 200$ mm × 100 mm = 7.87″ × 3.94″, close to 8″ × 4″

∴ From Fig. 11.19: As a trial assume C = 0.95

Eq. 11.16: $Q = \dfrac{0.95(0.00785)}{\sqrt{1 - (10/20)^4}}\sqrt{2(9.81)0.0590} = 0.008\,29$ m³/s

$V_2 = Q/A_2 = 0.008\,29/0.007\,85 = 1.056$ m/s

Table A.1 at 20°C: $\nu = 1.003\times10^{-6}$ m²/s ; $\mathbf{R} = D_2V_2/\nu = 0.1(1.056)/(1.003\times10^{-6}) = 1.052\times10^5$

Fig. 11.19: $C = 0.983$. Thus $Q = (0.983/0.95)0.00829 = 0.00858$ m³/s = 8.58 L/s ◀

11.7.3 *The venturi meter of Fig. S11.4 has D_1 = 900 mm, D_2 = 450 mm, and Δz = 2.25 m. Assume the discharge coefficients of Fig. 11.19 are applicable. Find the flow rate of 20°C water when the mercury (s = 13.56) manometer reads R_m = 160 mm.*

SI

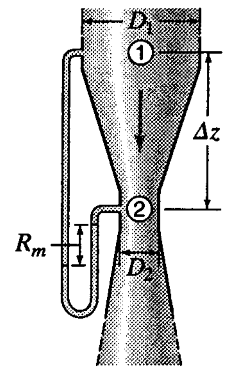


Figure S11.4

Venturi meter: $D_1 \times D_2$ = 900 mm × 450 mm = 35.4″ × 17.7″.

This is about midway between the top two curves on Fig. 11.19.

So assume $C = C_{max} = 0.988$.

Eq. 11.16: $$Q = \frac{0.988\,\pi\,0.225^2}{\sqrt{1-(45/90)^4}}\sqrt{2(9.81)0.16(13.56-1.00)} = 1.019 \text{ m}^3/\text{s}$$

$V_2 = Q/A_2 = 1.019/(\pi\,0.225^2) = 6.41$ m/s

Table A.1 for water at 20°C: $\nu = 1.003\times10^{-6}$ m²/s

Eq. 7.6: $$\mathbf{R} = \frac{D_2V_2}{\nu} = \frac{0.45(6.41)}{1.003\times10^{-6}} = 2.87\times10^6$$

Fig. 11.19: C = 0.988 (as assumed); so $Q = 1.019$ m³/s ◄

11.7.4 *The venturi meter of Fig. S11.4 has D_1 = 4 in, D_2 = 2 in, and Δz = 12 in. Assume the discharge coefficients of Fig. 11.19 are applicable. Find the reading R_m of the mercury (s = 13.56) manometer when oil (s = 0.92) with a kinematic viscosity of 0.0005 ft²/sec is flowing at a rate of 0.40 cfs.*

BG

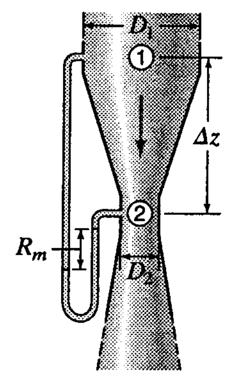


Figure S11.4

$D_1 \times D_2$ = 4″ × 2″ is about 3/4 of the way from 1/2″ × 1/4″ to 8″ × 4″ on Fig. 11.19.

So assume $C = C_{max} = 0.983$; $A_2 = \pi(1/12)^2 = 0.0218$ ft²

Eqs. 11.16 and 11.17: $$Q = \frac{0.983(0.0218)}{\sqrt{1-(2/4)^4}}\sqrt{2(32.2)\frac{R_m}{12}\left(\frac{13.56}{0.92}-1\right)} = 0.40 \text{ cfs}$$

from which R_m = 4.42 inches ; $V_2 = Q/A_2 = 0.40/0.0218 = 18.33$ fps

Eq. 7.6: $$\mathbf{R} = \frac{D_2V_2}{\nu} = \frac{(2/12)18.33}{0.0005} = 6110\ ; \quad \text{Fig. 11.19: } C = 0.940$$

Thus, from Eqs 11.16 and 11.17: $R_m = 4.42(0.983/0.940)^2 = 4.84$ inches ◄

Sec. 11.7: Venturi Meter – *Problems 11.26–11.29*

11.26 *The venturi meter of Fig. S11.4 has $D_1 = 10$ in, $D_2 = 5$ in, and $\Delta z = 2$ ft. Assume the discharge coefficients of Fig. 11.19 are applicable and the diverging cone angle is 12°. (a) Find the flow rate of 72°F water when the mercury (s = 13.55) manometer reads $R_m = 5$ in. Find also the head loss (b) from inlet to throat, and (c) across the entire meter.*

BG

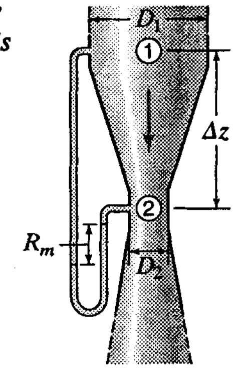


Figure S11.4

(*a*) Eq. 11.17: $[(z_1 + p_1/\gamma) - (z_2 + p_2/\gamma)] = (5/12)[(13.55/1.00) - 1] = 5.23$ ft

$A_2 = (\pi/4)(5/12)^2 = 0.1363$ ft².

From Fig. 11.19: As a first trial assume $C = 0.95$

Eq. 11.16: $Q = \dfrac{0.95(0.1363)}{\sqrt{1 - (5/10)^4}}\sqrt{2(32.2)5.23} = 2.46$ cfs

$Q = A_2V_2 = 2.46$ ft³/s ; $V_2 = 2.46/0.1363 = 18.01$ fps ; $D_2''V_2 = 5(18.01) = 90.0$

Fig. 11.19 for 18″ × 5″, $D_2''V_2 = 90.0$ and $T = 72°$F (given): $C = 0.985$

Curve is flat in this area of Fig 11.19, so $C = 0.985$ is O.K.

Thus $Q = (0.985/0.95)2.46 = 2.55$ ft³/sec ◄

(*b*) $V_2 = Q/A_2 = 2.55/0.1363 = 18.67$ fps ; $V_1 = V_3 = V_2(5/10)^2 = 4.67$ fps

Eq. 11.14: $h_{L_{1-2}} = \left(\dfrac{1}{0.985^2} - 1\right)\left[1 - \left(\dfrac{5}{10}\right)^4\right]\dfrac{18.67^2}{2(32.2)} = 0.1557$ ft ◄

(*c*) Fig 8.20 for $\alpha = 12°$: $k' = 0.219$. Therefore

Eq. 8.78: $h_{L_{2-3}} = \dfrac{0.219(V_2 - V_3)^2}{2g} = \dfrac{0.219(18.67 - 4.67)^2}{2(32.2)} = 0.533$ ft

Total head loss $= 0.1557 + 0.533 = 0.688$ ft ◄

11.27 *The venturi meter of Fig. S11.4 has D_1 = 250 mm, D_2 = 125 mm, and Δz = 600 mm. Assume the discharge coefficients of Fig. 11.19 are applicable and the diverging cone angle is 12°. (a) Find the flow rate of 20°C water when the mercury (s = 13.55) manometer reads R_m = 12.8 mm. Find also the head loss (b) from inlet to throat, and (c) across the entire meter.*

SI

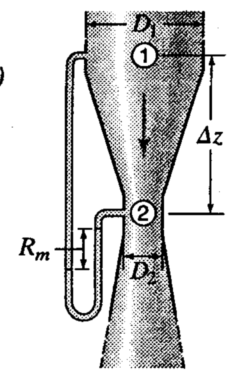


Figure S11.4

(*a*) Eq. 11.17: $[(z_1 + p_1/\gamma) - (z_2 + p_2/\gamma)] = 0.0128[(13.55/1.00) - 1) = 0.1606$ m

$A_2 = \pi(0.125)^2/4 = 0.012\,27\ \text{m}^2$.

$T = 20°\text{C} = 68°\text{F}$ (close enough to 72°F for Fig. 11.19)

From Fig. 11.19: As a first trial assume $C = 0.95$

$$\text{Eq. 11.16:}\quad Q = \frac{0.95(0.012\,27)}{\sqrt{1 - (12.5/25)^4}}\sqrt{2(9.81)0.1606} = 0.0214\ \text{m}^3/\text{s}$$

$V_2 = Q/A_2 = 0.0214/0.012\,27 = 1.742$ m/s. Table A.1 for water at 20°C: $\nu = 1.003\times10^{-6}\ \text{m}^2/\text{s}$

$\mathbf{R}_2 = (0.125)1.742/(1.003\times10^{-6}) = 2.17\times10^5$; Fig. 11.19: $C = 0.985$

Curve is flat in this area of Fig 11.19, so $C = 0.985$ is O.K.

Thus $Q = (0.985/0.95)0.0214 = 0.0222\ \text{m}^3/\text{s}$ ◄

(*b*) $V_2 = Q/A_2 = 0.0222/0.012\,27 = 1.806$ m/s ; $V_1 = V_3 = V_2(12.5/25)^2 = 0.452$ m/s

$$\text{Eq. 11.14:}\quad h_{L_{1\text{-}2}} = \left(\frac{1}{0.985^2} - 1\right)\left[1 - \left(\frac{12.5}{25}\right)^4\right]\frac{1.806^2}{2(9.81)} = 0.002\,34\ \text{m}$$ ◄

(*c*) Fig 8.20 for $\alpha = 12°$: $k' = 0.219$ Therefore

$$\text{Eq. 8.78:}\quad h_{L_{2\text{-}3}} = \frac{0.219(V_2 - V_3)^2}{2g} = \frac{0.219(1.806 - 0.452)^2}{2(9.81)} = 0.0205\ \text{m}$$

Total head loss = 0.002 34 + 0.0205 = 0.0228 m ◄

11.28 *Repeat Prob. 11.26 when the venturi meter is set horizontally (i.e., Δz = 0), with all other data the same.*

BG

When Eq. 11.17 is substituted into Eq. 11.26 to eliminate $\Delta(p/\gamma + z)$, the result is the same regardless of meter orientation. ∴ Answers are the same as for Prob. 11.26. ◄

11.29 *Repeat Prob. 11.27 when the venturi meter is set horizontally (i.e., Δz = 0), with all other data the same.*

SI

When Eq. 11.17 is substituted into Eq. 11.27 to eliminate $\Delta(p/\gamma + z)$, the result is the same regardless of meter orientation. ∴ Answers are the same as for Prob. 11.27. ◄

Sec. 11.8: Flow Nozzle -- ***Exercise*** *(1)*

11.8.1 *A 100 mm ISA flow nozzle (Figs. 11.21 and 11.22) is used to measure the flow of 40°C water through a 200-mm pipe. What would be the reading on a mercury manometer for the following flow rates: (a) 1.5 L/s; (b) 15 L/s; (c) 150 L/s?*

SI

Eq. 11.17: $\Delta(p/\gamma + z) = R_m[(13.56/1.00) - 1] = 12.56R_m$; Substitute into Eq. 11.18:

$Q = KA_2\sqrt{2(9.81)12.56R_m} = K(\pi 0.05^2)\sqrt{246R_m} = 0.1233K\sqrt{R_m}$. $\therefore$ $R_m = [Q/(0.1233K)]^2$

Table A.1 for 40°C water: $\nu = 0.658\times10^{-6}$ m²/s. $A_1 = (\pi/4)(0.2\text{ m})^2 = 0.0314$ m²

Case	Q (m³/s)	$V_1 = Q/A_1$ (m/s)	$\mathbf{R} = D_1V_1/\nu$	K (Fig. 11.22*)	R_m, m
(a)	0.0015	0.0477	1.45×10^4	0.978	0.000 1548
(b)	0.015	0.477	1.45×10^5	1.005	0.014 65
(c)	0.15	4.77	1.45×10^6	1.005	1.465

*For $D_2/D_1 = 10/20 = 0.5$ ▲

Sec. 11.8: Flow Nozzle -- ***Problems*** *11.30–11.31*

11.30 *An 80 mm ISA flow nozzle (Figs. 11.21 and 11.22) is used to measure the flow of 40°C water through a 160-mm pipe. What would be the reading on a mercury manometer for the following flow rates: (a) 3.8 L/s; (b) 38 L/s; (c) 380 L/s. (d) For which of the preceding three flows is the mercury manometer practical?*

SI

Eq. 11.17: $\Delta(p/\gamma + z) = R_m[(13.56/1.00) - 1] = 12.56R_m$; Substitute into Eq. 11.18:

$Q = KA_2\sqrt{2(9.81)12.56R_m} = K(\pi 0.04^2)\sqrt{246R_m} = 0.0789K\sqrt{R_m}$. $\therefore$ $R_m = [Q/(0.0789\text{K})]^2$

Table A.1 for 40°C water: $\nu = 0.658\times10^{-6}$ m²/s. $A_1 = (\pi/4)(0.16\text{ m})^2 = 0.0201$ m²

Case	Q (m³/s)	$V_1 = Q/A_1$ (m/s)	$\mathbf{R} = D_1V_1/\nu$	K (Fig. 11.22*)	R_m, m
(a)	0.0038	0.1890	4.60×10^4	1.001	0.00231
(b)	0.038	1.890	4.60×10^5	1.005	0.230
(c)	0.38	18.90	4.60×10^6	1.005	23.0

*For $D_2/D_1 = 8/16 = 0.5$ ▲

(d) The manometer is only practical for flow (b), because (a) is too small for good accuracy and (c) is too large to conveniently read. ◄

11.31 *A 7-in ISA flow nozzle is used to measure the flow of 20°F crude oil (s = 0.855) through a 10-in-diameter pipe (fig. P11.31). If a mercury manometer shows a reading of 6.9 in, find the flow rate.*

BG

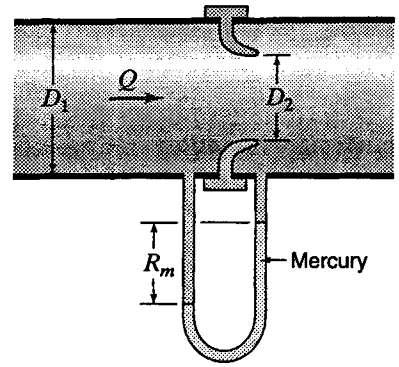


Figure P11.31

Figure 11.22 for $D_2/D_1 = 7/10 = 0.70$: Assume $K = 1.08$.

$A_2 = (\pi/4)(7/12)^2 = 0.267\ \text{ft}^2$

Eqs. 11.17 and 11.18: $Q = 1.08(0.267)\sqrt{2(32.2)\frac{6.9}{12}\left(\frac{13.56}{0.855} - 1.0\right)}$

$= 6.77$ cfs

Check **R** to confirm that the assumed K is correct.

$V_2 = Q/A_2 = 6.77/0.267 = 25.3$ fps

Fig A.2 at 20°F: $\nu = 0.00028\ \text{ft}^2/\text{sec}$

Eq. 7.6: $\mathbf{R} = DV/\nu = (7/12)25.3/0.000\,28 = 5.28\times10^4$

Fig. 11.22 for $D_2/D_1 = 0.70$: $K = 1.062$; Thus $Q = (1.062/1.08)6.77 = 6.66$ cfs ◄

Sec. 11.9: Orifice Meter -- *Exercise (1)*

11.9.1 *Repeat Exer. 11.8.1 for a VDI orifice.*
Exer. 11.8.1: A 100 mm flow nozzle is used to measure the flow of 40°C water through a 200-mm pipe. What would be the reading on a mercury manometer for the following flow rates: (a) 1.5 L/s; (b) 15 L/s; (c) 150 L/s?

SI

Eq. 11.17: $\Delta(p/\gamma + z) = R_m[(13.56/1.00) - 1] = 12.56R_m$; Substitute into Eq. 11.20:

$Q = KA_o\sqrt{2(9.81)\,12.56R_m} = K(\pi 0.05^2)\sqrt{246R_m} = 0.1233K\sqrt{R_m}$. $\therefore$ $R_m = [Q/(0.1233K)]^2$

Table A.1 for 40°C water: $\nu = 0.658\times10^{-6}\ \text{m}^2/\text{s}$; $A_1 = (\pi/4)(0.1\ \text{m})^2 = 0.007\,85\ \text{m}^2$

Case	Q (m³/s)	$V_1 = Q/A_1$ (m/s)	$\mathbf{R} = D_1V_1/\nu$	K (Fig. 11.24*)	R_m, m
(*a*)	0.0015	0.0477	2.90×10^4	0.647	0.000 365
(*b*)	0.015	0.4775	2.90×10^5	0.622	0.038 3
(*c*)	0.15	4.775	2.90×10^6	0.622	3.83

* For $D_o/D_1 = 10/20 = 0.5$ ▲

Sec. 11.9: Orifice Meter -- *Problems 11.32–11.34*

11.32 *Repeat Prob. 11.30 for a VDI orifice.*

Prob. 11.30: An 80 mm flow nozzle is used to measure the flow of 40°C water through a 160-mm pipe. What would be the reading on a mercury manometer for the following flow rates: (a) 3.8 L/s; (b) 38 L/s; (c) 380 L/s. (d) For which of the preceding three flows is the mercury manometer practical?

SI

Eq. 11.17: $\Delta(p/\gamma + z) = R_m[(13.56/1.00) - 1.00] = 12.56R_m$; Substitute into Eq. 11.20:

$Q = KA_o\sqrt{2(9.81)12.56R_m} = K(\pi 0.04^2)\sqrt{246R_m} = 0.0789K\sqrt{R_m}$. $\therefore$ $R_m = [Q/(0.0789\text{K})]^2$

Table A.1 for 40°C water: $\nu = 0.658\times10^{-6}$ m²/s. $A_1 = (\pi/4)(0.16 \text{ m})^2 = 0.0201$ m²

Case	Q (m³/s)	$V_1 = Q/A_1$ (m/s)	$\mathbf{R} = D_1V_1/\nu$	K (Fig. 11.24*)	R_m, m
(*a*)	0.0038	0.1890	4.60×10^4	0.630	0.005 86
(*b*)	0.038	1.890	4.60×10^5	0.623	0.599
(*c*)	0.38	18.90	4.60×10^6	0.623	59.9

* For $D_o/D_1 = 8/16 = 0.5$ ▲

(*d*) The manometer is only practical for flow (*b*), because (*a*) is too small for good good accuracy and (*c*) is too large to conveniently read. ◄

11.33 *Repeat Prob. 11.31 for a VDI orifice.*

Prob. 11.31: A 7-in flow nozzle is used to measure the flow of 20°F crude oil (s = 0.855) through a 10-in-diameter pipe. If a mercury manometer shows a reading of 6.9 in, find the flow rate.

BG

Fig. 11.24 for $D_o/D_1 = 7/10 = 0.70$: Assume $K = 0.70$. $A_o = (\pi/4)(7/12)^2 = 0.267$ ft²

Eqs. 11.17 and 11.20: $Q = 0.7(0.267)\sqrt{2(32.2)(6.9/12)[(13.56/0.855) - 1.0]} = 4.39$ cfs

Check $\mathbf{R}$ to confirm that the correct K is correct.

$V_o = Q/A_o = 4.39/0.267 = 16.42$ fps ; Fig. 2.4 at 20°F: $\nu = 0.00028$ ft²/sec

Eq. 7.4: $\mathbf{R} = DV/\nu = (7/12)16.42/0.000\,28 = 3.42\times10^4$

Fig. 11.24 for $D_o/D_1 = 0.70$: $K = 0.716$; $Q = (0.716/0.70)4.39 = 4.49$ cfs ◄

11.34 *An 8-in-diameter pipe carries 2.7 cfs of water at 72°F. Find the differential head across the following types of meters: (a) an 8-in by 4-in VDI orifice; (b) an 8-in by 4-in ISA flow nozzle; (c) a 8-in by 4-in venturi meter.*

BG

$A_1 = (\pi/4)(8/12)^2 = 0.349 \text{ ft}^2$; $A_2 = A_1/4 = 0.0873 \text{ ft}^2$.

$V_1 = Q/A_1 = 2.7/0.349 = 7.73$ fps ; $V_2 = 4V_1 = 30.9$ fps.

Table A.1 for water at 72°F, by interpolation: $\nu = 10.33 \times 10^{-6} \text{ ft}^2/\text{sec}$

Differential heads:

(*a*) Orifice meter: Approach $\mathbf{R} = D_1V_1/\nu = (8/12)7.73/(10.33 \times 10^{-6}) = 4.99 \times 10^5$

$D_o/D_1 = 4/8 = 0.5$. Fig. 11.24: $K = 0.622$

Eq. 11.20: $2.7 = 0.622(0.0873)\sqrt{2(32.2)\Delta h}$; $\Delta h = 38.4$ ft ◀

(*b*) Flow nozzle: Approach $\mathbf{R} = 4.99 \times 10^5$

$D_2/D_1 = 4/8 = 0.5$. Fig. 11.22: $K = 1.005$

Eq. 11.18: $2.7 = 1.005(0.0873)\sqrt{2(32.2)\Delta h}$; $\Delta h = 14.72$ ft ◀

(*c*) Venturi meter: Throat $\mathbf{R} = D_2V_2/\nu = (4/12)30.9/(10.33 \times 10^{-6}) = 9.98 \times 10^5$

Fig. 11.19 for 8″ × 4″: $C = 0.985$

Eq. 11.16: $2.7 = \dfrac{0.985(0.0873)}{\sqrt{1 - (0.5)^4}}\sqrt{2(32.2)\Delta h}$; $\Delta h = 13.47$ ft ◀

Sec. 11.10: Flow Measurement of Compressible Fluids -- *Exercises (8)*

11.10.1 *Natural gas, for which $R = 3100$ ft·lb/(slug·°R) and $k = 1.3$, flows through a venturi tube. The pipe diameter is 18 in and the throat diameter is 9 in. The initial pressure of the gas is 160 psia at 64°F. What is the weight flow rate of the gas if the throat pressure is 104 psia and the meter coefficient is 0.98?*

BG

$p_2/p_1 = 104/160 = 0.65$; $A_2 = (\pi/4)(9/12)^2 = 0.442 \text{ ft}^2$; $D_2/D_1 = 9/18 = 0.5$

Fig. 11.25 for $p_2/p_1 = 0.65$, and $D_2/D_1 = 0.5$: $Y = 0.778$

Eq. 2.5: $\gamma_1 = \dfrac{gp_1}{RT} = \dfrac{32.2(160)144}{3100(460 + 64)} = 0.457$ pcf

Eq. 11.23: $G = 0.98(0.778)0.442\sqrt{2(32.2)0.457\dfrac{(160 - 104)144}{1 - 0.5^4}} = 169.4$ lb/sec ◀

11.10.2 *Helium [$R = 12{,}420$ ft·lb/(slug·°R), $k = 1.66$] is stored in a tank at 60 psia and 76°F. The gas flows out through a ¾-in-diameter orifice for which $C_v = 0.98$ and $C_c = 0.62$ for liquids. Find the weight flow rate if $Y = 0.95$ and the pressure outside the tank is 44 psia.*

BG

Eq. 2.5: $\gamma_1 = \dfrac{gp_1}{RT_1} = \dfrac{32.2(60)144}{12{,}420(460 + 76)} = 0.0418$ pcf ; $A_2 = \pi(0.75/12)^2/4 = 0.003\,07 \text{ ft}^2$

$D_1 = \infty$, so $\sqrt{1 - (D_o/D_1)^4} = 1$; Eq. 11.23 becomes $G = CYA_2\sqrt{2g\gamma_1(p_1 - p_2)}$ (1)

Sec. 10.11: $C = C_d$ has the same value as for incompressible fluid at the same **R**.

From Eq. 11.11: $C = C_d = C_cC_v = 0.98(0.62)$

Thus in (1): $G = 0.98(0.62)0.95(0.003\,07)\sqrt{2(32.2)0.0418(60 - 44)144} = 0.1394$ lb/sec ◀

11.10.3 *A tank contains air at 200 psia and 100°F. It flows out through an orifice with an area of 1.5 in², and enters a space where the pressure is 80 psia. Assuming C_d = 0.60, find the weight flow rate.*

BG

Table A.5 for air: $R = 1715\ \text{ft}^2/(\text{sec}^2\cdot{}^\circ\text{R})$. $p_2/p_1 = 80/200 = 0.40$; $D_o/D_1 = D_o/\infty = 0$

From Fig. 11.25: $Y = 0.825$. Eq. 2.5: $\gamma_1 = \dfrac{32.2(200)144}{1715(460 + 100)} = 0.966$ pcf

Eq. 11.23 for orifice: $G = 0.60(0.825)\dfrac{1.5}{144}\sqrt{2(32.2)0.966\dfrac{(200 - 80)144}{1 - 0}} = 5.34$ lb/sec ◄

11.10.4 *A tank contains air at 1460 kPa abs and 44°C. It flows out through an orifice with an area of 1250 mm², and enters a space where the pressure is 713 kPa abs. Assuming C_d = 0.62, find the weight flow rate.*

SI

Table A.5 for air: $R = 287\ \text{m}^2/(\text{s}^2\cdot\text{K})$. $p_2/p_1 = 713/1460 = 0.488$; $D_o/D_1 = D_o/\infty = 0$

From Fig. 11.25: $Y = 0.850$. Eq. 2.5: $\gamma_1 = \dfrac{9.81(1\,460\,000)}{287(273 + 44)} = 157.4\ \text{N/m}^3$

Eq. 11.23 for orifice: $G = 0.62(0.850)\dfrac{12.5}{100^2}\sqrt{2(9.81)157.4\dfrac{(1460 - 713)1000}{1 - 0}} = 31.6$ N/s ◄

11.10.5 *Repeat Exer. 11.10.3, but discharging into a space where the pressure is 15 psia.*
Exer. 11.10.3: p_1 = 200 psia, T_1 = 100°F, $D_1 = \infty$, A_o = 1.5 in², C_d = 0.60; find G of air through orifice.

BG

Table A.5 for air: $R = 1715\ \text{ft}^2/(\text{sec}^2\cdot{}^\circ\text{R})$. $D_o/D_1 = D_o/\infty = 0$; $p_2/p_1 = 15/200 = 0.0750$

To extrapolate Fig. 11.25 for $D_o/D_1 = 0$:

line passes through $(p_2/p_1, Y) = (1, 1)$ and $(0.4, 0.825)$, so $Y = \dfrac{0.175}{0.6}\left(\dfrac{p_2}{p_1}\right) + \left(1 - \dfrac{0.175}{0.6}\right)$

Thus, when $p_2/p_1 = 0.0750$, $Y = 0.730$. Eq. 2.5: $\gamma_1 = \dfrac{32.2(200)144}{1715(460 + 100)} = 0.966$ pcf

Eq. 11.23 for orifice: $G = 0.60(0.730)\dfrac{1.5}{144}\sqrt{2(32.2)0.966\dfrac{(200 - 15)144}{1 - 0}} = 5.87$ lb/sec ◄

11.10.6 *Repeat Exer. 11.10.4, but discharging into a space where the pressure is 105 kPa abs.*
Exer. 11.10.4: p_1 = 1460 kN/m² abs, T_1 = 44°C, $D_1 = \infty$, A_o = 1250 mm², C_d = 0.62; find G of air through orifice.

SI

Table A.5 for air: $R = 287\ \text{m}^2/(\text{s}^2\cdot\text{K})$. $D_o/D_1 = D_o/\infty = 0$; $p_2/p_1 = 105/1460 = 0.0719$

To extrapolate Fig. 11.25 for $D_o/D_1 = 0$:

line passes through $(p_2/p_1, Y) = (1, 1)$ and $(0.4, 0.825)$, so $Y = \dfrac{0.175}{0.6}\left(\dfrac{p_2}{p_1}\right) + \left(1 - \dfrac{0.175}{0.6}\right)$

Thus, when $p_2/p_1 = 0.0719$, $Y = 0.729$. Eq. 2.5: $\gamma_1 = \dfrac{9.81(1\,460\,000)}{287(273 + 44)} = 157.4\ \text{N/m}^3$

Eq. 11.23 for orifice: $G = 0.62(0.729)\dfrac{12.5}{100^2}\sqrt{2(9.81)157.4\dfrac{(1460 - 105)1000}{1 - 0}} = 36.6$ N/s ◄

11.10.7 *Find (a) the critical pressure and the corresponding throat velocity in a suitable nozzle, assuming air at 70°F and p_1 = 100 psia. Neglect the velocity of approach. Refer to Secs. 13.3, 13.5, and 13.6. (b) What will the values be if D_2/D_1 = 0.80?*

BG

(*a*) Table A.5 for air: $R = 1715$ ft²/(sec² · °R), $k = 1.40$

Neglecting velocity of approach, $p_1 = p_o$

From Eq. 13.33: $p* = 0.528p_o = 0.528p_1 = 0.528(100) = 52.8$ psia ◀

From Eq. 13.36: $T* = T_1(p*/p_1)^{(k-1)k} = (460 + 70)0.528^{0.4/1.4} = 442°$ R

Sec. 13.5 for critical conditions: M = 1. ∴ (Eq. 7.10 and 13.10) $V = c$

Eq. 13.15: $c = \sqrt{kRT} = \sqrt{1.4(1715)442}$ so $V = 1030$ fps ◀

(*b*) If $D_2/D_1 = 0.80$, $p*$ will be the same and $V = c$ will be a little larger, perhaps about 1045 fps ◀◀

11.10.8 *Find (a) the critical pressure and the corresponding throat velocity in a suitable nozzle, assuming air at 20°C and p_1 = 700 kPa abs. Neglect the velocity of approach. Refer to Secs. 13.3, 13.5, and 13.6. (b) What will the values be if D_2/D_1 = 0.80?*

SI

(*a*) Table A.5 for air: $R = 287$ m²/(s² · K), $k = 1.40$. Neglecting velocity of approach, $p_1 = p_o$

From Eq. 13.33: $p* = 0.528p_o = 0.528p_1 = 0.528(700) = 370$ kN/m² abs ◀

From Eq. 13.36: $T* = T_1(p*/p_1)^{(k-1)k} = (273 + 20)0.528^{0.4/1.4} = 244$ K

Sec. 13.5 for critical conditions: M = 1. ∴ (Eq. 7.10 and 13.10) $V = c$

Eq. 13.15: $c = \sqrt{kRT} = \sqrt{1.4(287)244}$ so $V = 313$ m/s ◀

(*b*) If $D_2/D_1 = 0.80$, $p*$ will be the same and $V = c$ will be a little larger, perhaps about 318 m/s ◀◀

Sec. 11.10: Flow Measurement of Compressible Fluids -- *Problems* 11.35–11.40

11.35 *Air at 70°F and 104 psia flows through a venturi tube with a coefficient of 0.98. The pressure at the throat is 67.6 psia, the inlet area is 0.60 ft², and the throat area is 0.15 ft². Given k for the air is 1.40. (a) Using Eq. (11.22), find the ideal weight flow rate. (b) Evaluate Y from Fig. 11.25, and use it to find the actual weight flow rate. (c) What is the throat velocity?*

BG

Table A.5 for air: $R = 1715$ ft²/(sec²·°R), $c_p = 6000$ ft²/(sec²·°R)

Eq. 2.5: $\gamma_1 = 32.2(104)144/[1715(460 + 70)] = 0.531$ pcf ; $p_1 = 104(144) = 14{,}976$ psf abs

(*a*) Eq. 11.22:

$$G_{ideal} = 0.15\sqrt{2(32.2)\frac{1.4}{0.4}(14{,}976)0.531\left(\frac{67.6}{104}\right)^{\frac{2}{1.4}}\frac{1-\left(\frac{67.6}{104}\right)^{\frac{0.4}{1.4}}}{1-\left(\frac{0.15}{0.60}\right)^2\left(\frac{67.6}{104}\right)^{\frac{2}{1.4}}}} = 51.1 \text{ lb/sec} \quad ◀$$

(*b*) Fig. 11.25 for $D_2/D_1 = 0.5$ and $p_2/p_1 = 0.65$: $Y = 0.779$ ◀

Eq. 11.23: $G = 0.98(0.779)0.15\sqrt{2(32.2)0.531(104 - 67.6)144/(1 - 0.5^4)} = 50.0$ lb/sec ◀

(*c*) Eq. 11.21: $V^2/2 = 6000(460 + 70)\sqrt{1 - 0.65^{0.4/1.4}} = 3.68 \times 10^5$, $V = 858$ fps ◀

11.36 *Air at 20°C and 700 kPa abs flows through a venturi tube with a coefficient of 0.98. The pressure at the throat is 420 kPa abs, the inlet area is 0.060 m², and the throat area is 0.015 m². Given k for the air is 1.40. (a) Using Eq. (11.22), find the ideal weight flow rate. (b) Evaluate Y from Fig. 11.25, and use it to find the actual weight flow rate. (c) What is the throat velocity?*

SI

Air, Table A.5: $R = 287\ \text{m}^2/(\text{s}^2\cdot\text{K})$, $c_p = 1003\ \text{m}^2/(\text{s}^2\cdot\text{K})$. Eq. 2.5: $\gamma_1 = \dfrac{9.81(700\,000)}{287(273 + 20)} = 81.7\ \text{N/m}^3$

(*a*) Eq. 11.22:

$$G_{\text{ideal}} = 0.015\sqrt{2(9.81)\frac{1.4}{0.4}(700\,000)81.7\left(\frac{420}{700}\right)^{\frac{2}{1.4}}\frac{1-\left(\frac{420}{700}\right)^{\frac{0.4}{1.4}}}{1-\left(\frac{0.015}{0.060}\right)^2\left(\frac{420}{700}\right)^{\frac{2}{1.4}}}} = 244\ \text{N/s} \quad ◄$$

(*b*) Fig. 11.25 for $D_2/D_1 = 0.5$ and $p_2/p_1 = 0.60$: $Y = 0.744$ ◄

$$\text{Eq. 11.23:}\quad G = 0.98(0.744)0.015\sqrt{2(9.81)81.7\frac{(700-420)1000}{1-0.5^4}} = 239\ \text{N} \quad ◄$$

(*c*) Eq. 11.21: $V^2/2 = 1003(273 + 20)(1 - 0.60^{0.4/1.4}) = 3.99\times10^4$; $V = 283\ \text{m/s}$ ◄

11.37 *Using the same data as Prob. 11.35, what would be the value of Y and the weight flow rate for a square-edged orifice with C = 0.59?*

Prob. 11.35: Air flows, $A_1 = 0.60\ ft^2$, $A_o = 0.15\ ft^2$, $T_1 = 70°F$, $p_1 = 104$ psia, $p_2 = p_o = 67.6$ psia.

BG

Table A.5 for air: $R = 1715\ \text{ft}^2/(\text{sec}^2\cdot°\text{R})$. Eq. 2.5: $\gamma_1 = \dfrac{32.2(104)144}{1715(460 + 70)} = 0.531$ pcf

Fig. 11.25 for square-edged orifice with $D_o/D_1 = 0.5$ and $p_2/p_1 = 0.65$: $Y = 0.891$ ◄

$$\text{Eq. 11.23:}\quad G = 0.59(0.891)0.15\sqrt{2(32.2)0.531\frac{(104-67.6)144}{1-0.5^4}} = 34.5\ \text{lb/sec} \quad ◄$$

11.38 *Using the same data as Prob. 11.36, what would be the value of Y and the weight flow rate for a square-edged orifice with C = 0.62?*

Prob. 11.36: Air flows, $A_1 = 0.060\ m^2$, $A_o = 0.015\ m^2$, $T_1 = 20°C$, $p_1 = 700\ kN/m^2$ abs, $p_2 = p_o = 420\ kN/m^2$ abs.

SI

Table A.5 for air: $R = 287\ \text{m}^2/(\text{s}^2\cdot\text{K})$. Eq. 2.5: $\gamma_1 = \dfrac{9.81(700\,000)}{287(273 + 20)} = 81.7\ \text{N/m}^3$

Fig. 11.25 for square-edged orifice with $D_o/D_1 = 0.5$ and $p_2/p_1 = 0.60$: $Y = 0.876$ ◄

$$\text{Eq. 11.23:}\quad G = 0.62(0.876)0.015\sqrt{2(9.81)81.7\frac{(700-420)1000}{1-0.5^4}} = 178.2\ \text{N/s} \quad ◄$$

11.39 *Air flows through a 200-mm by 100-mm venturi meter. At inlet the air temperature is 15°C and the gage pressure is 150 kPa. Determine the flow rate if a mercury manometer reads 180 mm. Assume an atmospheric pressure of 101.3 kPa abs.*

SI

Venturi meter: $D_1 \times D_2 = 200 \text{ mm} \times 100 \text{ mm} = 7.9'' \times 3.9''$; $D_2/D_1 = 10/20 = 0.5$

$p_1 - p_2 = \gamma_M R_m = 13.56(9807 \text{ N/m}^3)0.18 \text{ m} = 23\,900 \text{ N/m}^2 = 23.9 \text{ kN/m}^2$

$p_1 = p_{gage} + p_{at} = 150 + 101.3 = 251.3 \text{ kN/m}^2$ abs

$p_2 = p_1 - (p_1 - p_2) = 251.3 - 23.9 = 227.4 \text{ kN/m}^2$ abs ; $p_2/p_1 = 227.4/251.3 = 0.905$

Fig. 11.25 for $p_2/p_1 = 0.905$ and $D_2/D_1 = 0.5$: $Y = 0.940$; Table A.5 for air: $R = 287 \text{ m}^2/(\text{s}^2\cdot\text{K})$

Eq. 2.5: $\gamma_1 = \dfrac{gp_1}{RT_1} = \dfrac{(9.81)251\,300 \text{ N/m}^2 \text{ abs}}{287(273 + 15)} = 29.8 \text{ N/m}^3$

Fig. 11.19 for $D_2 \times D_1 = 7.9'' \times 3.9''$ and $\mathbf{R}_2$ unknown: Assume $C = C_{max} = 0.985$

Eq. 11.23: $G = 0.985(0.940)\dfrac{\pi(0.1)^2}{4}\sqrt{2(9.81)29.8\dfrac{(23.9)1000}{1 - 0.5^4}} = 28.1 \text{ N/s}$

Check $\mathbf{R}_2$:

From Eqs. 2.2 and 2.6: $\dfrac{p_2}{p_1} = \left(\dfrac{v_1}{v_2}\right)^n = \left(\dfrac{\rho_2}{\rho_1}\right)^k \quad \therefore \quad \dfrac{\rho_2}{\rho_1} = \left(\dfrac{p_2}{p_1}\right)^{1/k} = 0.905^{1/1.4} = 0.931$

So $\rho_2 = 0.931\rho_1 = 0.931(\gamma_1/g) = 0.931(29.8/9.81) = 2.83 \text{ kg/m}^3$

Eq. 2.7: $T_2/T_1 = (p_2/p_1)^{(k-1)/k} = 0.905^{0.4/1.4} = 0.972$

So $T_2 = 0.972T_1 = 0.972(273 + 15) = 279.9 \text{ K} = 6.9°\text{C}$

Table A.2 for air at 6.9°C (noting, per Sec. 2.11, that μ is virtually independent of p), by interpolation:

$\mu_2 = 17.45 \times 10^{-6} = \text{N}\cdot\text{s/m}^2$

From Eq. 4.16*b*: $V_2 = \dfrac{G}{\rho_2 g A_2} = \dfrac{28.1 \text{ N/s}}{(2.83 \text{ kg/m}^3)9.81 \text{ m/s}^2(\pi 0.1^2/4) \text{ m}^2} = 128.9 \text{ m/s}$

At throat: $\mathbf{R}_2 = \dfrac{D_2 V_2 \rho_2}{\mu_2} = \dfrac{0.1(128.9)2.83}{17.45 \times 10^{-6}} = 2.09 \times 10^6$

Fig. 11.19 for $\mathbf{R}_2 = 2.09 \times 10^6$ and $D_1 \times D_2 = 7.9'' \times 3.9''$: $C = 0.985$

$\therefore$ Assumed C was correct; $G = 28.1 \text{ N/s}$ ◄

11.40 *Air flows through a 50 mm diameter orifice from a tank at 1500 kPa abs and 40°C into a space where the pressure is 500 kPa abs. (a) Compute the weight flow rate assuming C_d = 0.60. Refer to Sec. 13.8. Repeat for external pressures of (b) 750, (c) 1000, and (d) 1250 kPa abs.*

SI

Table A.5 for air: $R = 287\ \text{m}^2/(\text{s}^2\cdot\text{K})$.

(*a*) $p_2/p_1 = 500/1500 = 0.333$.

Sec. 13.8: Because $p_2/p_1 < 0.528 = p_2^*/p_1$, sonic velocity occurs at the throat, use Eq. 13.45.

From Eq. 13.45: $G = C_d\dot{m}g = 0.60(9.81)\dfrac{\pi(0.025)^2 1500}{\sqrt{273+40}}\sqrt{\dfrac{1.4}{287}\left(\dfrac{2}{2.4}\right)^{2.4/0.4}} = 0.0396$ kN/s ◄

(*b*) Sec. 13.8: If $p_2 < 0.528p_1 = 0.528(1500) = 792$ kN/m² abs, then sonic flow occurs at the throat and Eq. 13.45 applies (as above). Thus for $p_2 = 750$ kN/m² abs: $G = 0.0396$ kN/s ◄

(*c*) If $p_2 > 792$ kN/m² abs, then use Eq. 11.23, for which

Eq. 2.5: $\gamma_1 = \dfrac{gp_1}{RT_1} = \dfrac{9.81(1500)}{287(273+40)} = 0.1638\ \text{kN/m}^3$

So for $p_2 = 1000$ kN/m² abs:

Fig. 11.25 for $D_o/D_1 = 0$ and $p_2/p_1 = 1000/1500 = 0.667$: $Y = 0.902$

Eq. 11.23: $G = 0.60(0.902)\pi 0.025^2\sqrt{2(9.81)0.1638(1500 - 1000)/(1 - 0)} = 0.0426$ kN/s ◄

(*d*) Similarly for $p_2 = 1250$ kN/m² abs: $p_2/p_1 = 0.833$, $Y = 0.951$, $G = 0.0318$ kN/s ◄

Sec. 11.11: Thin-Plate Weirs -- *Exercises* (5)

11.11.1 *A 2.5-ft-high rectangular sharp-crested weir extends across 6-ft-wide rectangular channel. When the head is 1.12 ft, determine the flow rate by neglecting the velocity of approach.*

BG

Eq. 11.27: $Q = 3.32LH^{3/2} = 3.32(6)1.12^{3/2} = 23.6$ cfs ◄

11.11.2 *An 0.75-m-high rectangular sharp-crested weir extends across a 2.2-m-wide rectangular channel. When the head is 320 mm, determine the flow rate by neglecting the velocity of approach.*

SI

Eq. 11.27: $Q = 1.83LH^{3/2} = 1.83(2.2)0.32^{3/2} = 0.729\ \text{m}^3/\text{s}$ ◄

11.11.3 *Suppose the rectangular weir of Exer. 11.11.1 is contracted at both ends. (a) Find the flow rate for a head of 1.12 ft by the Francis formula. (b) What would be the maximum value of H for which the Francis formula could be used?*

Exer. 11.11.1: P = 2.5 ft, L = 6 ft.

BG

(*a*) Eq. 11.27 with end contractions: $Q = 3.32[6.00 - 2(0.1)1.12]1.12^{3/2} = 22.7$ cfs ◄

(*b*) Fig. 11.27: For Francis formula, require $L > 3H$; $H_{max} = L/3 = 6/3 = 2.00$ ft ◄

11.11.4 *Suppose the rectangular weir of Exer. 11.11.2 is contracted at both ends. (a) Find the flow rate for a head of 320 mm by the Francis formula. (b) What would be the maximum value of H for which the Francis formula could be used?*

Exer. 11.11.2: $P = 0.75$ m, $L = 2.2$ m.

SI

(*a*) Eq. 11.27 with end contractions: $Q = 1.83[2.2 - 2(0.1)0.32]0.32^{3/2} = 0.708\ \text{m}^3/\text{s}$ ◄

(*b*) Fig. 11.27: For Francis formula, require $L > 3H$; $H_{max} = L/3 = 2.2/3 = 0.733$ m ◄

11.11.5 *(a) What is the rate of discharge of water over a 45° triangular weir when the head is 0.6 ft? (b) With the same head, what would be the increase in discharge obtained by doubling the notch angle, i.e., for a 90° weir? (c) What would be the head for a discharge of 1.5 cfs of water over a 60° triangular weir?*

BG

(*a*) Fig. 11.29 for $\theta = 45°$ and $H = 0.6$ ft: $C_d = 0.593$

Eq. 11.28: $Q = 0.593(8/12)\sqrt{2(32.2)}[\tan(45°/2)]0.6^{5/2} = 0.293$ cfs ◄

(*b*) Fig. 11.29 for $\theta = 90°$ and $H = 0.6$ ft: $C_d = 0.583$

Eq. 11.28: $Q = 0.583(8/15)\sqrt{2(32.2)}[\tan(90°/2)]0.6^{5/2} = 0.696$ cfs ◄

Discharge increase = (0.696 - 0.293)/0.293 = 1.375 = 137.5% ◄

(*c*) Fig. 11.29 for $\theta = 60°$ and unknown H: Assume $C_d = 0.581$ (the minimum).

Eq. 11.28: $1.5 = 0.581(8/15)\sqrt{2(32.2)}[\tan(60°/2)]H^{5/2}$; $H = 1.018$ ft

Fig. 11.29 for $\theta = 60°$ and $H = 1.02$ ft, by extrapolation: $C_d = 0.582$

Eq. 11.28 with $C_d = 0.582$: $H = 1.017$ ft ◄

Sec. 11.11: Thin-Plate Weirs -- *Problems 11.41–11.45*

11.41 *Using the Rehbock formula, plot a family of curves of C_d versus H/P with H as a parameter. These curves give a complete picture of the variation of C_d for sharp-crested rectangular weirs. Include H/P values of 0.1, 0.2, 0.5, 1.0, 2.0, and H values of 0.2, 1.0, and 5.0 ft.*

BG

Eq. 11.25*a*:

H/P	C_d		
	$H = 0.2$ ft	1.0 ft	5.0 ft
0.1	0.629	0.616	0.614
0.2	0.637	0.624	0.622
0.5	0.661	0.648	0.646
1.0	0.701	0.688	0.686
2.0	0.781	0.768	0.766
	▲	▲	▲

11.42 *For the Cipolletti weir of Sec. 11.11, confirm that the side slopes of the trapezoid are 4V:1H by setting the reduction in discharge due to contraction equal to the increase in discharge due to the triangular area added.*

N

Eq. 11.24: The decrease in flow due to two end contractions is $\Delta Q = C_d(2/3)\sqrt{2g}[2(0.1)H]H^{3/2}$

This deficiency is made up by the equivalent triangular weir of angle θ, per Eq. 11.28. Equating these:

$C_d(2/3)\sqrt{2g}0.2H^{5/2} = C_d(8/15)\sqrt{2g}[\tan(\theta/2)]H^{5/2}$

Assuming C_d is the same for both weirs: $\tan(\theta/2) = 0.25$, or side slope = 4V:1H QED ◄

11.43 *Develop in general terms an expression for the percent of error in Q over a triangular weir if there is a small error in the measurement of the vertex angle. Assume there is no error in the weir coefficient. Compute the percent error in Q if there is a 1° error in the measurement of the total vertex angle of a triangular weir having a total vertex angle of 60°.*

N

Eq. 11.28: $Q = C_d(8/15)\sqrt{2g}H^{5/2}\tan(\theta/2)$

$dQ/d\theta = C_d(8/15)\sqrt{2g}H^{5/2}d/d\theta[\tan(\theta/2)] = C_d(8/15)\sqrt{2g}H^{5/2}(1/2)\sec^2(\theta/2)$

Dividing: $\dfrac{dQ}{Q} = \dfrac{\sec^2(\theta/2)\,d\theta}{2\tan(\theta/2)} = \dfrac{d\theta}{2\sin(\theta/2)\cos(\theta/2)} = \dfrac{d\theta}{\sin\theta}$ ◄

For $\theta = 60°$ and $d\theta = 1°$: $dQ/Q = \dfrac{1°}{\sin 60°} = \dfrac{\pi/180}{0.866} = 0.0202 = 2.02\%$ ◄

11.44 *A 60° V-notch weir and a rectangular weir with suppressed end contractions (crest length = 2 ft) are both used to measure a flow rate of approximately 0.25 cfs. Assuming C_d is known to be minimum for each weir, compute the percentage of error in Q that would result from an error of 0.01 ft in the respective head measurements.*

BG

For V-notch weir: Eq. 11.28: $Q = C_d(8/15)\sqrt{2g}[\tan(\theta/2)]H^{5/2}$

Differentiating: $dQ/dH = C_d(8/15)\sqrt{2g}[\tan(\theta/2)](5/2)H^{3/2}$

Dividing by Eq. 11.28: $dQ/Q = 2.5dH/H$

Fig. 11.29: Minimum C_d is 0.581. With $Q = 0.25$ cfs and $\theta = 60°$,

Eq. 11.28: $0.25 = 0.581(8/15)\sqrt{2(32.2)}[\tan(60°/2)]H^{5/2}$; $H = 0.498$ ft

For $\Delta H = 0.01$ ft: $dQ/Q = 2.5(0.01/0.498) = 0.0502 = 5.02\%$ ◄

For suppressed rectangular weir: Eq. 11.27: $Q = 3.32LH^{3/2}$

Differentiating: $dQ/dH = 3.32L(3/2)H^{1/2}$; Dividing by Eq. 11.27: $dQ/Q = 1.5(dH/H)$

Sec. 11.11: Minimum C_d is about 0.62, which is the value used in Eq. 11.27.

Eq. 11.27 with $Q = 0.25$ cfs and $L = 2$ ft: $0.25 = 3.32(2.0)H^{3/2}$; $H = 0.1123$ ft

For $\Delta H = 0.01$ ft: $dQ/Q = 1.5(0.01/0.1123) = 0.1335 = 13.35\%$ ◄

11.45 *A 60° V-notch weir and a rectangular weir with end contractions (crest length = 0.6 m) are both used to measure a flow rate of approximately 7 L/s. Assuming C_d is known to be minimum for each weir, compute the percentage of error in Q that would result from an error of 5 mm in the respective head measurements.*

SI

For V-notch weir: Eq. 11.28: $Q = C_d(8/15)\sqrt{2g}[\tan(\theta/2)]H^{5/2}$

Differentiating: $dQ/dH = C_d(8/15)\sqrt{2g}[\tan(\theta/2)](5/2)H^{3/2}$; dividing by Eq. 11.28: $dQ/Q = 2.5dH/H$

Fig. 11.29: Minimum C_d is 0.581. With $Q = 0.007$ m³/s and $\theta = 60°$,

Eq. 11.28: $0.007 = 0.581(8/15)\sqrt{2(32.2)}[\tan(60°/2)]H^{5/2}$; $H = 0.1509$ m

For $\Delta H = 0.005$ m: $dQ/Q = 2.5(0.005/0.1509) = 0.0828 = 8.28\%$ ◄

For rectangular weir: Eq. 11.27: $Q = 1.83LH^{3/2}$

Differentiating: $dQ/dH = 1.83L(3/2)H^{1/2}$; Dividing by Eq. 11.27: $dQ/Q = 1.5(dH/H)$

Sec. 11.11: Minimum C_d is about 0.62. With $Q = 0.007$ m³/s and $L = 0.6$ m,

Eq. 11.27: $0.007 = 1.83(0.6)H^{3/2}$; $H = 0.0344$ m

For $\Delta H = 0.005$ m: $dQ/Q = 1.5(0.005/0.0344) = 0.218 = 21.8\%$ ◄

Sec. 11.12: Streamlined Weirs and Free Outfall -- *Exercises (6)*

11.12.1 *What is the flow rate per unit width over a broad crested weir which rises 1.0 ft above the bed of a horizontal channel when the head is 3.0 ft above the crest?*

BG

Eq. 11.30: $Q = L\sqrt{g}(2/3)^{3/2}E^{3/2} = L\sqrt{32.2}(2/3)^{3/2}(3 + V_0^2/2g)^{3/2}$

$Q/L = q = 3.09(3 + V_0^2/2g)^{3/2}$, but $q = 4V_0$ or $V_0 = q/4$

$\therefore\ q = 3.09[3 + q^2/(32 \times 32.2)]^{3/2}$; by trial $q = 18.92$ cfs/ft ◄

11.12.2 *What is the flow rate per unit width over a broad crested weir which rises 0.3 m above the bed of a horizontal channel when the head is 0.9 m above the crest?*

SI

Eq. 11.30: $Q = L\sqrt{g}(2/3)^{3/2}E^{3/2} = L\sqrt{9.81}(2/3)^{3/2}(0.90 + V_0^2/2g)^{3/2}$

$Q/L = q = 1.705(0.90 + V_0^2/2g)^{3/2}$, but $q = 1.20V_0$ or $V_0 = q/1.20$

$\therefore\ q = 1.705[0.90 + q^2/(2.88 \times 9.81)]^{3/2}$; by trial $q = 1.715\ \text{m}^3/\text{s}$ per m ◄

11.12.3 *Find the water depth just upstream of a 1.5-ft-high broad-crested weir in a channel 6 ft wide when the flow is 18 cfs.*

BG

$q = 18/6 = 3$ cfs/ft ; $y_2 = y_c = (q^2/g)^{1/3} = (3^2/32.2)^{1/3} = 0.654$ ft

$V_2 = q/y_2 = 3/0.654 = 4.59$ fps ; $y_0 + V_0^2/2g = 1.5 + 0.654 + (4.59^2/2g)$; but $V_0 = 3/y_0$

$\therefore\ y_0 + (3/y_0)^2/(2 \times 32.2) = 2.48$; by trial, $y_0 = 2.46$ ft ◄

11.12.4 *Find the water depth just upstream of an 0.4-m-high broad-crested weir in a channel 1.8 m wide when the flow is 0.54 m^3/s.*

SI

$q = 0.54/1.8 = 0.30\ \text{m}^3/\text{s}$ per m ; $y_2 = y_c = (q^2/g)^{1/3} = (0.30^2/9.81)^{1/3} = 0.209$ m

$V_2 = q/y_2 = 0.30/0.209 = 1.433$ m/s ; $y_0 + V_0^2/(2g) = 0.4 + 0.209 + 1.433^2/(2g)$; $V_0 = 0.3/y_0$

$\therefore\ y_0 + 1/(218y_0^2) = 0.714$; by trial, $y_0 = 0.705$ m ◄

11.12.5 *Subcritical flow in a rectangular open channel 6 ft wide ends in a free overfall. If the water depth at the brink measures 2.70 ft, estimate the flow rate.*

BG

Sec. 11.12: $y_c = y_b/0.72 = 2.7/0.72 = 3.75$ ft

From Eq. 10.23: $q = \sqrt{gy_c^3} = \sqrt{32.2(3.75)^3} = 41.2$ cfs/ft ; $Q = bq = 6(41.2) = 247$ cfs ◄

11.12.6 *Subcritical flow in a rectangular open channel 2 m wide ends in a free overfall. If the water depth at the brink measures 0.75 m, estimate the flow rate.*

SI

Sec. 11.12: $y_c = y_b/0.72 = 0.75/0.72 = 1.042$ m

From Eq. 10.23: $q = \sqrt{gy_c^3} = \sqrt{9.81(1.042)^3} = 3.33\ \text{m}^3/\text{s}$ per m; $Q = bq = 2(3.33) = 6.66\ \text{m}^3/\text{s}$ ◄

Sec. 11.12: Streamlined Weirs and Free Outfall -- *Problems* 11.46–11.47

11.46 *Plot C_d versus H/P for broad-crested weirs using Eq. (11.31). Include H/p values of 0.1, 0.2, 0.5, 1.0, 2.0, and H values of 0.2, 1.0, and 5.0 ft.*

BG

Successive trial and error solutions are required.

Given P/H and H: $P = H(P/H)$

Fig. 11.30: $y_0 + V_0^2/2g = P + y_c + V_c^2/(2g) = P + 1.5y_c$; $y_0 = P + H$

i.e., $P + H + [1/(2g)]\,[q/(P + H)]^2 = 1.5(q^2/g)^{1/3}$; $E = H + V_0^2/(2g)$

i.e., $E = H + [1/(2g)][q/(P + H)]^2 = 1.5(q^2/g)^{1/3}$

Given P and H, solve this by trial for q, and hence find E.

Then find C_d using Eq. 11.31. Repeat for other P/H and H values.

Trial and error results:

H/P	C_d		
	H = 0.2 ft	1.0 ft	5.0 ft
0.1	0.579	0.578	0.579
0.2	0.581	0.581	0.581
0.5	0.594	0.593	0.592
1.0	0.614	0.614	0.614
2.0	0.656	0.656	0.652
	▲	▲	▲

These repetitive trial and error solutions can be performed effectively using a spreadsheet or Mathcad.

11.47 *All the weir crests discussed in this chapter produce flow rates which vary as the head to some power greater than 1. In certain cases, such as in the outlet of a constant-velocity sedimentation chamber, it is desirable to employ a weir form in which Q varies directly with H. The proportional-flow weir is set flush with the bottom of the channel, as shown in Fig. P11.47, while the sides taper inward, following the hyperbola $x\sqrt{y} = k$, a constant. Commencing with the head $h = H - y$, on the element of area $dA = 2x\,dy = 2(k/\sqrt{y})dy$, prove that the discharge equation for such a weir may be written as $Q = C_d\pi k\sqrt{2g}H$, and evaluate k in terms of the width B and the velocity V in the rectangular approach channel.*

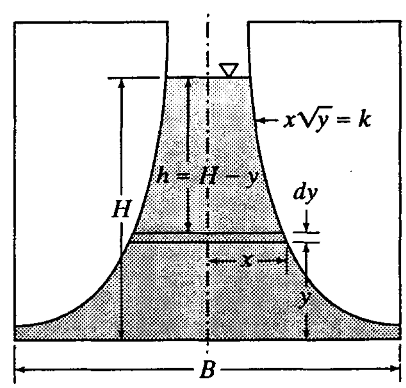


Figure P11.47

N

$$dQ_i = V dA = \sqrt{2gh}(2x dy) = \sqrt{2g}\sqrt{H - y}\,2k\,dy/\sqrt{y}$$

$$Q_i = 2k\sqrt{2g}\int_0^H \left[\frac{H - y}{y}\right]^{1/2} dy$$

To integrate, let $\sqrt{(H - y)/y} = \tan\theta$; Then,

$\sqrt{y/H} = \cos\theta$; $y = H\cos^2\theta$; $dy = -2H\cos\theta\sin\theta\,d\theta$

At $y = 0$, $\tan\theta = \infty$, $\theta = \pi/2$; at $y = H$, $\tan\theta = 0$, $\theta = 0$

Then, $Q_i = 2k\sqrt{2g}\int_{\pi/2}^{0} -2H\cos\theta\sin\theta\tan\theta\,d\theta = 2k\sqrt{2g}\int_{\pi/2}^{0} -2H\sin^2\theta\,d\theta$

$= 2k\sqrt{2g}(-2H)[\theta/2 - (1/4)\sin 2\theta]_{\pi/2}^{0} = \pi H k\sqrt{2g}$

$Q = C_d Q_i = C_d\pi k\sqrt{2g}H$ Q.E.D. ◄

$Q = C_d\pi k\sqrt{2g}H = VBH$; so $k = VB/(\pi C_d\sqrt{2g})$ ◄

Sec. 11.14: Sluice Gate -- *Exercises (2)*

11.14.1 *A rectangular channel contains a sluice gate which extends across the full width of the channel with an opening of 0.24 ft. Given C_d = 0.60 and free flow, find the flow rate per unit width and the sluice coefficient when the upstream and downstream depths are 2.05 ft and 0.15 ft respectively.*

BG

Eq. 11.35: $V_{2i} = \frac{1}{\sqrt{1 - (0.15/2.10)^2}} \sqrt{2(32.2)(2.05 - 0.15)} = 11.09$ fps

$q_i = aV_{2i} = 0.24(11.09) = 2.66$ cfs/ft ; $q = C_d q_i = 0.6(2.66) = 1.597$ cfs/ft ◄

From Eq. 11.36: $K_s = \frac{q}{a\sqrt{2gy_1}} = \frac{1.597}{0.24\sqrt{2(32.2)2.10}} = 0.579$ ◄

11.14.2 *A rectangular channel contains a sluice gate which extends across the full width of the channel with an opening of 100 mm. Given C_d = 0.60 and free flow, find the flow rate per unit width and the sluice coefficient when the upstream and downstream depths are 850 mm and 60 mm respectively.*

SI

Eq. 11.35: $V_{2i} = \frac{1}{\sqrt{1 - (0.06/0.85)^2}} \sqrt{2(9.81)(0.85 - 0.06)} = 3.96$ m/s

$q_i = aV_{2i} = 0.10(3.96) = 0.396$ m³/s per m ; $q = C_d q_i = 0.6(0.396) = 0.237$ m³/s per m ◄

From Eq. 11.36: $K_s = \frac{q}{a\sqrt{2gy_1}} = \frac{0.237}{0.10\sqrt{2(9.81)0.85}} = 0.581$ ◄

Sec. 11.14: Sluice Gate -- *Problems 11.48–11.50*

11.48 *An 8-ft-wide rectangular channel contains a sluice gate which extends across the full width of the channel with an opening of 0.55 ft. Assuming C_d = 0.60, C_c = 0.62, and free flow, find the flow rate and the sluice coefficient when the upstream depth is 4.8 ft.*

BG

$y_2 = C_c a = 0.62(0.55) = 0.341$ ft ; Eq. 11.35: $V_{2i} = \sqrt{\frac{2(32.2)(4.8 - 0.341)}{1 - (0.341/4.8)^2}} = 16.99$ fps

$Q = C_d Q_i = C_d A V_{2i} = C_d a V_{2i} = C_d a B V_{2i} = 0.6(0.55)8(16.99) = 44.9$ cfs ◄

From Eq. 11.36: $K_s = \frac{Q}{A\sqrt{2gy_1}} = \frac{44.9}{8(0.55)\sqrt{2(32.2)4.8}} = 0.580$ ◄

11.49 *A 2.0-m-wide rectangular channel contains a sluice gate which extends across the full width of the channel with an opening of 0.15 m. Assuming C_d = 0.60, C_c = 0.62, and free flow, find the flow rate and the sluice coefficient when the upstream depth is 1.25 m.*

SI

$y_2 = C_c a = 0.62(0.15) = 0.0930$ m ; Eq. 11.35: $V_{2i} = \sqrt{\frac{2(9.81)(1.25 - 0.0930)}{1 - (0.0930/1.25)^2}} = 4.78$ m/s

$Q = C_d Q_i = C_d A V_{2i} = C_d a B V_{2i} = 0.6(0.15)2.0(4.78) = 0.860$ m³/s ◄

From Eq. 11.36: $K_s = \frac{0.860}{2.0(0.15)\sqrt{2(9.81)1.25}} = 0.579$ ◄

11.50 *Refer to Sample Problem 5.12. (a) If C_v = 0.98, what is the flow rate? (b) If C_c = 0.62, what is the height of the opening? (c) What is K_s?*

Sample Prob. 5.12: For no energy losses, V_{1i} = 2.12 m/s, V_{2i} = 5.29 m/s.

SI

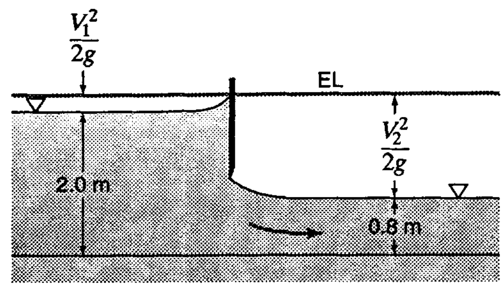


Figure S5.12

(*a*) First trial (using Sec. 11.6):

$V_2 = C_v V_{2i} = 0.98(5.29) = 5.18$ m/s

$V_1 = V_2(y_2/y_1) = 5.18(0.8/2.0) = 2.07$ m/s

$V_1^2/(2g) = 0.219$ m

Second trial: (using Fig. S5.11):

$V_{2i}^2/2g = 2.0 + 0.219 - 0.8 = 1.419$ m

$V_{2i} = 5.28$ m/s; $V_2 = 0.98(5.28) = 5.17$ m/s

This is little change from the first trial, so expect little further change. Close enough.

$q = y_2 V_2 = 0.8(5.17) = 4.14$ m³/s per m of width ◄

(*b*) Height of gate opening, $a = y_2/C_c = 0.8/0.62 = 1.290$ m ◄

(*c*) From Eq. 11.36: $K_s = \frac{Q}{A\sqrt{2gy_1}} = \frac{q}{a\sqrt{2gy_1}} = \frac{4.14}{1.290\sqrt{2(9.81)2.0}} = 0.512$ ◄

Chapter 12

Unsteady Flow Problems

PROBLEM SELECTION GUIDE

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
12.2	*Discharge With Varying Head*						
	X[1] 12.2.1	BG	Medium	Medium	2	12.2.2	Integration
	12.2.2	SI	Medium	Medium	2	12.2.1	Integration
	12.2.3	BG	Medium	Medium	1	12.2.4	Integration
	12.2.4	SI	Medium	Medium	1	12.2.3	Integration
	12.2.5	BG	Medium	Medium	1	12.2.6	Integration
	12.2.6	SI	Medium	Medium	1	12.2.5	Integration
	12.2.7	BG	Medium	Short	1	12.2.8	Integration
	12.2.8	SI	Medium	Short	1	12.2.7	Integration
	P 12.1	BG	Medium	Medium	3	12.2	Integration
	12.2	SI	Medium	Medium	3	12.1	Integration
	12.3	N	Medium	Medium	2		Integration
	12.4	BG	Medium	Medium	1	12.5	Integration; plot
	12.5	SI	Medium	Medium	1	12.4	Integration; plot
	12.6	BG	Hard	Long	1	12.7	Numerical integration
	12.7	SI	Hard	Long	1	12.6	Numerical integration
	12.8	BG	Hard	Long	1		Integration (using table of integrals)
12.3	*Unsteady Flow of Incompressible Fluids in Pipes*						
	X 12.3.1	BG	Medium	Medium	1		
	12.3.2	BG	Medium	Short	1		
	P 12.9	BG	Medium	Medium	2	12.10-12	Differentiation
	12.10	SI	Medium	Medium	2	12.9-12	Differentiation
	12.11	BG	Medium	Medium	2	12.9-12	Differentiation
	12.12	SI	Medium	Medium	2	12.9-11	Differentiation
	12.13	BG	Hard	Medium	1		Integration
	12.14	BG	Medium	Long	1	12.15	Assume/verify laminar flow; integration
	12.15	SI	Medium	Long	1	12.14	Assume/verify laminar flow; integration
	12.16	BG	Hard	Long	1	12.17	Integration
	12.17	SI	Hard	Long	1	12.16	Integration
	12.18	BG	Hard	Long	2	12.19	Integration
	12.19	SI	Hard	Long	2	12.18	Integration

/cont...

[1] For all Exercises (identified by "X"), answers are given in Appendix F of the textbook.
† Answers are sensitive to values (f) if they are read from a graph.
X = Exercise, P = (end-of-chapter) Problem, S = Sample Problem. ☐ = could use computing aids.

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
12.4	*Approach to Steady Flow*						
	X 12.4.1	BG	Easy	Short	1	12.4.2	Neglect minor losses (h', Sec 8.27)
	12.4.2	SI	Easy	Short	1	12.4.1	Neglect h' (Sec 8.27)
	12.4.3	SI	Easy	Medium	1	12.4.4,5,6	Neglect h' (Sec 8.27)
	12.4.4	BG	Easy	Medium	1	12.4.3,5,6	Include h' (Sec 8.27)
	12.4.5	BG	Easy	Short	1	12.4.3,4,6	Include h' (Sec 8.27)
	12.4.6	SI	Easy	Short	1	12.4.3,4,5	Include h' (Sec 8.27)
	P 12.20	BG	Medium	Medium	1	12.21,22	Neglect h' (Sec 8.27); plot
	12.21	BG	Medium	Medium	1	12.20,22	Include h' (Secs 8.21 & 8.27); plot
	12.22	SI	Medium	Medium	1	12.20,21	Neglect h' (Sec 8.27); plot
	12.23	BG	Medium	Long	1		Neglect h' (Sec 8.27)
	12.24	BG	Hard	Long	1	12.25	Integration; plot
	12.25	SI	Hard	Long	1	12.24	Integration; plot
	12.26	BG	Hard	Long	1		Integration; plot
	12.27	BG	Hard	Long	1	12.28	Neglect h' (Sec 8.27); assume/verify
	"						flow is laminar (Secs 8.1 & 8.7) and
	"						vel. head is negligible; integration; plot.
	12.28	SI	Hard	Long	1	12.27	Neglect h' (Sec 8.27); assume/verify
	"						flow is laminar (Sec 8.7) and velocity
	"						head is negligible; integration; plot.
	12.29	BG	V Hard	Long	3	12.30	Assume/verify laminar flow; integration.
	12.30	SI	V Hard	Long	3	12.29	Assume/verify laminar flow; integration.
12.5	*Velocity of Pressure Wave in Pipes*						
	X 12.5.1	BG	Easy	Short	3	12.5.2	
	12.5.2	SI	Easy	Short	3	12.5.1	
	12.5.3	BG	Easy	Short	1	12.5.4	
	12.5.4	SI	Easy	Short	1	12.5.3	
12.6	*Water Hammer*						
	X 12.6.1	BG	Easy	Short	2	12.6.2	Instantaneous closure
	12.6.2	SI	Easy	Short	2	12.6.1	Instantaneous closure
	12.6.3	BG	Medium	Medium	2	12.6.4	Rapid closure
	12.6.4	SI	Medium	Medium	2	12.6.3	Rapid closure
	12.6.5	BG	Medium	Medium	2	12.6.6	Slow closure
	12.6.6	SI	Medium	Medium	2	12.6.5	Slow closure
	P 12.31	BG	Medium	Medium	4	12.32	
	12.32	SI	Medium	Medium	4	12.31	
	12.33	BG	Medium	Medium	1		Uses Sec 8.5
	12.34	SI	Medium	Medium	2		† Slow closure; uses Sec 8.16 (or 8.15)
	12.35	BG	Medium	Medium	1		Slow closure
	12.36	BG	Medium	Medium	2	12.37	Rapid closure
	12.37	SI	Medium	Medium	2	12.36	Rapid closure

/cont...

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
12.7	***Surge Tanks***						
	X 12.7.1	BG	Easy	Short	1	12.7.2	Neglect h' & $V^2/2g$; find z_{max}
	12.7.2	SI	Easy	Short	1	12.7.1	Neglect h' & $V^2/2g$; find D_s
	P 12.38	N	Hard	Medium	1		Integration, differentiation.
	12.39	SI	Medium	Short	1	12.40	Include h' & $V^2/2g$; find Q_0
	12.40	SI	Medium	Short	1	12.39	Neglect h' & $V^2/2g$; find Q_0
	12.41	BG	Hard	Medium	1	12.42-3,46-7	☐ Incl. h' & $V^2/2g$, find z_{max}. T&E
	12.42	BG	Hard	Medium	1	12.41-3,46-7	☐ Negl. h' & $V^2/2g$, find z_{max}. T&E
	12.43	BG	Hard	Medium	1	12.41-2,46-7	☐ Incl. h' & $V^2/2g$, find z_{max}. T&E
	12.44	BG	Hard	Medium	1	12.45	☐ Include h' & $V^2/2g$, find D_s. T & E
	12.45	BG	Hard	Medium	1	12.44	☐ Neglect h' & $V^2/2g$, find D_s. T & E
	12.46	SI	Hard	Medium	1	12.41-3,47	☐ Include h' & $V^2/2g$, find z_{max}. T&E
	12.47	SI	Hard	Medium	1	12.41-3,46	☐ Neglect h' & $V^2/2g$, find z_{max}. T&E

Chapter 12
UNSTEADY-FLOW PROBLEMS

Sec. 12.2: Discharge With Varying Head -- *Exercises (8)*

12.2.1 *(a) A ship lock has vertical sides. Water enters or leaves through a conduit area of cross-sectional area A. The flow through the conduit is given by $Q = C_d A(2gz)^{1/2}$, where z is the variable difference in elevation between the water surface in the lock and that outside. Prove that the time required for the water level in the lock to drop from z_1 to z_2 is*

$$t = \frac{2A_s}{C_d A\sqrt{2g}}(z_1^{1/2} - z_2^{1/2})$$

(Note: If the lock is being filled, the signs must be reversed.) (b) Suppose the lock is 400 ft long by 100 ft wide, and the discharge coefficient for the conduit is 0.50. If the water surface in the lock is initially 40 ft below the level of the surface of the water upstream, how large must the conduit be if the lock is to be filled in 10 min?

BG

(*a*) In Eq 12.2: $Q_i = 0$; $Q_o = C_d A(2gz)^{1/2}$

$$\text{thus } t = \int_{z_1}^{z_2} \frac{A_s dz}{-C_d A(2gz)^{1/2}} = \frac{A_s}{C_d A(2g)^{1/2}}\int_{z_1}^{z_2}\frac{dz}{-z^{1/2}}$$

$$= \frac{A_s}{C_d A(2g)^{1/2}}\left[-2z^{1/2}\right]_{z_1}^{z_2} = \frac{2A_s}{C_d A\sqrt{2g}}\left(z_1^{1/2} - z_2^{1/2}\right) \quad \text{Q.E.D.} \blacktriangleleft$$

(*b*) From the above Eq: $10(60) = \dfrac{2(400)100}{0.50A\sqrt{2(32.2)}}(40^{1/2} - 0^{1/2})$ from which $A = 210 \text{ ft}^2$ ◀

12.2.2 *(a) Repeat Exer. 12.2.1(a) (b) Suppose the lock is 100 m long by 30 m wide, and the discharge coefficient for the conduit is 0.50. If the water surface in the lock is initially 12 m below the level of the surface of the water upstream, how large must the conduit be if the lock is to be filled in 10 min?*

SI

(*a*) See the solution to Exer. 12.2.1(*a*).

(*b*) From the above Eq: $10(60) = \dfrac{2(100)30}{0.50A\sqrt{2(9.81)}}(12^{1/2} - 0^{1/2})$ from which $A = 15.64 \text{ m}^2$ ◀

12.2.3 *The tank in Fig. X12.2.3 has the shape of the frustrum of a cone with a 2-ft² orifice (C_d = 0.62) in the bottom. Given D_1 = 44 ft, D_2 = 20 ft, z_0 = 36 ft, and that the water level outside the tank is constant at section 2, how long will it take the water level in the tank to drop from section 1 to section 2? (Note: The tank diameter = ky, and y = z + h_2, where z is the variable distance between surface levels.)*

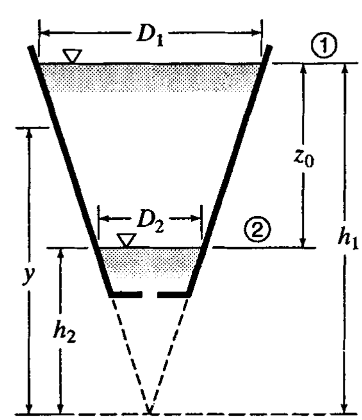


Figure X12.2.3

BG

$$\frac{h_2}{z_0} = \frac{D_2}{D_1 - D_2} \quad \text{so} \quad h_2 = \frac{D_2 z_0}{D_1 - D_2} = \frac{20(36)}{44 - 20} = 30 \text{ ft}$$

$z = y - h_2$; $y = z + h_2 = z + 30$; $k = D/y = (44 - 20)/36 = 2/3$

$A_s = (\pi/4)(2y/3)^2 = (\pi/9)y^2 = (\pi/9)(z + 30)^2$

$Q_i = 0$; $Q_o = C_d A\sqrt{2gz} = 0.62(2)\sqrt{2gz}$

$$\text{Eq 12.2: } t = \int \frac{A_s dz}{Q_i - Q_o} = \int_{36}^{0} \frac{(\pi/9)(z + 30)^2 dz}{0 - (0.62)2\sqrt{2gz}}$$

$$= -\frac{(\pi/9)}{(0.62)2\sqrt{2g}}\int_{36}^{0}[z^{3/2} + 60z^{1/2} + 900z^{-1/2}]dz = -\frac{(\pi/9)}{1.24\sqrt{2g}}\left[\frac{z^{2.5}}{2.5} + \frac{60z^{1.5}}{1.5} + \frac{900z^{0.5}}{0.5}\right]_{36}^{0} = 791 \text{ sec} \blacktriangleleft$$

12.2.4 *The cone-shaped tank in Fig X12.2.3 has the shape of the frustrum of a cone with a 0.2-m² orifice (C_d = 0.60) in the bottom. Given D_1 = 12 m, D_2 = 5.6 m, z_0 = 10.4 m, and that the water level outside the tank is constant at section 2, how long will it take the water level in the tank to drop from section 1 to section 2? (Note: The tank diameter = ky, and y = z + h_2, where z is the variable distance between surface levels.)*

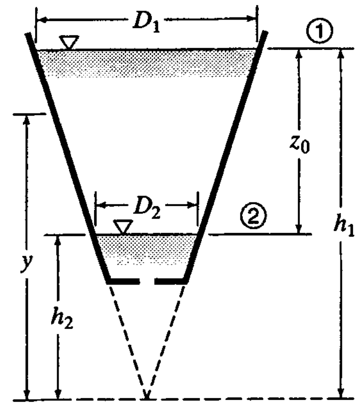


Figure X12.2.3

SI

$$\frac{h_2}{Z_0} = \frac{D_2}{D_1 - D_2}; \quad h_2 = \frac{D_2 z_0}{D_1 - D_2} = \frac{5.6(10.4)}{12 - 5.6} = 9.1\text{ m}$$

$$z = h_2 - y; \quad y = z + h_2 = z + 9.1; \quad k = d/y = (12 - 5.6)/10.4 = 0.615$$

$$A_s = (\pi/4)d^2 = (\pi/4)(0.615y)^2 = 0.297y^2 = 0.297(z + 9.1)^2\text{ m}^2$$

$$Q_i = 0; \quad Q_o = C_d A\sqrt{2gz} = 0.60(0.2)\sqrt{2(9.81)}z^{1/2} = 0.531z^{1/2}$$

$$\text{Eq. 12.2:}\quad t = \int\frac{A_s dz}{Q_i - Q_o} = \int_{10.4}^{0}\frac{0.297(z + 9.1)^2 dz}{0 - 0.531z^{1/2}} = -0.560\int_{10.4}^{0}[z^{1.5} + 18.2z^{0.5} + 82.8z^{-0.5}]dz$$

$$= -0.560\left[\frac{z^{2.5}}{2.5} + 18.2\frac{z^{1.5}}{1.5} + 82.8\frac{z^{0.5}}{0.5}\right]_{10.4}^{0} = 605\text{ s} \quad ◀$$

12.2.5 *Given the same tank and C_d as in Exer. 12.2.3, but now with the water surface outside constant at section 1 instead and the tank initially empty, how long will it take for the water level in the tank to rise from section 2 to section 1?*

Exer. 12.2.3: D_1 = 44 ft, D_2 = 20 ft, z_0 = 36 ft, 2-ft² orifice (C_d = 0.62). (Note: The tank diameter = ky, and y = z + h_2, where z is the variable distance between surface levels.)

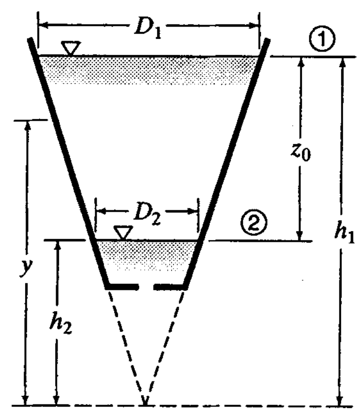


Figure X12.2.3

BG

$$\frac{h_2}{z_0} = \frac{D_2}{D_1 - D_2} \quad\text{so}\quad h_2 = \frac{D_2 z_0}{D_1 - D_2} = \frac{20(36)}{44 - 20} = 30\text{ ft}$$

$$A_s = (\pi/4)(2y/3)^2 = (\pi/9)y^2 = (\pi/9)(66 - z)^2$$

$$z = h_1 - y; \quad y = h_1 - z = 30 + 36 - z = 66 - z$$

$$k = d/y = (44 - 20)/36 = 2/3$$

$$Q_i = C_d A\sqrt{2gz} = 0.62(2)\sqrt{2gz}; \quad Q_o = 0. \qquad \text{Eq 12.2:}\quad t = \int\frac{A_s dz}{Q_i - Q_o} = \int_{36}^{0}\frac{(\pi/9)(66 - z)^2(-dz)}{(0.62)2(2gz)^{1/2}}$$

$$= -\frac{(\pi/9)}{(0.62)2\sqrt{2g}}\int_{36}^{0}[4356z^{-1/2} - 132z^{1/2} + z^{3/2}]dz = -\frac{(\pi/9)}{1.24\sqrt{2g}}\left[\frac{4356z^{0.5}}{0.5} - \frac{132z^{1.5}}{1.5} + \frac{z^{2.5}}{2.5}\right]_{36}^{0} = 1276\text{ sec} \quad ◀$$

12.2.6 *Given the same tank and C_d as in Exer. 12.2.4, but now with the water surface outside constant at section 1 instead and the tank initially empty, how long will it take for the water level in the tank to rise from section 2 to section 1?*

Exer. 12.2.4: D_1 = 12 m, D_2 = 5.6 m, z_0 = 10.4 m, 0.2-m² orifice (C_d = 0.60). (Note: The tank diameter = ky, and y = z + h_2, where z is the variable distance between surface levels.)

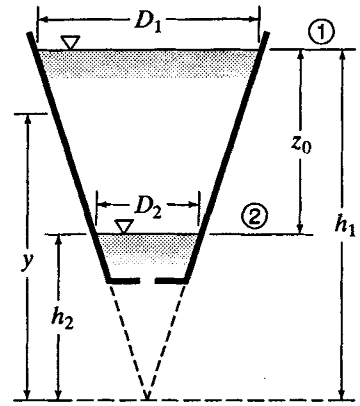


Figure X12.2.3

SI

$$\frac{h_2}{z_0} = \frac{D_2}{D_1 - D_2}; \quad h_2 = \frac{D_2 z_0}{D_1 - D_2} = \frac{5.6(10.4)}{12 - 5.6} = 9.1\text{ m}$$

$$z = h_1 - y; \quad y = h_1 - z = 9.1 + 10.4 - z = 19.5 - z$$

$$k = d/y = (12 - 5.6)/10.4 = 0.615$$

$$A_s = (\pi/4)d^2 = (\pi/4)(0.615y)^2 = 0.297y^2 = 0.297(19.5 - z)^2$$

$$Q_i = C_d A\sqrt{2gz} = 0.6(0.2)\sqrt{2(9.81)}z^{1/2} = 0.531\sqrt{z}; \quad Q_o = 0$$

Eq 12.2: $$t = \int \frac{A_s dz}{Q_i - Q_o} = \int_{10.4}^{0} \frac{0.297(19.5 - z)^2(-dz)}{0.531z^{1/2} - 0} = -0.560\int_{10.4}^{0}[380z^{-0.5} - 39z^{0.5} + z^{1.5}]dz$$

$$= -0.560\left[380\frac{z^{0.5}}{0.5} - 39\frac{z^{1.5}}{1.5} + \frac{z^{2.5}}{2.5}\right]_{10.4}^{0} = 962\text{ s} \quad ◄$$

12.2.7 *The tank in Fig. X12.2.7 has vertical sides, b_1 = 12 ft, b_2 = 24 ft, z_0 = 14 ft, and its dimension normal to the plane of the paper is a = 6 ft. The vertical dividing plate has a submerged orifice 0.7 ft² in area (C_d = 0.65). How long will it take for the two water surfaces to equalize?*

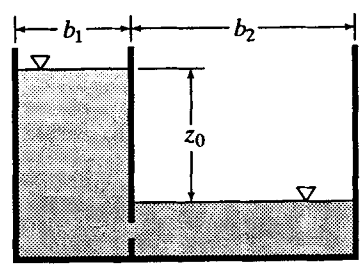


Figure X12.2.7

BG

$$dV = A_1 dz_1 = A_2 dz_2; \quad A_1/A_2 = b_1/b_2 = 12/24 = 1/2 = dz_2/dz_1$$

$$\therefore dz_1 = 2\,dz_2; \quad dz = dz_1 + dz_2 = 3dz_2 = 1.5dz_1; \quad dz_1 = (2/3)dz$$

Eq 12.2: $$t = \int \frac{A_1 dz_1}{C_d A\sqrt{2gz}} = \int \frac{b_1 a(2/3)dz}{C_d A\sqrt{2gz}}$$

$$= \frac{(12)6(2/3)}{0.65(0.7)\sqrt{2 \times 32.2}}\int_0^{14}\frac{dz}{z^{1/2}} = 13.15\left[\frac{z^{0.5}}{0.5}\right]_0^{14} = 98.4\text{ sec} \quad ◄$$

12.2.8 *The tank in Fig. X12.2.7 has vertical sides, b_1 = 3.6 m, b_2 = 7.2 m, z_0 = 4.2 m, and its dimension normal to the plane of the paper is a = 1.8 m. The vertical dividing plate has a submerged orifice 0.063 m² in area (C_d = 0.65). How long will it take for the two water surfaces to equalize?*

See Fig. X12.2.7 with Solution 12.2.7.

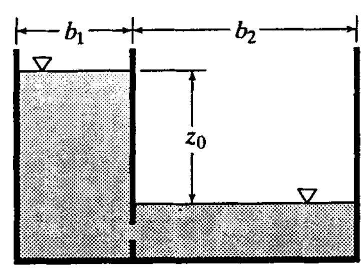


Figure X12.2.7

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$$dV = A_1 dz_1 = A_2 dz_2; \quad A_1/A_2 = b_1/b_2 = 3.6/7.2 = 1/2 = dz_2/dz_1$$

$$\therefore dz_1 = 2\,dz_2$$

$$dz = dz_1 + dz_2 = 3dz_2 = 1.5dz_1; \quad dz_1 = (2/3)dz$$

Eq 12.2: $$t = \int \frac{A_1 dz_1}{C_d A\sqrt{2gz}} = \int \frac{b_1 a(2/3)dz}{C_d A\sqrt{2gz}} = \frac{(3.6)1.8(2/3)}{0.65(0.063)\sqrt{2 \times 9.81}}\int_0^{4.2}\frac{dz}{z^{1/2}} = 23.8\left[\frac{z^{0.5}}{0.5}\right]_0^{4.2} = 97.6\text{ s} \quad ◄$$

Sec. 12.2: Discharge With Varying Head -- *Problems 12.1–12.8*

12.1 *(a) Outflow from a reservoir with vertical sides is over a spillway [$Q = C_W LH^{3/2}$, Eq. (11.26)]. Initially there is a steady inflow so that the height of the water surface above the spillway level is z_1. If the inflow is suddenly cut off, prove that the time required for the water level to fall from z_1 to z_2 is $t = (2A_s/C_W L)(z_2^{-0.5} - z_1^{-0.5})$. (Note: $z = H$.) (b) How long will it take theoretically for the outflow to cease entirely? What factors make this theoretical answer unrealistic? (c) Given the crest of the overflow spillway is 100 ft long, the value of C_w in Eq. (11.26) is 3.45, and the constant water surface area is 700,000 ft². With no inflow, find the time required for the water surface to fall from 3 ft to 1 ft above the spillway level.*

BG

(*a*) By Eq 12.2 with $Q_i = 0$ and $Q_o = C_W Lz^{3/2}$:

$$t = \int_{z_1}^{z_2} \frac{A_s dz}{-C_W Lz^{3/2}} = \frac{A_s}{C_W L}\left[\frac{2}{z^{1/2}}\right]_{z_1}^{z_2} = \frac{2A_s}{C_W L}\left[\frac{1}{z_2^{1/2}} - \frac{1}{z_1^{1/2}}\right] \quad \text{Q.E.D.} \quad ◄$$

(*b*) As $z_2 \to 0$, $t \to \infty$, so theoretically the outflow would never cease. ◄ Factors which make this theoretical answer unrealistic include surface tension, surface ripples, evaporation, and leakage/seepage losses.

(*c*) Eq 12.25 with $H = z$: $Q_o = C_W Lz^{3/2}$; substituting this into Eq 12.2 with $Q_i = 0$:

$$t = \int_{z_1}^{z_2} \frac{A_s dz}{-C_W Lz^{3/2}} = \frac{2A_s}{C_W L}\left[\frac{1}{z_2^{1/2}} - \frac{1}{z_1^{1/2}}\right] = \frac{2(700{,}000)}{3.45(100)}\left[\frac{1}{1^{1/2}} - \frac{1}{3^{1/2}}\right] = 1715 \text{ sec} = 28.6 \text{ min} \quad ◄$$

12.2 *Repeat parts (a) and (b) of Prob. 12.1. (c) Given the crest of the overflow spillway is 30m long, the value of C_W in Eq. (11.26) is 1.91, and the constant water surface area is 65 000 m². With no inflow find the time required for the water surface to fall from 900 mm to 300 mm above the spillway level.*

Prob. 12.1: (a) Outflow from a reservoir with vertical sides is over a spillway [$Q = C_W LH^{3/2}$, Eq. (11.26)]. Initially there is a steady inflow so that the height of the water surface above the spillway level is z_1. If the inflow is suddenly cut off, prove that the time required for the water level to fall from z_1 to z_2 is $t = (2A_s/C_W L)(z_2^{-0.5} - z_1^{-0.5})$. (Note: $z = H$.) (b) How long will it take theoretically for the outflow to cease entirely? What factors make this theoretical answer unrealistic?

SI

(*a*) By Eq 12.2 with $Q_i = 0$ and $Q_o = C_W Lz^{3/2}$:

$$t = \int_{z_1}^{z_2} \frac{A_s dz}{-C_W Lz^{3/2}} = \frac{A_s}{C_W L}\left[\frac{2}{z^{1/2}}\right]_{z_1}^{z_2} = \frac{2A_s}{C_W L}\left[\frac{1}{z_2^{1/2}} - \frac{1}{z_1^{1/2}}\right] \quad \text{Q.E.D.} \quad ◄$$

(*b*) As $z_2 \to 0$, $t \to \infty$, so theoretically the outflow would never cease. ◄ Factors which make this theoretical answer unrealistic include surface tension, surface ripples, evaporation, and leakage/seepage losses.

(*c*) Eq 12.25 with $H = z$: $Q_o = C_W Lz^{3/2}$; substituting this into Eq 12.2 with $Q_i = 0$:

$$t = \int_{z_1}^{z_2} \frac{A_s dz}{-C_W Lz^{3/2}} = \frac{2A_s}{C_W L}\left[\frac{1}{z_2^{1/2}} - \frac{1}{z_1^{1/2}}\right] = \frac{2(65\,000)}{1.91(30)}\left[\frac{1}{0.3^{1/2}} - \frac{1}{0.9^{1/2}}\right] = 1751 \text{ sec} = 29.2 \text{ min} \quad ◄$$

12.3 *Figure P12.3 shows a tank with vertical sides containing liquid with a surface area of A_s. The liquid discharges through an orifice under a head z which varies from the initial height h to zero as the tank empties down to the orifice level. (a) Neglecting friction losses, what is the cumulative kinetic energy of the jet during the time required for the liquid surface to drop from h to zero? (b) How does this kinetic-energy summation compare with the total energy of the mass of fluid initially in the tank above the orifice level?*

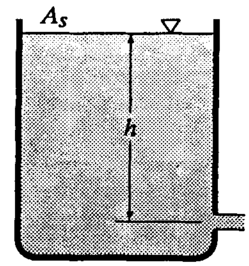

Figure P12.3

N

(*a*) $V = \sqrt{2gz}$; $Q_o = AV = A\sqrt{2gz}$; Eq 12.1: $A_s dz = -Q_o\,dt$

Sec 5.1: Cumulative K.E. $= \int(\Delta Wt \times V^2/2g)dt = \int \gamma(-Q_o dt)V^2/2g$

$$= -\gamma\int_h^0 A_s dz(2gz/2g) = \gamma A_s \int_0^h z dz = \gamma A_s (z^2/2)]_0^h = \gamma A_s h^2/2 \quad ◄$$

(*b*) Total potential energy in tank above orifice = weight × ave. height $= (\gamma A_s h)h/2 = \gamma A_s h^2/2$

This is the same as the cumulative K.E. ◄

12.4 *How long will it take for the tank in Prob. 12.3 to empty down to the orifice level if A_s is 8 ft², h is 16 ft, and the jet diameter is 5 in? Plot a graph of h versus time, using increments of 4 ft.*

See Fig. P12.3 with Solution 12.3. Prob. 12.3: A_s = surface area; head z on orifice varies from h to zero.

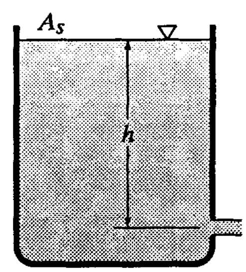

Figure P12.3

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$Q_i = 0$, $Q_o = A(2gz)^{1/2} = A(2g)^{1/2}z^{1/2}$

$$\text{Eq 12.2: } t = \int_{z_1}^{z_2}\frac{A_s dz}{0 - A(2g)^{1/2}z^{1/2}} = \frac{-A_s}{A(2g)^{1/2}}\int_{z_1}^{z_2} z^{-1/2}dz$$

$$= \frac{-A_s}{A(2g)^{1/2}}[2z^{1/2}]_{z_1}^{z_2} = \frac{2A_s}{A\sqrt{2g}}\left[\sqrt{z_1} - \sqrt{z_2}\right]$$

$2A_s/(A\sqrt{2g}) = 2(8)/[0.1363\sqrt{2(32.2)}] = 14.62 \text{ sec}\cdot\text{ft}^{-1/2}$

z_1 ft	$h = z_2$ ft	t sec	Σt sec
–	16	0	0
16	12	7.84	7.84
12	8	9.29	17.13
8	4	12.11	29.2
4	0	29.2	58.5
	▲		▲

Time to empty = 58.5 sec ◄

12.5 *How long will it take for the tank in Prob. 12.3 to empty down to the orifice level if A_s is 0.72 m^2, h is 5 m, and the jet diameter is 125 mm? Plot a graph of h versus time, using increments of 1 m.*

See Fig. P12.3 with Solution 12.3. Prob. 12.3: A_s = surface area; head z on orifice varies from h to zero.

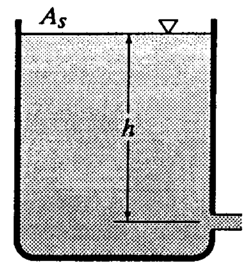

Figure P12.3

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$Q_i = 0, \quad Q_o = A(2gz)^{1/2} = A(2g)^{1/2} z^{1/2}$

$$\text{Eq 12.2:}\quad t = \int_{z_1}^{z_2} \frac{A_s dz}{0 - A(2g)^{1/2} z^{1/2}} = \frac{-A_s}{A(2g)^{1/2}} \int_{z_1}^{z_2} z^{-1/2} dz$$

$$= \frac{-A_s}{A(2g)^{1/2}} [2z^{1/2}]_{z_1}^{z_2} = \frac{2A_s}{A\sqrt{2g}} [\sqrt{z_1} - \sqrt{z_2}]$$

$2A_s/(A\sqrt{2g}) = 2(0.72)/[0.01227\sqrt{2(9.81)}) = 26.5 \text{ s·m}^{-1/2}$

z_1 m	$h = z_2$ m	t sec	$\sum t$ sec
–	5	0	0
5	4	6.25	6.25
4	3	7.10	13.35
3	2	8.42	21.8
2	1	10.97	32.7
1	0	26.5	59.2
	▲		▲

Time to empty = 59.2 s ◄

12.6 *Water enters a reservoir at such a rate that the height z of water above the level of the spillway crest is 3 ft. The spillway ($Q = C_W LH^{3/2}$, $H = z$) is 100 ft long, and the value of C_W is 3.50. The area of the water surface for various water levels is as follows:*

z, ft:	*3.00*	*2.50*	*2.00*	*1.50*	*1.25*	*1.00*
A_s*, ft²:*	*850,000*	*800,000*	*700,000*	*550,000*	*510,000*	*470,000*

If the inflow is suddenly reduced to 150 cfs, what will be the water level for equilibrium? How long will it take, theoretically, for equilibrium to be attained? How long will it take for the level to drop from 3 to 1 ft above the spillway level?

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$Q_o = 3.50(100)z^{3/2}$; when $Q_o = Q_i = 150$ cfs, $z = 0.568$ ft ◄

and the time for equilibrium is theoretically infinite. ◄

From Eq 12.2: $t = \int \frac{A_s dz}{350z^{3/2} - 150}$ (z gets smaller)

z (ft)	Δz (ft)	A_s (ft²)	$\bar{A}_s$ (ft²)	$\bar{A}_s(\Delta z)$ (ft³)	$Q_o = 350z^{3/2}$ (cfs)	$\bar{Q}_o$ (cfs)	$\bar{Q}_o - 150$ (cfs)	$\frac{\bar{A}_s(\Delta z)}{\bar{Q}_o - 150}$ (sec)
3.0		850,000			1819			
	0.5		825,000	412,500		1601	1451	284
2.5		800,000			1383			
	0.5		750,000	375,000		1187	1037	362
2.0		700,00			990			
	0.5		625,000	312,500		816	667	469
1.5		550,000			643			
	0.25		530,000	132,500		566	416	318
1.25		510,000			489			
	0.25		490,000	122,500		420	270	454
1.00		470,000			350			
								Σ = 1888 sec

Total time for z to change from 3 ft to 1.0 ft is 1888 seconds = 31.4 minutes. ◄

12.7 *Work Prob. 12.6 using the same numbers but changing ft to m, ft² to m², and cfs to m³/s.*

Prob. 12.6: Water enters a reservoir at such a rate that the height z of water above the level of the spillway crest is 3 ft. The spillway ($Q = C_W L H^{3/2}$, $H = z$) is 100 ft long, and the value of C_W is 3.50. The area of the water surface for various water levels is as follows:

z, ft:	*3.00*	*2.50*	*2.00*	*1.50*	*1.25*	*1.00*
A_s*, ft²:*	*850,000*	*800,000*	*700,000*	*550,000*	*510,000*	*470,000*

If the inflow is suddenly reduced to 150 cfs, what will be the water level for equilibrium? How long will it take, theoretically, for equilibrium to be attained? How long will it take for the level to drop from 3 to 1 ft above the spillway level?

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From Eqs. 11.27: $C_W = 3.50(1.83/3.32) = 1.929$; so $Q_o = 1.929(100)z^{3/2} = 192.9\,z^{3/2}$

When $Q_o = Q_i = 150\,\text{m}^3/\text{s}$, $z = (150/194)^{2/3} = 0.846\,\text{m}$ ◄ and theoretically $t_{\text{equilib}} = \infty$ ◄

From Eq. 12.2: $t = \displaystyle\int \frac{A_s\,dz}{192.9z^{3/2} - 150}$ (z gets smaller)

z (m)	Δz (m)	A_s (m²)	$\bar{A}_s$ (m²)	$\bar{A}_s(\Delta z)$ (m³)	$Q_o = 192.9z^{3/2}$ (m³/s)	$\bar{Q}_o$ (m³/s)	$\bar{Q}_o - 150$ (m³/s)	$\dfrac{\bar{A}_s(\Delta z)}{\bar{Q}_o - 150}$ (sec)
3		850 000			1002			
	0.5		825 000	412 500		883	733	563
2.5		800 000			763			
	0.5		750 000	375 000		654	504	744
2		700 000			547			
	0.5		625 000	312 500		460	300	1042
1.5		550 000			354			
	0.25		530 000	132 500		312	162.0	818
1.25		510 000			270			
	0.25		490 000	122 500		231	81.3	1507
1.00		470 000			192.9			
								Σ = 4670 sec

Total time for z to change from 3 m to 1.0 m is approximately 4670 s = 77.9 min. ◄

12.8 *A reservoir has an overflow spillway with a 40-ft-long crest, and a value of $C_W = 3.50$ [Eq. (11.26)]. For the range of water levels considered here, the area of the water surface is essentially constant at 600,000 ft^2. Initially, the water surface is 3 ft below the level of the spillway crest. If a flow of 500 cfs is suddenly discharged into this reservoir, what will be the height z of water in the reservoir (above spillway crest) for equilibrium? How much time will be required for this equilibrium height to be reached? How much time will be required for the water surface to reach 2 ft above the spillway level? (Note: This last part can be solved by integration after substituting x^3 for $z^{3/2}$ and consulting integral tables. However, it will be easier to solve it graphically, either by plotting and actually measuring the area under the curve, or by computing the latter by some method such as Simpson's rule.)*

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Eq. 11.26 with $H = z$: $Q_o = 3.50(40)z^{3/2} = 140z^{3/2}$. For equilibrium

$Q_i = Q_o = 500$ cfs, from which $z = (500/140)^{2/3} = 2.34$ ft ◄

The time required to reach this equilibrium height is theoretically infinite, but surface ripples and minor irregularities in flow cause practical equilibrium to be reached in a finite time. ◄ However, this does point out that true equilibrium is not quickly obtained. This should be borne in mind in experimental work.

Time for water level to reach spillway crest = hA/Q = 3(600,000/500) = 3600 sec. Time for water level to rise from crest to 2 ft above it is

Using Eq 12.2: $t = 600{,}000\int_0^2 \frac{dz}{500 - 140z^{3/2}}$. Letting $z^{3/2} = x^3$, then $z^{1/2} = x$, $z = x^2$, and $dz = 2xdx$

$\therefore\ t = 2(600{,}000)\int_0^{\sqrt{2}} \frac{x\,dx}{500 - 140x^3}$ (refer to a published table of integrals), using: $-1.529 = \left(\frac{500}{-140}\right)^{1/3}$

$$t = \frac{1{,}200{,}000}{6(-140)1.529}\left[\ln\frac{1.529^2 + 1.529x + x^2}{(x - 1.529)^2} + 2\sqrt{3}\tan^{-1}\frac{2x + 1.529}{-1.529\sqrt{3}}\right]_0^{\sqrt{2}}$$

$t = 935[(6.21 - 3.55) - (0 - 1.814)] = 4180$ sec

Total time = 3600 + 4180 = 7780 seconds = 129.7 minutes = 2.16 hours ◄

Sec. 12.3: Unsteady Flow of Incompressible Fluids in Pipes -- *Exercises (2)*

12.3.1 *Verify that the neglect of the accelerative head was justified in Sample Problem 12.1. Find its magnitude at z = 5 ft from values of V and t at z = 4.5 and 5.5 ft.*

Sample Prob. 12.1: Pipe length L = 10 ft. Energy equation from water surface to jet, neglecting h_a is $z - V^2/2g = [(fL/D) + \Sigma k]V^2/2g = 1.42V^2/2g$ from which $V = 5.16z^{1/2}$ and the time for the surface to fall from z = 8 ft is $t = -35.5[(2/3)z^{3/2}]_8^z$.

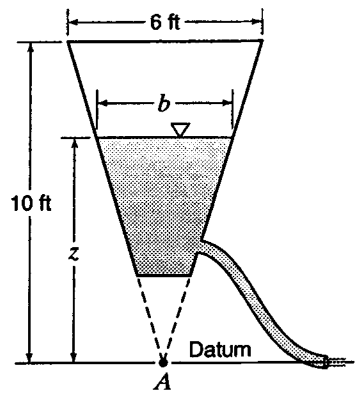

Figure S12.1

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Energy equation from water surface to jet, including h_a:

$$\text{Eq. 12.6a:}\quad z - \frac{V^2}{2g} = \left(\frac{fL}{D} + \Sigma k\right)\frac{V^2}{2g} + h_a\ ; \quad z = 2.42\frac{V^2}{2g} + h_a$$

$$\text{where } h_a = \frac{L}{g}\frac{dV}{dt}$$

Assuming the solution of Sample Prob. 12.1 to be approximately correct:

$$\text{At } z = 5.5\text{ ft:}\quad V = 5.16(5.5)^{1/2} = 12.10\text{ fps},\quad t = -35.5\left[\frac{2}{3}z^{3/2}\right]_8^{5.5} = 230\text{ sec}$$

$$\text{At } z = 4.5\text{ ft:}\quad V = 5.16(4.5)^{1/2} = 10.94\text{ fps},\quad t = -35.5\left[\frac{2}{3}z^{3/2}\right]_8^{4.5} = 310\text{ sec}$$

$$\therefore\ \text{at } z = 5\text{ ft},\ h_a = \frac{L}{g}\frac{dV}{dt} \approx \frac{L}{g}\frac{\Delta V}{\Delta t} = \frac{10}{32.2}\left(\frac{12.10 - 10.94}{230 - 310}\right) = -0.004\,52\text{ ft} \quad ◄$$

This is 0.09% of z = 5 ft. So, in comparison with other terms in the energy equation, h_a is negligible as assumed. ◄

12.3.2 *Assume the speed of the pump in Sample Prob. 12.2 is instead reduced instantaneously from 1650 to 1150 rpm. What then is the deceleration of the flow rate immediately after the change in pump speed?*

Sample Prob. 12.2: Q = 1600 gpm at n = 1650 rpm, and $h \propto n^2$. From energy and continuity: $50 + h_p = 32.0(V_2^2/2g) + (822/g)dV_2/dt$ (a). At original n = 1650: V_2 = 18.16 fps, h_p = 113.8 ft.

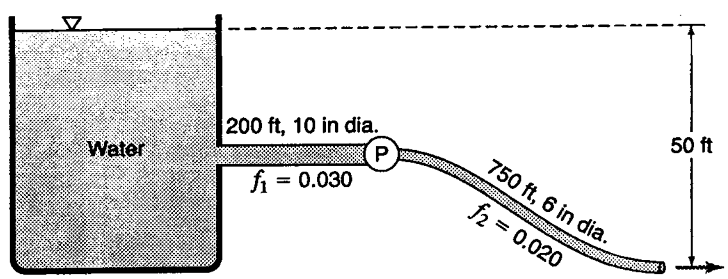

Fig. S12.2

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$$h_p' = h_p\left(\frac{n_2}{n_1}\right)^2 113.8\left(\frac{1150}{1650}\right)^2 = 55.3\text{ ft.}$$ Substituting this into Eq (*a*) of Sample Prob. 12.2:

$$50 + 55.3 = 32.0\frac{V_2^2}{2g} + \frac{822}{g}\frac{dV_2}{dt}\quad \text{where } V_2 = 18.16\text{ fps.}\quad \therefore\ \frac{dV_2}{dt} = \frac{105.3 - 163.8}{(822/32.2)} = -2.29\text{ fps/sec}$$

$$dQ/dt = A(dV/dt) = (\pi/4)(0.5)^2(-2.29) = -0.450\text{ cfs/sec} \quad ◄$$

Sec. 12.3: Unsteady Flow of Incompressible Fluids in Pipes -- *Problems 12.9–12.19*

12.9 *Attached to the tank in Fig. P12.9 is a flexible 1.5-in-diameter hose (f = 0.020) 250 ft long. The tank is hoisted in such a manner that h = 25 + 2t, where h is the head in feet and t is the time in seconds. (a) Find as accurately as you can the flow rate at t = 10 sec. (b) Suppose h were decreasing at the same rate. What would be the flow rate when h = 45 ft?*

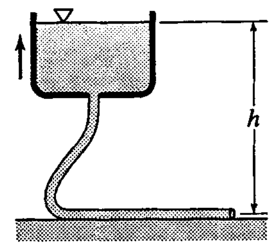

Figure P12.9

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Find steady state velocity for $t = 10$ sec; then $h = 25 + 2(10) = 45$ ft.

Eq. 12.6*a*: $45 - V^2/2g = 0.020(250 \times 12/1.5)(V^2/2g)$

$V = 8.41$ fps at steady state

Now find the effect of changing head:

Given $h = 25 + 2t$, $\therefore$ $dh/dt = 2.0$ ft/sec

Eq 12.6*a*: $h - 40(V^2/2g) = V^2/2g + (L/g)(dV/dt)$; $dh/dt = 41(2V/2g)(dV/dt) + (L/g)(d^2V/dt^2)$

Neglect the last term (of higher order); $\therefore$ $dh/dt = 2.0 = 41(V/g)(dV/dt)$

$dV/dt = 2g/(41V) = 0.1868$ fps/sec

(*a*) Substituting back into Eq 12.6*a*: $45 = 41(V^2/2g) + (250/32.2)(0.1868)$

$41(V^2/2g) = 45 - 1.451 = 43.5$; $V = 5.27$ fps, $Q = 0.1015$ cfs ◀

(and neglected higher order term = 0.0325 fps)

(*b*) For decreasing *h*, one sign (of dV/dt) is reversed: $41(V^2/2g) = 45 + 1.451 = 46.5$

$V = 8.54$ fps, $Q = 0.1048$ cfs ◀

12.10 *Attached to the tank in Fig. P12.9 is a flexible 40-mm-diameter hose (f = 0.024) 65 m long. The tank is hoisted in such a manner that h = 10 + 0.5t, where h is the head in meters and t is the time in seconds. (a) Find as accurately as you can the flow rate at t = 10 s. (b) Suppose h were decreasing at the same rate. What would be the flow rate when h = 15 m?*

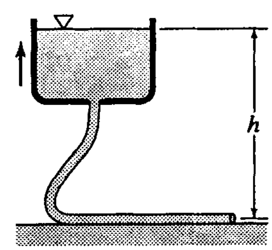

Figure P12.9

SI

Find steady state velocity for $t = 10$ s; then $h = 10 + 0.5(10) = 15$ m

Eq. 12.6*a*: $15 - V^2/2g = 0.024(65/0.040)(V^2/2g)$; $V = 2.21$ m/s at steady state

Now find effect of changing head. Given $h = 10 + 0.5t$, $\therefore$ $dh/dt = 0.5$ m/s

Eq 12.6*a*: $h - 39(V^2/2g) = V^2/2g + (L/g)(dV/dt)$; $dh/dt = 40(2V/2g)(dV/dt) + (L/g)(d^2V/dt^2)$

Neglect the last term (of higher order);

$\therefore$ $0.5 = 40(V/g)(dV/dt)$; $dV/dt = 0.5g/(40V) = 0.0452$ m/s per s

(*a*) Substituting back into Eq 12.6*a*:

$15 = 40(V^2/2g) + (65/9.81)(0.0452)$; $40(V^2/2g) = 15 - 0.300 = 14.70$

$V = 2.69$ m/s, $Q = 0.003\,37$ m^3/s ◀

(*b*) For decreasing *h*, one sign (of dV/dt) is reversed: $40(V^2/2g) = 15 + 0.300 = 15.30$

$V = 2.74$ m/s, $Q = 0.003\,44$ m^3/s ◀

12.11 *Repeat Prob. 12.9 for the case of a 6-in-diameter hose with all other data remaining the same.*

Prob. 12.9: The flexibile hose has f = 0.020, L = 250 ft. The tank is hoisted so that h = 25 + 2t, where h = ft, t = sec. (a) Find as accurately as possible Q when t = 10 sec. (b) Find Q when h = 45 ft if h were decreasing at the same rate.

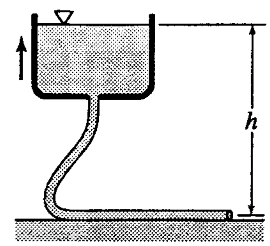

Figure P12.9

BG

Find steady state velocity for t = 10 sec; then $h = 25 + 2(10) = 45$ ft

Eq. 12.6*a*: $45 - \frac{V^2}{2g} = 0.020\frac{250}{0.5}\frac{V^2}{2g}$; V = 16.23 fps at steady state

Now find the effect of changing head

Given: $h = 25 + 2t$, $\therefore\ dh/dt = 2$ fps

Eq 12.6*a*: $h = 11(V^2/2g) + (L/g)(dV/dt)$

Differentiating with respect to t and neglecting the higher order term:

$dh/dt = 2.0 = 11(2V/2g)(dV/dt)$; $dV/dt = 2g/11V = 0.361$ fps/sec

(*a*) Substituting back into Eq. 12.6*a*: $45 = 11(V^2/2g) + (250/32.2)(0.361)$; $11(V^2/2g) = 45 - 2.80 = 42.2$

V = 15.72 fps, Q = 3.09 cfs ◄

(*b*) For decreasing h, one sign (of dV/dt) is reversed: $45 = 11(V^2/2g) + (250/32.2)(-0.361)$;

V = 16.73 fps, Q = 3.28 cfs ◄

12.12 *Repeat Prob. 12.10 for the case of a 150-mm-diameter hose with all other data remaining the same.*

Prob. 12.10: The flexibile hose has f = 0.024, L = 65 m. The tank is hoisted so that h = 10 + 0.5t, where h = m, t = s. (a) Find as accurately as possible Q when t = 10 s. (b) Find Q when h = 15 m if h were decreasing at the same rate.

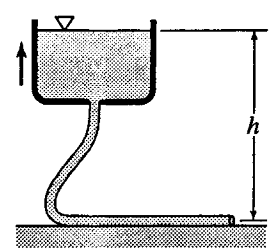

Figure P12.9

SI

Find steady state velocity for t = 10 sec: then $h = 70 + 0.5t = 15$ m

Eq. 12.6*a*: $15 - \frac{V^2}{2g} = 0.024\frac{65}{0.15}\frac{V^2}{2g}$; V = 5.08 m/s, at steady state

Now find the effect of changing head. Given: $h = 10 + 0.5t$, $\therefore\ dh/dt = 0.5$

Eq 12.6*a*: $h = 11.4(V^2/2g) + (L/g)(dV/dt)$

Differentiating with respect to t and neglecting the higher order term:

$dh/dt = 0.5 = 11.4(2V/2g)(dV/dt)$; $dV/dt = 0.5g/(11.4V) = 0.0847$ m/s per s

(*a*) Substituting back into Eq. 12.6*a*: $15 = 11.4(V^2/2g) + (65/9.81)(0.0847)$; V = 4.98 m/s,

$Q = 0.0881\ m^3/s$ ◄

(*b*) For decreasing h, one sign (of dV/dt) is reversed: $15.0 = 11.4(V^2/2g) + (65/9.81)(-0.0847)$;

V = 5.18 m/s, $Q = 0.0915\ m^3/s$ ◄

12.13 *Work Sample Prob. 12.2 for the case where the pipe lengths are 400 and 1500 ft rather than 200 and 750 ft. All other data are the same.*

Sample Prob. 12.2: $Q = 1600$ gpm at $n = 1650$ rpm, suppose n is increased instantaneously to 2000 rpm, and $h \propto n^2$. At original $n = 1650$: $V_2 = 18.16$ fps, $h_p = 113.8$ ft. Find Q as a function of t.

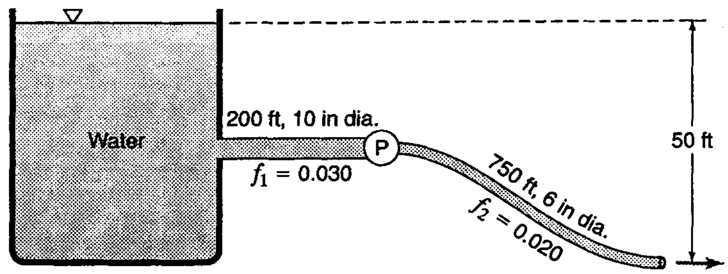

Fig. S12.2

BG

Subsituting 400 and 1500 for 200 and 750 in Sample Prob. 12.2:

Energy equation will now be: $50 + h_p = 62.9\dfrac{V_2^2}{2g} + \dfrac{1644}{g}\dfrac{dV_2}{dt}$

Original steady flow condition: $V = 18.16$ fps

$$h_p = 62.9(V_2^2/2g) - 50 = 322 - 50 = 272 \text{ ft}$$

After pump speed is increased to 2000 rpm; $h_p' = 272(2000/1650)^2 = 400$ ft

Thus $50 + 400 = 62.9\dfrac{V_2^2}{2g} + \dfrac{1644}{g}\dfrac{dV_2}{dt}$

Expressing this equation in terms of Q: $450 = 62.9\dfrac{Q^2}{2gA_2^2} + \dfrac{1644}{gA_2}\dfrac{dQ}{dt} = 25.3Q^2 + 260\dfrac{dQ}{dt}$

Dividing all terms by 25.3 (the coefficient of Q^2) yields:

$$dt = \frac{10.26dQ}{17.75 - Q^2} = \frac{10.26dQ}{4.21^2 - Q^2}; \quad t = \int_0^t dt = \int_{3.57}^{Q} \frac{10.26Q}{4.21^2 - Q^2}$$

$$t = \frac{10.26}{8.43}\left[\ln\frac{4.21 + Q}{4.21 - Q}\right]_{3.57}^{Q} = 1.217\ln\frac{4.21 + Q}{4.21 - Q} - 3.03$$

$$\frac{t + 3.03}{1.217} = \ln\frac{4.21 + Q}{4.21 - Q}; \quad e^{0.821t + 2.49} = \frac{4.21 + Q}{4.21 - Q}; \quad Q = 4.22\frac{e^{0.821t + -2.49} - 1}{e^{0.821t + 2.49} + 1}$$ ◀

12.14 *A 1-in-diameter smooth brass pipe 1000 ft long drains an open 4-ft-diameter cylindrical tank that contains oil having ρ = 1.8 slugs/ft^3 and μ = 0.0006 lb·sec/ft^2 (Fig. P12.14). The pipe discharges at elevation 100 ft. Find the time required for the oil level to drop from elevation 120 to elevation 108 ft.*

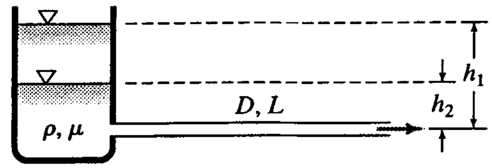

Figure P12.14

BG

Check if flow is always laminar. When $V = V_{max}$, the elevation of oil in the tank is at 120 ft, then

$120 - f(L/D)(V^2/2g) = 100$; i.e., $20 = f(L/D)(V^2/2g)$

Assuming laminar flow, Eqs. 8.29 and 8.1: $f = 64/\mathbf{R}$, $\mathbf{R} = DV\rho/\mu$

Substituting: $20 = \dfrac{64\mu}{DV\rho}\left(\dfrac{L}{D}\right)\dfrac{V^2}{2g} = \dfrac{64(0.0006)}{(1/12)V(1.8)}\,\dfrac{1000}{(1/12)}\,\dfrac{V^2}{2(32.2)} = 47.7\,V$; $V = V_{max} = 0.419$ fps

$\mathbf{R}_{max} = (1/12)0.419(1.8)/0.0006 = 104.8$; $\mathbf{R}_{max} < 2000$, so the flow is always laminar.

In general, from the above: elev − 100 = 47.7 V

$V = \dfrac{\text{elev} - 100}{47.7} = \dfrac{h}{47.7}$ (defining h = elev − 100); $Q = AV = 0.00545(h/47.7)$

Eq 12.1: $Q\,dt = -A\,dh$, $\therefore$ $dt = -\dfrac{(\pi 4^2/4)dh}{0.00545(h/47.7)} = -110{,}000\,dh/h$

$\int_0^t dt = -110{,}000\int_{20}^{8} dh/h = -110{,}000[\ln h]_{20}^{8}$

$t = -110{,}000(2.08 - 3.00) = 100{,}800$ seconds = 28.0 hours ◄

12.15 *A 25-mm-diameter smooth brass pipe 300 m long drains an open 1.2-m-diameter cylindrical tank which contains oil having ρ = 950 kg/m^3 and μ = 0.03 N·s/m^2 (Fig. P12.14). The pipe discharges at elevation 30 m. Find the time required for the oil level to drop from elevation 36 to elevation 32.5 m.*

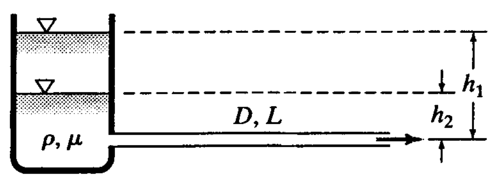

Figure P12.14

SI

Check if flow is always laminar. When $V = V_{max}$, the elevation of oil in the tank is at 36 m, then

$36 - f(L/D)V^2/2g = 30$; i.e. $6 = f(L/D)V^2/2g$

Assuming laminar flow, Eqs. 8.29 and 8.1: $f = 64/\mathbf{R}$; $\mathbf{R} = DV\rho/\mu$

Substituting: $6 = \dfrac{64\mu}{DV\rho}\,\dfrac{L}{D}\,\dfrac{V^2}{2g} = \dfrac{64(0.03)}{(0.025)V(950)}\,\dfrac{300}{(0.025)}\,\dfrac{V^2}{2(9.81)} = 49.4\,V$

$V = V_{max} = 0.1213$ m/s; $\mathbf{R}_{max} = (0.025)0.1213(950)/0.03 = 96.1$

$\mathbf{R}_{max} < 2000$, so the flow is always laminar.

In general: elev − 30 = 49.4 V; $V = \dfrac{\text{elev} - 30}{49.4} = \dfrac{h}{49.4}$ (where h = elev − 30)

$Q = AV = (\pi/4)(0.025)^2(h/49.4) = 0.000491(h/49.4)$

Eq 12.1: $Q\,dt = -A\,dh$, $\therefore$ $dt = -\dfrac{(0.6)^2\,\pi\,dh}{0.000\,491(h/49.4)} = -113{,}900\dfrac{dh}{h}$

$\int_0^t dt = -113{,}900\int_6^{2.5} dh/h = -113{,}900[\ln h]_6^{2.5}$

$t = -113{,}900(0.916 - 1.792) = 99\,700$ sec = 27.7 hours ◄

12.16 *Figure P12.16 depicts a large reservoir being drained with a pipe system (f = 0.025, L = 2000 ft, D = 9 in, z_0 = 45 ft). The flow rate is 3.5 cfs when the pump is initially rotating at 180 rpm. If the pump speed is increased instantaneously to 220 rpm, determine the flow rate as a function of time. Assume that the head h_p developed by the pump is proportional to the square of the rotative speed; that is $h_p \propto n^2$.*

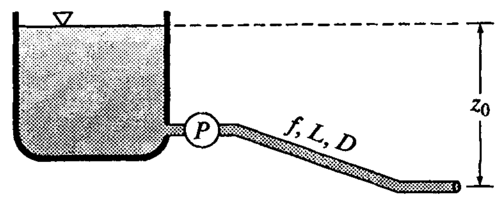

Figure P12.16

BG

Note to instructor: This problem is more challenging!

Initial condition (energy from water surface to jet) neglecting minor losses: $z_0 - h_f + h_p = V^2/2g$

$45 - 0.025(2000/0.75)V^2/2g + h_p = V^2/2g$, where $V = Q/A = Q/[\pi(9/12)^2/4] = 3.5/0.442 = 7.92$ fps

from which $h_p = 20.9$ ft; $h_p' = 20.9(220/180)^2 = 31.3$ ft

New conditions (per Eq. 12.6):

$$45 - 66.7(V^2/2g) + 31.3 = V^2/2g + (L/g)(dV/dt)$$

Eventually $dV/dt = 0$, when $V^2/2g = (45 + 31.3)/(66.7 + 1) = 1.127$ ft

$V = 8.52$ fps when steady flow is eventually achieved

In general: $76.3 - 67.7(V^2/2g) = (2000/32.2)(dV/dt)$; $dt = 59.1\, dV/(72.6 - V^2)$

Integrate left side from zero to t ; integrate right side from 7.92 to V_t. Select various V_t's and find the corresponding t's.

$$\int dt = t = \frac{59.1}{17.04}\left[\ln\frac{8.52 + V}{8.52 - V}\right]_{7.92}^{V_t} = 3.47\ln\frac{8.52 + V_t}{8.52 - V_t} - 11.49 ; \quad Q = AV_t = 0.442V_t$$

V_t (fps)	t (sec)	Q (cfs)
7.92	0	3.50
7.95	0.1700	3.51
8.00	0.498	3.53
8.15	1.707	3.60
8.40	5.64	3.71
8.45	7.50	3.73
8.52	∞	3.76
	▲	▲

12.17 *Figure P12.16 depicts a large reservoir being drained with a pipe system (f = 0.022, L = 825 m, D = 250 mm, z_0 = 14 m). The flow rate is 120 L/s when the pump is initially rotating at 200 rpm. If the pump speed is increased instantaneously to 250 rpm, determine the flow rate as a function of time. Assume that the head h_p developed by the pump is proportional to the square of the rotative speed; that is $h_p \propto n^2$.*

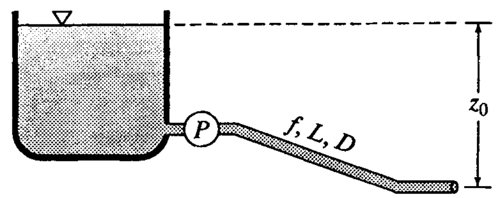

Figure P12.16

SI

Initial conditions (energy from water surface to jet) neglecting minor losses: $z - h_f + h_p = V^2/2g$

$14.00 - 0.022(825/0.250)V^2/2g + h_p = V^2/2g$, where $V = Q/A = 0.120/(\pi 0.250^2/4) = 2.44$ m/s

from which $h_p = 8.42$ m; $h_p' = 8.42(250/200)^2 = 13.15$ m

New conditions (per Eq. 12.6):

$$14 - 72.6(V^2/2g) + 13.15 = V^2/2g + (L/g)(dV/dt)$$

Eventually $dV/dt = 0$, when $V^2/2g = (14 + 13.15)/(72.6 + 1) = 0.369$ m

$V = 2.69$ m/s when steady flow is eventually achieved.

In general: $$27.2 - 73.6\frac{V^2}{2g} = \frac{825}{9.81}\frac{dV}{dt};\quad dt = \frac{84.1dV}{27.2 - 3.75V^2} = \frac{22.4dV}{7.24 - V^2}$$

Integrate left side from zero to t; integrate right side from 2.44 to V_t.

Select various V_t's and find the corresponding t's.

$$\int dt = t = \frac{22.4}{5.38}\left[\ln\frac{2.69 + V}{2.69 - V}\right]_{2.44}^{V_t} = 4.17\ln\frac{2.69 + V_t}{2.69 - V_t} - 12.75\ ;\quad Q = (\pi 0.250^2/4)V_t = 0.0491V_t$$

V_t (m/s)	t (s)	Q (m^3/s)
2.44	0	0.1200
2.50	1.021	0.1227
2.55	2.33	0.1252
2.60	4.20	0.1276
2.65	7.60	0.1301
2.69	∞	0.1321
	▲	▲

12.18 *(a) Repeat Prob. 12.16 with all the data the same except use an 18-in-diameter pipe rather than a 9-in pipe. (b) Repeat also for the case of an 8-in-diameter pipe.*

Prob. 12.16: The large reservoir in Fig. P12.16 (see with Solution 12.16) is being drained with a pipe system (f = 0.025, L = 2000 ft, z_0 = 45 ft). Initially Q = 3.5 cfs when the pump n = 180 rpm. If n is increased instantaneously to 220 rpm, determine the variation of Q with time. Assume $h_p \propto n^2$.

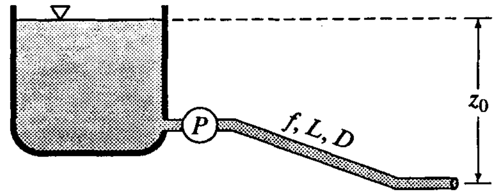

Figure P12.16

BG

(*a*) 18-inch-dia pipe:

Initial conditions (energy from water surface to jet): $45 - 0.025(2000/1.5)(V^2/2g) + h_p = V^2/2g$

$V = Q/A = 3.5/(\pi 1.5^2/4) = 1.981$ fps; $h_p = 34.3(V^2/2g) - 45 = -42.9$ ft

Thus a pump is not required; actually a head of 42.9 ft is available to drive a turbine when the flow rate is 3.5 cfs.

(*b*) 8-inch-dia pipe:

Initial conditions: $45 - 0.025[2000/(8/12)](V^2/2g) + h_p = V^2/2g$

where $V = Q/A = 3.5/[\pi(8/12)^2/4] = 3.5/0.349 = 10.03$ fps

$h_p = 76(V^2/2g) - 45 = 73.6$ ft; $h_p' = 73.6(220/180)^2 = 110.0$ ft

New conditions (per Eq. 12.6): $45 - 75(V^2/2g) + 110.0 = V^2/2g + (L/g)(dV/dt)$

Eventually $dV/dt = 0$, when $V^2/2g = (45 + 110.0)/(75 + 1) = 2.04$ ft

$V = 11.46$ fps when steady flow is eventually achieved

In general: $155.0 - 76(V^2/2g) = (2000/32.2)(dV/dt)$

$dt = 52.6\,dV/(131.4 - V^2)$; $\int_0^t dt = t = \int_{10.03}^{V_t} 52.6\,dV/(131.4 - V^2)$

$$t = \frac{52.6}{22.9}\left[\ln\frac{11.46 + V}{11.46 - V}\right]_{10.03}^{V_t} = 2.30\ln\frac{11.46 + V_t}{11.46 - V_t} - 6.22\ ; \quad Q = AV_t = 0.349V_t$$

V_t (fps)	t (sec)	Q (cfs)
10.03	0	3.50
10.25	0.412	3.58
10.50	0.969	3.67
11.00	2.71	3.84
11.46	∞	4.00
	▲	▲

12.19 *(a) Repeat Prob. 12.17 with all the data the same except use a 500-mm-diameter pipe rather than a 250-mm pipe. (b) Repeat also for the case of a 200-mm-diameter pipe.*

Prob. 12.17: The large reservoir in Fig. P12.17 (see with Solution 12.16) is being drained with a pipe system (f = 0.022, L = 825 m, z_0 = 14 m). Initially Q = 120 L/s when the pump n = 200 rpm. If n is increased instantaneously to 250 rpm, determine the variation of Q with time. Assume $h_p \propto n^2$.

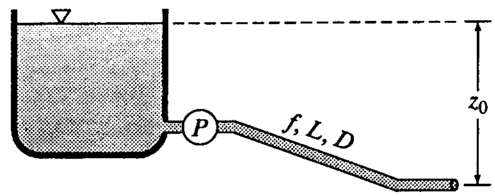

Figure P12.16

SI

(*a*) 500-mm-dia pipe:

Initial conditions (energy from water surface to jet): $14.0 - 0.022(825/0.50)(V^2/2g) + h_p = V^2/2g$

where $V = Q/A = 0.120/(\pi 0.5^2/4) = 0.611$ m/s ; $h_p = 37.3(V^2/2g) - 14.0 = -13.29$ m

Thus a pump is not required; actually a head of 13.29 m is available to drive a turbine when the flow rate is 0.120 m³/s.

(*b*) 200-mm-dia pipe:

Initial conditions: $z - h_f + h_p = V^2/2g$, or $14.0 - 0.022(825/0.20)(V^2/2g) + h_p = V^2/2g$

where $V = Q/A = 0.12/[\pi(0.20/2)^2] = 0.12/0.0314 = 3.82$ m/s

$h_p = 91.8(V^2/2g) - 14.0 = 54.2$ m; $h_p' = 54.2(250/200)^2 = 84.7$ m

New conditions (per Eq. 12.6): $14.0 - 91.8(V^2/2g) + 84.7 = V^2/2g + (L/g)(dV/dt)$

Eventually $dV/dt = 0$, when $V^2/2g = (14.0 + 84.7)/(91.8 + 1) = 1.065$ m

$V = 4.57$ m/s when steady flow is eventually achieved

In general: $98.7 - 92.8(V^2/2g) = (825/9.81)(dV/dt)$ or $20.9 - V^2 = 17.79\dfrac{dV}{dt}$

i.e., $dt = 17.79\,dV/(20.9 - V^2)$; $\int_0^t dt = t = \int_{3.82}^{V_t} 17.79\,dV/(20.9 - V^2)$

$$t = \frac{17.79}{2(4.57)}\left[\ln\frac{4.57 + V}{4.57 - V}\right]_{3.82}^{V_t} = 1.946\ln\frac{4.57 + V_t}{4.57 - V_t} - 4.70\ ; \quad Q = AV_t = 0.0314V_t$$

V_t (m/s)	t (s)	Q (cfs)
3.82	0	0.1200
4.00	0.576	0.1257
4.20	1.462	0.1319
4.40	3.02	0.1382
4.57	∞	0.1436
	▲	▲

Sec. 12.4: Approach to Steady Flow -- *Exercises* (6)

12.4.1 *A 4-in-diameter drain pipe (flush entrance, f = 0.020) 3000 ft long will freely discharge reservoir water at an elevation 100 ft below the reservoir water surface. Initially there is no flow, since there is a plug in the pipe outlet. What will be the steady flow velocity in this pipe, and how long after the plug is removed will the flow velocity be half the steady velocity? See Sec. 8.27 regarding the neglect of minor losses.*

BG

Sec. 8.27: $L/D = 3000/(4/12) = 9000 > 1000$, ∴ neglect minor losses.

Sec. 12.4: $K = fL/D + 1 + 0 = 0.020(9000) + 1 = 181$

Eq. 12.7: $V_s = \sqrt{2gH/K} = \sqrt{2(32.2)100/181} = 5.96$ fps ◄

Eq. 12.8 for $V = 0.5V_s$: $t = \dfrac{3000}{181(5.96)}\ln\left(\dfrac{1 + 0.5}{1 - 0.5}\right) = 3.05$ sec ◄

12.4.2 *A 150-mm-diameter drain pipe (flush entrance, f = 0.030) 500 m long will freely discharge reservoir water at an elevation 60 m below the reservoir water surface. Initially there is no flow, since there is a plug in the pipe outlet. What will be the steady flow velocity in this pipe, and how long after the plug is removed will the flow velocity be half the steady velocity? See Sec. 8.27 regarding the neglect of minor losses.*

SI

Sec. 8.27: $L/D = 500/0.15 = 3333 > 1000$, ∴ neglect minor losses.

Sec. 12.4: $K = fL/D + 1 + 0 = 0.030(3333) + 1 = 101$

Eq. 12.7: $V_s = \sqrt{2gH/K} = \sqrt{2(9.81)60/101} = 3.41$ m/s ◄

Eq. 12.8 for $V = 0.5V_s$: $t = \frac{500}{101(3.41)}\ln\left(\frac{1+0.5}{1-0.5}\right) = 1.593$ s ◄

12.4.3 *A 250-mm-diameter pipe (flush entrance, f = 0.026) 1000 m long can deliver water from a higher reservoir to a lower one (submerged discharge) (Fig. X12.4.3). The difference between the two water surface elevations is 75 m. After a valve near the downstream end is suddenly opened, what will be the steady flow velocity in this pipe, and how long after the valve is opened will the flow velocity be one-third and two-thirds of the steady velocity?*

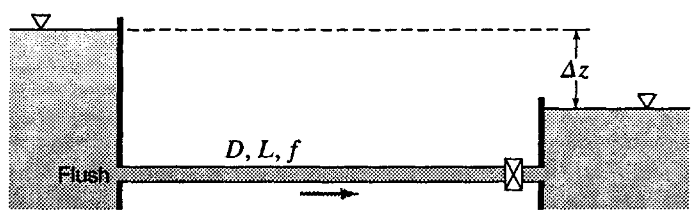

Figure X12.4.3

SI

Sec. 8.27: $L/D = 1000/0.25 = 4000 > 1000$, ∴ neglect minor losses.

Sec. 12.4: $K = fL/D + 0 + 0 = 0.026(4000) = 104$

Eq. 12.7: $V_s = \sqrt{2gH/K} = \sqrt{2(9.81)75/104} = 3.76$ m/s ◄

Eq. 12.8 for $V = (1/3)V_s = 1.254$ m/s: $t = \frac{1000}{104(3.76)}\ln\left(\frac{1+0.333}{1-0.333}\right) = 1.772$ s ◄

Eq. 12.8 for $V = (2/3)V_s = 2.51$ m/s: $t = \frac{1000}{104(3.76)}\ln\left(\frac{1+0.667}{1-0.667}\right) = 4.11$ s ◄

12.4.4 *A 4-in-diameter drain pipe (flush entrance, f = 0.022) 300 ft long will freely discharge reservoir water at an elevation 40 ft below the reservoir water surface. Initially there is no flow, since there is a plug in the pipe outlet. What will be the steady flow velocity in this pipe, and how long after the plug is removed will the flow velocity be 50% and 75% of the steady velocity?*

BG

Sec. 8.27: $L/D = 300/(4/12) = 900 < 1000$, ∴ include minor losses.

Sec. 12.4: $K = fL/D + 1 + 0.5 = 0.022(900) + 1.5 = 21.3$

Eq. 12.7: $V_s = \sqrt{2gH/K} = \sqrt{2(32.2)40/21.3} = 11.00$ fps ◄

$t = \frac{300}{21.3(11.00)}\ln\left(\frac{1+0.5}{1-0.5}\right) = 1.407$ sec ◄

Eq. 12.8 for $V = 0.75V_s = 8.25$ fps: $t = \frac{300}{21.3(11.00)}\ln\left(\frac{1+0.75}{1-0.75}\right) = 2.49$ sec ◄

12.4.5 *A 6-in-diameter pipe (reentrant entrance, $f = 0.028$) 475 ft long can deliver water from a higher reservoir to a lower one (submerged discharge). The difference between the two water surface elevations is 25 ft. After a valve near the downstream end is suddenly opened, what will be the steady flow velocity in this pipe, and how long after the valve is opened will the flow velocity be 90% of the steady velocity?*

BG

Sec. 8.27: $L/D = 475/(6/12) = 950 < 1000$, ∴ include minor losses.

Sec. 12.4: $K = fL/D + 0 + (0.5 + 1) = 0.028(950) + 1.5 = 28.1$

Eq. 12.7: $V_s = \sqrt{2gH/K} = \sqrt{2(32.2)25/28.1} = 7.57$ fps ◄

Eq. 12.8 for $V = 0.9V_s$: $t = \dfrac{475}{28.1(7.57)}\ln\left(\dfrac{1+0.9}{1-0.9}\right) = 6.58$ sec ◄

12.4.6 *A 150-mm-diameter pipe (reentrant entrance, $f = 0.023$) 140 m long can deliver water from a higher reservoir to a lower one (submerged discharge). The difference between the two water surface elevations is 12 m. After a valve near the downstream end is suddenly opened, what will be the steady flow velocity in this pipe, and how long after the valve is opened will the flow velocity be 90% of the steady velocity?*

SI

Sec. 8.27: $L/D = 140/0.15 = 933 < 1000$, ∴ include minor losses.

Sec. 12.4: $K = fL/D + 0 + (0.5 + 1) = 0.023(933) + 1.5 = 23.0$

Eq. 12.7: $V_s = \sqrt{2gH/K} = \sqrt{2(9.81)12/23.0} = 3.20$ m/s ◄

Eq. 12.8 for $V = 0.9V_s$: $t = \dfrac{140}{23.0(3.20)}\ln\left(\dfrac{1+0.9}{1-0.9}\right) = 5.61$ s. ◄

Sec. 12.4: Approach to Steady Flow -- *Problems 12.20–12.30*

12.20 *A reservoir is to be drained by a 6-in-diameter pipe (flush entrance, $f = 0.020$) 3500 ft long. The pipe outlet is at an elevation 110 ft below the reservoir water surface. Initially there is no flow since there is a plug at the pipe outlet. Plot Q versus t for time after the plug is removed.*

BG

Sec. 8.27: $L/D = 3500/0.50 = 7000 > 1000$, ∴ neglect minor losses, i.e., $\Sigma k \approx 0$

Sec. 12.4: $K = fL/D + 1 + 0 = 0.02(3500)/(6/12) + 1 = 141$

Eq. 12.7: $V_s = \sqrt{2gH/K} = \sqrt{2(32.2)110/141} = 7.09$ fps

Eq 12.8: $t = \dfrac{3500}{141(7.09)}\ln\dfrac{V_s+V}{V_s-V} = 3.50\ln\dfrac{7.09+V}{7.09-V}$; $Q = AV = (\pi/4)D^2V = (\pi/4)(6/12)^2V = 0.1963\,V$

V (fps)	t (sec)	Q (cfs)
0	0	0
1	0.995	0.1963
3	3.16	0.589
5	6.15	0.9817
6.0	8.71	1.178
7.09	∞	1.392
	▲	▲

12.21 *Repeat Prob. 12.20 for the case where the pipe length is 400 ft rather than 3500 ft.*

Prob. 12.20: A reservoir is to be drained by a 6-in-diameter pipe (flush entrance, f = 0.020). The pipe outlet is at an elevation 110 ft below the reservoir water surface. Initially there is no flow since there is a plug at the pipe outlet. Plot Q versus t for time after the plug is removed.

BG

Sec. 8.27: $L/D = 400/0.50 = 800 < 1000$, $\therefore$ minor losses are <u>not</u> negligible ;

Sec. 8.21: $k_e = 0.5$ (flush entrance).

Sec. 12.4: $K = fL/D + 1 + \Sigma k = 0.02(400)/(6/12) + 1 + 0.5 = 17.5$

Eq. 12.7: $V_s = \sqrt{2gH/K} = \sqrt{2(32.2)110/17.5} = 20.1$ fps

Eq 12.8: $t = \dfrac{400}{17.5(20.1)} \ln \dfrac{V_s + V}{V_s - V} = 1.136 \ln \dfrac{20.1 + V}{20.1 - V}$

Continuity: $Q = AV = (\pi/4)D^2V = (\pi/4)(6/12)^2V = 0.1963\ V$

V (fps)	t (sec)	Q (cfs)
0	0	0
5	0.577	0.982
10	1.239	1.963
15	2.188	2.95
18	3.283	3.53
20.1	∞	3.95
	▲	▲

12.22 *A reservoir is to be drained by a 200-mm-diameter pipe (f = 0.028) 700 m long. The elevation of the pipe outlet is 50 m below the reservoir water surface. Initially there is no flow because a valve at the pipe outlet is closed. Plot Q (L/s) versus t for time after the valve is suddenly opened.*

SI

Sec. 8.27: $L/D = 700/0.20 = 3500 > 1000$, $\therefore$ neglect minor losses, i.e., $\Sigma k \approx 0$

Sec. 12.4: $K = fL/D + 1 + 0 = 0.028(700)/0.2 + 1 = 99.0$

Eq. 12.7: $V_s = \sqrt{2gH/K} = \sqrt{2(9.81)50/99.0} = 3.15$ m/s

Eq 12.8: $t = \dfrac{700}{99.0(3.15)} \ln \dfrac{V_s + V}{V_s - V} = 2.25 \ln \dfrac{3.15 + V}{3.15 - V}$

Continuity: $Q = AV = (\pi/4)D^2V = (\pi/4)(0.2^2)V = 0.0314V$

V (m/s)	t (s)	Q (m³/s)	Q (L/s)
0	0	0	0
1.0	1.478	0.0314	31.4
2.0	3.37	0.0628	62.8
3.0	8.37	0.0942	94.2
3.1	10.94	0.0974	97.4
3.15	∞	0.0989	98.9
	▲		▲

12.23 *A reservoir with its water surface at elevation 850 ft may be drained with a long pipe that discharges at elevation 50 ft. The pipe consists of 4000 ft of 4-in-diameter pipe (f = 0.020) followed by 500 ft of 2-in-diameter pipe (f = 0.020). Initially a valve at the discharge end is closed and the water in the pipe is at rest. The valve is then suddenly opened and as time ensues the flow rate gradually increases. Determine the time rate of change of the flow rate (cfs/sec) at the instant when the flow rate is 0.40 cfs.*

BG

$\dfrac{L_1}{D_1} = \dfrac{4000}{4/12} = 12{,}000;\ \dfrac{L_2}{D_2} = \dfrac{500}{2/12} = 3000$; minor losses ($\Sigma k$) are negligible in both lines, so $k_L = \dfrac{fL}{D}$

$$H_1 - H_2 = H = k_L\frac{V^2}{2g} + \frac{L_1}{g}\frac{dV_1}{dt} + \frac{L_2}{g}\frac{dV_2}{dt};\qquad z_1 - \left(z_2 + \frac{V_2^2}{2g}\right) = \left(\frac{fL}{D}\right)_1\frac{V_1^2}{2g} + \left(\frac{fL}{D}\right)_2\frac{V_2^2}{2g} + \frac{L_1}{g}\frac{dV_1}{dt} + \frac{L_2}{g}\frac{dV_2}{dt}$$

$$850 - 50 = \frac{(0.02)4000}{4/12}\frac{V_1^2}{2g} + \left(\frac{(0.02)500}{2/12} + 1\right)\frac{V_2^2}{2g} + \frac{L_1}{g}\frac{dV_1}{dt} + \frac{L_2}{g}\frac{dV_2}{dt}$$

$$800 = 240\frac{V_1^2}{2g} + 61\frac{V_2^2}{2g} + \frac{L_1}{g}\frac{dV_1}{dt} + \frac{L_2}{g}\frac{dV_2}{dt}$$

Continuity: $A_1V_1 = A_2V_2;\ (\pi/4)(4/12)^2V_1 = (\pi/4)(2/12)^2V_2\ ;\ 16V_1 = 4V_2\ ;\ V_2 = 4V_1;\ dV_2/dt = 4dV_1/dt$

$\therefore$ Putting all velocities in terms of V_1: $\quad 800 = 240\dfrac{V_1^2}{2g} + 61\dfrac{(4V_1)^2}{2g} + \dfrac{4000}{g}\dfrac{dV_1}{dt} + \dfrac{500}{g}4\dfrac{dV_1}{dt}$

$$800g = 120V_1^2 + 30.5(16)V_1^2 + \frac{dV_1}{dt}(4000 + 2000) = 608V_1^2 + 6000\frac{dV_1}{dt}$$

But $Q = A_1V_1 = \dfrac{\pi}{4}\left(\dfrac{4}{12}\right)^2V_1\ ;\ \dfrac{\pi}{4}\left(\dfrac{4}{12}\right)^2\dfrac{dV_1}{dt} = \dfrac{dQ}{dt}\ ;\ V_1 = Q\dfrac{4}{\pi}\left(\dfrac{12}{4}\right)^2$

$$\therefore\ 800g = 608Q^2\left(\frac{4}{\pi}\right)^2\left(\frac{12}{4}\right)^4 + 6000\frac{dQ}{dt}\frac{4}{\pi}\left(\frac{12}{4}\right)^2$$

When $Q = 0.4$ cfs: $\quad 6000\dfrac{dQ}{dt}\left(\dfrac{4}{\pi}\right)9 = 800(32.2) - 608(0.4)^2\left(\dfrac{4}{\pi}\right)^2 81 = 12{,}990\ ;\ \dfrac{dQ}{dt} = 0.1889$ cfs/sec ◄

12.24 *The pipeline for draining a large reservoir consists of 200 ft of 6-in-diameter pipe (f = 0.030) followed by 500 ft of 10-in-diameter pipe (f = 0.020). The elevation of the outlet is 100 ft below the reservoir water surface (Fig. P12.24). A valve at the outlet, initially closed, is quickly opened. Derive an equation similar to Eq. (12.8) that is applicable to this situation, and plot flow rate versus time. Neglect minor losses.*

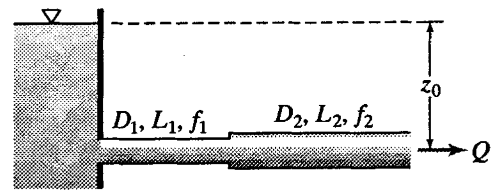

Figure P12.24

BG

Eq. 12.6a: $$h - f_1\frac{L_1}{D_1}\frac{V_1^2}{2g} - f_2\frac{L_2}{D_2}\frac{V_2^2}{2g} = \frac{V_2^2}{2g} + \frac{L_1}{g}\frac{dV_1}{dt} + \frac{L_2}{g}\frac{dV_2}{dt}$$

By continuity: $V_1 = V_2(D_2/D_1)^2$; $dV_1 = dV_2(D_2/D_1)^2$

Eliminate V_1 (to get Eq. 12.6a in terms of V_2 only):

$$h - \left[f_1\frac{L_1}{D_1}\left(\frac{D_2}{D_1}\right)^4 + f_2\frac{L_2}{D_2} + 1\right]\frac{V_2^2}{2g} = \frac{L_1(D_2/D_1)^2 dV_2}{g\,dt} + \frac{L_2}{g}\frac{dV_2}{dt}$$

For steady state condition when $dV_2/dt = 0$ and $V_2 = V_{2s}$, substituting known values yields steady-flow:

$$100 - \left[0.03\left(\frac{200}{0.5}\right)\left(\frac{10}{6}\right)^4 + 0.02\left(\frac{500}{10/12}\right) + 1\right]\frac{V_{2s}^2}{2g} = 0 \quad \text{or} \quad V_{2s} = 7.81 \text{ fps}$$

In general, subtracting the steady-state equation and rearranging:

$$dt = 2\left[\frac{L_1(D_2/D_1)^2 + L_2}{1 + f_1\frac{L_1}{D_1}\left(\frac{D_2}{D_1}\right)^4 + f_2\frac{L_2}{D_2}}\right]\frac{dV_2}{V_{2s}^2 - V_2^2}\ ; \quad \text{integrating,} \quad t = \left[\frac{L_1(D_2/D_1)^2 + L_2}{1 + f_1\frac{L_1}{D_1}\left(\frac{D_2}{D_1}\right)^4 + f_2\frac{L_2}{D_2}}\right]\frac{1}{V_{2s}}\ln\frac{V_{2s} + V_2}{V_{2s} - V_2}$$

$$t = \left[\frac{200(10/6)^2 + 500}{1 + 0.03(200/0.5)(10/6)^4 + 0.02\left(\frac{500}{10/12}\right)}\right]\frac{1}{7.81}\ln\frac{7.81 + V_2}{7.81 - V_2} = 1.280\ln\frac{7.81 + V_2}{7.81 - V_2} \quad ◀$$

Continuity: $Q = A_1V_1 = (\pi/4)D_1^2V_1 = (\pi/4)(6/12)^2V_1 = 0.1963\,V_1$; $V_1 = V_2(D_2/D_1)^2 = 2.78V_2$

V_1(fps)	V_2(fps)	t(sec)	Q(cfs)
0	0	0	0
8.33	3	1.037	1.636
13.89	5	1.942	2.73
19.44	7	3.72	3.82
20.8	7.5	4.99	4.09
21.5	7.75	7.12	4.23
21.7	7.81	∞	4.26

▲ ▲

12.25 *The pipeline for draining a large reservoir consists of 60 m of 150-mm-diameter pipe (f = 0.030) followed by 150 m of 250-mm-diameter pipe (f = 0.020). The elevation of the outlet is 30 m below the reservoir water surface (Fig. 12.24). A valve at the outlet, initially closed, is quickly opened. Derive an equation similar to Eq. (12.8) that is applicable to this situation, and plot flow rate (L/s) versus time. Neglect minor losses.*

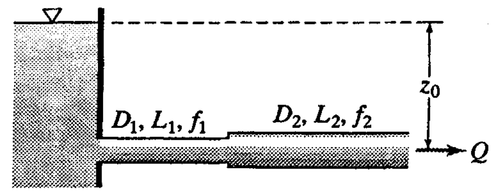

Figure P12.24

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Eq. 12.6*a*: $h - f_1\dfrac{L_1}{D_1}\dfrac{V_1^2}{2g} - f_2\dfrac{L_2}{D_2}\dfrac{V_2^2}{2g} = \dfrac{V_2^2}{2g} + \dfrac{L_1}{g}\dfrac{dV_1}{dt} + \dfrac{L_2}{g}\dfrac{dV_2}{dt}$

By continuity: $V_1 = V_2(D_2/D_1)^2$; $dV_1 = dV_2(D_2/D_1)^2$

Eliminate V_1 (to get Eq. 12.6*a* in terms of V_2 only):

$$h - \left[f_1\frac{L_1}{D_1}\left(\frac{D_2}{D_1}\right)^4 + f_2\frac{L_2}{D_2} + 1\right]\frac{V_2^2}{2g} = \frac{L_1(D_2/D_1)^2 dV_2}{g\,dt} + \frac{L_2}{g}\frac{dV_2}{dt}$$

For steady state condition when $dV_2/dt = 0$ and $V_2 = V_{2s}$, substituting known values yields steady-flow:

$$30 - \left[0.03\left(\frac{60}{0.150}\right)\left(\frac{250}{150}\right)^4 + 0.02\left(\frac{150}{0.250}\right) + 1\right]\frac{V_{2s}^2}{2g} = 0 \quad \text{or} \quad V_{2s} = 2.36 \text{ m/s}$$

In general, subtracting the steady-state equation and rearranging:

$$dt = 2\left[\frac{L_1(D_2/D_1)^2 + L_2}{1 + f_1\dfrac{L_1}{D_1}\left(\dfrac{D_2}{D_1}\right)^4 + f_2\dfrac{L_2}{D_2}}\right]\frac{dV_2}{V_{2s}^2 - V_2^2}\ ; \quad \text{integrating,} \quad t = \left[\frac{L_1(D_2/D_1)^2 + L_2}{1 + f_1\dfrac{L_1}{D_1}\left(\dfrac{D_2}{D_1}\right)^4 + f_2\dfrac{L_2}{D_2}}\right]\frac{1}{V_{2s}}\ln\frac{V_{2s} + V_2}{V_{2s} - V_2}$$

$$t = \left[\frac{60(250/150)^2 + 150}{1 + 0.03\left(\dfrac{60}{0.150}\right)\left(\dfrac{250}{150}\right)^4 + 0.02\left(\dfrac{150}{0.250}\right)}\right]\frac{1}{2.36}\ln\frac{2.36 + V_2}{2.36 - V_2} = 1.270\ln\frac{2.36 + V_2}{2.36 - V_2} \quad ◀$$

Continuity: $Q = A_1V_1 = (\pi/4)D_1^2V_1 = (\pi/4)(0.150)^2V_1 = 0.017\,67\,V_1$ m³/s

$V_1 = V_2(D_2/D_1)^2 = V_2(0.250/0.150)^2 = 2.78V_2$

V_1 (m/s)	V_2 (m/s)	t (s)	Q (m³/s)	Q (L/s)
0	0	0	0	0
2.78	1	2.94	0.0491	49.1
4.17	1.5	3.52	0.0736	73.6
5.56	2	4.63	0.0982	98.2
6.25	2.25	6.13	0.1104	110.4
6.39	2.3	6.89	0.1129	112.9
6.53	2.35	9.06	0.1154	115.4
6.56	2.36	∞	0.1159	115.9

▲ ▲

12.26 ***Water in a large reservoir may be released through a 12-in-diameter pipe (f = 0.020) 350 ft long. Losses at the pipe entrance are negligible; a nozzle (C_v = 0.95) at the outlet end produces a 6-in-diameter jet. The jet discharges at an elevation 50 ft below the reservoir water surface. Initially, there is a tight-fitting plug in the nozzle, which is then removed. For this situation derive an equation similar to Eq. (12.8) and plot flow rate versus time.***

BG

Using Eq. 11.14 for nozzle h_L, with Eq. 12.6*a*: $h - k_L\dfrac{V^2}{2g} - \left[\dfrac{1}{C_v^2} - 1\right]\left[1 - \left(\dfrac{D_j}{D}\right)^4\right]\dfrac{V_j^2}{2g} = \dfrac{V_j^2}{2g} + \dfrac{L}{g}\dfrac{dV}{dt}$

Sec. 12.4: $k_L = fL/D + \Sigma k = 0.020(350)/(12/12) + 0 = 7.0$

But $V_j = V(A/A_j) = V(D/D_j)^2$, so $h = C\dfrac{V^2}{2g} + \dfrac{L}{g}\dfrac{dV}{dt}$

where $C = k_L + \left(\dfrac{D}{D_j}\right)^4\left\{1 + \left[\dfrac{1}{C_v^2} - 1\right]\left[1 - \left(\dfrac{D_j}{D}\right)^4\right]\right\} = 7 + \left(\dfrac{12}{6}\right)^4\left\{1 + \left[\dfrac{1}{0.95^2} - 1\right]\left[1 - \left(\dfrac{6}{12}\right)^4\right]\right\} = 24.6$

For steady-state conditions when $dV/dt = 0$ and $V = V_s$:

$$h = CV_s^2/2g, \quad \text{or} \quad V_s = \sqrt{2gh/C} = \sqrt{2(32.2)50/24.6} = 11.44 \text{ fps}$$

Eliminating h between the above two equations and rearranging:

$$dt = \frac{2L}{C}\frac{dV}{V_s^2 - V^2} \quad \text{and integrating} \quad t = \frac{L}{CV_s}\ln\frac{V_s + V}{V_s - V}$$

so that $t = \dfrac{350}{24.6(11.44)}\ln\dfrac{V_s + V}{V_s - V} = 1.243\ln\dfrac{11.44 + V}{11.44 - V}$ ◄

Continuity: $Q = AV = (\pi/4)D^2V = (\pi/4)(12/12)^2V = 0.785V$ and $V_j = V/(D/D_j) = V(12/6)^2 = 4V$

V_j (fps)	V (fps)	t (sec)	Q (cfs)
0	0	0	0
8	2	0.500	1.57
16	4	0.954	3.14
24	6	1.70	4.71
32	8	2.35	6.28
40	10	2.82	7.85
45.7	11.44	∞	8.98
		▲	▲

12.27 *A large open tank containing oil (s = 0.85, μ = 0.0005 lb·s/ft²) is connected to a 3-in-diameter smooth pipe 4000 ft long. The elevation of the discharge point is 10 ft below the liquid surface in the tank (Fig. P12.27). A closed valve at the discharge end of the pipe is opened suddenly. Plot the ensuing flow rate versus time.*

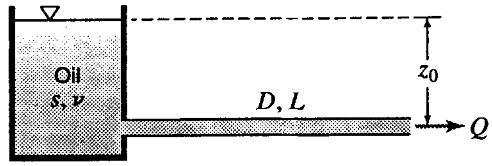

Figure P12.27

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Sec. 8.27: $L/D = 4000/(3/12) = 16{,}000 > 1000$,

$\therefore$ neglect minor losses. Assume laminar flow:

Eq. 8.28: $h_L = 10 = 32(\mu/\gamma)(L/D^2)V_s \quad \therefore \; V_s = \dfrac{10(0.85 \times 62.4)(0.25)^2}{32(0.0005)(4000)} = 0.518$ fps

Eq. 8.1: $\mathbf{R} = \dfrac{DV\rho}{\mu} = \dfrac{0.25(0.518)(0.85 \times 1.940)}{0.0005} = 427$

$\mathbf{R} < 2000$, therefore flow is laminar, assumption was O.K.

Assuming the velocity head is negligible:

From Eqs. 8.28 and 12.6*a*: $h - 32\dfrac{\mu}{\gamma}\dfrac{L}{D^2}V = \dfrac{L}{g}\dfrac{dV}{dt}$, or $h - \alpha V = \beta\dfrac{dV}{dt}$,

where $\alpha = 32\dfrac{\mu}{\gamma}\dfrac{L}{D^2} = \dfrac{32(0.0005)4000}{(0.85 \times 62.4)(0.25)^2} = 19.31$ and $\beta = L/g = 4000/32.2 = 124.2$

Rearranging: $dt = \dfrac{\beta dV}{h - \alpha V} = \dfrac{dV}{(h/\beta) - (\alpha/\beta)V}$; integrating: $t = \dfrac{-1}{(\alpha/\beta)}\ln[(h/\beta) - (\alpha/\beta)V]_0^V$

from which $t = -6.43\ln[0.0805 - 0.1554V]_0^V = -6.43\ln\left(\dfrac{0.0805 - 0.1554V}{0.0805}\right)$ or $t = 6.43\ln(1 - 1.931V)$

Continuity: $Q = AV = (\pi/4)D^2V = (\pi/4)(2/12)^2V = 0.0218\,V$

V (fps)	t (sec)	Q (cfs)
0	0	0
0.10	1.38	0.0049
0.20	3.14	0.0098
0.30	5.57	0.0147
0.40	9.52	0.0196
0.45	13.07	0.0221
0.518	∞	0.0254

▲ ▲

Check velocity head assumption: $\dfrac{V^2/2g}{h_L} = \dfrac{V^2/2g}{\alpha V} = \dfrac{V}{2g\alpha} = \dfrac{V}{2100}$

As V varies from 0 to 0.518 fps, $(V^2/2g)/h_L$ varies from 0 to 0.000 417

$\therefore$ $V^2/2g < 0.042\ \%$ of h_L, and velocity head <u>is</u> negligible (as assumed).

12.28 *A large open tank containing oil (s = 0.82, $\nu = 6.5 \times 10^{-5}$ m^2/s) is connected to a 150-mm-diameter pipe 450 m long. The elevation of the discharge point is 2.2 m below the liquid surface in the tank (Fig. P12.27). A closed valve at the discharge end of the pipe is opened suddenly. Plot the ensuing flow rate (L/s) as a function of time.*

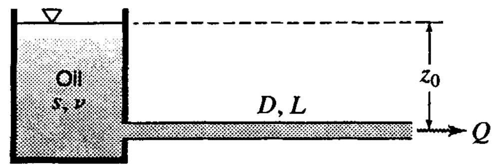

Figure P12.27

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Sec. 8.27: L/D = 450/0.15 = 3000 > 1000,

∴ neglect minor losses.

Assuming laminar flow (Eq. 8.28) for h_L, and assuming the velocity head is negligible:

Eqs. 12.6*a* and 8.28: $H - 32\,\nu \dfrac{L}{gD^2} V = \dfrac{L}{g}\dfrac{dV}{dt}$; $2.2 - 32(0.000\,065)\dfrac{450}{9.81(0.15)^2} V = \dfrac{450}{9.81}\dfrac{dV}{dt}$

i.e. $2.2 - 4.24V = 45.9 dV/dt$. At steady state, $2.2 - 4.24 V_s = 0$ so steady-flow velocity V_s = 0.519 m/s

Rearranging, $dt = \dfrac{45.9 dV}{2.2 - 4.24V}$ and integrating: $t = -\dfrac{1}{1.928}[\ln(1 - 1.928\,V)]_0^V = -0.519 \ln(1 - 1.928\,V)$

V (m/s)	t (s)	Q (m/s)	Q (L/s)
0	0	0	0
0.1	0.1111	1.77×10^{-3}	1.77
0.2	0.253	3.54×10^{-3}	3.54
0.3	0.448	5.30×10^{-3}	5.30
0.4	0.765	7.07×10^{-3}	7.07
0.5	1.721	8.84×10^{-3}	8.84
0.51	2.12	9.01×10^{-3}	9.01
0.519	∞	9.17×10^{-3}	9.17
	▲		▲

$\mathbf{R}_{max} = DV/\nu = 0.15(0.519)/(6.5 \times 10^{-5}) = 1198$. **R** < 2000, so flow <u>is</u> laminar (as assumed).

Check velocity head assumption: $\max \dfrac{V^2/2g}{h_L} = \dfrac{V^2/2g}{4.24V} = \dfrac{V}{2g(4.24)} = \dfrac{0.519}{2(9.81)4.24} = 0.006\,24$

∴ $V^2/2g$ < 0.63% of h_L, and velocity head is negligible, as assumed. Both assumptions are valid, so answers are valid.

12.29 *A vertical pipe full of oil is allowed to drain by removing a plug from its lower end. (a) Obtain an equation for the varying flow velocity as a function of time t, the pipe diameter D, and the specific weight γ and viscosity μ of the oil. Assume that the head loss is given by the equation of established laminar flow and that surface-tension effects are negligible. (b) Obtain an equation for the distance x of the oil surface from the top of the pipe, in terms of the same variables. (c) Given that the pipe is 10 ft long, its diameter is 1 in, and the oil has specific gravity 0.88 and viscosity 0.004 lb·s/ft², find the time required to drain the pipe after the plug is removed. Check whether the laminar flow assumption is reasonable.*

BG

(*a*) Method 1: Energy equation from (1) to (2): $h + V^2/2g - h_L = V^2/2g + (h/g)(dV/dt)$

Given established laminar flow: Eq. 8.28: $h_L = 32\dfrac{\mu}{\gamma}\dfrac{h}{D^2}V = khV$ where $k = \dfrac{32\mu}{\gamma D^2}$

Thus $h - khV = \left(\dfrac{h}{g}\right)\dfrac{dV}{dt}$, or $\dfrac{dV}{dt} = g(1 - kV)$; $dt = \dfrac{1}{g}\left[\dfrac{dV}{1 - kV}\right] = \dfrac{dV}{g[1 - (32u/\gamma D^2)V]}$

Method 2 (Newton's Law, $\Sigma F_z = ma$):

At any time t liquid column of height h remains in the pipe.

Gravity force = $\rho(Ah)g$ acting down

Friction force = $\tau_0(\pi D)h$ acting up, where by Eq. 2.10, $\tau_0 = \mu(du/dy)_{wall}$

For laminar flow: Substituting: $r = (r_0 - y)$ into laminar flow velocity profile Eq. 8.16:

$u = (V_c/r_0^2)(2yr_0 - y^2)$. Differentiating: $du/dy = 2(V_c/r_0^2)(r_0 - y)$ so when $y = 0$ then

$(du/dy)_{wall} = 2V_c/r_0 = 2(2V)/(D/2) = 8V/D$. Thus (Eq. 2.9) $\tau_0 = \mu(8V/D)$

Σ Forces = (mass)(accel); $\rho g\left(\dfrac{\pi D^2}{4}\right)h - \mu\dfrac{8V}{D}\pi Dh = \rho\left(\dfrac{\pi D^2}{4}h\right)\dfrac{dV}{dt}$

$\dfrac{dV}{dt} = g - kVg$ since $32\mu = k\gamma D^2$ and $\gamma = \rho g$. From which again $dt = \dfrac{1}{g}\left[\dfrac{dV}{1 - kV}\right]$

Integrating: $t = -(1/gk)[\ln(1 - kV)]_0^V = -(1/gk)[\ln(1 - kV) - \ln(1)]$

$= -(1/gk)\ln(1 - kV)$ or $V = (1 - e^{-gkt})/k$ ◄

(*b*) Writing $V = dx/dt$, where x is the distance of the oil surface from the top of the tank:

Rearranging: $kdx = (1 - e^{-gkt})dt$. Integrating: $kx = t + (1/gk)e^{-gkt} + C$

$x = 0$ when $t = 0$, $\therefore C = -1/gk$. Thus $x = t/k + (1/gk^2)(e^{-gkt} - 1)$ ◄

(*c*) $k = \dfrac{32\mu}{\gamma D^2} = \dfrac{32(0.004)}{(0.88 \times 62.4)(1/12)^2} = 0.336$ sec/ft

Note that for $t > 2$, $e^{-gkt} < 5 \times 10^{-10}$ (negligible), so that $V = dx/dt = 1/k$ and $t = kx + 1/gk$

For $x = 10$ ft: $t = 0.336(10) + 1/(32.2 \times 0.336) = 3.45$ sec ◄

$V_{max} = V_{t = 3.45\,sec} = 1/k = 1/0.336 = 2.98$ fps

$\mathbf{R}_{max} = DV_{max}\rho/\mu = (1/12)2.98(0.88 \times 1.940)/0.004 = 106.0$; $\mathbf{R} < 2000$, so flow is laminar. ◄

12.30 *Repeat parts (a) and (b) of Prob. 12.29. (c) Given that the pipe is 2.8 m long, its diameter is 20 mm, and the oil has specific gravity 0.88 and viscosity 0.25 N·s/m², find the time required to drain the pipe after the plug is removed. Check whether the laminar flow assumption is reasonable.*

Refer to Prob. statement 12.29

SI

(*a*,*b*) Same as Solution 12.29(*a*) and (*b*).

$$k = \frac{32\mu}{\gamma D^2} = \frac{32(0.25)}{(0.88 \times 9810)(0.020)^2} = 2.32 \text{ s/m}$$

Note that for $t > 1.1$, $e^{-gkt} < 10^{-10}$ (negligible), so that $V = dx/dt = 1/k$ and $t = kx + 1/gk$

For $x = 2.8$ m: $t = 2.32(2.8) + 1/(9.81 \times 2.32) = 6.53$ s ◀

$V_{max} = V_{t=6.53\,s} = 1/k = 1/2.32 = 0.432$ m/s

$\mathbf{R}_{max} = DV_{max}\rho/\mu = = (0.020)0.432(0.88 \times 1000)/0.25 = 30.4$; $\mathbf{R} < 2000$, so flow is laminar. ◀

Sec. 12.5: Velocity of Pressure Wave in Pipes -- ***Exercises (4)***

12.5.1 *(a) Calculate the celerity of a pressure wave in open fresh water at 70°F. (b) What will the celerity be if the water is contained in a cast iron pipe with 12.00 in inside diameter and 3/8-in wall thickness? (c) Express the celerity in the pipe as a percentage of the celerity in open water.*

BG

(*a*) Table A.1 at 70°F: $\rho = 1.936$ slug/ft³, $E_v = 320{,}000$ psi

Eq. 12.9: $c = \sqrt{E_v/\rho} = \sqrt{320{,}000(144)/1.936} = 4879$ fps ◀

(*b*) Sec. 12.5 for cast iron: $E \approx 15{,}000{,}000$ psi

$$\text{Eq. 12.10:}\quad c_j = \frac{4879}{\sqrt{1 + \left(\dfrac{12}{0.375}\right)\left(\dfrac{320{,}000}{15{,}000{,}000}\right)}} = 3761 \text{ fps}$$ ◀

(*c*) Ratio = 3761/4879 = 0.771 = 77.1% ◀

12.5.2 *(a) Calculate the celerity of a pressure wave in open fresh water at 20°C. (b) What will the celerity be if the water is contained in a cast iron pipe with 489 mm inside diameter and 14.3 mm wall thickness? (c) Express the celerity in the pipe as a percentage of the celerity in open water.*

SI

(*a*) Table A.1 at 20°C: $\rho = 998.2$ kg/m³, $E_v = 2.18 \times 10^6$ kN/m²

Eq. 12.9: $c = \sqrt{E_v/\rho} = \sqrt{2.18 \times 10^6(1000)/998.2} = 1478$ m/s ◀

(*b*) Sec. 12.5 for cast iron: $E \approx 103 \times 10^6$ kN/m²

$$\text{Eq. 12.10:}\quad c_j = \frac{1478}{\sqrt{1 + \left(\dfrac{489}{14.3}\right)\left(\dfrac{2.18 \times 10^6}{103 \times 10^6}\right)}} = 1126 \text{ m/s}$$ ◀

(*c*) Ratio = 1126/1478 = 0.762 = 76.2% ◀

12.5.3 *Find the celerity of a pressure wave in benzene (Appendix A, Table A.4) contained in an 8-in diameter steel pipe having a wall thickness of 0.32 in.*

BG

Table A.4 for benzene: $s = 0.88, \ E_v = 150{,}000$ psi

Eq. 12.9: $c = \sqrt{E_v/\rho} = \sqrt{150{,}000 \times 144/(0.88 \times 1.940)} = 3557$ fps

Eq. 12.10: $c_j = \dfrac{3557}{\sqrt{1 + \left(\dfrac{8}{0.32}\right)\left(\dfrac{150{,}000}{30 \times 10^6}\right)}} = 3354$ fps ◄

12.5.4 *Find the celerity of a pressure wave in benzene (Appendix A, Table A.4) contained in an 200-mm diameter steel pipe having a wall thickness of 8.0 mm.*

SI

Table A.4 for benzene: $s = 0.88, \ E_v = 1030 \times 10^6$ N/m^2

Eq. 12.9: $c = \sqrt{1030 \times 10^6/(0.88 \times 1000)} = 1082$ m/s

Eq. 12.10: $c_j = \dfrac{1082}{\sqrt{1 + \left(\dfrac{200}{8.0}\right)\left(\dfrac{1030 \times 10^6}{207 \times 10^9}\right)}} = 1020$ m/s ◄

Sec. 12.6: Water Hammer – *Exercises (6)*

12.6.1 *Given that the pipe in Fig. 12.5a is 1500 ft long, that the water is initially flowing through it at 2 fps, and that the pressure waves travel along it at 3750 fps, find the maximum water hammer pressure at the valve, and at points 500 ft and 1000 ft from the valve, (a) if the valve is completely closed instantaneously, and (b) if the valve is partially closed instantaneously, reducing the flow velocity to 0.6 fps.*

Fig. 12.5a: The valve is at the outlet end of the straight pipe.

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(*a*) Complete closure: $\Delta V = V_2 - V = 0 - 2 = -2$ fps

Eq. 12.12: $\Delta p = -\rho c_j \Delta V = -(62.4/32.2)(3750)(-2) = 14{,}530$ psf $= 100.9$ psi at all locations ◄

(*b*) Partial closure: $\Delta V = V_2 - V = 0.6 - 2 = -1.4$ fps

Eq. 12.12: $\Delta p = -\rho c_j \Delta V = -(62.4/32.2)3750(-1.4) = 10{,}170$ psf $= 70.7$ psi at all locations ◄

12.6.2 *Given that the pipe in Fig. 12.5a is 300 m long, that the water is initially flowing through it at 0.8 m/s, and that the pressure waves travel along it at 1000 m/s, find the maximum water hammer pressure at the valve, and at points 100 m and 200 m from the valve, (a) if the valve is completely close instantaneously, and (b) if the valve is partially closed instantaneously, reducing the flow velocity to 0.2 m/s.*

Fig. 12.5a: The valve is at the outlet end of the straight pipe.

SI

(*a*) Complete closure: $\Delta V = V_2 - V = 0 - 0.8 = -0.8$ m/s

Eq. 12.12: $\Delta p = -1000$ kg/m^3(1000 m/s)(−0.8 m/s) = 800 kN/m^2 = 800 kPa at all locations ◄

(*b*) Partial closure: $\Delta V = V_2 - V = 0.2 - 0.8 = -0.6$ m/s

Eq. 12.12: $\Delta p = -\rho c_j \Delta V = -1000(1000)(-0.6) = 600$ kN/m^2 = 600 kPa at all locations ◄

12.6.3 *Repeat Exer. 12.6.1 for valve closure times of 0.36 sec.*

Exer. 12.6.1: Straight pipe $L = 1500$ ft, initial $V = 2$ fps, $c_j = 3750$ fps. Find the maximum waterhammer Δp at the outlet valve and at 500 ft and 1000 ft from it (a) for complete closure, and (b) for partial closure which reduces V to 0.6 fps.

BG

Eq. 12.11: $T_r = 2(1500)/3750 = 0.8$ sec $> t_c = 0.36$ sec, so closure is "rapid."

Eq. 12.15: $x_0 = 3750(0.36)/2 = 675$ ft.

(*a*) Complete closure: $\Delta V = V_2 - V = 0 - 2 = -2$ fps

Eq. 12.12: $\Delta p = -\rho c_j \Delta V = -(62.4/32.2)(3750)(-2) = 14{,}530$ psf $= 100.9$ psi

At outlet valve, $x = L = 1500$ ft: $x > x_0$, ∴ have full $\Delta p = 100.9$ psi ◄

At $x_2 = L - 500 = 1000$ ft: $x_2 > x_0$, ∴ have full $\Delta p = 100.9$ psi ◄

At $x_1 = L - 1000 = 500$ ft: $x_1 < x_0$, ∴ have partial Δp

$\Delta p_{500} = (x/x_0)\Delta p = (500/675)100.9 = 74.8$ psi ◄

(*b*) Partial closure: $\Delta V = V_2 - V = 0.6 - 2.0 = -1.4$ fps

Eq. 12.12: $\Delta p = -\rho c_j \Delta V = -(62.4/32.2)3750(-1.4) = 10{,}170$ psf $= 70.7$ psi

At valve and x_2: have full $\Delta p = 70.7$ psi ◄

At x_1: $\Delta p_{500} = (500/675)70.65 = 52.3$ psi ◄

12.6.4 *Repeat Exer. 12.6.2 for valve closure times of 0.33 s.*

Exer. 12.6.2: Straight pipe $L = 300$ m, initial $V = 0.8$ m/s, $c_j = 1000$ m/s. Find the maximum waterhammer Δp at the outlet valve and at 100 m and 200 m from it (a) for complete closure, and (b) for partial closure which reduces V to 0.2 m/s.

SI

Eq. 12.11: $T_r = 2L/c_j = 2(300)/1000 = 0.6$ s > 0.33 s $= t_c$, so closure is "rapid."

Eq. 12.15: $x_0 = c_j t_c/2 = 1000(0.33)/2 = 165$ m

(*a*) Complete closure: $\Delta V = V_2 - V = 0 - 0.8 = -0.8$ m/s

Eq. 12.12: $\Delta p = -\rho c_j \Delta V = -(1000\text{ kg/m}^3)(1000\text{ m/s})(-0.8\text{ m/s}) = 800\text{ kN/m}^2 = 800$ kPa

At outlet valve, $x = L = 300$ m: $x > x_0$, ∴ have full $\Delta p = 800$ kPa ◄

At $x_2 = L - 100 = 200$ m: $x_2 > x_0$, ∴ have full $\Delta p = 800$ kPa ◄

At $x_1 = L - 200 = 100$ m: $x_1 < x_0$, ∴ have partial Δp

$\Delta p_{100} = (x/x_0)\Delta p = (100/165)800 = 485$ kPa ◄

(*b*) Partial closure: $\Delta V = V_2 - V = 0.2 - 0.8 = -0.6$ m/s

Eq. 12.12: $\Delta p = -\rho c_j \Delta V = -1000(1000)(-0.6) = 600\text{ kN/m}^2 = 600$ kPa

At valve and x_2: full have $\Delta p = 600$ kPa ◄

At x_1: $\Delta p_{100} = (100/165)600 = 364$ kPa ◄

12.6.5 *Repeat Exer. 12.6.1 for valve closure times of 1.00 sec.*

Exer. 12.6.1: Straight pipe L = 1500 ft, initial V = 2 fps, c_j = 3750 fps. Find the maximum waterhammer Δp at the outlet valve and at 500 ft and 1000 ft from it (a) for complete closure, and (b) for partial closure which reduces V to 0.6 fps.

BG

Eq. 12.11: T_r = 2(1500)/3750 = 0.8 sec < t_c = 1.0 sec, so closure is "slow."

Eq. 12.15: $x_0 = c_j t_c/2$ = 3750(1)/2 = 1875 ft

(*a*) Complete closure: $\Delta V = V_2 - V = 0 - 2.0 = -2.0$ fps

Eq. 12.12: $\Delta p = -\rho c_j \Delta V = -(62.4/32.2)3750(-2)$ = 14,530 psf = 100.9 psi

As $L < x_0$, at valve Eq. 12.16: $\Delta p' = (L/x_0)\Delta p$ = (1500/1875)100.9 = 80.7 psi ◄

Assuming a straight line variation within x_0.

At $x_2 = L - 500$ = 1000 ft: Δp = (1000/1500)80.7 psi = 53.8 psi ◄

At $x_1 = L - 1000$ = 500 ft: Δp = (500/1500)80.7 psi = 26.9 psi ◄

(*b*) Partial closure: $\Delta V = V_2 - V = 0.6 - 2.0 = -1.4$ fps

At valve, Eq. 12.16: $\Delta p' = -\dfrac{2L\rho\Delta V}{t_c} = -\dfrac{2(1500)62.4(-1.4)}{(1.0)32.2}$ = 8140 psf = 56.5 psi ◄

Assuming a straight line variation within x_0.

At x_2 = 1000 ft, Δp = (1000/1500)56.5 = 37.7 psi ◄

At x_1 = 500 ft, Δp = (500/1500)56.5 = 18.84 psi ◄

12.6.6 *Repeat Exer. 12.6.2 for valve closure times of 0.84 s.*

Exer. 12.6.2: Straight pipe L = 300 m, initial V = 0.8 m/s, c_j = 1000 m/s. Find the maximum waterhammer Δp at the outlet valve and at 100 m and 200 m from it (a) for complete closure, and (b) for partial closure which reduces V to 0.2 m/s.

SI

Eq. 12.11: T_r = 2(300)/1000 = 0.6 s < t_c = 0.84 s, so closure is "slow."

Eq. 12.15: $x_0 = c_j t_c/2$ = 1000(0.84)/2 = 420 m

(*a*) Complete closure: $\Delta V = V_2 - V = 0 - 0.8 = -0.8$ m/s

As $L < x_0$, at valve, Eq. 12.16:

$\Delta p' = -2L\rho\Delta V/t_c = -2(300)1000(-0.8)/0.84$ = 571 000 N/m^2 = 571 kPa at valve ◄

Assuming a straight line variation within x_0.

At $x_2 = L - 100$ = 200 m: Δp = (200/300)571 = 381 kPa ◄

At $x_1 = L - 200$ = 100 m: Δp = (100/300)571 = 190.5 kPa ◄

(*b*) Partial closure: $\Delta V = V_2 - V = 0.2 - 0.8 = -0.6$ m/s

At valve, Eq. 12.16: $\Delta p' = -2L\rho\Delta V/t_c = -2(300)1000(-0.6)/0.84$ N/m^2 = 429 kPa ◄

Assuming a straight line variation within x_0.

At x_2 = 200 m: Δp = (200/300)429 = 286 kPa ◄

At x_1 = 100 m: Δp = (100/300)429 = 142.9 kPa ◄

Sec. 12.6: Water Hammer – ***Problems*** *12.31–12.37*

12.31 *(a) What is the celerity of a pressure wave in water in a 6-ft-diameter pipe with 0.65-in steel walls? (b) If the pipe is 4200 ft long, what is the time required for a pressure wave to make the round trip from the valve? (c) If the initial water velocity is 7 fps, what will be the rise in pressure at the valve if the time of closure is less than the time of a round trip? (d) If the valve is closed at such a rate that the velocity in the pipe decreases uniformly with respect to time and closure is completed in a time $t_c = 5L/c_j$, approximately what will be the increase in pressure head at the valve when the first pressure unloading wave reaches the valve?*

BG

(*a*) Sec. 12.5: For water c = 4720 fps and E_v = 300,000 psi, for steel $E = 30 \times 10^6$ psi.

Eq 12.10: $$c_j = \frac{4720}{\sqrt{1 + \frac{6 \times 12}{0.65}\left(\frac{300{,}000}{30 \times 10^6}\right)}} = 3251 \text{ fps} \quad \blacktriangleleft$$

(*b*) Eq 12.11: $T_r = 2L/c_j = 2(4200)/3251 = 2.58$ sec ◀

(*c*) Eq 12.14: $\Delta p = (\gamma/g)c_j V = (62.4/32.2)(3251)7 = 44{,}100 \text{ psf} = 306$ psi ◀

(*d*) If velocity changes uniformly from 7.0 fps to 0 in time $5L/c_j$, the change in V in $t = T_r = 2L/c_j$ is $\Delta V = V_2 - V = (2/5)(0 - 7.0) = -2.80$ fps

when, from Eq. 12.12: $\Delta p'/\gamma \approx -c_j \Delta V/g = -3251(-2.80)/32.2 = 283$ ft ◀

Alternatively, from Eq. 12.16 with $t_c = 5L/c_j$:

$$\frac{\Delta p'}{\gamma} = -\frac{2L\,\Delta V}{g t_c} = -\frac{2}{5}\frac{c_j \Delta V}{g} = -\frac{2(3251)(-7.0)}{5(32.2)} = 283 \text{ ft} \quad \blacktriangleleft$$

12.32 *(a) What is the celerity of a pressure wave in water in a 1.8-m diameter pipe with 125-mm concrete walls? (b) If the pipe is 1250 m long, what is the time required for a pressure wave to make the round trip from the valve? (c) If the initial water velocity is 2.75 m/s, what will be the rise in pressure at the valve if the time of closure is less than the time of a round trip? (d) If the valve is closed at such a rate that the velocity in the pipe decreases uniformly with respect to time and closure is completed in a time $t_c = 5L/c_j$, approximately what will be the increase in pressure head at the valve when the first pressure unloading wave reaches the valve?*

SI

(*a*) Sec. 12.5: For water c = 1440 m/s and $E_v = 2.07 \times 10^6$ kPa, for concrete $E = 207 \times 10^6$ kPa.

Eq 12.10: $$c_j = \frac{1440}{\sqrt{1 + \frac{1.8}{0.125}\left(\frac{2.07 \times 10^6}{20.7 \times 10^6}\right)}} = 922 \text{ m/s} \quad \blacktriangleleft$$

(*b*) Eq 12.11: $T_r = 2L/c_j = 2(1250)/922 = 2.71$ s ◀

(*c*) Eq 12.14: $\Delta p = (\gamma/g)c_j V = (9810/9.81)922(2.75) = 2.54 \times 10^6 \text{ N/m}^2 = 2.54$ MPa ◀

(*d*) If the velocity changes uniformly from 2.75 m/s to 0 in time $5L/c_j$, the change in V in $t = T_r = 2L/c_j$ is $\Delta V = V_2 - V = (2/5)(0 - 2.75) = -1.100$ m/s

when, from Eq. 12.12: $\Delta p'/\gamma \approx -c_j \Delta V/g = -922(-1.100)/9.81 = 103.4$ m ◀

Alternatively, from Eq. 12.16 with $t_c = 5L/c_j$:

$$\frac{\Delta p'}{\gamma} = -\frac{2L\,\Delta V}{g t_c} = -\frac{2}{5}\frac{c_j \Delta V}{g} = -\frac{2(922)(-2.75)}{5(9.81)} = 103.4 \text{ m} \quad \blacktriangleleft$$

12.33 *Using Eqs. (12.12) and (12.16) and the data for Figs. 12.8 and 12.9 as given in footnote 3, compute the water-hammer pressure for each case and compare the answers with the actual measurements. Also, for the given data, compute f.*

Footnote 3: $L = 3060$ ft, $D = 2.06$ in, $c_j = 4371$ fps, $V = 1.11$ fps, $T_r = 1.40$ sec, $h_L = 5.1$ ft, $t_c = 1$ sec (rapid complete closure, Fig. 12.8) or 3 sec (slow complete closure, Fig. 12.9).

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For Fig. 12.8 (rapid complete closure):

Eq 12.14: $\Delta p = \rho c_j V = 1.940(4371)1.11 = 9410\,\text{psf}$. Thus $\Delta p/\gamma = 9410/62.4 = 150.8\,\text{ft}$ ◄

This compares very well with the slightly more than 151 ft recorded in Fig 12.8 ◄

For Fig. 12.9 (slow complete closure):

Eq 12.16: $\Delta p' \approx -2L\rho\Delta V/t_c = -2(3060)1.940(-1.11)/3 = 4390$. Thus $\Delta p'/\gamma \approx 4390/62.4 = 70.4$ ft ◄

This compares fairly well with the 55 ft or so recorded in Fig 12.9. ◄

From conditions before valve closure, Eq 8.13: $5.1 = f\dfrac{3060}{(2.06/12)}\dfrac{1.11^2}{2(32.2)}$; $f = 0.01495$ ◄

12.34 *Water at 15°C is flowing through a 300-mm-diameter welded-steel pipe 2400 m long that drains a reservoir under a head of 50 m (Fig. P12.34). The pipe wall has a thickness of 8.5 mm. (a) If a valve at the end of the pipe is completely closed in 10 s, approximately what water-hammer pressure will be developed? (b) If the steady-state flow is instantaneously reduced to one-half its original value, what water-hammer pressure would you expect?*

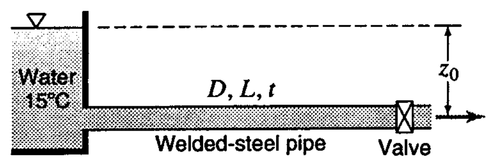

Figure P12.34

SI

Table A.1 at 15°C: $\nu = 1.139 \times 10^{-6}$ m/s². Table 8.1 for welded steel: $e = 0.046$ mm

$$\text{Eq. 8.56}a\text{:}\quad V = -2\sqrt{\frac{(2)9.81(0.3)50}{2400}}\log\left[\frac{0.046/300}{3.7} + \frac{2.51(1.139)}{10^6(2400)}\sqrt{\frac{2400}{2(9.81)0.3(50)}}\right] = 3.07\text{ m/s}$$

Sec. 12.5: For water $c = 1440$ m/s and $E_v = 2.07 \times 10^6$ kPa, for steel $E = 207 \times 10^6$ kPa.

$$\text{Eq. 12.10:}\quad c_j = \frac{1440}{\sqrt{1 + \dfrac{300}{8.5}\left(\dfrac{2.07 \times 10^6}{207 \times 10^6}\right)}} = 1238\text{ m/s}$$

(*a*) Eq. 12.11: $T_r = 2(2400)/1238 = 3.88\text{ s} \le t_c = 10$ s, so closure is "slow."

Eq 12.16: $\Delta p' \approx -2L\rho\Delta V/t_c = -2(2400)1000(-3.07)/10\text{ N/m}^2 = 1473\text{ kN/m}^2$ ◄

(*b*) Eq 12.12: $\Delta p = -\rho c_j \Delta V = -(1000)1238(0.5 - 1)3.07\text{ N/m}^2 = 1900\text{ kN/m}^2$ ◄

12.35 *For the situation described in Sample Prob. 12.5, find the water-hammer pressure at the valve if a flow of 90 cfs is reduced to 30 cfs in 3.2 sec. Under these conditions, what would be the maximum water-hammer pressures at points 500 and 1500 ft from the reservoir?*

Sample Prob. 12.5: Length of straight pipe to valve, $L = 2000$ *ft,* $D = 4$ *ft,* $c_j = 3200$ *fps.*

BG

$V_1 = Q/A = 90/(\pi 2^2) = 7.16$ fps, $V_2 = 30/(\pi 2^2) = 2.39$ fps. $\Delta V = 2.39 - 7.16 = -4.77$ fps over 3.2 sec

Eq 12.11: $T_r = 2L/c_j = 2(2000)/3200 = 1.250$ sec $< t_c = 3.2$ sec, so closure is "slow."

At valve, Eq 12.16: $\Delta p' \approx -2(2000)1.940(-4.77)/3.2 = 11{,}580$ psf $= 80.4$ psi ◄

500 ft from reservoir: $\Delta p' \approx (500/2000)80.4 = 20.1$ psi ◄

1500 ft from reservoir: $\Delta p' \approx (1500/2000)80.4 = 60.3$ psi ◄

12.36 *In Fig. P12.36 the total length of pipe is 10,000 ft, it diameter is 42 in, and its wall thickness is 0.89 in. Assume* $E = 32{,}000{,}000$ *psi and* $E_v = 300{,}000$ *psi. If the initial velocity for steady flow is 8 fps and the valve at G is partially closed so as to reduce the flow to half of the initial velocity in 3.6 sec, find (a) the maximum pressure rise due to the water hammer; (b) the location of the point of maximum total pressure.*

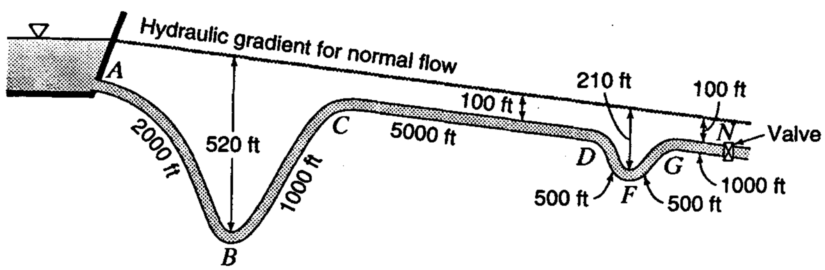

Figure P12.36

BG

(*a*) Sec. 12.5: $c = 4720$ fps. Eq 12.10: $c_j = \dfrac{4720}{\sqrt{1 + \dfrac{42}{0.89}\left(\dfrac{300{,}000}{32{,}000{,}000}\right)}} = 3930$ fps

Eq 12.12: $\Delta p = -\rho c_j \Delta V = -1.940(3930)(4 - 8) = 30{,}500$ psf $= 212$ psi ◄

(*b*) Eq. 12.11: $T_r = 2(10{,}000)/3930 = 5.09$ sec $> t_c = 3.6$ sec, so closure is "rapid"

Eq. 12.15: $x_0 = c_j t_c/2 = 3930(3.6)/2 = 7070$ ft

Thus water hammer pressure rise,

at B ($x = 2000$ ft $< x_0$): $\Delta p = 212(2000/7070) = 59.9$ psi

at E ($x = 8500$ ft $> x_0$): $\Delta p = 212(100\%) = 212$ psi

Total pressure = static pressure plus water hammer pressure = $\gamma h + \Delta p$

Total pressure at B: $520(62.4/144) + 59.9 = 285$ psi

at E: $210(62.4/144) + 212 = 303$ psi

∴ maximum total pressure occurs at point E ◄

12.37 *Refer to Fig. P12.36, but take all the dimensions given in feet to be in meters instead. This 10-km-long pipe has a diameter of 1.35 m and a wall thickness of 21 mm. Assume E = 205 GPa and E_v = 2 GPa. The initial steady flow velocity is 5 m/s. The valve at G is then partially closed so as to reduce the velocity to 1 m/s in 14 s. Find (a) the maximum pressure rise due to water hammer and (b) the location of the point of maximum total pressure.*

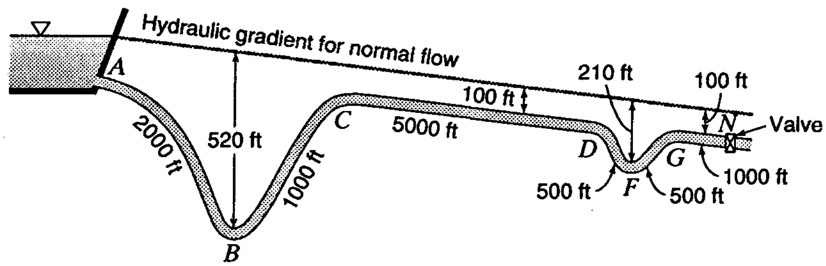

Figure P12.36

SI

(*a*) Sec. 12.5: c = 1440 m/s. Eq. 12.10: $c_j = \dfrac{1440}{\sqrt{1 + \dfrac{1350}{21}\left(\dfrac{2}{205}\right)}} = 1129$ m/s

Eq. 12.12: $\Delta p = -\rho c_j \Delta V = -1000(1129)(1-5) = 4520$ kN/m^2 (or kPa) ◄

(*b*) Eq. 12.11: $T_r = 2(10{,}000)/1129 = 17.72$ s $> t_c = 14$ s, so closure is "rapid."

Eq. 12.15: $x_0 = c_j t_c/2 = 1129(14)/2 = 7900$ m

Thus the water hammer pressure rise,

at B (x = 2000 m $< x_0$): $\Delta p = 4520(2000/7900) = 1143$ kN/m^2 (or kPa)

at E (x = 8500 m $> x_0$): $\Delta p = 4520(100\%) = 4520$ kN/m^2 (or kPa)

Total pressure = static pressure plus water hammer pressure = $\gamma h + \Delta p$

Total pressure at B = 9.81(520) + 1143 = 6240 kPa

at E = 9.81(210) + 4520 = 6580 kPa

∴ maximum total pressure occurs at point E ◄

Sec. 12.7: Surge Tanks -- *Exercises (2)*

12.7.1 *A 48-in-diameter steel pipe (f = 0.020, flush inlet) 4000 ft long delivers 275 cfs of water to a power plant. It is protected from water hammer by a 7-ft-diameter surge tank located at the downstream end just before the control valve (Fig. X12.7.1). Without using trial and error, obtain a safe, preliminary estimate of the largest height (above the reservoir surface elevation) to which a surge would rise. Neglect all velocity heads and minor losses; in the surge tank (only) neglect fluid friction and inertial effects.*

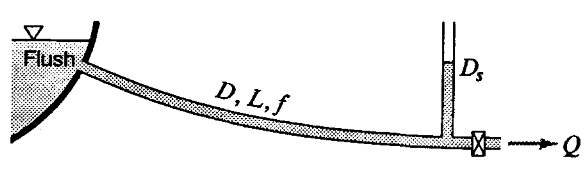

Figure X12.7.1

BG

From Eq. 12.17: $K = fL/D + 0 + 0 = 0.020(4000/4) = 20$

From Eq. 12.24: $z_{max} < \dfrac{LA}{KA_s} = \dfrac{L}{K}\left(\dfrac{D}{D_s}\right)^2 = \dfrac{4000}{20}\left(\dfrac{4}{7}\right)^2 = 65.3$ ft ◄

12.7.2 *A 900-mm-diameter steel pipe (f = 0.018, flush inlet) 850 m long delivers 4 m^3/s of water to a power plant. It is protected from water hammer by a surge tank located at the downstream end just before the control valve Fig. X12.7.1). Without using trial and error, obtain a safe, preliminary estimate of the smallest surge tank diameter that would be required to prevent surges from exceeding 12 m above the reservoir water surface elevation. Neglect all velocity heads and minor losses; in the surge tank (only) neglect fluid friction and inertial effects.*

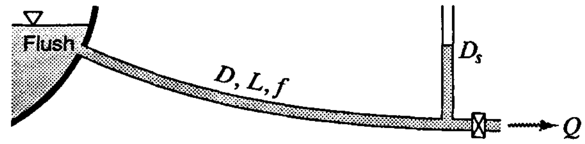

Figure X12.7.1

SI

From Eq. 12.17: $K = fL/D + 0 + 0 = (0.018)850/0.9 = 17.0$

From Eq. 12.24: $\left(\frac{D_s}{D}\right)^2 = \frac{A_s}{A} < \frac{L}{Kz_{max}} = \frac{850}{17.0(12)} = 4.17$ so $\frac{D_s}{D} < \sqrt{4.17} = 2.04$

$D_s < 2.04D = 2.04(0.9) = 1.837$ m ◀

Sec. 12.7: Surge Tanks -- *Problems* 12.38–12.47

12.38 *Derive Eq. (12.20).*

N

Substitute into the original differential equation for K from Eq. 12.17 and for dz/dt from Eq. 12.19:

$$-z = K\frac{V^2}{2g} + \frac{L}{g}\frac{dV}{dz}\frac{AV}{A_s} \quad \text{or} \quad z + \frac{K}{2g}V^2 + \frac{LA}{2gA_s}\frac{2V\,dV}{dz} = 0$$

Define $N = K/2g$; $M = (L/2g)(A/A_s)$; and $V^2 = x$ so that $2V\,dV = dx$

That gives: $z + Nx + M\,dx/dz = 0$ or $dx/dz + (N/M)x + (1/M)z = 0$

Now define $\alpha = N/M$, $\beta = 1/M$; then $dx/dz + \alpha x + \beta z = 0$

Multiply by $e^{\alpha z}$: $dx/dz\, e^{\alpha z} + \alpha x e^{\alpha z} + \beta z e^{\alpha z} = 0$

i.e., $d/dz(xe^{\alpha z}) + e^{\alpha z}\beta z = 0$ or $d(xe^{\alpha z}) + e^{\alpha z}\beta z\,dz = 0$. Integrate: $\int d(xe^{\alpha z}) + \int e^{\alpha z}\beta z\,dz = 0$

i.e., $xe^{\alpha z} + \beta(e^{\alpha z}/\alpha^2)(\alpha z - 1) + C = 0$ or $x = (\beta/\alpha^2)(1 - \alpha z) - Ce^{-\alpha z}$

But $\frac{\beta}{\alpha^2} = \frac{M}{N^2} = \frac{M(2g)^2}{K^2} = \frac{LA(2g)}{A_sK^2}$, $\alpha = \frac{N}{M} = \frac{KA_s}{LA}$, and $x = V^2$

$$\therefore\ V^2 = \frac{LA(2g)}{A_sK^2}\left(1 - \frac{KA_sz}{LA}\right) - C\exp\left(-\frac{KA_s}{LA}z\right) = \frac{2g}{K}\left(\frac{LA}{KA_s} - z\right) - C\exp\left(-\frac{KA_s}{LA}z\right) \quad \text{(Eq. 12.20) Q.E.D.}$$ ◀

12.39 *A 1.5-m-diameter steel pipeline 1000 m long (flush inlet, f = 0.020) carries water at Q_0 (m^3/s) from a reservoir to a power plant (fig. P12.39). When the valve at the outlet end is closed instantaneously, water rises in the 2.5-m-diameter simple surge tank immediately adjacent to the valve. Determine the maximum allowable initial discharge Q_0 so that the resulting surge will not rise more than 12 m above the reservoir surface. In the surge tank only, neglect the velocity head, minor losses, fluid friction, and inertial effects.*

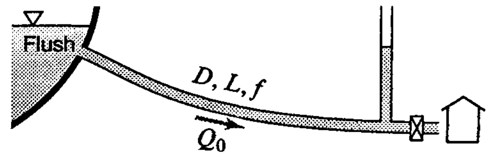

Figure P12.39

SI

Fig. 8.13: Flush inlet: $\Sigma k = 0.5$; $K = (0.02)1000/1.5 + 1 + 0.5 = 14.83$. $\dfrac{A_s}{A} = \left(\dfrac{D_s}{D}\right)^2 = 2.78$

Eq. 12.25: $V_0^2 = \dfrac{-2(9.81)}{14.83}\left[12 + \dfrac{1000}{14.83(2.78)}\ln\left(1 - \dfrac{14.83}{1000}(2.78)12\right)\right] = 6.02$; $V_0 = 2.45$ m/s

so $Q_0 = AV_0 = (\pi/4)(1.5)^2 2.45 = 4.34\ m^3/s$ ◄

12.40 *Repeat Prob. 12.39 while also neglecting the velocity head and minor losses in the pipeline.*

Prob. 12.39: Pipeline L = 1000 m, D = 1.5 m, f = 0.020, flush inlet so $\Sigma k = 0.5$; $D_s = 2.5$, $t_c = 0$. Find the initial Q_0 so that $z_0 = 12$ m. In the surge tank only, neglect the velocity head, minor losses, fluid friction, and inertial effects.

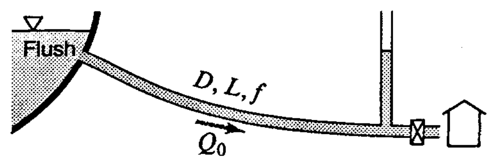

Figure P12.39

SI

Sec. 12.7: Neglecting minor losses and $V^2/2g$ in the pipeline: $K = fL/D = 0.02(1000)/1.5 = 13.33$

$\dfrac{A_s}{A} = \left(\dfrac{D_s}{D}\right)^2 = 2.78$. Eq 12.25: $V_0^2 = \dfrac{-2(9.81)}{13.33}\left[12 + \dfrac{1000}{13.33(2.78)}\ln\left(1 - \dfrac{13.33}{1000}(2.78)12\right)\right] = 5.70$

$V_0 = 2.39$ m/s so $Q_0 = AV_0 = (\pi/4)(1.5)^2 2.39 = 4.22\ m^3/s$ ◄

12.41 *Refer to Fig. 12.10. A 42-in-diameter steel pipe MN 3600 ft long (flush inlet, f = 0.017) supplies water to a small power plant. The discharge is 200 cfs, JN = 100 ft, and the elevations of J and the valve N are respectively 130 ft and 145 ft below the reservoir water surface. To protect against instaneous closure of the valve, what height would be required for the simple 6.5-ft-diameter surge tank if it is not to overflow? In the surge tank only, neglect the velocity head, minor losses, fluid friction, and inertial effects.*

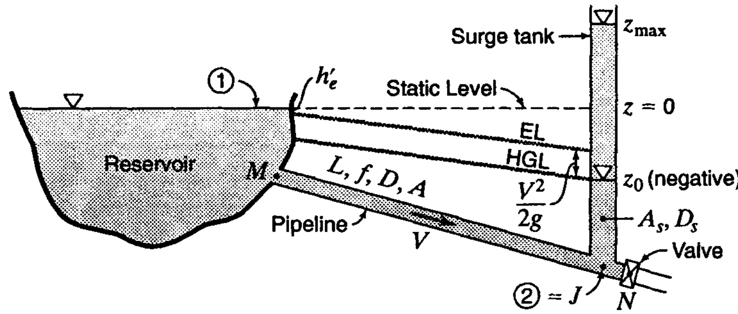

Figure 12.10

BG

Given: $Q = 200\text{ cfs}, \ f = 0.017, \ k_e = 0.5$ (Fig. 8.13)

$A_s/A = (D_s/D)^2 = 3.45$; $A = (\pi/4)(3.5)^2 = 9.62\text{ ft}^2$; $L = MJ = 3600 - 100 = 3500\text{ ft}$

$V_0 = Q/A = 200/9.62 = 20.8\text{ fps}$; $V_0^2/2g = 20.8^2/(2 \times 32.2) = 6.71\text{ ft}$

From Eq. 12.17: $K = (fL/D) + 1 + \Sigma k = 0.017(3500/3.5) + 1 + 0.5 = 18.5$

$$\frac{KA_s}{LA} = \frac{18.5(3.45)}{3500} = \frac{1}{54.9} \quad \text{and} \quad KV_0^2/2g = 18.5(6.71) = 124.1\text{ ft} \quad (= -z_0)$$

From Eq. 12.24: $z_{max} < LA/(KA_s) = 54.9\text{ ft}$

From Eq. 12.23: $$1 - \frac{z_{max}}{54.9} - \exp\left[-\frac{1}{54.9}(124.1 + z_{max})\right] = 0$$

So by trial and error:

Trial z_{max}	Left side = error	
50	0.046 7	
53	−0.005 79	
52	0.011 71	
52.5	0.002 96	
52.7	0.000 543	
52.69	−0.000 368	Good enough

$z_{max} = 52.7$ ft. $\therefore$ height of surge tank = 130 + 52.7 = 182.7 ft ◄

12.42 *Repeat Prob. 12.41 while also neglecting the velocity head and minor losses in the pipeline.*

Prob. 12.41: Refer to Fig. 12.10. A 42-in-diameter steel pipe MN 3600 ft long (flush inlet, f = 0.017) supplies water to a small power plant. The discharge is 200 cfs, JN = 100 ft, and the elevations of J and the valve N are respectively 130 ft and 145 ft below the reservoir water surface. To protect against instaneous closure of the valve, what height would be required for the simple 6.5-ft-diameter surge tank if it is not to overflow? In the surge tank only, neglect the velocity head, minor losses, fluid friction, and inertial effects.

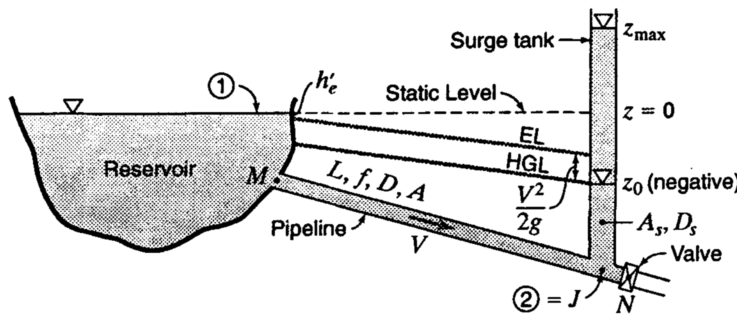

Figure 12.10

BG

Given: $Q = 200$ cfs, $f = 0.017$, $k_e = 0.5$ (Fig. 8.13)

$A_s/A = (D_s/D)^2 = 3.45$; $A = (\pi/4)(3.5)^2 = 9.62$ ft^2; $L = MJ = 3600 - 100 = 3500$ ft

$V_0 = Q/A = 200/9.62 = 20.8$ fps; $V_0^2/2g = 20.8^2/(2 \times 32.2) = 6.71$ ft

From Eq. 12.17: $K = fL/D = 0.017(3500)/3.5 = 17$

$$\frac{KA_s}{LA} = \frac{17(3.45)}{3500} = \frac{1}{59.7} \quad \text{and} \quad KV_0^2/2g = 17(6.71) = 114.1 \text{ ft} \quad (= -z_0)$$

From Eq. 12.24: $z_{max} < LA/(KA_s) = 59.7$ ft

From Eq. 12.23: $$1 - \frac{z_{max}}{59.7} - \exp\left[-\frac{1}{59.7}(114.1 + z_{max})\right] = 0$$

Solve for z_{max} by trial and error:

Trial z_{max}	Left side = error	
55	0.020 6	
60	−0.058 5	
57	−0.011 00	
56	0.047 9	
56.5	−0.003 10	
56.3	0.000 0588	
56.31	−0.000 0992	Close enough

$\therefore$ $z_{max} = 56.3$ ft and height of surge tank $= 130 + 56.3 = 186.3$ ft ◀

12.43 *Repeat Prob. 12.41 for a surge tank diameter of 10 ft.*

Prob. 12.41: Refer to Fig. 12.10. A 42-in-diameter steel pipe MN 3600 ft long (flush inlet, f = 0.017) supplies water to a small power plant. The discharge is 200 cfs, JN = 100 ft, and the elevations of J and the valve N are respectively 130 ft and 145 ft below the reservoir water surface. To protect against instaneous closure of the valve, what height would be required for the simple 6.5-ft-diameter surge tank if it is not to overflow? In the surge tank only, neglect the velocity head, minor losses, fluid friction, and inertial effects.

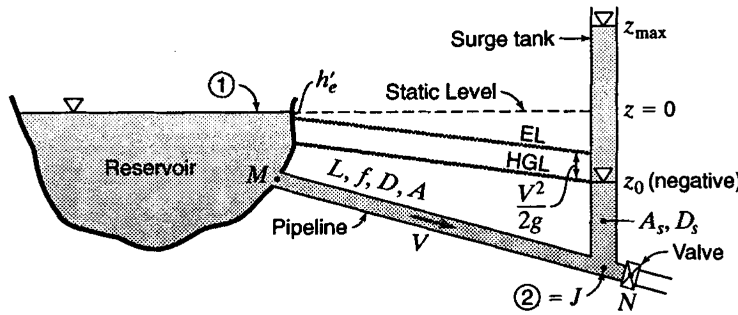

Figure 12.10

BG

Given: $Q = 200$ cfs, $f = 0.017$, $k_e = 0.5$ (Fig. 8.13)

$$\frac{A_s}{A} = \left(\frac{D_s}{D}\right)^2 = \frac{100}{12.25} = 8.16; \quad A = \pi\frac{3.5^2}{4} = 9.62\text{ ft}^2; \quad L = MJ = 3600 - 100 = 3500\text{ ft}$$

$$V_0 = Q/A = 200/9.62 = 20.8\text{ fps}; \quad V_0^2/2g = 20.8^2/(2\times 32.2) = 6.71\text{ ft}$$

From Eq. 12.17: $K = (fL/D) + 1 + \Sigma k = 0.017(3500)/3.5 + 1 + 0.5 = 18.5$

$$\frac{KA_s}{LA} = \frac{18.5(8.16)}{3500} = \frac{1}{23.2} \quad\text{and}\quad \frac{KV_0^2}{2g} = 18.5(6.71) = 124.1\text{ ft} \quad (= -z_0)$$

From Eq. 12.24: $z_{max} < LA/(KA_s) = 23.2$ ft

From Eq. 12.23: $1 - \frac{z_{max}}{23.2} - \exp\left[-\frac{1}{23.2}(124.1 + z_{max})\right] = 0$

Trial z_{max}	Left side = error	
20	0.135	
23	0.005 83	
23.2	−0.002 78	
23.13	0.000 232	Close enough

$z_{max} = 23.1$ ft. ∴ height of surge tank $= 130 + 23.1 = 153.1$ ft ◀

12.44 *Using the data of Prob. 12.41, find the diameter of surge tank that will produce a surge requiring a tank height of 165 ft.*

Prob. 12.41: Refer to Fig. 12.10. A 42-in-diameter steel pipe MN 3600 ft long (flush inlet, f = 0.017) supplies water to a small power plant. The discharge is 200 cfs, JN = 100 ft, and the elevations of J and the valve N are respectively 130 ft and 145 ft below the reservoir water surface. To protect against instaneous closure of the valve, what height would be required for the simple 6.5-ft-diameter surge tank if it is not to overflow? In the surge tank only, neglect the velocity head, minor losses, fluid friction, and inertial effects.

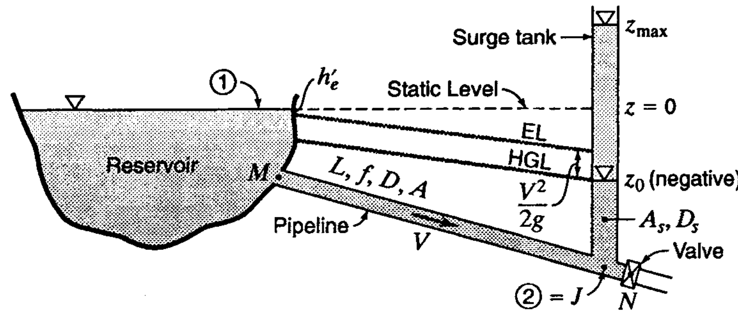

Figure 12.10

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Given: $Q = 200$ cfs, $f = 0.017$, $k_e = 0.5$ (Fig. 8.13)

$D = 42/12 = 3.5$ ft; $A = (\pi/4)(3.5)^2 = 9.62$ ft^2; $L = MJ = 3600 - 100 = 3500$ ft

$z_{max} = 165 - 130 = 35$ ft. $V_0 = Q/A = 200/9.62 = 20.8$ fps ; $V_0^2/2g = 20.8^2/(2 \times 32.2) = 6.71$ ft

From Eq. 12.17: $K = (fL/D) + 1 + \Sigma k = 0.017(3500/3.5) + 1 + 0.5 = 18.5$

For Eq. 12.23: $\dfrac{K}{LA} = \dfrac{18.5}{3500(9.62)} = \dfrac{1}{1820}$ and $(KV_0^2/2g + z_{max}) = 18.5(6.71) + 35 = 159.1$ ft

So Eq. 12.23: $1 - \dfrac{A_s}{1820}(35) = \exp\left[-\dfrac{A_s}{1820}(159.1)\right]$ or $1 - \dfrac{A_s}{52.0} = \exp\left[-\dfrac{A_s}{11.44}\right]$

But $A_s = \dfrac{\pi D_s^2}{4} = 0.785 D_s^2$ so $1 - \dfrac{0.785 D_s^2}{52.0} - \exp\left[-\dfrac{0.785 D_s^2}{11.44}\right] = 0$ or $1 - \dfrac{D_s^2}{66.2} - \exp\left[-\dfrac{D_s^2}{14.56}\right] = 0$

Per Eq. 12.24, here: $D_s^2/66.2 < 1$, i.e., $D_s < \sqrt{66.2} = 8.13$ ft

Trial D_s	Left side = error
8.0	0.021
8.1	−0.001 89
8.09	0.000 662
8.092	−0.000 034 15

Close enough $\therefore D_s = 8.09$ ft ◄

12.45 *Using the data of Prob. 12.42, find the diameter of surge tank that will produce a surge requiring a tank height of 165 ft.*

Prob. 12.41: Refer to Fig. 12.10. A 42-in-diameter steel pipe MN 3600 ft long (flush inlet, f = 0.017) supplies water to a small power plant. The discharge is 200 cfs, JN = 100 ft, and the elevations of J and the valve N are respectively 130 ft and 145 ft below the reservoir water surface. To protect against instaneous closure of the valve, what height would be required for the simple 6.5-ft-diameter surge tank if it is not to overflow? In the surge tank only, neglect the velocity head, minor losses, fluid friction, and inertial effects.

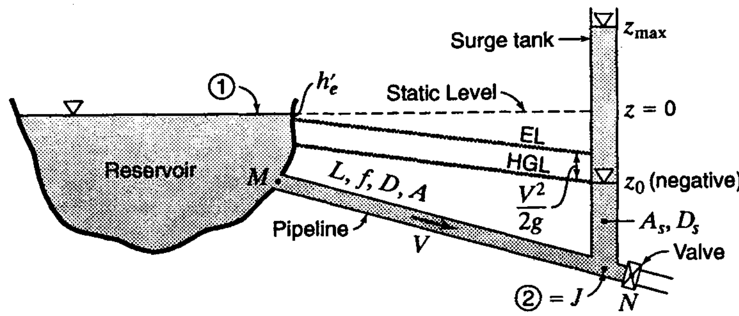

Figure 12.10

Prob. 12.42: Also neglect the velocity head and minor losses in the pipeline.

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Given: $Q = 200$ cfs, $f = 0.017$

$D = 42/12 = 3.5$ ft; $A = (\pi/4)(3.5)^2 = 9.62$ ft²; $L = MJ = 3600 - 100 = 3500$ ft

$z_{max} = 165 - 130 = 35$ ft. $V_0 = Q/A = 200/9.62 = 20.8$ fps ; $V_0^2/2g = 20.8^2/(2 \times 32.2) = 6.71$ ft

From Eq. 12.17: $K = fL/D = 0.017(3500/3.5) = 17$

For Eq. 12.23: $\frac{K}{LA} = \frac{17}{3500(9.62)} = \frac{1}{1981}$ and $(KV_0^2/2g + z_{max}) = 17(6.71) + 35 = 149.1$ ft

So Eq. 12.23: $1 - \frac{A_s}{1981}(35) = \exp\left[-\frac{A_s}{1981}(149.1)\right]$ or $1 - \frac{A_s}{56.6} = \exp\left[-\frac{A_s}{13.29}\right]$

But $A_s = \frac{\pi D_s^2}{4} = 0.785 D_s^2$ so $1 - \frac{0.785 D_s^2}{56.6} - \exp\left[-\frac{0.785 D_s^2}{13.28}\right] = 0$ or $1 - \frac{D_s^2}{72.1} - \exp\left[-\frac{D_s^2}{16.92}\right] = 0$

Per. Eq. 12.24, here: $D_s^2/72.1 < 1$, i.e., $D_s < \sqrt{72.1} = 8.49$ ft

Trial D_s	Left side = error
8.4	0.005 36
8.45	−0.005 59
8.420	0.000 992
8.425	−0.000 102
8.4245	0.000 007 62

Close enough $\therefore D_s = 8.42$ ft ◀

12.46 *Refer to Fig. 12.10. A 1-m-diameter steel pipe MN 1070 m long (flush inlet, f = 0.016) supplies water to a small power plant. The discharge is 2.45 m^3/s, JN = 20 m, and the elevations of J and the valve N are respectively 25 m and 30 m below the reservoir water surface. To protect against instantaneous closure of the valve, what height would be required for the simple 3.5-m-diameter surge tank if it is not to overflow? In the surge tank only, neglect the velocity head, minor losses, fluid friction, and inertial effects.*

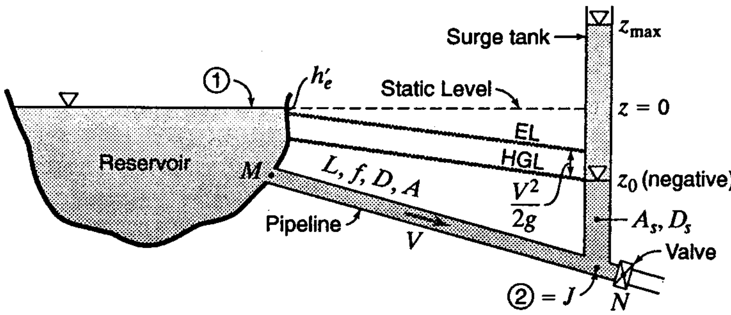


Figure 12.10

SI

Given: $Q = 2.45\ m^2/s$, $f = 0.016$, $k_e = 0.5$ (Fig. 8.13)

$A_s/A = D_s^2/D^2 = 3.5^2/1 = 12.25$; $L = MJ = 1070 - 20 = 1050$ m

$A = \pi 1^2/4 = 0.785\ m^2$; $V_0 = Q/A = 2.45/0.785 = 3.12$ m/s ; $V_0^2/2g = 3.12^2/(2 \times 9.81) = 0.496$ m

From Eq. 12.17: $K = fL/D + 1 + \Sigma k = 0.016(1050/1) + 1 + 0.5 = 18.3$

For Eq. 12.23: $\frac{KA_s}{LA} = \frac{18.3(12.25)}{1050} = \frac{1}{4.68}$ and $KV_0^2/2g = 18.3(0.496) = 9.08$ m $(= -z_0)$

So Eq. 12.23: $1 - \frac{1}{4.68} z_{max} - \exp\left[-\frac{1}{4.68}(9.08 + z_{max})\right] = 0$

Per Eq. 12.24, here: $z_{max}/4.68 < 1$, i.e. $z_{max} < 4.68$ m

Trial z_{max}	Left side = error	
4.4	0.004 31	
4.5	−0.015 9	
4.45	−0.005 77	
4.42	0.002 76	
4.423	−0.000 329	Close enough

$z_{max} = 4.42$ m. ∴ height of surge tank = 25 + 4.42 = 29.42 m ◀

12.47 *Repeat Prob. 12.46 while also neglecting the velocity head and minor losses in the pipeline.*

Prob. 12.46: Refer to Fig. 12.10. A 1-m-diameter steel pipe MN 1070 m long (flush inlet, f = 0.016) supplies water to a small power plant. The discharge is 2.45 m³/s, JN = 20 m, and the elevations of J and the valve N are respectively 25 m and 30 m below the reservoir water surface. To protect against instantaneous closure of the valve, what height would be required for the simple 3.5-m-diameter surge tank if it is not to overflow? In the surge tank only, neglect the velocity head, minor losses, fluid friction, and inertial effects.

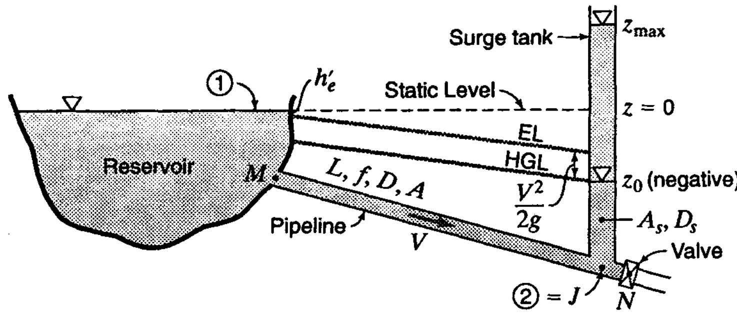


Figure 12.10

SI

Given: $Q = 2.45\ \text{m}^3/\text{s}$, $f = 0.016$

$A_s/A = D_s^2/D^2 = 3.5^2/1 = 12.25$; $L = MJ = 1070 - 20 = 1050$ m

$A = \pi 1^2/4 = 0.785\ \text{m}^2$; $V_0 = Q/A = 2.45/0.785 = 3.12$ m/s ; $V_0^2/2g = 3.12^2/(2\times 9.81) = 0.496$ m

From Eq. 12.17: $K = fL/D = 0.016(1050/1) = 16.8$

For Eq. 12.23: $\dfrac{KA_s}{LA} = \dfrac{16.8(12.25)}{1050} = \dfrac{1}{5.10}$ and $\dfrac{KV_0^2}{2g} = 16.8(0.496) = 8.33$ m $(= -z_0)$

So Eq. 12.23: $1 - \dfrac{1}{5.10}z_{max} - \exp\left[-\dfrac{1}{5.10}(8.33 + z_{max})\right] = 0$

Per Eq. 12.24, here: $z_{max}/5.10 < 1$, i.e. $z_{max} < 5.10$ m

Trial z_{max}	Left side = error	
5	−0.0533	
4.5	0.0371	
4.7	0.001 06	
4.8	−0.017	
4.71	−0.000 753	
4.705	0.000 15	
4.706	−0.000 0295	Close enough

$z_{max} = 4.71$ m. ∴ height of surge tank = 25 + 4.71 = 29.7 m ◄

Chapter 13

Steady Flow of Compressible Fluids

PROBLEM SELECTION GUIDE

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
13.1	*Thermodynamic Considerations*						
	X[1]13.1.1	BG	Easy	Short	1	13.1.2	
	13.1.2	SI	Easy	Short	1	13.1.1	
	13.1.3	BG	Medium	Medium	1	13.1.4	Uses $pv = RT$ (Sec. 2.7)
	13.1.4	SI	Medium	Medium	1	13.1.3	Uses $pv = RT$ (Sec. 2.7)
	13.1.5	SI	Easy	Short	1		
	P 13.1	SI	Medium	Long	1		Uses $pv = RT$, and a graphical approach
13.3	*Speed of Sound*						
	X 13.3.1	BG	Easy	Short	1	13.3.2	
	13.3.2	SI	Easy	Short	1	13.3.1	
13.5	*Stagnation Properties*						
	X 13.5.1	BG	Medium	Medium	1	13.5.2	
	13.5.2	SI	Medium	Medium	1	13.5.1	
	13.5.3	BG	Medium	Medium	1	13.5.4	
	13.5.4	SI	Medium	Medium	1	13.5.3	
	P 13.2	BG	Medium	Medium	1		Optionally uses $pv = RT$ (Sec. 2.7)
13.6	*Isentropic Flow*						
	X 13.6.1	BG	Easy	Medium	2	13.6.2	
	13.6.2	SI	Easy	Medium	2	13.6.1	
	13.6.3	BG	Easy	Short	1	13.6.4	Check if $V > c$
	13.6.4	SI	Easy	Short	1	13.6.3	Check if $V > c$
	P 13.3	N	Medium	Medium	1		Derivation.
	13.4	BG	Medium	Medium	1		Optionally uses $pv = RT$ (Sec. 2.7)
13.7	*Effect of Area Variation on One-Dimensional Compressible Flow*						
	X 13.7.1	N	Easy	Short	1		Derivation.

/cont...

[1] For all Exercises (identified by "X"), answers are given in Appendix F of the textbook.
† Answers are sensitive to values (β) that must be measured from a figure.
X = Exercise, P = (end-of-chapter) Problem, S = Sample Problem. □ = could use computing aids.

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
13.8	***Compressible Flow Through a Converging Nozzle***						
	X 13.8.1	N	Medium	Medium	1		Derivation
	13.8.2	N	Medium	Medium	1		Derivation. Differentiation.
	13.8.3	BG	Medium	Medium	1	13.8.4	
	13.8.4	SI	Medium	Medium	1	13.8.3	
	13.8.5	BG	Medium	Medium	1	13.8.6	
	13.8.6	SI	Medium	Medium	1	13.8.5	
	P 13.5	BG	Hard	Long	1	13.6-7	Uses $pv = RT$ (Sec. 2.7)
	13.6	SI	Hard	Long	1	13.5-7	Uses $pv = RT$ (Sec. 2.7)
	13.7	SI	Hard	Long	1	13.5-6	Uses $pv = RT$ (Sec. 2.7)
	13.8	BG	Hard	Long	1		Uses $pv = RT$ (Sec. 2.7)
	13.9	BG	Hard	Long	1		☐ Uses $pv^k = C$ and $pv = RT$. T&E
13.9	***Isentropic Flow Through a Converging-Diverging Nozzle***						
	X 13.9.1	BG	Medium	Short	1	13.9.2	Uses $pv^k = C$ and $pv = RT$ (Sec. 2.7)
	13.9.2	SI	Medium	Short	1	13.9.1	Uses $pv^k = C$ and $pv = RT$ (Sec. 2.7)
	13.9.3	BG	Medium	Medium	1	S13.5	Uses $pv = RT$ (Sec. 2.7)
	13.9.4	BG	Easy	Short	1		Uses $pv = RT$ (Sec. 2.7)
	13.9.5	BG	Easy	Short	2	13.9.6	
	13.9.6	SI	Easy	Short	2	13.9.5	
	P 13.10	BG	Hard	Long	1	13.11	☐ Uses $pv = RT$ (Sec. 2.7). T & E.
	13.11	SI	Hard	Long	1	13.10	☐ Uses $pv = RT$ (Sec. 2.7). T & E.
	13.12	BG	Medium	Medium	2	13.13	Uses $pv = RT$. A check is required.
	13.13	SI	Medium	Medium	2	13.12	Uses $pv = RT$. A check is required.
13.10	***One-Dimensional Shock Wave***						
	X 13.10.1	BG	Medium	Medium	2	13.10.2	
	13.10.2	SI	Medium	Medium	2	13.10.1	
	P 13.14	SI	Hard	Long	2		Uses $pv = RT$ (Sec. 2.7)
	13.15	BG	Hard	Long	1		Uses $pv = RT$ (Sec. 2.7)
13.11	***The Oblique Shock Wave***						
	X 13.11.1	BG	Easy	Short	1		†
	13.11.2	BG	Easy	Short	1	13.11.3	
	13.11.3	SI	Easy	Short	1	13.11.2	
13.12	***Isothermal Flow***						
	P 13.16	BG	Hard	Long	1	13.17	Integration; uses Secs. 2.7, 13.13
	13.17	SI	Hard	Long	1	13.16	Integration; uses Secs. 2.7, 13.13

/cont...

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
13.13	***Isothermal Flow in a Constant-Area Duct***						
	X 13.13.1	BG	Medium	Medium	4	13.13.2	Uses $pv = RT$ (Sec. 2.7)
	13.13.2	SI	Medium	Medium	4	13.13.1	Uses $pv = RT$ (Sec. 2.7)
	P 13.18	BG	Medium	Long	1	S13.7	Uses Sec. 2.7
	13.19	BG	Medium	Medium	1	13.20	☐ Uses $pv = RT$; assume/confirm **R**.
	13.20	SI	Medium	Medium	1	13.19	☐ Uses $pv = RT$; assume/confirm **R**.
	13.21	BG	Hard	Long	1		☐ Uses Secs 2.7, 2.11, 8.12-15;
	"						assume and correct **R**.
13.14	***Adiabatic Flow in a Constant-Area Duct***						
	P 13.22	BG	Hard	V Long	3	S13.9	☐ Uses Secs 2.7-8,7.4; long T&E, plots
	13.23	BG	Medium	Long	1	13.24	Uses Secs. 2.7 and 8.12-13.
	13.24	SI	Medium	Long	1	13.23	Uses Secs. 2.7 and 8.12-13.

Chapter 13
STEADY FLOW OF COMPRESSIBLE FLUIDS

Sec. 13.1: Thermodynamic Considerations -- *Exercises (5)*

13.1.1 BG *Compute the change in enthalpy of 15 slugs of oxygen if its temperature is increased from 120°F to 155°F.*

Table A.5 for oxygen: $c_p = 5437$ ft·lb/(slug·°R)

From Eq. 13.1: $\Delta h = c_p(T_2 - T_1) = 5437(155 - 120) = 190{,}300$ ft·lb/slug

Thus change in enthalpy $= 190{,}300(15) = 2.85 \times 10^6$ ft·lb ◄

13.1.2 SI *Compute the change in enthalpy of 250 kg of oxygen if its temperature is increased from 50°C to 70°C.*

Table A.5 for oxygen: $c_p = 909$ N·m/(kg·K)

From Eq. 13.1: $\Delta h = c_p(T_2 - T_1) = 909(70 - 50) = 18\,180$ N·m/kg

Thus change in enthalpy $= 18\,180(250) = 4.55 \times 10^6$ N·m (or J) ◄

13.1.3 *Suppose 15 slugs of oxygen are compressed isentropically to 80 percent of its original volume. Find the final temperature and pressure, the work required, and the change in enthalpy. Assume T_1 = 120°F and p_1 = 200 psia.*

BG

pv^k = constant (isentropic, Sec. 13.1), and $pv = RT$ (Eq. 2.4)

Eliminating p: $(RT/v)v^k$ = constant $= RTv^{k-1}$

Since also R = constant, $\therefore Tv^{k-1}$ = constant; $k = 1.40$ (Table A.5 for oxygen)

From which: $T_2 = T_1(v_1/v_2)^{k-1} = (120 + 460)(1/0.8)^{0.4} = 580(1.25)^{0.4} = 634.2°R = 174.2°F$ ◄

From Eq. 2.4: $R = \dfrac{p_1 v_1}{T_1} = \dfrac{p_2 v_2}{T_2}$; so $\dfrac{200 v_1}{580} = \dfrac{p_2(0.8 v_1)}{634}$; $p_2 = 273$ psia ◄

Table A.5 for oxygen: $c_v = 3883$, $c_p = 5437$ ft·lb/(slug·°R)

Work required = change of internal energy (as isentropic); using Eq. 13.2

$= \Delta i \times m = c_v(\Delta T) \times 15$ where $\Delta T = 174.2 - 120 = 54.2°F = 54.2°R$

$= 3883(54.2)15 = 3.15 \times 10^6$ ft·lb ◄

Using Eq. 13.11: change in enthalpy, $\Delta h = 5437(54.2)15 = 4.42 \times 10^6$ ft·lb increase ◄

13.1.4 *Suppose 250 kg of oxygen are compressed isentropically to 80 percent of its original volume. Find the final temperature and pressure, the work required, and the change in enthalpy. Assume T_1 = 50°C and p_1 = 1400 kN/m² abs.*

SI

pv^k = constant (isentropic, Sec. 13.1), and $pv = RT$ (Eq. 2.4)

Eliminating p: $(RT/v)v^k$ = constant $= RTv^{k-1}$

Since also R = constant, $\therefore Tv^{k-1}$ = constant; $k = 1.40$ (Table A.5 for oxygen)

From which: $T_2 = T_1(v_1/v_2)^{k-1} = (50 + 273)(1/0.8)^{0.4} = 323(1.25)^{0.4} = 353.2$ K $= 80.2°C$ ◄

From Eq. 2.4: $R = \dfrac{p_1 v_1}{T_1} = \dfrac{p_2 v_2}{T_2}$; so $\dfrac{1400 v_1}{323} = \dfrac{p_2(0.8 v_1)}{353.2}$; $p_2 = 1913$ kPa abs ◄

Table A.5 for oxygen: $c_v = 649$, $c_p = 909$ N·m/(kg·K) /cont...

Work required = change of internal energy (as isentropic); Using Eq. 13.2

$= \Delta i \times m = c_v(\Delta T) \times 250$ where $\Delta T = 80.2 - 50 = 30.2°C = 30.2$ K

$= 649(30.2)250 = 4.89 \times 10^6$ N·m $= 4.89 \times 10^6$ J ◄

Using Eq. 13.1: change in enthalpy, $\Delta h = 909(30.2)250 = 6.85 \times 10^6$ N·m $= 6.85 \times 10^6$ J increase ◄

13.1.5 *Using the data of Sample Prob. 13.2 compute $\Delta(p/\rho)$ and thus show that $\Delta h = \Delta i + \Delta(p/\rho)$.*
Sample Prob. 13.2: Δh = 130 100 J/kg, Δi = 92 900 J/kg, T_2 = 423 K.

SI

Table A.5 for air: R = 287 J/(kg·K)

$p_1/\rho_1 = RT_1 = 287(20 + 273) = 84\,100$ J/kg; $p_2/\rho_2 = RT_2 = 287(423) = 121\,400$ J/kg

$\Delta(p/\rho) = p_2/\rho_2 - p_1/\rho_1 = 121\,400 - 84\,100 = 37\,300$ J/kg ◄

$\Delta i + \Delta(p/\rho) = 92\,900 + 37\,300 = 130\,200$ which checks accurately ◄ with $\Delta h = 130\,100$.

Sec. 13.1: Thermodynamic Considerations -- *Problem 13.1*

13.1 *Using the data of Sample Prob. 13.2, determine the work done in compressing the air by finding the area under a pressure-vs-volume curve. Compute and tabulate volumes and pressures using volume increments that are 10 percent of the original volume.*
Sample Prob. 13.2: 15 kg of air are compressed isentropically. Originally T_1 = 20°C, p_1 = 95 kPa abs.

SI

Table A.5 for air: R = 287 N·m/(kg·K), k = 1.40

Using Eq. 2.4: $v_1 = RT_1/p_1 = 287(20 + 273)/95\,000 = 0.885$ m³/kg

After compression, from Sample Prob. 13.2: $T = T_1(v_1/v)^{k-1} = 293(v_1/v)^{0.4}$;

$v = v_1/(v_1/v)$; $\mathcal{V} = mv = 15v$; $p = RT/v$

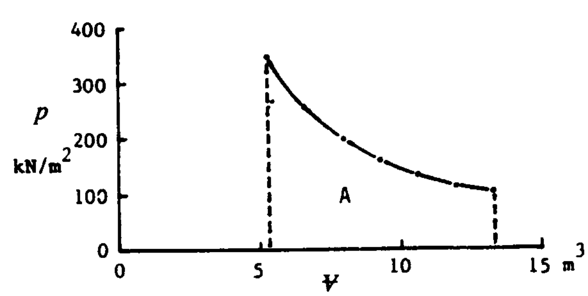
v_1/v	T, K	v, m³/kg	$\mathcal{V}$, m³	p, N/m²
100/100	293	0.885	13.28	95 000
100/90	306	0.797	11.95	110 100
100/80	320	0.708	10.62	129 900
100/70	338	0.620	9.29	156 500
100/60	359	0.531	7.97	194 200
100/50	387	0.443	6.64	251 000
100/40	423	0.354	5.31	343 000
			▲	▲

Measure or calculate area A under p-$\mathcal{V}$ curve.

Work done in compression = A

$= 1.397 \times 10^6$ N·m ◄

(here calculated by Weddle's Rule)



Sec. 13.3: Speed of Sound -- *Exercises (2)*

13.3.1 BG *Use Sec. 13.3 to calculate the sonic velocity in air at sea level and at elevations 5000, 10,000, 20,000, and 30,000 ft. Assume standard atmosphere (Appendix A, Table A.3).*

Eq. 13.15 (Sec. 13.3): $c = \sqrt{kRT}$; Table A.5: In all cases $k = 1.40$ and $R = 1715$ ft²/(sec²·°R)

Table A.3: Temps = 59°F, 41.2°F, 23.4°F, −12.3°F, and −47.8°F

Substitute for T = (460 + Temp) and compute the resulting c's.

Elev (ft)	Temp (°F)	c (ft/sec)
0	59.0	1116
5,000	41.2	1097
10,000	23.4	1077
20,000	−12.3	1037
30,000	−47.8	995

▲

13.3.2 SI *Use Sec. 13.3 to calculate the sonic velocity in air at sea level and at elevations 2000 and 10 000 m, expressing the answers in SI units. Assume standard atmosphere (Appendix A, Table A.3).*

Eq. 13.15 (Sec. 13.3): $c = \sqrt{kRT}$; Table A.5: In all cases $k = 1.40$ and $R = 287$ m²/(s²·K)

Table A.3: Temps = 15.0, 2.0, and −49.9°C

Substitute for T = (273 + Temp °C) and compute the resulting c's.

Elev (m)	Temp (°C)	c (m/s)
0	15.0	340
2 000	2.0	332
10 000	−49.9	299

▲

Sec. 13.5: Stagnation Properties -- *Exercises (4)*

13.5.1 BG *Air flows past an object at 600 ft/sec. Determine the stagnation pressures and temperatures in the standard atmosphere at elevations of sea level, 5000 and 30,000 ft.*

Table A.5 for air: $R = 1715$ ft²/(sec²·°R), $c_p = 6000$ ft²/(sec²·°R); Eq. 13.10: $\mathbf{M} = V/c = 600/c$

From Eq. 13.25: $T_o = T + (V^2/2c_p) = T + [600^2/(2 \times 6000)] = T + 30$

Eq. 13.30: $p_o = p + \frac{1}{2}\rho V^2 \frac{(1 + \mathbf{M}^2/4)}{144} = p + \frac{1}{2}\rho(600^2)\frac{(1 + \mathbf{M}^2/4)}{144} = p + 1250\rho(1 + \mathbf{M}^2/4)$

Elev (ft)	*T (°F)	*p (psia)	*ρ (slug/ft³)	*c (ft/sec)	M	T_o (°F)	p_o (psia)
0	59.0	14.70	0.002 38	1116	0.537	89.0	17.89
5000	41.2	12.23	0.002 05	1097	0.547	71.2	14.98
30,000	−47.8	4.37	0.000 891	995	0.603	−17.8	5.59

*From Table A.3 for standard atmosphere. ▲ ▲

13.5.2 *Air flows past an object at 200 m/s. Determine the stagnation pressures and temperatures in the standard atmosphere at elevations of sea level, 2000 and 10 000 m.*

SI

Table A.5 for air: $R = 287\ \text{m}^2/(\text{s}^2\cdot\text{K})$, $c_p = 1003\ \text{m}^2/(\text{s}^2\cdot\text{K})$; Eq. 13.10: $\mathbf{M} = V/c = 200/c$

From Eq. 13.25: $T_o = T + (V^2/2c_p) = T + [200^2/(2 \times 1003)] = T + 19.94$

Eq. 13.30: $p_o = p + \frac{1}{2}\rho V^2 \frac{(1 + \mathbf{M}^2/4)}{1000} = p + \frac{1}{2}\rho(200^2)\frac{(1 + \mathbf{M}^2/4)}{1000} = p + 20\rho(1 + \mathbf{M}^2/4)$

Elev (m)	*T (°C)	*p (kPa abs)	*ρ (kg/m³)	*c (m/s)	M	T_o (°C)	p_o (kPa abs)
0	15.00	101.3	1.225	340.3	0.588	34.9	127.9
2 000	2.00	79.5	1.007	332.5	0.601	21.9	101.5
10 000	−49.90	26.5	0.414	299.5	0.668	−30.0	35.7
						▲	▲

*From Table A.3 for standard atmosphere.

13.5.3 *Air at 250 psia is moving at 500 ft/sec in a high-pressure wind tunnel at a temperature of 100°F. Find the stagnation pressure and temperature. Note the magnitude of the sonic velocity for the 250-psia 100°F air.*

BG

Table A.5 for air: $R = 1715\ \text{ft}^2/(\text{sec}^2\cdot{}^\circ\text{R})$, $c_p = 6000\ \text{ft}^2/(\text{sec}^2\cdot{}^\circ\text{R})$, k = 1.40

Eq. 13.15: $c = \sqrt{kRT} = \sqrt{(1.40)1715(460 + 100)} = 1160$ ft/sec ◀

Eq. 13.10: $\mathbf{M} = V/c = 500/1160 = 0.431$; $\mathbf{M} < 1$ so we can use Eq. 13.27

Sec. 13.5: $T_o = T + V^2/(2c_p) = (460 + 100) + 500^2/(2 \times 6000) = 581°\text{R} = 121°\text{F}$ ◀

From Eq. 13.27: $p_o = 250[1 + 0.5(1.40 - 1)0.431^2]^{1.4/0.4} = 284$ psia ◀

<u>Alternatively</u>, obtain $\rho = 0.0375\ \text{slug/ft}^3$ from Eq. 2.4, then find p_o from Eq. 13.30.

13.5.4 *Air at 1750 kPa abs is moving at 150 m/s in a high-pressure wind tunnel at a temperature of 40°C. Find the stagnation pressure and temperature. Note the magnitude of the sonic velocity for this moving air.*

SI

Table A.5 for air: $R = 287\ \text{m}^2/(\text{s}^2\cdot\text{K})$, $c_p = 1003\ \text{m}^2/(\text{s}^2\cdot\text{K})$, k = 1.40

Eq. 13.15: $c = \sqrt{kRT} = \sqrt{1.40(287)(273 + 40)} = 355$ m/s ◀

Eq. 13.10: $\mathbf{M} = V/c = 150/355 = 0.423$; $\mathbf{M} < 1$ so we can use Eq. 13.27

Sec. 13.5: $T_o = T + V^2/(2c_p) = (273 + 40) + 150^2/(2 \times 1003) = 324\text{K} = 51.2°\text{C}$ ◀

From Eq. 13.27: $p_o = 1750[1 + 0.5(1.40 - 1)0.423^2]^{1.4/0.4} = 1979$ kPa abs ◀

<u>Alternatively</u>, obtain $\rho = 19.48\ \text{kg/m}^3$ from Eq. 2.4, then find p_o from Eq. 13.30.

Sec. 13.5: Stagnation Properties -- *Problem 13.2*

13.2 *Find the stagnation pressure and temperature in air flowing at 88 ft/sec if the static pressure and static temperature are 14.7 psia and 50°F respectively.*

BG

Table A.5 for air: $R = 1715\ \text{ft}^2/(\text{sec}^2\cdot{}^\circ\text{R})$, $c_p = 6000\ \text{ft}^2/(\text{sec}^2\cdot{}^\circ\text{R})$, k = 1.40

Eq. 13.15: $c = \sqrt{kRT} = \sqrt{1.40(1715)(460 + 50)} = 1107$ ft/sec

Eq. 13.10: $\mathbf{M} = V/c = 88/1107 = 0.0795$; $\mathbf{M} < 1$ so we can use Eq. 13.27

Sec. 13.25: $T_o = T + V^2/(2c_p) = (460 + 50) + 88^2/(2 \times 6000) = 511°\text{R} = 50.6°\text{F}$ ◀

From Eq. 13.27: $p_o = 14.7[1 + 0.5(1.40 - 1)0.0795^2]^{1.4/0.4} = 14.77$ psia ◀

<u>Alternatively</u>, obtain $\rho = 0.00242\ \text{slug/ft}^3$ from Eq. 2.4, then find p_o from Eq. 13.30.

Sec. 13.6: Isentropic Flow -- *Exercises (4)*

13.6.1 *Air at a pressure of 150 psia and a temperature of 100°F expands in a suitable nozzle to 15 psia. (a) If the flow is frictionless and adiabatic and the initial velocity is negligible, find the final velocity by Eq. 13.35. (b) Find the final temperature at the end of the expansion through use of Eq. 13.23.*

BG

Table A.5 for air: R = 1715 ft²/(sec²·°R), c_p = 6000 ft²/(sec²·°R), k = 1.40

(*a*) Eq. 13.35: $\dfrac{V_2^2 - V_1^2}{2} = RT_1\dfrac{k}{k-1}\left[1 - (p_2/p_1)^{(k-1)/k}\right]$ where $V_1 \approx 0$ (given)

$V_2^2/2 = 1715(100 + 460)(1.40/0.40)[1 - (15/150)^{0.4/1.4}]$; V_2 = 1800 ft/sec ◄

(*b*) Eq. 13.23: $V_2^2 - V_1^2 = 2c_p(T_1 - T_2)$

$1800^2 - 0 = 2(6000)(560 - T_2)$; T_2 = 290°R = −170°F ◄

13.6.2 *Air at a pressure of 1000 kPa and a temperature of 40°C expands in a suitable nozzle to 100 kPa. (a) If the flow is frictionless and adiabatic and the initial velocity is negligible, find the final velocity by Eq. 13.35. (b) Find the final temperature at the end of the expansion through use of Eq. 13.23.*

SI

Table A.5 for air: R = 287 m²/(s²·K), c_p = 1003 m²/(s²·K), k = 1.40

(*a*) Eq. 13.35: $\dfrac{V_2^2 - V_1^2}{2} = RT_1\dfrac{k}{k-1}\left[1 - (p_2/p_1)^{(k-1)/k}\right]$ where $V_1 \approx 0$ (given)

$V_2^2/2 = 287(40 + 273)(1.40/0.40)[1 - (100/1000)^{0.4/1.4}]$; V_2 = 551 m/s ◄

(*b*) Eq. 13.23: $V_2^2 - V_1^2 = 2c_p(T_1 - T_2)$

$551^2 - 0 = 2(1003)(313 - T_2)$; T_2 = 161.9 K = −111.1°C ◄

13.6.3 *Carbon dioxide flows isentropically. At a point in the flow the velocity is 50 ft/sec and the temperature is 125°F. At a second point on the same streamline the temperature is 80°F. What is the velocity at the second point? Check if your answer is valid.*

BG

Table A.5 for carbon dioxide: R = 1123 ft²/(sec²·°R), k = 1.28,

The flow is isentropic, therefore (Sec. 13.6) it is adiabatic.

Eq. 13.24: $V_2^2 - 50^2 = \dfrac{2(1.28)}{0.28}1123(460 + 125)\left(1 - \dfrac{460 + 80}{460 + 125}\right)$; V_2 = 682 ft/sec

Check to see if sonic velocity is exceeded. Eq. 13.15: $c = \sqrt{kRT} = \sqrt{1.28(1123)(460+80)}$ = 881 ft/sec

Since 628 = $V_2 < c$ = 881, the flow is subsonic, and V_2 = 682 ft/sec is correct. ◄

13.6.4 *Carbon dioxide flows isentropically. At a point in the flow the velocity is 15 m/s and the temperature is 50°C. At a second point on the same streamline the temperature is 25°C. What is the velocity at the second point? Check if your answer is valid.*

SI

Table A.5 for carbon dioxide: R = 188 m²/(s²·K), k = 1.28

The flow is isentropic, therefore (Sec. 13.6) it is adiabatic.

Eq. 13.24: $V_2^2 - 15^2 = \dfrac{2(1.28)}{0.28}188(273 + 50)\left(1 - \dfrac{273 + 25}{273 + 50}\right)$; V_2 = 208 m/s

Check to see if sonic velocity is exceeded. Eq. 13.15: $c = \sqrt{kRT} = \sqrt{1.28(188)(273+25)}$ = 268 m/s

Since 208 = $V_2 < c$ = 286, the flow is subsonic, and V_2 = 208 m/s is correct. ◄

Sec. 13.6: Isentropic Flow -- *Problems 13.3–13.4*

13.3 N *Derive Eq. 13.35 for isentropic flow by integrating the one-dimensional Euler equation.*

Euler Eq. 13.9: $dp/\rho + VdV = 0$ (1)

Sec. 13.1 for isentropic flow: pv^k = constant = $C = p/\rho^k$ because $v = 1/\rho$ (Eq. 2.2)

$p = C\rho^k;\ dp = Ck\rho^{k-1}d\rho$ (2)

Substitute (2) into (1) and integrate: $Ck\rho^{k-2}d\rho + VdV = 0$

$$\left.\frac{Ck\rho^{k-1}}{k-1}\right|_{\rho_1}^{\rho_2} + \left.\frac{V^2}{2}\right|_{V_1}^{V_2} = 0;\quad \frac{V_2^2 - V_1^2}{2} = \frac{Ck}{k-1}\left(\rho_1^{k-1} - \rho_2^{k-1}\right) = \frac{k}{k-1}C\rho_1^{k-1}\left[1 - \left(\frac{\rho_2}{\rho_1}\right)^{k-1}\right]$$

We note that $\dfrac{p}{\rho} = C\rho^{k-1}$ and $\left(\dfrac{p}{C}\right)^{(k-1)/k} = \rho^{k-1}$

So substituting: $$\frac{V_2^2 - V_1^2}{2} = \frac{p_1}{\rho_1}\frac{k}{k-1}\left[1 - \left(\frac{p_2}{p_1}\right)^{(k-1)/k}\right]$$ (Eq. 13.35) Q.E.D. ◄

13.4 BG *Carbon dioxide flows isentropically. At a point in the flow the velocity is 150 ft/sec, the temperature is 125°F, and the pressure is 20 psia. Determine the pressure and temperature on the nose of a streamlined object placed in the flow at that point.*

Table A.5 for carbon dioxide: $R = 1123$ ft²/(sec²·°R), $k = 1.28$, $c_p = 5132$ ft²/(sec²·°R)

Eq. 13.15: $c = \sqrt{kRT} = \sqrt{1.28(1123)(125 + 460)} = 917$ ft/sec

Eq. 13.10: $\mathbf{M} = V/c = 150/917 = 0.1636$

From Eq. 13.27: $p_o = 20[1 + 0.5(1.28 - 1)0.1636^2]^{1.28/0.28} = 20.3$ psia ◄

Alternatively, obtain $\rho = 0.004\,38$ slug/ft³ from Eq. 2.4, then find p_o from Eq. 13.30.

Sec. 13.5: $T_o = T + V^2/(2c_p) = (125 + 460) + 150^2/(2 \times 5132) = 587°\text{R} = 127.2°\text{F}$ ◄

Temperature and pressure increases are small (0.345 psi, 2.19°F).

Sec. 13.7: Effect of Area Variation on One-Dimensional Compressible Flow -- *Exercise (1)*

13.7.1 N *Show in detail the development of Eq. (13.39) from Eqs. (13.38) and (13.37).*

Eq. 13.37: $\dfrac{dA}{A} + \dfrac{d\rho}{\rho} + \dfrac{dV}{V} = 0$; Eq. 13.38: $c^2\dfrac{d\rho}{\rho} + VdV = 0$ or $\dfrac{d\rho}{\rho} = -\dfrac{VdV}{c^2}$

Substituting into Eq. 13.37: $\dfrac{dA}{A} = \dfrac{VdV}{c^2} - \dfrac{dV}{V} = \dfrac{dV}{V}\left[\dfrac{V^2}{c^2} - 1\right] = \dfrac{dV}{V}(\mathbf{M}^2 - 1)$ using Eq. 13.10

Rearranging: $$\frac{dV}{V} = \frac{1}{\mathbf{M}^2 - 1}\frac{dA}{A}$$ (Eq. 13.39) Q.E.D. ◄

Sec. 13.8: Compressible Flow Through a Converging Nozzle -- *Exercises (6)*

13.8.1 N *Start with Eq. 13.40 and derive Eq. 13.43.*

Eq. 13.40: $\frac{V_2^2}{2} = \frac{p_2}{\rho_2}\frac{k}{k-1}\left[(p_O/p_2)^{(k-1)/k} - 1\right]$; Eq. 13.5: $\dot{m} = (\rho A V)_2$

$\therefore\ \dot{m}^2 = \rho_2^2 A_2^2 V_2^2 = A_2^2 2 p_2 \rho_2 \frac{k}{k-1}\left[(p_O/p_2)^{(k-1)/k} - 1\right]$

Isentropic flow, Sec. 13.1: pv^k = const; Eq. 2.2: $v = 1/\rho$

Eliminating v: p/ρ^k = const, from which $\frac{p_O}{\rho_O^k} = \frac{p_2}{\rho_2^k}$, thus $\rho_2 = \left(\frac{p_2}{p_O}\right)^{1/k}\rho_O$

Substituting for ρ_2: $\dot{m}^2 = A_2^2 2 p_2 (p_2/p_O)^{1/k}\rho_O \frac{k}{k-1}\left[(p_O/p_2)^{(k-1)/k} - 1\right]$

and noting that $p_2(p_2/p_O)^{1/k} = p_O^{-1/k} p_2^{(k+1)/k}$, we divide outside the bracketed term by $(p_2/p_O)^{(k+1)/k}$ and multiply inside by the same quantity to get

$$\dot{m}^2 = A_2^2 2 p_O \rho_O \frac{k}{k-1}\left[\left(\frac{p_2}{p_O}\right)^{2/k} - \left(\frac{p_2}{p_O}\right)^{(k+1)/k}\right] \quad \text{i.e.,}\quad \dot{m} = A_2\sqrt{\frac{2k}{k-1}p_O\rho_O\left[\left(\frac{p_2}{p_O}\right)^{2/k} - \left(\frac{p_2}{p_O}\right)^{(k+1)/k}\right]}$$

(Eq. 13.43) Q.E.D. ◄

13.8.2 N *Differentiate Eq. 13.43 with respect to p_2/p_O and set to zero to find the value of p_2/p_O for which $\dot{m}$ is a maximum. The answer should correspond to Eq. 13.42.*

Eq. 13.43: $\dot{m} = A_2\sqrt{\frac{2k}{k-1}p_O\rho_O\left[\left(\frac{p_2}{p_O}\right)^{2/k} - \left(\frac{p_2}{p_O}\right)^{(k+1)/k}\right]}$ so $\frac{\dot{m}^2}{A^2 p_O \rho_O}\frac{k-1}{2k} = \left(\frac{p_2}{p_O}\right)^{2/k} - \left(\frac{p_2}{p_O}\right)^{(k+1)/k}$

$$d/d\left(\frac{p_2}{p_O}\right) \Rightarrow 0 = \frac{2}{k}\left(\frac{p_2}{p_O}\right)^{(2/k)-1} - \frac{k+1}{k}\left(\frac{p_2}{p_O}\right)^{\left(\frac{k+1}{k}\right)-1} = \frac{2}{k}\left(\frac{p_2}{p_O}\right)^{(2-k)/k} - \frac{k+1}{k}\left(\frac{p_2}{p_O}\right)^{1/k}$$

$$\frac{k+1}{k}\left(\frac{p_2}{p_O}\right)^{1/k} = \frac{2}{k}\left(\frac{p_2}{p_O}\right)^{(2-k)/k};\quad \left(\frac{p_2}{p_O}\right)^{(1/k)-(2-k)/k} = \frac{2}{k}\left(\frac{k}{k+1}\right);\quad \left(\frac{p_2}{p_O}\right)^{(k-1)/k} = \frac{2}{k+1}$$

$$\frac{p_2{*}}{p_O} = \left(\frac{2}{k+1}\right)^{k/(k-1)}$$ (Eq. 13.42) Q.E.D. ◄

13.8.3 BG *Carbon dioxide within a tank at 40 psia and 80°F discharges through a convergent nozzle into a 14.2 psia atmosphere. Find the velocity, pressure, and temperature at the nozzle outlet. Assume isentropic conditions.*

Table A.5 for CO_2: R = 1123 ft²/(sec²·°R), k = 1.28 ; Eq. 13.42: $p_2^*/p_O = (2/2.28)^{1.28/0.28} = 0.549$

Sec. 13.8: If $p_b > 0.549 p_O$ = 0.549(40) = 22.0 psia, the flow is subsonic.

But (given) p_b = 14.2 psia < 22.0 psia, ∴ the flow is sonic.

Sec. 13.8: For sonic flow, $p_2/p_O = p_2^*/p_O$. ∴ The pressure at the nozzle outlet is 22.0 psia ◄

From Eq. 13.36: $T_2 = (80 + 460)(22.0/40)^{0.28/1.28} = 474°R = 13.68°F$ ◄

Eq. 13.24 with $V_1 = V_O$ = zero: $V_2^2 = 2(1.28/0.28)(1123)(540)[1 - (474/540)]$; V_2 = 825 ft/sec ◄

13.8.4 *Carbon dioxide within a tank at 280 kPa abs and 25°C discharges through a convergent nozzle into a 98 kPa abs atmosphere. Find the velocity, pressure, and temperature at the nozzle outlet. Assume isentropic conditions.*

SI

Table A.5 for carbon dioxide: $R = 188\ \text{m}^2/(\text{s}^2\cdot\text{K})$, $k = 1.28$

Eq. 13.42: $p_2^*/p_O = (2/2.28)^{1.28/0.28} = 0.549$

Sec. 13.8: If $p_b > 0.549p_O = 0.549(280) = 153.8\ \text{kN/m}^2$ abs, the flow is subsonic.

But (given) $p_b = 98\ \text{kN/m}^2$ abs < 153.8, ∴ the flow is sonic.

Sec. 13.8: For sonic flow, $p_2/p_O = p_2{*}/p_O$. ∴ The pressure at the nozzle outlet is 153.8 kPa abs ◄

From Eq. 13.36: $T_2 = (25 + 273)(153.8/280)^{0.28/1.28} = 261\text{K} = -11.60°\text{C}$ ◄

Eq. 13.24 with $V_1 = V_O =$ zero: $V_2^2 = 2(1.28/0.28)(188)(298)[1 - (261/298)]$; $V_2 = 251$ m/s ◄

13.8.5 *In Exer. 13.8.3 if the pressure and temperature within the tank had been 20 psia and 100°F, what would have been the velocity, pressure, and temperature at the nozzle outlet? Assume isentropic conditions.*
Exer. 13.8.3: Carbon dioxide flows through a convergent nozzle, $p_b = 14.2$ *psia.*

BG

Table A.5 for carbon dioxide: $R = 1123\ \text{ft}^2/(\text{sec}^2\cdot°\text{R})$, $k = 1.28$

Eq. 13.42: $p_2^*/p_O = (2/2.28)^{1.28/0.28} = 0.549$

Sec. 13.8: If $p_b > 0.549p_O = 0.549(20) = 10.99$ psia, the flow is subsonic.

But (given) $p_b = 14.2$ psia > 10.99 psia, ∴ the flow is subsonic.

Sec. 13.8: With subsonic flow $p_2 = p_b = 14.2$ psia ◄

From Eq. 13.36: $T_2 = (100 + 460)(14.2/20)^{0.28/1.28} = 520°\text{R} = 59.6°\text{F}$ ◄

Eq. 13.24 with $V_1 = V_O =$ zero: $V_2^2 = 2(1.28/0.28)(1123)(560)[1 - (520/560)]$; $V_2 = 644$ ft/sec ◄

13.8.6 *In Exer. 13.8.4 if the pressure and temperature within the tank had been 140 kPa abs and 40°C, what would have been the velocity, pressure, and temperature at the nozzle outlet? Assume isentropic conditions.*
Exer. 13.8.4: Carbon dioxide flows through a convergent nozzle, $p_b = 98$ *kPa abs.*

SI

Table A.5 for carbon dioxide: $R = 188\ \text{m}^2/(\text{s}^2\cdot\text{K})$, $k = 1.28$

Eq. 13.42: $p_2^*/p_O = (2/2.28)^{1.28/0.28} = 0.549$

Sec. 13.8: If $p_b > 0.549p_O = 0.549(140) = 76.9\ \text{kN/m}^2$ abs, the flow is subsonic.

But (given) $p_b = 98\ \text{kN/m}^2$ abs > 76.9, ∴ the flow is subsonic.

Sec. 13.8: With subsonic flow $p_2 = p_b = 98\ \text{kN/m}^2$ abs ◄

From Eq. 13.36: $T_2 = (40 + 273)(98/140)^{0.28/1.28} = 290\text{K} = 16.51°\text{C}$ ◄

Eq. 13.24 with $V_1 = V_O = 0$: $V_2^2 = 2(1.28/0.28)(188)(313)[1 - (290/313)]$; $V_2 = 201$ m/s ◄

Sec. 13.8: Compressible Flow Through a Converging Nozzle -- *Problems 13.5–13.9*

13.5 *Air flows at 150°F from a large tank through a 1.5-in-diameter converging nozzle (Fig. P13.5). Within the tank the pressure is 85 psia. Calculate the flow rate for back pressures of 10, 30, 50, and 70 psia. Assume isentropic conditions. Plot $\dot{m}$ as a function of p_b. Assume that the temperature within the tank is 150°F in all cases. Compute also the temperature at the nozzle outlet for each condition.*

BG

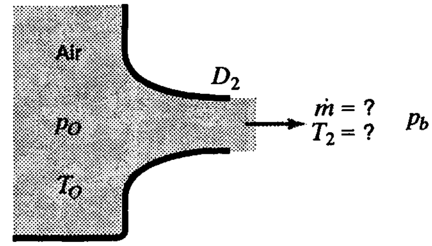


Figure P13.5

Table A.5 for air: $R = 1715\ \text{ft}^2/(\text{sec}^2\cdot{}^\circ\text{R})$, $k = 1.40$

Eq. 13.33 or 13.42: $p_2^*/p_o = (2/2.4)^{1.4/0.4} = 0.528$

Sec. 13.8: If $p_b > 0.528(85) = 44.9$ psia the flow is subsonic and $p_2 = p_b$

If $p_b < 0.528(85) = 44.9$ psia the flow is sonic and $p_2 = 44.9$ psia.

For sonic flows (i.e., $p_b < 44.9$ psia) $\dot{m}_{max}$ is given by Eq. 13.45. $A_2 = (\pi/4)(1.5/12)^2 = 0.012\,27\ \text{ft}^2$.

$$\text{Eq. 13.45:}\quad \dot{m}_{max} = \frac{0.012\,27(85)144}{\sqrt{460+150}}\sqrt{\frac{1.4}{1715}\left(\frac{2}{2.4}\right)^{2.4/0.4}} = 0.1006\ \text{slug/sec} \quad ◀$$

For subsonic flows (i.e., $p_b > 44.9$ psia) $\dot{m}$ is given by Eq. 13.43

From Eq. 2.4: $\rho_o = p_o/RT_o = 85(144)/(1715\times 610) = 0.011\,70\ \text{slug/ft}^3$

$$\text{Eq. 13.43:}\quad \dot{m} = 0.012\,27\sqrt{\frac{2(1.4)}{0.4}85(144)0.011\,70\left[\left(\frac{p_2}{85}\right)^{2/1.4} - \left(\frac{p_2}{85}\right)^{2.4/1.4}\right]} \quad \text{where } p_2 = p_b$$

Temperatures, from Eq. 13.36: $T_2 = (460+150)(p_2/85)^{0.4/1.4}$ °R

Results:

p_o (psia)	T_o (°F)	p_b (psia)	p_2 (psia)	Flow Eq.	$\dot{m}$ or $\dot{m}_{max}$ (slug/sec)	T_2 (°F)
85	150	10	44.9	sonic 13.45	0.1006	48.3
85	150	30	44.9	sonic 13.45	0.1006	48.3
85	150	50	50	subsonic 13.43	0.0998	64.2
85	150	70	70	subsonic 13.43	0.0786	117.1
85	150	85	85	subsonic 13.43	0	150.0

▲ ▲

13.6 *Air flows at 65°C from a large tank through a 40-mm-diameter converging nozzle (Fig. P13.5). Within the tank the pressure is 600 kPa abs. Calculate the flow rate for back pressures of 50, 200, 350, and 500 kPa abs. Assume isentropic conditions. Plot $\dot{m}$ as a function of p_b. Assume that the temperature within the tank is 65°C in all cases. Compute also the temperature at the nozzle outlet for each condition.*

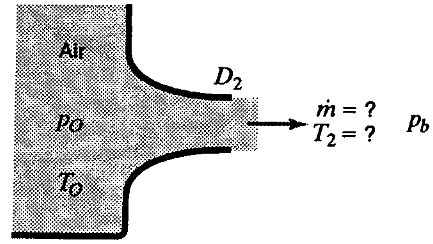


Figure P13.5

SI

Table A.5 for air: $R = 287\ \text{m}^2/(\text{s}^2\cdot\text{K})$, $k = 1.40$

Eq. 13.33 or 13.42: $p_2^*/p_O = (2/2.4)^{1.4/0.4} = 0.528$

Sec. 13.8: If $p_b > 0.528(600) = 317$ kPa abs the flow is subsonic and $p_2 = p_b$

If $p_b < 0.528(600) = 317$ kPa abs the flow is sonic and $p_2 = 317$ kPa abs.

For sonic flows (i.e., $p_b < 317$ kPa abs) $\dot{m}_{max}$ is given by Eq. 13.45. $A_2 = (\pi/4)0.04^2 = 0.001\,257\ \text{m}^2$

Eq. 13.45: $$\dot{m}_{max} = 0.001\,257\frac{600\,000}{\sqrt{273+65}}\sqrt{\frac{1.4}{287}\left(\frac{2}{2.4}\right)^{2.4/0.4}} = 1.658\ \text{kg/s}$$ ◄

For subsonic flows (i.e., $p_b > 317$ kPa abs) $\dot{m}$ is given by Eq. 13.43

From Eq. 2.4: $\rho_O = p_O/RT_O = 600\,000/(287 \times 338) = 6.19\ \text{kg/m}^3$

Eq. 13.43: $$\dot{m} = 0.001\,257\sqrt{\frac{2(1.4)}{0.4}(600\,000)6.19\left[\left(\frac{p_2}{600}\right)^{2/1.4} - \left(\frac{p_2}{600}\right)^{2.4/1.4}\right]}$$ where $p_2 - p_b$

Temperatures, from Eq. 13.36: $T_2 = (273+65)(p_2/600)^{0.4/1.4}$ K

Results:

p_O (kPa abs)	T_O (°C)	p_b (kPa abs)	p_2 (kPa abs)	Flow Eq.	$\dot{m}$ or $\dot{m}_{max}$ (kg/s)	T_2 (°C)
600	65	50	317	sonic 13.45	1.658	8.67
600	65	200	317	sonic 13.45	1.658	8.67
600	65	350	350	subsonic 13.43	1.647	16.76
600	65	500	500	subsonic 13.43	1.267	47.8
600	65	600	600	subsonic 13.43	0	65.0
					▲	▲

13.7 *Air flows at 25°C from a large tank through a 100-mm-diameter converging nozzle (Fig. P13.5). Within the tank the pressure is 50 kPa abs. Calculate the flow rate for back pressures of 30, 20, and 10 kPa abs. Assume isentropic conditions. Plot $\dot{m}$ as a function of p_b. Assume that the temperature within the tank is 25°C in all cases. Compute also the temperature at the nozzle outlet for each condition.*

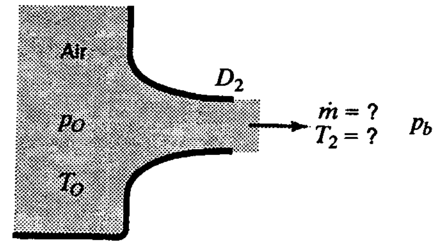


Figure P13.5

SI

Table A.5 for air: $R = 287 \text{ m}^2/(\text{s}^2\cdot\text{K})$, $k = 1.40$

Eq. 13.33 or 13.42: $p_2^*/p_O = (2/2.4)^{1.4/0.4} = 0.528$

Sec. 13.8: If $p_b > 0.528(50) = 26.4$ kPa abs the flow is subsonic and $p_2 = p_b$

If $p_b < 0.528(50) = 26.4$ kPa abs the flow is sonic and $p_2 = 26.4$ kPa abs.

For sonic flows (i.e., $p_b < 26.4$ kPa abs) $\dot{m}_{max}$ is given by Eq. 13.45. $A_2 = (\pi/4)0.10^2 = 0.007\,85 \text{ m}^2$

$$\text{Eq. 13.45:}\quad \dot{m}_{max} = 0.007\,85\,\frac{50\,000}{\sqrt{273+25}}\sqrt{\frac{1.4}{287}\left(\frac{2}{2.4}\right)^{2.4/0.4}} = 0.919 \text{ kg/s}$$

For subsonic flows (i.e., $p_b > 26.4$ kPa abs) $\dot{m}$ is given by Eq. 13.43.

From Eq. 2.4: $\rho_O = p_O/RT_O = 50\,000/(287\times 298) = 0.585 \text{ kg/m}^3$

$$\text{Eq. 13.43:}\quad \dot{m} = 0.007\,85\sqrt{\frac{2(1.4)}{0.4}50\,000(0.585)\left[\left(\frac{p_2}{50}\right)^{2/1.4} - \left(\frac{p_2}{50}\right)^{2.4/1.4}\right]}$$

Results:

p_O (kPa, abs)	T_O (°C)	p_b (kPa, abs)	p_2 (kPa, abs)	Flow Eq.	$\dot{m}$ or $\dot{m}_{max}$ (kg/s)	T_2 (°C)
50	25	10	26.4	sonic 13.45	0.919	−24.6
50	25	20	26.4	sonic 13.45	0.919	−24.6
50	25	30	30	subsonic 13.43	0.909	−15.47
					▲	▲

13.8 *Air within a tank at 120°F flows isentropically through a 2-in-diameter convergent nozzle into a 14.2-psia atmosphere. Find the mass flow rate for air pressures within the tank of 5, 10, 20, 40, and 50 psia.*

BG

Table A.5 for air: $R = 1715\ \text{ft}^2/(\text{sec}^2\cdot°\text{R}),\ k = 1.40$

Eq. 13.33 or 13.42: $p_2^*/p_O = (2/2.4)^{1.4/0.4} = 0.528;\ p_2 = 14.2$ psia (given)

Sec. 13.8: If $p_b/p_O > 0.528$, i.e., $p_O < 14.2/0.528 = 26.9$ psia, the flow is subsonic and $p_2 = p_b$,

$\dot{m}$ is given by Eq. 13.43.

Sec. 13.8: If $p_b/p_O \le 0.528$, i.e., $p_O \ge 14.2/0.528 = 26.9$ psia, the flow is sonic and $p_2 = 0.528p_O$

For sonic flow (i.e., $p_O \ge 26.9$ psia), $\dot{m}_{max}$ is given by Eq. 13.45. $A_2 = (\pi/4)(2/12)^2 = 0.0218\ \text{ft}^2$.

Eq. 13.45: $$\dot{m}_{max} = \frac{0.0218(144p_O)}{\sqrt{580}}\sqrt{\frac{1.4}{1715}\left(\frac{2}{2.4}\right)^{2.4/0.4}} = 0.002\,16p_O\ \text{slug/sec}$$ ◄

For subsonic flows (i.e., $p_O < 26.9$ psia) $\dot{m}$ is given by Eq. 13.43.

From Eq. 2.4: $\rho_O = p_O/RT_O = 144p_O/(1715 \times 580)$

Eq. 13.43: $$\dot{m} = 0.0218\sqrt{2\left(\frac{1.4}{0.4}\right)\frac{(144p_O)^2}{1715(580)}\left[\left(\frac{14.2}{p_O}\right)^{2/1.4} - \left(\frac{14.2}{p_O}\right)^{2.4/1.4}\right]}\ \text{slug/sec}$$

If $p_O < 14.2$ psia (negative gage pressure) reverse flow will occur. Results are:

p_O (psig)	p_O (psia)	T_O (°F)	p_b (psia)	p_2 (psia)	Flow Eq.	$\dot{m}$ or $\dot{m}_{max}$ (slug/sec)
0	14.2	120	14.2	14.2	–	0
5	19.2	120	14.2	14.2	subsonic 13.43	0.0371
10	24.2	120	14.2	14.2	subsonic 13.43	0.0518
12.7	26.9	120	14.2	14.2	sonic 13.45	0.0580
20	34.2	120	14.2	18.07	sonic 13.45	0.0738
40	54.2	120	14.2	28.6	sonic 13.45	0.1169

▲

13.9 *Refer to Sample Prob. 13.4. If the pressure in the tank is 5 psig, confirm by calculation that $\dot{m}$ = 0.0374 slug/sec, p_2 = 13.50 psia, and T_2 = 494°R.*

Sample Prob. 13.4: D = 2 in, T_O = 80°F = 540°R, p_b = 13.50 psia, p_2^*/p_O = 0.528.

BG

Table A.5 for air: $R = 1715\ \text{ft}^2/(\text{sec}^2\cdot°\text{R})$, $k = 1.40$

$p_O = 5$ psig; $p_O = 5 + p_{atm} = 5 + 13.5 = 18.50$ psia. $A_2 = (\pi/4)(2/12)^2 = 0.0218\ \text{ft}^2$.

From Eq. 2.4: $\rho_O = p_O/RT_O = 18.50(144)/(1715 \times 540) = 0.002\,88\ \text{slug/ft}^3$

$p_b/p_O = 13.5/18.5 = 0.730 > 0.528 = p_2{*}/p_O$

so flow is subsonic, ($p_2 = p_b$), and $\dot{m}$ is given by Eq. 13.43.

$$\dot{m} = 0.0218\sqrt{2\frac{1.4}{0.4}(18.5)144(0.002\,88)\left[\left(\frac{13.5}{18.5}\right)^{2/1.4} - \left(\frac{13.5}{18.5}\right)^{2.4/1.4}\right]} = 0.0374\ \text{slug/sec} \quad ◀$$

(1) $V_2^2/2 = (p_2/\rho_2)[k/(k-1)][(p_O/p_2)^{(k-1)/k} - 1]$ (Eq. 13.40)

(2) $\dot{m} = \rho_2 A_2 V_2$ (Eq. 13.5)

(3) $\rho_2 = p_2/RT_2$ (from Eq. 2.4)

(4) $p_O/\rho_O^k = p_2/\rho_2^k$ (from pv^k = const and $v = 1/\rho$, Sec. 13.1 and Eq. 2.2)

From (4): $p_2 = p_O\dfrac{\rho_2^k}{\rho_O^k} = (18.5 \times 144)\dfrac{\rho_2^k}{0.002\,88^{1.4}} = 9{,}620{,}000\rho_2^k$ (5)

From (2) $\rho_2 = \dot{m}/(A_2V_2) = 0.0374/(0.0218V_2) = 1.716/V_2$ (6)

Substitute for ρ_2 from (6) into (5): $p_2 = 9{,}620{,}000(1.716/V_2)^{1.4} = 20.5 \times 10^6/V_2^{1.4}$ (7)

Substitute into (1): $V_2^2 = 2\dfrac{20.5 \times 10^6(1.4/0.4)}{V_2^{1.4}(1.716/V_2)}\left[\left(\dfrac{18.5(144)V_2^{1.4}}{20.5 \times 10^6}\right)^{0.4/1.4} - 1\right]$

or $V_2^{2.4} = 83.6 \times 10^6[0.0776V_2^{0.4} - 1]$; by T and E $V_2 = 747$ ft/sec

From (2): $\rho_2 = \dot{m}/(A_2V_2) = 0.0374/(0.0218 \times 747) = 0.002\,30\ \text{slug/ft}^3$

From (4): $p_2 = (18.5)0.002\,30^{1.4}/0.002\,88^{1.4} = 13.50$ psia ◀

From (3): $T_2 = p_2/(R\rho_2) = (13.50 \times 144)/(1715 \times 0.002\,30) = 494°\text{R}$ ◀

Sec. 13.9: Isentropic Flow Through a Converging-Diverging Nozzle -- *Exercises (6)*

13.9.1 *Air enters a converging-diverging nozzle at a pressure of 120 psia and a temperature of 90°F. Neglecting the entrance velocity and assuming a frictionless process, find the Mach number at the cross section where the pressure is 35 psia.*

BG

Table A.5 for air: $R = 1715\ \text{ft}^2/(\text{sec}^2\cdot°\text{R})$, $k = 1.40$

From Eq. 13.36: $T_2 = T_O(p_2/p_O)^{(k-1)/k} = (460 + 90)(35/120)^{0.4/1.4} = 387°\text{R}$

Eq. 13.24 with $V_1 = 0$: $V_2^2 = 2(1.4/0.40)1715(460 + 90)[1 - (387/550)]$; $V_2 = 1400$ ft/sec

Eq. 13.15: $c = \sqrt{kRT_2} = \sqrt{1.4(1715)387} = 964$ ft/sec

Eq. 13.10: $\mathbf{M} = V/c = 1400/964 = 1.453$ ◀

13.9.2 *Air enters a converging-diverging nozzle at a pressure of 830 kPa abs and a temperature of 32°C. Neglecting the entrance velocity and assuming a frictionless process, find the Mach number at the cross section where the pressure is 240 kPa abs.*

SI

Table A.5 for air: $R = 287\ \text{m}^2/(\text{s}^2\cdot\text{K})$, $k = 1.40$

From pv^k = const and $pv = RT$: $T_2 = T_O(p_2/p_O)^{(k-1)/k} = (273 + 32)(240/830)^{0.4/1.4} = 214$ K

Eq. 13.24 with $V_1 = 0$: $V_2^2 = 2(1.40/0.40)287(273 + 32)[1 - (214/305)]$; $V_2 = 428$ m/s

Eq. 13.15: $c = \sqrt{kRT} = \sqrt{1.4(287)214} = 293$ m/s

Eq. 13.10: $\mathbf{M} = V_2/c = 428/293 = 1.459$ ◄

13.9.3 *Work Sample Prob. 13.5 with all data the same except for the pressure within the tank, which is 100 rather than 50 psia.*
Sample Prob. 13.5: $T_O = 80°F = 540°R$, $D_2 = 2$ in, $A_2 = 0.0218\ ft^2$, $p_3 = p_b = 13.5$ psia. Find D_3, $\dot{m}$ (=0.0118 slug/sec), V_2, V_3, T_2, T_3. Assume isentropic flow.

BG

Table A.5 for air: $R = 1715\ \text{ft}^2/(\text{sec}^2\cdot°\text{R})$, $k = 1.40$

Pressure at throat (where $V_2 = c$): $p_2 = p_2^* = 0.528p_O = 0.528(100) = 52.8$ psia

Eq. 13.35 (isentropic, given): $V_3^2 = 2(1.4/0.4)1715(540)[1 - (13.5/100)^{0.4/1.4}]$; $V_3 = 1681$ ft/sec ◄

Eq. 13.45: $\dot{m}_{max} = 0.224$ slug/sec (exactly twice that of Sample Prob. 13.5) ◄

Eq. 13.24 with $V_1 = 0$: $1681^2 = 2(1.4/0.4)(1715)540(1 - T_3/540)$; $T_3 = 305°\text{R} = -155.3°\text{F}$ ◄

From Eq. 2.4: $\rho_3 = p_3/(RT_3) = (13.5 \times 144)/(1715 \times 305) = 0.003\,72$ slug/ft³

p/ρ^k = constant, (isentropic, Sec. 13.1), ∴ $52.8/\rho_2^{1.4} = 13.5/0.003\,72^{1.4}$; $\rho_2 = 0.009\,86$ slug/ft³

From Eq. 13.5: $V_2 = \dot{m}/(\rho_2 A_2) = 0.224/(0.009\,84 \times 0.0218) = 1039$ ft/sec ◄

Eq. 13.24 with $V_1 = 0$: $1039^2 = 2(1.4/0.4)(1715)540(1 - T_2/540)$; $T_2 = 450°\text{R} = -10.00°\text{F}$ ◄

Finally, from Eq. 13.5: $A_3 = \dot{m}/(\rho_3 V_3) = 0.224/(0.003\,72 \times 1681) = 0.0358\ \text{ft}^2 = 5.15\ \text{in}^2 = \pi D_3^2/4$

so $D_3 = 2.56$ in ◄

13.9.4 *Air is to flow through a converging-diverging nozzle at 0.6 slug/sec. At the throat the pressure, temperature, and velocity are to be 20 psia, 100°F and 500 ft/sec, respectively. At outlet the velocity is to be 200 ft/sec. Determine the throat diameter. Assume isentropic flow.*

BG

Table A.5 for air: $R = 1715$ ft·lb/(slug·°R)

From Eq. 2.4: $\rho_2 = p_2/(RT_2) = 20(144)/(1715 \times 560) = 0.003\,00$ slug/ft³

Eq. 13.5: $\dot{m} = 0.6 = \rho_2 A_2 V_2 = 0.003\,00(A_2)500$

$A_2 = 0.400\ \text{ft}^2 = 57.6\ \text{in}^2 = \pi D_2^2/4$; $D_2 = 8.57$ in ◄

13.9.5 *Air in a tank at a pressure of 140 psia and 70°F flows out into the atmosphere through a 1.00-in-diameter converging nozzle. (a) Find the mass flow rate. (b) If a diverging section with an outlet diameter of 1.50 in were attached to the converging nozzle, what then would be the flow rate? Neglect friction.*

BG

Table A.5 for air: $R = 1715\ \text{ft}^2/(\text{sec}^2\cdot{}^\circ\text{R})$, $k = 1.40$. Assume $p_b = p_{atm} = 14.70$ psia.

(*a*) Sec. 13.8: $p_b/p_O = 14.7/140 = 0.105 < 0.528$; so flow at the throat is sonic; use Eq. 13.45:

$$\dot{m}_{max} = \frac{\pi}{4}\left(\frac{1}{12}\right)^2 \frac{(140)144}{\sqrt{460 + 70}}\sqrt{\frac{1.4}{1715}\left(\frac{2}{2.4}\right)^{2.4/0.4}} = 0.0790 \text{ slug/sec} \quad ◄$$

(*b*) Sec. 13.9: With sonic flow (and no change in Δp), attaching a diverging nozzle will not change the flow rate. ◄

13.9.6 *Air in a tank at a pressure of 950 kPa abs and 20°C flows out into the atmosphere through a 25-mm-diameter converging nozzle. (a) Find the mass flow rate. (b) If a diverging section with an outlet diameter of 40 mm were attached to the converging nozzle, what then would be the flow rate? Neglect friction.*

SI

Table A.5 for air: $R = 287\ \text{m}^2/(\text{s}^2\cdot\text{K})$, $k = 1.40$. Assume $p_b = p_{atm} = 101$ kPa abs.

(*a*) Sec. 13.8: $p_b/p_O = 101/950 = 0.106 < 0.528$; so flow at the throat is sonic; use Eq. 13.45:

$$\dot{m}_{max} = \frac{\pi}{4}\left(\frac{2.50}{100}\right)^2 \frac{950\,000}{\sqrt{273 + 20}}\sqrt{\frac{1.4}{287}\left(\frac{2}{2.4}\right)^{2.4/0.4}} = 1.101 \text{ kg/s} \quad ◄$$

(*b*) Sec. 13.9: With sonic flow (and no change in Δp), attaching a diverging nozzle will not change the flow rate. ◄

Sec. 13.9: Isentropic Flow Through a Converging-Diverging Nozzle -- ***Problems*** *13.10–13.13*

13.10 *Air discharges from a large tank through a converging-diverging nozzle (Fig. P13.10). The throat diameter is 3.0 in and the exit diameter is 4.0 in. Within the tank the air pressure and temperature are 40 psia and 150°F respectively. Calculate the mass flow rate for back pressures of 39, 38, 36, and 30 psia. Assume no friction.*

BG

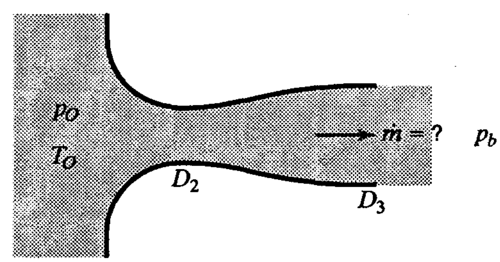


Figure P13.10

Table A.5 for air: $R = 1715\ \text{ft}^2/(\text{sec}^2\cdot{}^\circ\text{R})$, $k = 1.40$

From Eq. 2.4: $\rho_O = p_O/(RT_O) = 40(144)/(1715 \times 610)$

$= 005\ 51$ slug/ft^3

If sonic flow occurs, Eq. 13.45: $\dot{m}_{max} = \frac{\pi}{4}\left(\frac{3}{12}\right)^2 \frac{40(144)}{\sqrt{460 + 150}}\sqrt{\frac{1.4}{1715}\left(\frac{2.0}{2.4}\right)^{2.4/0.4}} = 0.1893$ slug/sec

For subsonic flow: Eq. 13.43: $\dot{m} = \frac{\pi}{4}\left(\frac{4}{12}\right)^2\sqrt{2\left(\frac{1.4}{0.4}\right)(40)144(0.005\,51)\left[\left(\frac{p_b}{40}\right)^{2/1.4} - \left(\frac{p_b}{40}\right)^{2.4/1.4}\right]}$

By T and E using Eq. 13.43, at threshold point with $\dot{m} = 0.1893$ slug/sec, $p_b = p_2 = 36.75$ psia.

p_b (psia)	40	39	38	36.75	36	30	
$\dot{m}$ (slug/sec), Eq. 13.43	0	0.1084	0.1512	0.1893	--	--	◄◄
$\dot{m}_{max}$ (slug/sec), Eq. 13.45	--	--	--	0.1893	0.1893	0.1893	◄◄

13.11 *Air discharges from a large tank through a converging-diverging nozzle (Fig. P13.10). The throat diameter is 75 mm and the exit diameter is 100 mm. Within the tank the air pressure and temperature are 290 kPa and 65°C, respectively. Calculate the flow rate for back pressures of 280, 270, 250, and 200 kPa abs. Assume no friction.*

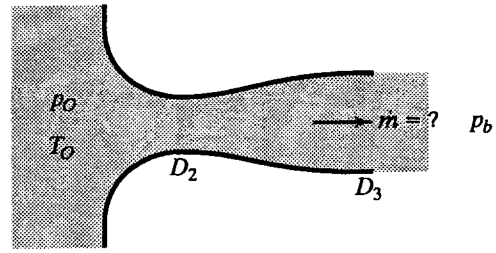


Figure P13.10

SI

Table A.5 for air: $R = 287\ \text{m}^2/(\text{s}^2\cdot\text{K})$, $k = 1.40$.

Eq. 2.4: $\rho_O = p_O/(RT_O) = 290\,000/(287\times 338) = 2.99$

If sonic flow occurs, Eq. 13.45: $\dot{m}_{max} = \frac{\pi}{4}\left(\frac{7.5}{100}\right)^2 \frac{290\,000}{\sqrt{273+65}}\sqrt{\frac{1.4}{287}\left(\frac{2}{2.4}\right)^{2.4/0.4}} = 2.82$ kg/s

For subsonic flow, Eq. 13.43: $\dot{m} = \pi 0.05^2\sqrt{2\left(\frac{1.4}{0.4}\right)290\,000\times 2.99\left[\left(\frac{p_b}{290}\right)^{2/1.4}-\left(\frac{p_b}{290}\right)^{2.4/1.4}\right]}$

By T and E using Eq. 13.43, at threshold point with $\dot{m} = 2.82$ kg/s, $p_b = p_2 = 266$ kPa abs.

p_b (kPa abs)	290	280	270	266	250	200	
$\dot{m}$ (kg/sec), Eq. 13.43	0	1.885	2.61	2.82	--	--	◄◄
$\dot{m}_{max}$ (kg/sec), Eq. 13.45	--	--	--	2.82	2.82	2.82	◄◄

13.12 *Repeat Exer. 13.9.5 for the case where the air within the tank is at 20 psia. Assume all other data to be the same.*

Exer. 13.9.5: Converging nozzle with D_2 = 1.00 in, air T_O = 70°F = 530°R. (a) Find $\dot{m}$. (b) Find $\dot{m}$ after a diverging section with D_3 = 1.50 in is attached.

BG

Table A.5 for air: $R = 1715\ \text{ft}^2/(\text{sec}^2\cdot{}^\circ\text{R})$, $k = 1.40$. Assume $p_b = p_{atm} = 14.70$ psia.

(*a*) Sec. 13.8: $p_b/p_O = 14.7/20 = 0.735 > 0.528$; so flow at the throat is subsonic, $p_2 = p_b$,

$\dot{m}$ is given by Eq. 13.43.

From Eq. 2.4: $\rho_O = p_O/(RT_O) = (20\times 144)/(1715\times 530) = 0.003\,17\ \text{slug/ft}^3$

Eq. 13.43: $\dot{m} = \frac{\pi}{4}\left(\frac{1}{12}\right)^2\sqrt{2\frac{1.4}{0.4}(20\times 144)0.003\,17[0.735^{2/1.4}-0.735^{2.4/1.4}]} = 0.01015$ slug/sec ◄

(*b*) Sec. 13.9: Attaching a diverging nozzle with subsonic flow (when the air behaves like an incompressible fluid) will increase the flow rate. It only changes A_2 in Eq. 13.43.

Thus in Eq. 13.43: $\dot{m} = 0.01015(1.5/1.0)^2 = 0.0228$ slug/sec providing subsonic flow still occurs in the throat. Check this by computing $\dot{m}_{max}$, which is the maximum $\dot{m}$ if p_O is not increased.

Eq. 13.45: $\dot{m}_{max} = \frac{\pi}{4}(1.0)^2\frac{20}{\sqrt{460+70}}\sqrt{\frac{1.4}{1715}\left(\frac{2}{2.4}\right)^{2.4/0.4}} = 0.011\,28$ slug/sec

0.0228 slug/sec exceeds $\dot{m}_{max} = 0.011\,28$, ∴ with the 1.5-in-diameter outlet the flow at the throat has become sonic, and $\dot{m} = 0.011\,28$ slug/sec. ◄

13.13 *Repeat Exer. 13.9.6 for the case where the air within the tank is at 140 kPa abs. Assume all other data to be the same.*

Exer. 13.9.6: Converging nozzle with $D_2 = 25$ mm, air $T_O = 20°C = 293$ K. (a) Find $\dot{m}$. (b) Find $\dot{m}$ after a diverging section with $D_3 = 40$ mm is attached.

SI

Table A.5 for air: $R = 287\ m^2/(s^2 \cdot K)$, $k = 1.40$. Assume $p_b = p_{atm} = 101$ kPa abs.

(*a*) Sec 13.8: $p_b/p_O = 101/140 = 0.721 > 0.528$; so flow at the throat is subsonic, $p_2 = p_b$, and

$\dot{m}$ is given by Eq. 13.43. From Eq. 2.4: $\rho_O = p_O/(RT_O) = 140000/(287 \times 293) = 1.665\ kg/m^3$

$$\text{Eq. 13.43:}\quad \dot{m} = \frac{\pi}{4}\left(\frac{2.50}{100}\right)^2 \sqrt{2\frac{1.4}{0.4}(140\,000)1.665\left[0.721^{2/1.4} - 0.721^{2.4/1.4}\right]} = 0.1482\ \text{kg/s} \quad ◄$$

(*b*) Sec. 13.9: Attaching a diverging nozzle with subsonic flow (when the air behaves like an incompressible fluid) will increase the flow rate. It only changes A_2 in Eq. 13.43.

Thus in Eq. 13.43: $\dot{m} = 0.1482(4/2.5)^2 = 0.379$ kg/s providing subsonic flow still occurs in the throat. Check this by computing $\dot{m}_{max}$, which is the maximum $\dot{m}$ if p_O is not increased.

$$\text{Eq. 13.45:}\quad \dot{m}_{max} = \frac{\pi}{4}\left(\frac{2.5}{100}\right)^2 \frac{140\,000}{\sqrt{273 + 20}} \sqrt{\frac{1.4}{287}\left(\frac{2}{2.4}\right)^{2.4/0.4}} = 0.1623\ \text{kg/s}$$

0.379 kg/s exceeds $\dot{m}_{max} = 0.1623$ kg/s, ∴ with the 40-mm-diameter outlet the flow at the throat has become sonic, and $\dot{m} = 0.1623$ kg/s. ◄

Sec. 13.10: One Dimensional Shock Wave -- *Exercises (2)*

13.10.1 *Just downstream of a normal shock wave the pressure, velocity, and temperature are 52 psia, 400 ft/sec and 120°F. Compute the Mach number upstream of the shock wave. Consider (a) air and (b) carbon dioxide as the working fluids.*

BG

(*a*) Table A.5 for air: $R = 1715\ ft^2/(s^2 \cdot °R)$, $k = 1.40$

Eq. 13.15: $c_2 = \sqrt{1.4(1715)(460 + 120)} = 1180$ ft/sec; Eq. 13.10: $M_2 = 400/1180 = 0.339$

Eq. 13.52: $\left(2 + 0.4M_1^2\right)/\left(2(1.4)M_1^2 - 0.4\right) = M_2^2 = (0.339)^2$; $M_1^2 = -26.1$ (impossible)

∴ This situation cannot happen with air. ◄

(*b*) Table A.5 for carbon dioxide: $R = 1123\ ft^2/(s^2 \cdot °R)$, $k = 1.28$

Eq. 13.15: $c_2 = \sqrt{1.28(1123)(460 + 120)} = 913$ ft/sec; $M_2 = 400/913 = 0.438$

Eq. 13.52: $\left(2 + 0.28M_1^2\right)/\left(2(1.28)M_1^2 - 0.28\right) = M_2^2 = (0.438)^2$; $M_1^2 = 9.72$; $M_1 = 3.12$ ◄

13.10.2 *Just downstream of a normal shock wave the pressure, velocity, and temperature are 360 kPa abs, 110 m/s and 50°C. Compute the Mach number upstream of the shock wave. Consider (a) air and (b) carbon dioxide as the working fluids.*

SI

(*a*) Table A.5 for air: $R = 287\ m^2/(s^2 \cdot K)$, $k = 1.40$

Eq. 13.15: $c_2 = \sqrt{1.40(287)(273 + 50)} = 360$ m/s; Eq. 13.10: $M_2 = 110/360 = 0.305$

From Eq. 13.52: $M_1^2 = (2 + 0.4M_2^2)/(2 \times 1.4M_2^2 - 0.4) = -14.66$ (impossible)

∴ This situation cannot happen with air. ◄

(*b*) Table A.5 for carbon dioxide: $R = 188\ m^2/(s^2 \cdot K)$, $k = 1.28$

Eq. 13.15: $c_2 = \sqrt{1.28(188)(273 + 50)} = 279$ m/s; Eq. 13.10: $M_2 = 110/279 = 0.395$

From Eq. 13.52: $M_1^2 = (2 + 0.28M_2^2)/(2 \times 1.28M_2^2 - 0.28) = 17.24$; $M_1 = 4.15$ ◄

Sec. 13.10: One Dimensional Shock Wave – ***Problems*** *13.14–13.15*

13.14 *Air discharges from a large tank through a converging-diverging nozzle with a 25-mm-diameter throat into the atmosphere. The gage pressure and temperature in the tank are 700 kPa and 40°C respectively, the barometric pressure is 995 millibars. (a) Find the nozzle tip diameter required for p_3 to be equal to the atmospheric pressure. For this case, what are the flow velocity, sonic velocity, and Mach number at the nozzle exit? (b) Determine the value of p_b that will cause the shock wave to be located at the nozzle exit.*

SI

Table A.5 for air: $R = 287\ \text{m}^2/(\text{s}^2\cdot\text{K})$, $c_p = 1003\ \text{m}^2/(\text{s}^2\cdot\text{K})$, $k = 1.40$

$p_{atm} = 995$ millibars $= 99.5$ kPa abs; $p_O = 700 + 99.5 = 799.5$ kPa abs; $T_O = 40 + 273 = 313$ K

(*a*) Assume sonic velocity at the throat:

Eq. 13.35 with $V_1 = 0$: $\dfrac{V_3^2}{2} = 287(313)\dfrac{1.4}{0.4}\left[1 - \left(\dfrac{99.5}{799.5}\right)^{0.4/1.4}\right]$; $V_3^2 = 282\,000$; $V_3 = 531$ m/s ◀

and Eq. 13.45: $\dot{m}_{max} = \dfrac{(\pi \times 0.0125^2)799\,500}{\sqrt{273 + 40}}\sqrt{\dfrac{1.4}{287}\left(\dfrac{2}{2.4}\right)^{2.4/0.4}} = 0.897$ kg/s

Eq. 13.23: $V_3^2 - V_1^2 = 2c_p(T_1 - T_3)$; $531^2 - 0 = 2(1003)(313 - T_3)$; $T_3 = 172.4$ K

From Eq. 2.4: $\rho_3 = \dfrac{p_3}{RT_3} = \dfrac{99\,500}{287(172.4)} = 2.01\ \text{kg/m}^3$

From Eq. 13.5: $A_3 = \dot{m}/(\rho_3 V_3) = 0.897/(2.01 \times 531) = 0.000\,839\ \text{m}^2 = 839\ \text{mm}^2 = (\pi/4)D_3^2$ so

$D_3 = 32.7$ mm ◀ Eq. 13.15: $c_3 = \sqrt{kRT_3} = \sqrt{1.4(287)172.4} = 263$ m/s ◀

Eq. 13.10: $\mathbf{M}_3 = V_3/c_3 = 531/263 = 2.02$ ◀

Thus flow is supersonic at the nozzle exit, sonic velocity occurs at the throat, and the shock wave is located downstream of the nozzle tip.

(*b*) Across a shock wave, Eq. 13.49: $p_2/p_1 = \left[2k\mathbf{M}_1^2 - (k - 1)\right]/(k + 1)$

where Section (1) is just inside the nozzle tip and Section (2) is just outside the tip.

Relating this to Fig. 13.6 we have $\dfrac{p_b}{p_3} = \dfrac{p_2}{p_1} = \dfrac{2(1.4)2.02^2 - 0.4}{2.4} = 4.59$

$p_b = 4.59p_3 = 4.59(99.5) = 456$ kPa abs ◀

13.15 *The pressure, velocity, and temperature just upstream of a normal shock wave in air are 10 psia, 2200 fps, and 23°F. Determine the pressure, velocity, and temperature just downstream of the wave.*

BG

Table A.5 for air: $R = 1715\ \text{ft}^2/(\text{sec}^2\cdot°\text{R})$, $k = 1.40$

From Eq. 2.4: $\rho_1 = p_1/(RT_1) = (10)144/(1715 \times 483) = 0.001\,738\ \text{slug/ft}^3$

Eq. 13.15: $c_1 = \sqrt{kRT_1} = \sqrt{1.40(1715)483} = 1077$ ft/sec. Eq. 13.10: $\mathbf{M}_1 = 2200/1077 = 2.04$

Eq. 13.49: $p_2 = p_1[2(1.4)2.04^2 - 0.4]/2.4 = 10(4.70) = 47.0$ psia ◀

From Eq. 13.47 and continuity: $p_1 - p_2 = \rho_1 V_1(V_2 - V_1)$

i.e., $V_2 = V_1 + \dfrac{p_1 - p_2}{\rho_1 V_1} = 2200 + \dfrac{(10 - 47.0)144}{(0.001\,738)2200} = 806$ ft/sec ◀

From Eqs. 13.48 and 2.2: $806^2 - 2200^2 = \dfrac{2(1.4)}{0.4}\left(\dfrac{10(144)}{0.001\,738} - \dfrac{47.0(144)}{\rho_2}\right)$; $\rho_2 = 0.004\,75\ \text{slug/ft}^3$

From Eq. 2.4: $T_2 = \dfrac{p_2}{R\rho_2} = \dfrac{47.0(144)}{1715(0.004\,75)} = 832°\text{R} = 372°\text{F}$ ◀

Sec. 13.11: The Oblique Shock Wave – *Exercises (3)*

13.11.1 *Assuming the tip of the model in Fig. 13.9 to be a point source of infinitesimal disturbance, find the air velocity if the temperature is −60°F and k = 1.4. If the actual Mach number is 1.38, what is the percentage error involved in the preceding assumption?*

BG

Table A.5 for air: $R = 1715 \text{ ft}^2/(\text{s}^2\cdot°\text{R})$, $k = 1.40$

Eq. 13.15: $c = \sqrt{kRT} = \sqrt{1.40(1715)(460 - 60)} = 980$ ft/sec

At $\mathbf{M} = 1.38$ (given), true $V = 1.38(980) = 1352$ ft/sec (from Eq. 13.10)

Fig. 13.9: $\beta = 49°$; Eq. 13.53: $\mathbf{M} = V/c = 1/\sin\beta = 1/\sin 49° = 1.325$

$\therefore\ V = c\mathbf{M} = (980)1.325 = 1299$ ft/sec ◄

% error is $(1.38 - 1.325)100/1.38 = 3.98\%$ (low) ◄

13.11.2 *A schlieren photograph of a bullet shows a Mach angle of 40°. The air is at a pressure of 14 psia and 50°F. Find the approximate speed of the bullet.*

BG

Table A.5 for air: $R = 1715 \text{ ft}^2/(\text{s}^2\cdot°\text{R})$, $k = 1.40$

Eq. 13.15: $c = \sqrt{kRT} = \sqrt{1.40(1715)(460 + 50)} = 1107$ ft/sec

Eq. 13.53: $\mathbf{M} = 1/\sin\beta = 1/\sin 40° = 1.556$; $\therefore\ V = c\mathbf{M} = 1107(1.556) = 1722$ ft/sec ◄

13.11.3 *A schlieren photograph of a bullet shows a Mach angle of 25°. The air is at a pressure of 9.5 kPa abs and 20°C. Find the approximate speed of the bullet.*

SI

Table A.5 for air: $R = 287 \text{ m}^2/(\text{s}^2\cdot\text{K})$, $k = 1.40$

Eq. 13.15: $c = \sqrt{kRT} = \sqrt{1.40(287)(273 + 20)} = 343$ m/s

Eq. 13.53: $\mathbf{M} = 1/\sin\beta = 1/\sin 25° = 2.37$; $\therefore\ V = c\mathbf{M} = 343(2.37) = 812$ m/s ◄

Sec. 13.12: Isothermal Flow -- *Problems 13.16–13.17*

13.16 *Air flows isothermally in a long pipe. At one section the pressure is 90 psia, the temperature is 80°F and the velocity is 100 fps. At a second section some distance from the first the pressure is 15 psia. Find the energy head loss due to friction, and determine the thermal energy (ft·lb/lb) that must have been added to or taken from the fluid between the two sections. The diameter of the pipe is constant.*

BG

Table A.5 for air: $R = 1715\ \text{ft}^2/(\text{sec}^2\cdot{}^\circ\text{R})$

From Eq. 2.4 for isothermal flow: $\rho_1/\rho_2 = p_1/p_2 = 90/15 = 6$

But from continuity $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$, thus as A is constant, $V_2 = 6V_1 = 6(100) = 600$ fps

From Eq. 13.54: $q_H = 600^2/2 - 100^2/2 = 175{,}000$ ft·lb/slug of air

$Q_H = 175{,}000/32.2 = 5430$ ft·lb/lb of air ◀

For head loss: From Sec. 13.13, ignoring z terms:

$$\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) = -\left(\frac{\tau P}{\rho A}ds\right) = -\frac{gh_f}{L}ds = -\frac{g\,dh_f}{dL}ds = -g\,dh_f$$

Integrating: $g h_f = RT\ln(p_1/p_2) + (V_1^2 - V_2^2)/2 = 1715(460+80)\ln 6 + (100^2 - 600^2)/2$

$= 1{,}659{,}000 - 175{,}000 = 1{,}484{,}000$ ft·lb/slug

$h_f = 1{,}484{,}000/32.2 = 46{,}100$ ft·lb/lb of air ◀

Note: The total energy at section (2) is greater than that at section (1) because external heat is added. Thus h_L represents a degradation of mechanical energy, but not a net loss of energy, because it is converted to another form of energy, namely internal heat.

13.17 *Air flows isothermally in a long pipe. At one section the pressure is 600 kPa abs, the temperature is 25°C, and the velocity is 30 m/s. At a second section some distance from the first the pressure is 100 kPa abs. Find the energy head loss due to friction, and determine the thermal energy (N·m/N) that must have been added to or taken from the fluid between the two sections. The diameter of the pipe is constant.*

SI

Table A.5 for air: $R = 287\ \text{m}^2/(\text{s}^2\cdot\text{K})$

From Eq. 2.4 for isothermal flow: $\rho_1/\rho_2 = p_1/p_2 = 600/100 = 6$

But from continuity $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$, thus as A is constant, $V_2 = 6V_1 = 6(30) = 180$ m/s.

From Eq. 13.54: $q_H = (180^2 - 30^2)/2 = 15\,750$ N·m/kg of air

$Q_H = 15\,750/9.81 = 1606$ N·m/N of air ◀

For head loss: From Sec. 13.13, ignoring z terms:

$$\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) = -\left(\frac{\tau P}{\rho A}ds\right) = -\frac{gh_f}{L}ds = -\frac{g\,dh_f}{dL}ds = -g\,dh_f$$

Integrating: $g h_f = RT\ln(p_1/p_2) + (V_1^2 - V_2^2)/2 = 287(273+25)\ln 6 + (30^2 - 180^2)/2$

$= 153\,200 - 15\,750 = 137\,500$ N·m/kg

$h_f = 137\,500/9.81 = 14\,020$ N·m/N of air ◀

Note: The total energy at section (2) is greater than that at section (1) because external heat is added. Thus h_L represents a degradation of mechanical energy, but not a net loss of energy, because it is converted to another form of energy, namely internal heat.

Sec. 13.13: Isothermal Flow in a Constant-Area Duct -- *Exercises (2)*

13.13.1 *Air flows isothermally through a long horizontal pipe of uniform diameter. At a section where the pressure is 100 psia, the velocity is 120 ft/sec. Because of fluid friction the pressure at a distance point is 40 psia. (a) What is the increase in kinetic energy per slug of air? (b) What is the amount of thermal energy in Btu per slug of air that must be transferred in order to maintain the temperature constant? (c) Is this heat transferred to the air in the pipe or removed from it? (d) If the temperature of the air is 100°F and the diameter of the pipe is 3 in, find the total heat transferred in Btu per hour.*

BG

Table A.5 for air: $R = 1715\ \text{ft}^2/(\text{sec}^2\cdot{}^\circ\text{R})$

Sec. 13.13: pV = const for isothermal flow in a constant-area pipe,

$\therefore\ V_2 = p_1V_1/p_2 = 100(120)/40 = 300$ ft/sec ; $V_1^2/2 = 120^2/2 = 7200$ ft·lb/slug, $V_2^2/2 = 45{,}000$ ft·lb/slug

(*a*) Eq. 13.54: Increase in K.E. of air = 45,000 − 7200 = 37,800 ft·lb/slug ◄

(*b*) Thermal energy/slug transferred = $q_H = \Delta KE$ = 37,800/778 = 48.6 Btu/slug ◄

(*c*) KE increases, ∴ (Sec. 13.12) heat is transferred to the air in the pipe. ◄

(*d*) From Eq. 2.4: $\rho_1 = p_1/(RT_1) = (100 \times 144)/[1715(460 + 100)] = 0.014\,99\ \text{slug/ft}^3$

Eq. 13.5: $\dot{m} = \rho_1V_1A = 0.014\,99(120)(\pi/4)(3/12)^2 = 0.0883$ slug/sec

Heat transfer rate = (48.6 Btu/slug)(0.0833 slug/sec) = 4.29 Btu/sec = 15,450 Btu/hr ◄

13.13.2 *Air flows isothermally through a long horizontal pipe of uniform diameter. At a section where the pressure is 700 kPa abs, the velocity is 35 m/s. Because of fluid friction the pressure at a distance point is 280 kPa abs. (a) What is the increase in kinetic energy per kg of air? (b) What is the amount of thermal energy in J/kg air that must be transferred in order to maintain the temperature constant? (c) Is this heat transferred to the air in the pipe or removed from it? (d) If the temperature of the air is 40°C and the diameter of the pipe is 75 mm, find the total heat transferred in joules per hour.*

SI

Table A.5 for air: $R = 287\ \text{m}^2/(\text{s}^2\cdot\text{K})$. Sec. 13.13: pV = const for isothermal flow in a const-A pipe,

$\therefore\ V_2 = p_1V_1/p_2 = 700(35)/280 = 87.5$ m/s ; $V_1^2/2 = 35^2/2 = 613$ N·m/kg, $V_2^2/2 = 3830$ N·m/kg

(*a*) Eq. 13.54: Increase in K.E. of air = 3830 − 613 = 3220 N·m/kg ◄

(*b*) Thermal energy/kg transferred = $q_H = \Delta KE$ = 3220 J/kg ◄

(*c*) KE increases, ∴ (Sec. 13.12) heat is transferred to the air in the pipe. ◄

(*d*) From Eq. 2.4: $\rho_1 = p_1/(RT_1) = 700\,000/[287(273 + 40)] = 7.79\ \text{kg/m}^3$

Eq. 13.5: $\dot{m} = \rho_1V_1A = 7.79(35)(\pi/4)(7.5/100)^2 = 1.205$ kg/s

Heat transfer rate = (3220 J/kg)(1.205 kg/s) = 3780 J/s = 13.95×10^6 J/hr ◄

Sec. 13.13: Isothermal Flow in a Constant-Area Duct -- ***Problems*** *13.18–13.21*

13.18 *Refer to Sample Prob. 13.7. Neglecting the logarithm term in Eq. 13.59, find the pressure and velocities at sections 100, 300, and 800 ft downstream of the section where the pressure is 80 psia. Plot the pressure and velocity as a function of distance along the duct.*

Sample Prob. 13.7: Air flows isothermally (65 °F) at 3.0 slug/sec through a horizontal 10 in × 14 in duct. $R_h = 0.243$ *ft,* $f = 0.0083$, $\rho_1 = 0.012\,79$ *slug/ft*^{3}, $V_1 = 242$ *fps.*

BG

Table A.5 for air: $R = 1715$ ft^2/(sec$^2\cdot$°R). Substitute into Eq. 13.59 (neglecting the logarithm term):

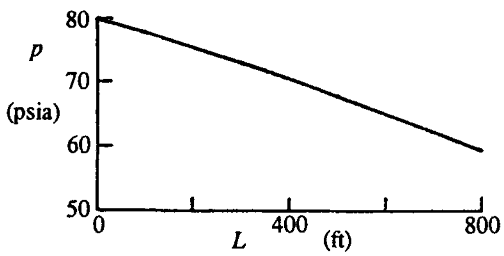
$$(80\times 144)^2 - p_2^2 = \frac{(3.0)^2 1715(460+65)}{(10\times 14/144)^2}\left[0.0083\frac{L}{4(0.243)}\right]; \quad p_2^2 = 1.327\times 10^8 - 73{,}200L$$

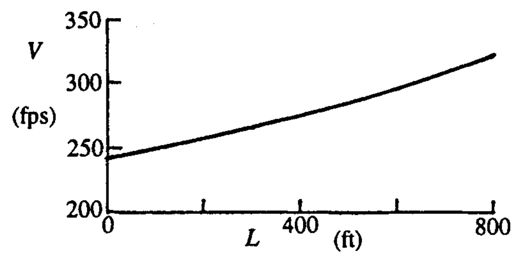
Isothermal, using Eq. 2.4: $p_1/\rho_1 = p_2/\rho_2, \;\therefore\; \rho_1/\rho_2 = p_1/p_2$

Continuity Eq. 13.5 with A = const: ρV = const, $\therefore\; V_2 = V_1(\rho_1/\rho_2) = V_1(p_1/p_2) = 242(80/p_2)$

Results:

L (ft)	p (psia)	V (ft/sec)
0	80.0	242
100	77.8	249
300	73.1	265
500	68.1	284
800	59.8	324





Note: We could achieve greater accuracy by considering the logarithmic term.

13.19 *Carbon dioxide flows isothermally at 100°F through a horizontal 6-in-diameter pipe (Fig. P13.39). At this temperature $\mu = 4.0 \times 10^{-7}$ lb·sec/ft². The pressure changes from 150.0 to 140.0 psig in a 100-ft length of pipe. Determine the mass flow rate if the atmospheric pressure is 14.5 psia and e for the pipe is 0.002 ft.*

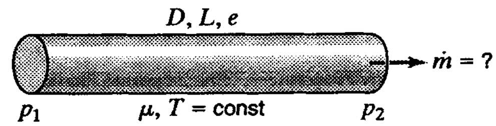


Figure P13.19

BG

Table A.5: $R = 1123$ ft²/(s²·°R); $A = (\pi/4)(6/12)^2 = 0.1963$ ft²

At section 1 (inlet), $p_1 = 150.0$ psig + 14.5 psi = 164.5 psia

From Eq. 2.4: $\rho_1 = (164.5 \times 144)/[1123(460+100)] = 0.0377$ slug/ft³;

$e/D = 0.002/0.5 = 0.004$; Assume $\mathbf{R} > 10^6$, Fig. 8.11 or Eq. 8.51 or 52: $f = 0.0284$

At section 2, $p_2 = 140.0$ psig + 14.5 psi = 154.5 psia.

Eq. 13.59: $(164.5 \times 144)^2 - (154.5 \times 144)^2 = \dfrac{\dot{m}^2 1123(460 + 100)}{0.1963^2}\left[0.0284\left(\dfrac{100}{0.5}\right) + 2\ln\left(\dfrac{164.5}{154.5}\right)\right]$

$5.61 \times 10^8 - 4.95 \times 10^8 = 1.632 \times 10^6 \dot{m}^2(5.70 + 0.1254) = 95.1 \times 10^6$

$\therefore\ \dot{m}^2 = (66.1 \times 10^6)/(94.7 \times 10^6) = 0.699$; $\dot{m} = 0.836$ slug/sec (based on the above assumption for **R**)

From Eq. 13.5: $V_1 = \dot{m}/(\rho_1 A) = 0.834/(0.0377 \times 0.1963) = 113.0$ ft/sec

Eq. 13.58: $\mathbf{R} = [0.5(113.0)0.0377]/(4.0 \times 10^{-7}) = 5.32 \times 10^6$

Hence the assumption for **R** was O.K. and $\dot{m} = 0.836$ slug/sec ◀

Alternative solutions: (1) Solve Eq. 13.59 and 8.51 simultaneously for f and $\dot{m}$ using Mathcad or similar mathematics software. (2) Eliminate f between these two equations, and solve for $\dot{m}$ using an equation solver on a programmable calculator.

13.20 *Carbon dioxide flows isothermally at 40°C through a horizontal 150-mm-diameter pipe (Fig. P13.19). At this temperature $\mu = 1.95 \times 10^{-5}$ N·s/m². The pressure changes from 1000 to 930 kPa gage in a 30-m length of pipe. Determine the mass flow rate if the atmospheric pressure is 100 kPa abs and e for the pipe is 0.60 mm.*

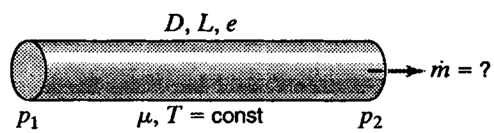


Figure P13.19

SI

Table A.5: $R = 188$ N·m/(kg·K) ; $A = (\pi/4)(15/100)^2 = 0.017\,67$ m²

At section 1 (inlet), $p_1 = 1000$ kN/m² gage + 100 kN/m² = 1100 kN/m² abs

From Eq. 2.4: $\rho_1 = 1\,100\,000/[188(273 + 40)] = 18.69$ kg/m³

$e/D = 0.002/0.5 = 0.004$; Assume $\mathbf{R} > 10^6$, Fig. 8.11 or Eq. 8.51 or 52: $f = 0.0284$

At section 2, $p_2 = 930$ kN/m² + 100 kN/m² = 1030 kN/m² abs.

Eq. 13.59: $(1\,100\,000)^2 - (1\,030\,000)^2 = \dfrac{\dot{m}^2 188(273 + 40)}{0.017\,67^2}\left(0.0284\dfrac{30}{0.15} + 2\ln\dfrac{1100}{1030}\right)$

$1.210 \times 10^{12} - 1.061 \times 10^{12} = 1.884 \times 10^8 \dot{m}^2(5.68 + 0.1315) = 1.095 \times 10^9 \dot{m}^2$

from which $\dot{m} = 11.67$ kg/s (based on the above assumption)

Eq. 13.5: $V_1 = \dot{m}/(\rho_1 A) = 11.67/(18.69 \times 0.017\,67) = 35.3$ m/s

Eq. 13.58: $\mathbf{R} = [0.15(35.3)18.69]/(1.95 \times 10^{-5}) = 5.08 \times 10^6$

Hence the assumption for **R** was O.K. and $\dot{m} = 11.67$ kg/s ◀

Alternative solutions: (1) Solve Eq. 13.59 and 8.51 simultaneously for f and $\dot{m}$ using Mathcad or similar mathematics software. (2) Eliminate f between these two equations, and solve for $\dot{m}$ using an equation solver on a programmable calculator.

13.21 *Methane gas is to be pumped through a 24-in-diameter welded-steel pipe connecting two compressor stations 25 miles apart. At the upstream station the pressure is not to exceed 60 psia, and at the downstream station it is to be at least 20 psia. Determine the maximum possible rate of flow (in cubic feet per day at 60°F and 1 atm). Assume isothermal flow at 60°F.*

BG

For methane: $R = 3100\ \text{ft}^2/(\text{sec}^2\cdot°\text{R})$ (Table A.5); $\mu = 2.40 \times 10^{-7}\ \text{lb}\cdot\text{sec/ft}^2$ (Fig. 2.3 at 60°F)

Absolute viscosity μ is independent of pressure over a wide range of pressures but kinematic viscosity v varies with pressure (see end of Sec. 2.11).

From Eqs. 2.11 and 2.4: $v_1 = \dfrac{\mu}{\rho_1} = \dfrac{\mu}{p_1/RT_1} = \dfrac{2.40 \times 10^{-7}}{(60 \times 144)/(3100 \times 520)} = 4.48 \times 10^{-5}\ \text{ft}^2/\text{sec}$

Table 8.1 for welded steel: $e = 0.000\,15$ ft; $e/D = 0.000\,15/2 = 0.000\,075$

As a first trial, assume $\mathbf{R} > 10^7$, in which case from Fig. 8.11 or Eq. 8.51 or 52, $f = 0.0116$

Eq. 13.59: $(60 \times 144)^2 - (20 \times 144)^2 = \dfrac{\dot{m}^2(3100)520}{\pi^2}\left[0.0116\dfrac{25(5280)}{2} + 2\ln\left(\dfrac{60}{20}\right)\right]$

$66{,}400{,}000 = \dot{m}^2 163{,}300(766 + 2.20) = \dot{m}^2 1.254 \times 10^8$; $\dot{m}^2 = 0.529$; $\dot{m} = 0.727$ slug/sec

Eq. 13.58: $\mathbf{R} = \dot{m}D/(\mu A) = (0.727 \times 2)/(2.40 \times 10^{-7} \times \pi) = 1.930 \times 10^6$

Thus, from Fig. 8.11: $f = 0.0124$; The initially-assumed f was too low.

Substituting $f = 0.0124$ into Eq. 13.59 gives $\dot{m} = 0.704$ slug/sec

From Eq. 2.4: $\rho_1 = p_1/RT_1 = (60 \times 144)/(3100 \times 520) = 0.005\,36\ \text{slug/ft}^3$

From Eq. 13.5: $Q_1 = \dot{m}/\rho_1 = 0.705/0.005\,36 = 131.3\ \text{ft}^3/\text{sec} = 11.34 \times 10^6\ \text{ft}^3/\text{day}$ at 60°F and 60 psia,

From Eqs. 2.4 and 13.5: $\dot{m} = \rho Q = pQ/RT$, i.e. here $pQ = \text{const}$

$\therefore$ at 60°F and 14.7 psia: $Q = Q_1(p_1/p) = 11.34 \times 10^6(60/14.7) = 46.3 \times 10^6\ \text{ft}^3/\text{day}$ ◄

<u>Alternative solutions</u>: (1) Solve Eq. 13.59 and 8.51 simultaneously for f and $\dot{m}$ using Mathcad or similar mathematics software. (2) Eliminate f between these two equations, and solve for $\dot{m}$ using an equation solver on a programmable calculator.

Sec. 13.14: Adiabatic Flow in a Constant-Area Duct -- *Problems* *13.22–13.24*

13.22 *Refer to Sample Prob. 13.9. Find the distance along the duct to (a) where $\rho_2 = 0.9\rho_1$; (b) where $\rho_2 = 0.7\rho_1$; (c) where subsonic adiabatic flow ends. Compute the corresponding values of p, V, T, and* **M** *and plot the first three as a function of distance along the duct.*

Sample Prob. 13.9: Air flows adiabatically at 3.0 slug/sec through a 10 in × 14 in rectangular duct (e = 0 on Fig. 8.11). p_1 = 80 psia, T_1 = 65°F = 525°R. R = 1715 ft·lb/(slug·°R), ρ_1 = 0.012 79 slug/ft³, $\mathbf{R}_1 = 7.93 \times 10^6$, f = 0.0083, C = 6,360,000 ft²/sec².

BG

Follow the procedure given in Sample Prob. 13.9, as follows:

(1) $\rho_2 = \rho_1(\rho_2/\rho_1)$

(2) Find p_2 from Eq. 13.63.

(3) Find $\int_1^2 \rho dp$ from Eq. 13.64.

(4) Find L from Eq. 13.65; check (from T_2) if **R** and f changes.

(5) $V_2 = \dot{m}/(\rho_2 A)$

(6) $T_2 = p_2/(\rho_2 R)$

In addition, for **M**, Eq. 7.10: $\mathbf{M} = V/\sqrt{E_v/\rho}$; Sec. 2.8: $E_v = kp$; combining these, $\mathbf{M} = V\sqrt{\rho/(1.4p)}$

For part (*c*), find by T and E the value of ρ_2 which yields $\mathbf{M}_2 = 1$

Results:

	ρ_2/ρ_1	ρ_2 (slug/ft³)	p_2 (psia)	$\int_1^2 \rho dp$	L (ft)	V_2 (ft/sec)	T_2 (°F)	$\mathbf{M}_2$
	1.0	0.012 79	80.0	0	0	242	65.0	0.215
(*a*)	0.9	0.011 51	71.8	−14.26	326	268	63.8	0.239
	0.8*	0.010 23	63.6	−27.1	613	302	62.2	0.270
(*b*)	0.7	0.008 95	55.4	−38.4	861	345	59.9	0.309
(*c*)	0.2341	0.002 99	15.74	−72.2	1454**	1031	−18.7	1.000
			▲			▲	▲	▲

* From Sample Prob. 13.9
** With $\mathbf{R} = 9.22 \times 10^6$, $f = 0.0082$

The required plots look like the adiabatic pressure, velocity, and temperature curves in Fig 13.12. ◄
Suggestion: Solve using a spreadsheet.

13.23 *Air flows adiabatically at 3.0 slug/sec in a 12-in-diameter horizontal pipe. At a certain section the pressure is 150 psia and the temperature 140°F. Determine the distance along the pipe to the section where $\rho_2 = 0.80\rho_1$. Assume e/D = 0.0004.*

BG

Table A.5 for air: $R = 1715$ ft²/(sec²·°R), $k = 1.40$. $A = (\pi/4)1^2 = 0.785$ ft².

From Eq. 2.4: $\rho_1 = p_1/RT_1 = 150(144)/[1715(460 + 140)] = 0.0210$ slug/ft³

From Eq. 13.5: $V_1 = \dot{m}/(\rho_1 A_1) = 3.0/(0.021 \times 0.785) = 182.0$ ft/sec

Table A.2 for air at 140°F: $\mu = 4.14 \times 10^{-7}$ lb·sec/ft².

Eq. 13.58: $\mathbf{R}_1 = DV_1\rho_1/\mu_1 = 1(182.0)0.0210/(4.14 \times 10^{-7}) = 9.23 \times 10^6$; $e/D = 0.0004$ (given)

At this **R** and *e/D* the pipe behaves as wholly rough and $f = 0.0160$ (Fig. 8.11 or Eq. 8.51 or 52).

Eq. 13.62: $$C = \frac{\dot{m}^2}{\rho_1^2 A^2} + \frac{2k}{k-1}\frac{p_1}{\rho_1} = \frac{3.0^2}{(0.0210^2)0.785^2} + \frac{2(1.4)}{0.4}\frac{150(144)}{0.0210} = 7.24 \times 10^6$$

Eq. 13.63 with $\rho_2 = 0.8\rho_1$:

$$p_2 = \frac{0.4}{2.8}\left(7.24 \times 10^6(0.8)0.0210 - \frac{3.0^2}{(0.785^2)(0.8 \times 0.0210)}\right) = 17{,}240 \text{ lb/ft}^2 \text{ abs} = 119.7 \text{ psia}$$

Eq. 13.64: $\int_1^2 \rho dp = -82.5$; putting this into Eq. 13.65 yields $L = 679$ ft ◄

13.24 *Air flows adiabatically at 50 kg/s in a 300-mm-diameter horizontal pipe. At a certain section the pressure is 1000 kPa abs and the temperature 60°C. Determine the distance along the pipe to the section where $\rho_2 = 0.80\rho_1$. Assume e/D = 0.0004.*

SI

Table A.5 for air: $R = 287$ m²/(s²·K), $k = 1.40$. $A = (\pi/4)(30/100)^2 = 0.0707$ m².

From Eq. 2.4: $\rho_1 = p_1/RT_1 = (1\,000\,000)/[287(273 + 60)] = 10.46$ kg/m³

From Eq. 13.5: $V_1 = \dot{m}/(\rho_1 A_1) = 50/(10.46 \times 0.0707) = 67.6$ m/s

Table A.2 for air at 60°C: $\mu = 2.00 \times 10^{-5}$ N·s/m²

Eq. 13.58: $\mathbf{R}_1 = DV_1\rho_1/\mu_1 = 0.3(67.6)10.46/(2.00 \times 10^{-5}) = 10.61 \times 10^6$

At this **R** and $e/D = 0.0004$ (given) the pipe behaves as wholly rough

and $f = 0.0160$ (Fig. 8.11 or Eq. 8.51 or 52).

Eq. 13.62: $$C = \frac{\dot{m}^2}{\rho_1^2 A^2} + \frac{2k}{k-1}\frac{p_1}{\rho_1} = \frac{50^2}{10.46^2(0.0707)^2} + \frac{2(1.4)}{0.4}\frac{1\,000\,000}{10.46} = 674\,000 \text{ m}^2/\text{s}^2$$

Eq. 13.63 with $\rho_2 = 0.8\rho_1$:

$$p_2 = \frac{0.4}{2.8}\left(674\,000(0.8)10.46 - \frac{50^2}{0.0707^2(0.8 \times 10.46)}\right) = 797\,000 \text{ Pa abs}$$

Eq. 13.64: $\int_1^2 \rho dp = -1.912 \times 10^6$; putting this in Eq. 13.65 yields $L = 134.9$ m ◄

Chapter 14

Ideal Flow Mathematics

PROBLEM SELECTION GUIDE

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features	
14.1	*Differential Equation of Continuity*							
	X[1] 14.1.1	N	Easy	Medium	6		Differentiation	
	14.1.2	N	Easy	Medium	7		Differentiation	
	P 14.1	N	Easy	Short	1			
	14.2	N	Easy	V Short	1			
	14.3	N	Easy	Short	1		Derivations	
14.2	*Irrotational Flow*							
	X 14.2.1	N	Easy	Medium	6		Differentiation	
	14.2.2	N	Easy	Medium	7		Differentiation	
14.3	*Circulation and Vorticity*							
	X 14.3.1	N	Easy	Medium	6	14.2.1	Differentiation	
	14.3.2	N	Easy	Medium	7	14.2.2	Differentiation	
14.4	*The Stream Function*							
	X 14.4.1	N	Medium	Short	1		Differentiation, integration	
	14.4.2	N	Medium	Medium	6		Differentiation, integration	
	14.4.3	N	Medium	Medium	7		Differentiation, integration	
	P 14.4	N	Easy	Short	3		Differentiation	Sketch
	14.5	N	Easy	Short	1		Differentiation	Sketch
	14.6	N	Medium	Medium	1		Differentiation	Plot
	14.7	N	Medium	Medium	2		Differentiation, integration.	Sketch
	14.8	N	Medium	Short	1		Differentiation	Sketch
14.5	*Basic Flow Fields*							
	X 14.5.1	SI	Easy	Short	2			
	14.5.2	SI	Medium	Medium	2		Differentiation (to check).	
	14.5.3	SI	Easy	Short	1		Differentiation	
	14.5.4	BG	Medium	Medium	1		Differentiation	
	P 14.9	N	Medium	Long	4		Differentiation	Plot
	14.10	SI	Medium	Long	1		Interpolation	Plot
	14.11	BG	Medium	Long	1		Differentiation	Plot
	14.12	N	Hard	Long	1		Differentiation	Sketch
	14.13	N	Hard	Long	5		Different'n, integration.	Plot & sketch

/cont...

[1] For all Exercises (identified by "X"), answers are given in Appendix F of the textbook.
† Answers are sensitive to the accuracy of sketched flowfields and to scaling from them.
X = Exercise, P = (end-of-chapter) Problem, S = Sample Problem.

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features	
14.6	***Velocity Potential***							
	X 14.6.1	N	Easy	Short	1		Differentiation	
	14.6.2	N	Medium	Short	1		Differentiation, integration	
	14.6.3	N	Medium	Medium	2	14.6.4	Differentiation, integration	
	14.6.4	N	Medium	Medium	2	14.6.3	Differentiation, integration	
14.7	***Orthogonality of Streamlines and Equipotential Lines***							
	X 14.7.1	N	Medium	Medium	6		Differentiation, integration	
	14.7.2	N	Medium	Medium	7		Differentiation, integration	
	P 14.14	BG	Hard	Medium	3	14.15	Differentiation	
	14.15	SI	Hard	Medium	3	14.14	Differentiation	
	14.16	BG	Medium	Short	1	14.17	Differentiation	
	14.17	SI	Medium	Short	1	14.16	Differentiation	
	14.18	BG	Medium	Medium	2	14.19	† Differentiation, integration	Sketch
	14.19	SI	Medium	Medium	2	14.18	† Differentiation, integration	Sketch
	14.20	N	Medium	Medium	1		Differentiation	Plot
	14.21	BG	Medium	Medium	1		Differentiation	Plot
	14.22	BG	Hard	Long	1		†	Sketch
	14.23	BG	Hard	Long	1	14.24	†	Sketch
	14.24	SI	Hard	Long	1	14.23	†	Sketch
	14.25	BG	Hard	Medium	1		Differentiation	Sketch
	14.26	N	Hard	Long	2		Proof, differentiation	Plot
	14.27	BG	Hard	Long	4			Plot
14.8	***Flow Through Porous Media***							
	X 14.8.1	BG	Easy	Short	1	14.8.2		
	14.8.2	SI	Easy	Short	1	14.8.1		
	14.8.3	BG	Easy	Short	1	14.8.4		
	14.8.4	SI	Easy	Short	1	14.8.3		
	P 14.28	BG	Medium	Medium	2	14.29	Optional integration	
	14.29	SI	Medium	Medium	2	14.28	Optional integration	

Chapter 14
IDEAL FLOW MATHEMATICS

Sec. 14.1: Differential Equation of Continuity -- *Exercises* (2)

14.1.1 *Which of the following incompressible flows satisfy continuity?*

(a) $u = 2$ (d) $u = 2y,\ v = 3x$

(b) $u = 2,\ v = 3$ (e) $u = 2y,\ v = -3x$

(c) $u = 2 + 3x,\ v = 4$ (f) $u = 3xy,\ v = 1.5x^2$

N

(a) $u = 2,\ v = 0\ ;\quad \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0 + 0 = 0,$ so continuity is satisfied.

(b) $u = 2,\ v = 3\ ;\quad \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0 + 0 = 0,$ so continuity is satisfied.

(c) $u = 2 + 3x,\ v = 4\ ;\quad \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 3 + 0 \neq 0,$ so continuity is not satisfied.

(d) $u = 2y,\ v = 3x\ ;\quad \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0 + 0 = 0,$ so continuity is satisfied.

(e) $u = 2y,\ v = -3x\ ;\quad \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0 + 0 = 0,$ so continuity is satisfied.

(f) $u = 3xy,\ v = 1.5x^2\ ;\quad \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 3y + 0 \neq 0,$ so continuity is not satisfied.

Flows (*a*), (*b*), (*d*), and (*e*) satisfy continuity ◄

14.1.2 *Which of the following incompressible flows satisfy continuity?*

(a) $u = 3y,\ v = 0$ (e) $u = 4xy + x^2,\ v = -2xy - 2y^2$

(b) $u = 3x,\ v = 3y$ (f) $u = -2xy + 2x^2,\ v = 4xy - y^2$

(c) $u = 3x,\ v = -3y$ (g) $u = -2xy - 2x^2 + 2y^2,\ v = 4xy - x^2 + y^2$

(d) $u = 4 + 2x,\ v = -6 - 2y$

N

(a) $u = 3y,\ v = 0\ ;\quad \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0 + 0 = 0,$ so continuity is satisfied.

(b) $u = 3x,\ v = 3y\ ;\quad \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 3 + 3 \neq 0,$ so continuity is not satisfied.

(c) $u = 3x,\ v = -3y\ ;\quad \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 3 - 3 = 0,$ so continuity is satisfied.

(d) $u = 4 + 2x,\ v = -6 - 2y\ ;\quad \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 2 - 2 = 0,$ so continuity is satisfied.

(e) $u = 4xy + x^2,\ v = -2xy - 2y^2\ ;\quad \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = (4y + 2x) + (-2x - 4y) = 0,$
so continuity is satisfied.

(f) $u = -2xy + 2x^2,\ v = 4xy - y^2\ ;\quad \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = (-2y + 4x) + (4x - 2y)$
$= 8x - 4y \neq 0,$ so continuity is not satisfied

(g) $u = -2xy - 2x^2 + 2y^2,\ v = 4xy - x^2 + y^2\ ;\quad \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = (-2y - 4x) + (4x + 2y) = 0,$
so continuity is satisfied

Flows (*a*), (*c*), (*d*), (*e*), and (*g*) satisfy continuity ◄

Sec. 14.1: Differential Equation of Continuity -- *Problems 14.1–14.3*

14.1 *Given a flow defined by $u = 3 + 2x$. If this flow satisfies continuity, what can be said about the density of the fluid?*

N

The flow satisfies continuity, so Eq. 14.1 holds. Flow in the y and z directions = 0, doesn't change with t,

so $\dfrac{\partial}{\partial y}(\rho u) = \dfrac{\partial}{\partial z}(\rho u) = \dfrac{\partial \rho}{\partial t} = 0$ and Eq. 14.1 becomes $\dfrac{\partial}{\partial x}(\rho u) = 0$, i.e., ρu = constant

i.e. $\rho = \dfrac{\text{constant}}{u} = \dfrac{\text{constant}}{3 + 2x}$; $\therefore$ the density ρ must decrease as x increases ◄

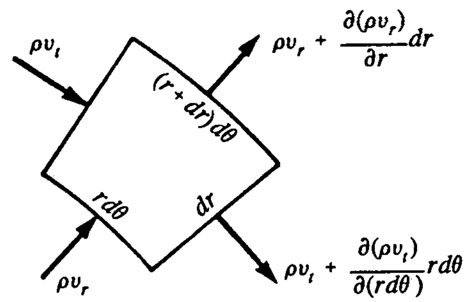
14.2 *Why are Eqs. (14.2) and (14.3) applicable to real fluids as well as ideal fluids?*

N

No mention is made of friction in their derivation, so the equations do not depend on the presence or absence of friction.

14.3 *Derive Eqs. (14.4) and (14.5)*

N



Flow in: $\rho v_r r\, d\theta + \rho v_t\, dr$

Flow out: $\left[\rho v_r + \dfrac{\partial(\rho v_r)}{\partial r}dr\right](r + dr)d\theta + \left[\rho v_t + \dfrac{\partial(\rho v_t)}{\partial(r\, d\theta)}r\, d\theta\right]dr$

Continuity: Flow in − Flow out = 0

Substituting, collecting terms, and simplifying:

$$-\rho v_r\, dr\, d\theta - \frac{\partial(\rho v_r)}{\partial r}dr\, r\, d\theta - \frac{\partial(\rho v_t)}{\partial(r\, d\theta)}r\, d\theta\, dr = 0$$

$$\frac{\rho v_r}{r} + \frac{\partial(\rho v_r)}{\partial r} + \frac{\partial(\rho v_t)}{\partial(r\, d\theta)} = 0 \quad \text{(14.4) for steady compressible flow}$$

And if ρ = constant, dividing through by ρ yields

$$\frac{v_r}{r} + \frac{\partial v_r}{\partial r} + \frac{\partial v_t}{\partial(r\, d\theta)} = 0 \quad \text{(14.5) for steady incompressible flow}$$

Sec. 14.2: Rotational and Irrotational Flow -- *Exercises (2)*

14.2.1 *Which of the flows of Exer. 14.1.1 are irrotational?*

N

(*a*) $u = 2,\ v = 0$; $\dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 0 - 0 = 0$, irrotational

(*b*) $u = 2,\ v = 3$; $\dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 0 - 0 = 0$, irrotational

(*c*) $u = 2 + 3x,\ v = 4$; $\dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 0 - 0 = 0$, irrotational

(*d*) $u = 2y,\ v = 3x$; $\dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 3 - 2 \neq 0$, rotational

(*e*) $u = 2y,\ v = -3x$; $\dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = -3 - 2 \neq 0$, rotational

(*f*) $u = 3xy,\ v = 1.5x^2$; $\dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 3x - 3x = 0$, irrotational

Flows (*a*), (*b*), (*c*), and (*f*) are irrotational ◄

14.2.2 N *Which of the flows of Exer. 14.1.2 are irrotational?*

(*a*) $u = 3y, \quad v = 0\,; \quad \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 0 - 3 \neq 0,$ rotational

(*b*) $u = 3x, \quad v = 3y\,; \quad \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 0 - 0 = 0,$ irrotational

(*c*) $u = 3x, \quad v = -3y\,; \quad \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 0 - 0 = 0,$ irrotational

(*d*) $u = 4 + 2x, \quad v = -6 - 2y\,; \quad \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 0 - 0 = 0,$ irrotational

(*e*) $u = 4xy + x^2, \quad v = -2xy - 2y^2\,; \quad \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = -2y - 4x \neq 0,$ rotational

(*f*) $u = -2xy + 2x^2, \quad v = 4xy - y^2\,; \quad \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 4y + 2x \neq 0,$ rotational

(*g*) $u = -2xy - 2x^2 + 2y^2, \quad v = 4xy - x^2 + y^2;$

$\dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = (4y - 2x) - (-2x + 4y) = 0,$ irrotational

Flows (*b*), (*c*), (*d*), and (*g*) are irrotational ◄

Sec. 14.3: Circulation and Vorticity -- *Exercises (2)*

14.3.1 N *Find the vorticity of each of the flows in Exer. 14.1.1.*

(*a*) $u = 2, \quad v = 0\,; \quad \xi = \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 0 - 0 = 0$ ◄

(*b*) $u = 2, \quad v = 3\,; \quad \xi = \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 0 - 0 = 0$ ◄

(*c*) $u = 2 + 3x, \quad v = 4\,; \quad \xi = \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 0 - 0 = 0$ ◄

(*d*) $u = 2y, \quad v = 3x\,; \quad \xi = \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 3 - 2 = 1$ ◄

(*e*) $u = 2y, \quad v = -3x\,; \quad \xi = \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = -3 - 2 = -5$ ◄

(*f*) $u = 3xy, \quad v = 1.5x^2\,; \quad \xi = \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 3x - 3x = 0$ ◄

14.3.2 N *Find the vorticity of each of the flows in Exer. 14.1.2.*

(*a*) $u = 3y, \quad v = 0\,; \quad \xi = \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 0 - 3 = -3$ ◄

(*b*) $u = 3x, \quad v = 3y\,; \quad \xi = \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 0 - 0 = 0$ ◄

(*c*) $u = 3x, \quad v = -3y\,; \quad \xi = \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 0 - 0 = 0$ ◄

(*d*) $u = 4 + 2x, \quad v = -6 - 2y\,; \quad \xi = \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 0 - 0 = 0$ ◄

(*e*) $u = 4xy + x^2, \quad v = -2xy - 2y^2\,; \quad \xi = \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = -2y - 4x$ ◄

(*f*) $u = -2xy + 2x^2, \quad v = 4xy - y^2\,; \quad \xi = \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 4y + 2x$ ◄

(*g*) $u = -2xy - 2x^2 + 2y^2, \quad v = 4xy - x^2 + y^2\,; \quad \xi = \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = (4y - 2x) - (-2x + 4y) = 0$ ◄

Sec. 14.4: The Stream Function -- *Exercises (3)*

14.4.1 N *The flow of an incompressible fluid is defined by u = 2, v = 8x. Does a stream function exist for this flow? If so, determine the expression for the stream function.*

$u = \partial\psi/\partial y = 2$; integrating: $\psi = 2y + f(x)$

$-v = \partial\psi/\partial x = -8x$; integrating: $\psi = -4x^2 + f(y)$

Comparing: $f(x) = -4x^2$ and $f(y) = 2y$

∴ A stream function <u>does</u> exist, it is $\psi = -4x^2 + 2y$ ◄

14.4.2 N *For each of the flows of Exer. 14.1.1, write an expression for the stream function if one exists.*

Sec. 14.4: Stream functions exist only for those flows that satisfy continuity.

Per Solution 14.1.1, continuity Eq. 14.3, $\partial u/\partial x + \partial v/\partial y = 0$, is satisfied for flows (*a*), (*b*), (*d*) and (*e*).

Flows (*c*) and (*f*) do not satisfy continuity and so do not have a stream function.

By integrating Eq. 14.14 we get:

(*a*) $\psi = 2y$ ◄ (*d*) $\psi = -1.5x^2 + y^2$ ◄

(*b*) $\psi = 2y - 3x$ ◄ (*e*) $\psi = 1.5x^2 + y^2$ ◄

14.4.3 N *For each of the flows of Exer. 14.1.2, write an expression for the stream function if one exists.*

Sec. 14.4: Stream functions exist only for those flows that satisfy continuity.

Per Solution 14.1.2, continuity Eq. 14.3, $\partial u/\partial x + \partial v/\partial y = 0$, is satisfied for flows (*a*), (*c*), (*d*), (*e*) and (*g*).

Flows (*b*) and (*f*) do not satisfy continuity and so do not have a stream function.

(*a*) $u = \partial\psi/\partial y = 3y$. Integrating: $\psi = (3/2)y^2 + f(x)$

$-v = \partial\psi/\partial x = 0$. Integrating: $\psi = f(y)$

Comparing: $f(x) = 0, f(y) = (3/2)y^2$. ∴ $\psi = (3/2)y^2$ ◄

(*c*) $u = \partial\psi/\partial y = 3x$. Integrating: $\psi = 3xy + f(x)$

$-v = \partial\psi/\partial x = 3y$. Integrating: $\psi = 3xy + f(y)$

Comparing: $f(x) = f(y) = 0$. ∴ $\psi = 3xy$ ◄

(*d*) $u = \partial\psi/\partial y = 4 + 2x$. Integrating: $\psi = 4y + 2xy + f(x)$

$-v = \partial\psi/\partial x = 6 + 2y$. Integrating: $\psi = 6x + 2xy + f(y)$

Comparing: $f(x) = 6x$, $f(y) = 4y$. ∴ $\psi = 6x + 2xy + 4y$ ◄

(*e*) $u = \dfrac{\partial\psi}{\partial y} = 4xy + x^2$. Integrating: $\psi = 2xy^2 + x^2y + f(x)$

$-v = \dfrac{\partial\psi}{\partial x} = 2xy + 2y^2$. Integrating: $\psi = x^2y + 2xy^2 + f(y)$

Comparing: $f(x) = f(y) = 0$. ∴ $\psi = x^2y + 2xy^2$ ◄

(*g*) $u = \dfrac{\partial\psi}{\partial y} = -2xy - 2x^2 + 2y^2$. Integrating: $\psi = -xy^2 - 2x^2y + (2/3)y^3 + f(x)$

$-v = \dfrac{\partial\psi}{\partial x} = -4xy + x^2 - y^2$. Integrating: $\psi = -2x^2y + (1/3)x^3 - xy^2 + f(y)$

Comparing: $f(x) = (1/3)x^3$, $f(y) = (2/3)y^3$. ∴ $\psi = (1/3)x^3 - 2x^2y - xy^2 + (2/3)y^3$ ◄

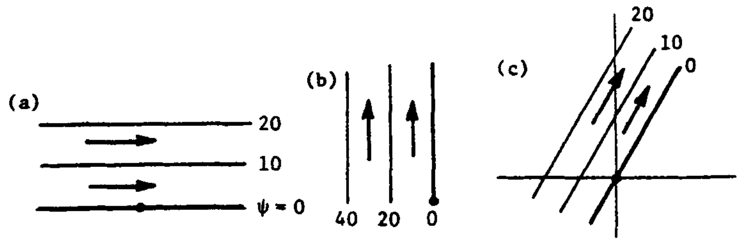
Sec. 14.4: The Stream Function -- ***Problems** 14.4–14.8*

14.4 *Sketch streamlines (ψ = 0, 1, 2, 3) for the following flow fields, note the values of u and v, and verify that continuity is satisfied in all cases. (a) $\psi = 10y$, (b) $\psi = -20x$, (c) $\psi = 10y - 20x$.*

N

Eq. 14.16: (*a*) $u = \partial\psi/\partial y = 10$, $v = 0$; (*b*) $u = 0$, $v = \partial\psi/\partial y = 20$; (*c*) $u = 10$, $v = 20$ ◀

In all cases $(\partial u/\partial x) + (\partial v/\partial y) = 0$, thus continuity (Eq. 14.3) is satisfied ◀



◀◀

14.5 *A flow field is described by the equation $\psi = 1.2xy$. Sketch the streamlines in one quadrant for ψ = 0, 1, 2, 3, 4.*

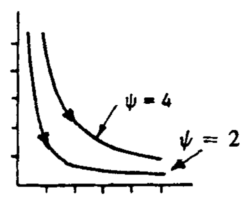
N

$\psi = 1.2xy$

Eq. 14.16: $u = \partial\psi/\partial y = 1.2x$

$v = -\partial\psi/\partial x = -1.2y$

Thus:



◀

14.6 *Plot the streamlines in the upper right-hand quadrant for the flow defined by $\psi = 1.5x^2 + y^2$ and determine the value of the velocity at x = 4, y = 2.*

N

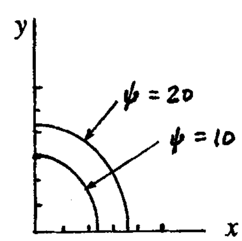
Eq. 14.16: $u = \partial\psi/\partial y = 2y$, $v = -\partial\psi/\partial x = -3x$

At (4,2): $u = 4$, $v = -12$

Velocity = $\sqrt{4^2 + 12^2} = 12.65$ ◀

To plot streamlines, rewrite as $y = \pm\sqrt{\psi - 1.5x^2}$.

Assume ψ, compute y's for different x's



◀

14.7 *The components of the velocities of a certain flow system are*

$$u = -\frac{Q}{2\pi}\left(\frac{x}{x^2+y^2}\right) + By + C \quad \text{and} \quad v = -A\left(\frac{y}{x^2+y^2}\right) + Dx + E$$

(a) Calculate a value of A consistent with continuous flow. (b) Sketch the streamlines for this flow system, assuming B = C = D = E = 0.

N

(*a*) To satisfy continuity (Eq. 14.3 for 2-D flow): $\partial u/\partial x + \partial v/\partial y = 0$

$$\frac{\partial u}{\partial x} = -\frac{Q}{2\pi}\frac{(x^2+y^2) - x(2x)}{(x^2+y^2)^2} = -\frac{Q}{2\pi}\frac{(y^2-x^2)}{(x^2+y^2)^2}$$

$$\frac{\partial v}{\partial y} = -A\frac{(x^2+y^2) - y(2y)}{(x^2+y^2)^2} = -A\frac{(x^2-y^2)}{(x^2+y^2)^2}$$

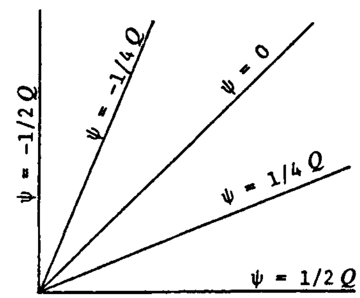
Substituting, $-Q/2\pi(y^2 - x^2) - A(x^2 - y^2) = 0$

thus $A = Q/2\pi$ ◄

(*b*) $u = -\dfrac{Q}{2\pi}\dfrac{x}{(x^2+y^2)}$; $v = -\dfrac{Q}{2\pi}\dfrac{y}{(x^2+y^2)}$

Eq. 14.14: $d\psi = -v\,dx + u\,dy = \dfrac{Q}{2\pi}\dfrac{y\,dx}{(x^2+y^2)} - \dfrac{Q}{2\pi}\dfrac{x\,dy}{(x^2+y^2)}$

Integrating, $\psi = \dfrac{Q}{2\pi}\tan^{-1}\left(\dfrac{x}{y}\right) - \tan^{-1}\left(\dfrac{y}{x}\right)$



◄

14.8 *A flow field is described by $\psi = x^2 - y$. Sketch the streamlines for ψ = 0, 1, and 2. Derive an expression for the velocity at any point in the flow field and determine the vorticity of the flow.*

N

From the equation for ψ, the flow field is a family of parabolas symmetric about the *y*-axis. The streamline $\psi = 0$ passes through the origin. ◄

From Eq. 14.16: $u = \partial\psi/\partial y = -1$, $v = -\partial\psi/\partial x = -2x$

Thus $V = \sqrt{u^2 + v^2} = \sqrt{1 + 4x^2}$ ◄

Eq. 14.12: Vorticity $\xi = (\partial v/\partial x) - \partial u/\partial y) = -2$ ◄ (Since $\xi \neq 0$, the flow is rotational)

Sec. 14.5: Basic Flow Fields -- *Exercises (4)*

14.5.1 *A source discharging 25 m³/s per m is located at the origin and a uniform flow with a velocity of 5 m/s from left to right is superimposed on the source flow. Determine the stream function of the flow (a) in polar and (b) in rectangular coordinates.*

SI

Per Section 14.5: $\psi_1 = 25\dfrac{\theta \text{ rad}}{2\pi} = \dfrac{25}{2\pi}\arctan\left(\dfrac{y}{x}\right)$; $\psi_2 = 5y = 5r\sin\theta$

(*a*) $\psi_{polar} = \dfrac{12.5\,\theta}{\pi} + 5r\sin\theta$ ◄ (*b*) $\psi_{rect} = \dfrac{12.5}{\pi}\arctan\left(\dfrac{y}{x}\right) + 5y$ ◄

14.5.2 *For the flow of Exer. 14.5.1, (a) find the location of the stagnation points and (b) find the velocity at x = 3 m, y = 4 m.*

Exer. 14.5.1: A source (25 m³/s per m) discharging at the origin is superimposed on a uniform flow (V = 5 m/s) from left to right.

SI

(*a*) Stagnation point is to the left of origin where source velocity = 5 m/s

i.e., where $V = 25/(2\pi r) = 5$; $r = 25/(10\pi) = 0.796$ m

Stagnation point is at $x = -0.796$ m, $y = 0$ (or $\theta = 180°$, $r = 0.796$ m) ◄

(*b*) At the point (3 m, 4 m):

From rectilinear flow: $u_1 = 5$ m/s, $v_1 = 0$

From source flow: $v_r = \dfrac{Q}{2\pi r} = \dfrac{25}{2\pi(5)} = 0.796$ m/s ;

$$u_2 = (3/5)v_r = (3/5)(0.796) = 0.477 \text{ m/s}$$

and $v_2 = (4/5)v_r = (4/5)(0.796) = 0.637$ m/s

$u = u_1 + u_2 = 5.48$ m/s ; $V = \sqrt{u^2 + v^2} = \sqrt{5.48^2 + 0.637^2} = 5.51$ m/s ◄

Check: (using Eq. 14.16 and $x = 3$ m, $y = 4$ m):

$$u = \frac{\partial\psi}{\partial y} = 5 + \frac{12.5}{\pi}\left(\frac{1}{1 + y^2/x^2}\right)\left(\frac{1}{x}\right) = 5.48 \text{ m/s}$$

$$v = -\frac{\partial\psi}{\partial x} = -\frac{12.5}{\pi}\left(\frac{1}{1 + y^2/x^2}\right)\left(-\frac{y}{x^2}\right) = 0.637 \text{ m/s}$$

14.5.3 *Refer to Exer. 14.5.1. Use Bernoulli's theorem to find the difference in pressure head between point A (−14 m, 0) and point B (0, 2 m).*

Exer. 14.5.1: A source (25 m³/s per m) discharging at the origin is superimposed on a uniform flow (V = 5 m/s) from left to right.

SI

Differentiate ψ_{rect} (Exer. 14.5.1) per Eq. 14.16:

$$u = \frac{\partial\psi}{\partial y} = \frac{12.5}{\pi}\frac{x}{(x^2 + y^2)} + 5 \;; \qquad v = -\frac{\partial\psi}{\partial x} = \frac{12.5}{\pi}\frac{y}{(x^2 + y^2)}$$

At point A (−14 m, 0): $u = 4.72$ m/s, $v = 0$; $V_A = 4.72$ m/s

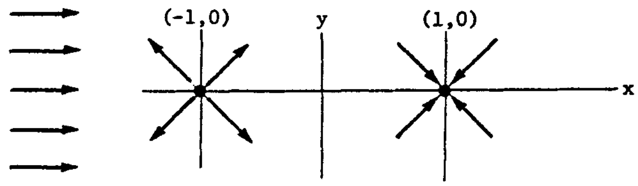
At point B (0, 2 m): $u = 5$ m/s, $v = 1.989$ m/s ; $V_B = \sqrt{5^2 + 1.989^2} = 5.38$ m/s

From Bernoulli's theorem (Eq. 4.10):

$$p_A/\gamma - p_B/\gamma = V_B^2/(2g) - V_A^2/(2g) = \frac{5.38^2 - 4.72^2}{2(9.81)} = 0.342 \text{ m}$$ ◄

14.5.4 *A source discharging 13 cfs/ft is at (−1, 0) and a sink taking in 13 cfs/ft is at (+1, 0). If a uniform flow with velocity 8 fps from left to right is superimposed on the source-sink combination, what is the length of the resulting closed body contour?*

BG



Differentiate Eq. 14.18 with respect to y (per Eq. 14.16):

$$u = \frac{\partial \psi}{\partial y} = U + \frac{q}{2\pi}\left[\frac{x + a}{(x + a)^2 + y^2} - \frac{x - a}{(x - a)^2 + y^2}\right]$$

Along the x-axis ($y = 0$), with $U = 8$ fps and $q = 13$ cfs/ft,

$$u = 8 + \frac{13}{2\pi(x + 1)} - \frac{13}{2\pi(x - 1)} = 0 \quad \text{at ends of body.}$$

Solving for x: $x = \pm 1.232$ ft. ∴ Length of body $= 2x = 2.46$ ft ◀

Sec. 14.5: Basic Flow Fields – ***Problems 14.9–14.13***

14.9 *A source of strength 8π is located at (2, 0). Another source of strength 16π is located at (−3, 0). For the combined flow field produced by these two sources: (a) find the location of the stagnation point; (b) plot the $\psi = 0$, $\psi = 4\pi$, $\psi = 8\pi$ lines; (c) find the values of ψ at (0, 2) and at (3, −1); (d) find the velocity at (−2, 5).*

N

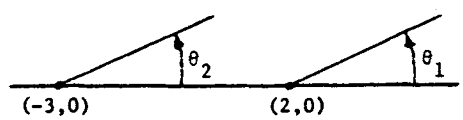
(*a*) In general (Sec. 14.5), $\psi = \dfrac{q_1\theta_1}{2\pi} + \dfrac{q_2\theta_2}{2\pi} = \dfrac{8\pi}{2\pi}\arctan\dfrac{y}{x-2} + \dfrac{16\pi}{2\pi}\arctan\dfrac{y}{x+3}$

Per Eq. 14.16 (differentiating and simplifying): $u = 4\dfrac{(x-2)}{(x-2)^2+y^2} + 8\dfrac{(x+3)}{(x+3)^2+y^2}$

Due to symmetry, stagnation point is on x-axis ($y = 0$). If $y = 0$, $u = \dfrac{4}{x-2} + \dfrac{8}{x+3}$

At stagnation point, $u = 0 = \dfrac{4}{x-2} + \dfrac{8}{x+3}$; thus $x = 1/3$ ◄

(*b*) ψ's at various points may be found by substituting coordinates of the point into the general expression for ψ (above). Lines of equal u may then be plotted by interpolation. ◄



(*c*) At (0, 2): $\psi = 4\arctan\dfrac{2}{-2} + 8\arctan\dfrac{2}{3} = 4(135°/57.3°) + 8(33.7°/57.3°) = 14.13$ ◄

At (3, −1): $\psi = 4\arctan\dfrac{-1}{1} + 8\arctan\dfrac{-1}{6} = 4(315°/57.3°) + 8(350.5°/57.3°) = 70.9$ ◄

(*d*) Differentiating ψ per Eq. 14.16 and simplifying: $u = \dfrac{\partial\psi}{\partial y} = 4\left[\dfrac{(x-2)}{(x-2)^2+y^2}\right] + 8\left[\dfrac{(x+3)}{(x+3)^2+y^2}\right]$

At (−2, 5): $u = 4\left[\dfrac{-4}{16+25}\right] + 8\left[\dfrac{1}{1+25}\right] = -0.0826$

Similarly, $v = -\dfrac{\partial\psi}{\partial x} = 4\left[\dfrac{y}{(x-2)^2+y^2}\right] + 8\left[\dfrac{y}{(x+3)^2+y^2}\right]$

At (−2, 5): $v = 4\left[\dfrac{5}{16+25}\right] + 8\left[\dfrac{5}{1+25}\right] = 2.03$

Adding vectorially, $V = \sqrt{u^2+v^2} = 2.03$, upwards at an angle of 2° 18′ from the vertical ◄

14.10 *Using the method described in Sample Prob. 14.4, plot the boundary of the body and a set of streamlines for a steady two-dimensional flow past a body such as that of Fig. 4.12, for b = 15 m using a scale of 1 cm = 2 m.*

SI

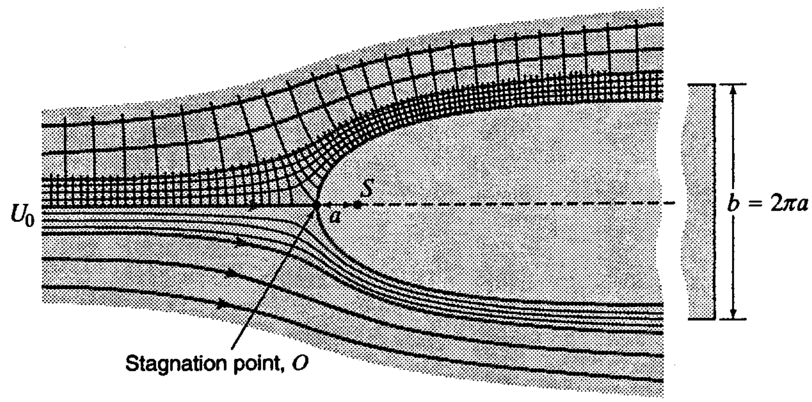


Figure 4.12

For Fig. 4.12,

$$\psi = Uy + \frac{q}{2\pi}\tan^{-1}\left(\frac{y}{x-a}\right)$$

Placing the source at the origin ($a = 0$), with $q = bU = 15U$

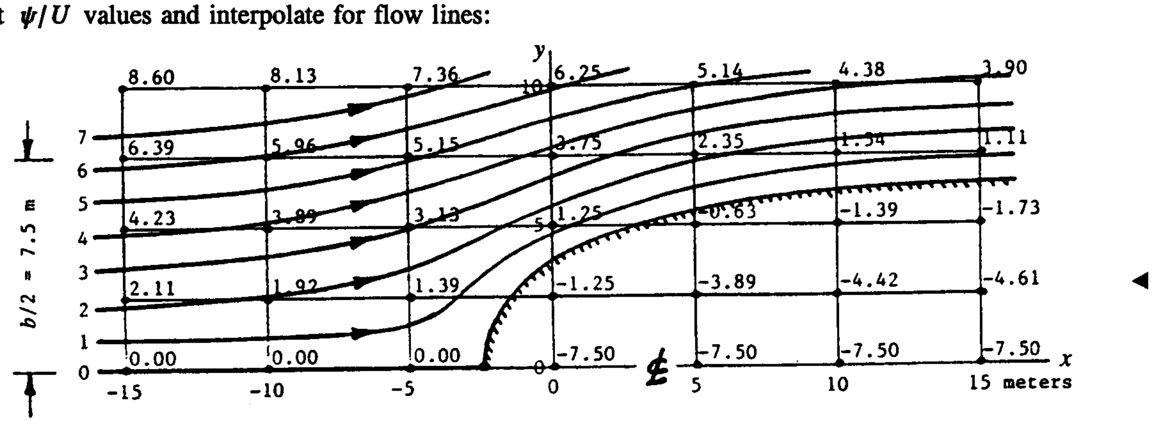
$$\psi = U[y + (15/2\pi)\tan^{-1}(y/x)]$$

Note from Fig. 4.12: Stagnation point O is on the x-axis at $x = -b/2\pi = -15/2\pi = -2.39$ m

x m	y m	y/x	$\tan^{-1}(y/x)$ degrees	radians $= N$	$\psi/U = y + 15N/2\pi$
−15	0	0	0	0	0
−15	2.5	−0.167	−9.4	−0.165	2.11
−15	5	−0.333	−18.4	−0.322	4.23
−15	7.5	−0.500	−26.6	−0.464	6.39
−15	10	−0.667	−33.7	−0.588	8.60
−5	2.5	−0.5	−26.6	−0.464	1.39
−5	5	−1.0	−45.0	−0.785	3.13
−5	7.5	−1.5	−56.3	−0.983	5.15
−5	10	−2.0	−63.4	−1.107	7.36
5	0	0	−180.0	−3.142	−7.50
5	2.5	0.5	−153.4	−2.678	−3.89
5	5	1.0	−135.0	−2.356	−0.63
5	7.5	1.5	−123.7	−2.159	2.35
5	10	2.0	−116.6	−2.034	5.14
etc.					

Note: Need $-180° \le N \le 0$ for consistency $[\tan(N - \pi) = \tan N]$.

Plot ψ/U values and interpolate for flow lines:



14.11 *Combine the uniform flow defined by u = 16 fps with the doublet 2qa = m, where q = 10 cfs/ft and a = 2 in. Sketch the streamlines for ψ = −3, −2, −1, −½, 0, ½, 1, 2, 3 cfs/ft. Use the scale of 1 in = 1 in.*

BG

From Eq. 14.18: $\psi = 16y + 1.592\left(\tan^{-1}\frac{y}{x+2} - \tan^{-1}\frac{y}{x-2}\right)$

Substitute various values of x and y and compute the corresponding ψ's. Plot the points and sketch the streamlines.

To find location of $\psi = 0$ on y-axis ($x = 0$):

$0 = 16y + 1.592\left(\tan^{-1}\frac{y}{2} - \tan^{-1}\frac{y}{-2}\right)$; by trial $y = 0.283$ in ◀

To find location of stagnation points:

$\psi = 0$, note that $\tan\theta = \theta$ for small angles. $\psi = 0 = 16y + 1.592\left(\frac{y}{x+2} - \frac{y}{x-2}\right)$

$16y(x^2 - 4) + 1.592y(x-2) - 1.592y(x+2) = 0$; $16(x^2 - 4) + 1.592(-4) = 0$

$x^2 - 4 = 6.37/16 = 0.398$; $x^2 = 4.398$; $x = \pm 2.10$ in ◀

The streamline pattern is similar to Fig. S14.4 (textbook p. 634). ◀

14.12 *A flow is defined by the stream function ψ = 15r sinθ − 30 ln r − (20/r)sin θ. Sketch this flow field. Calculate the velocities at r = 3 for θ = 0, 45, 90, 150, 210, and 315°.*

N

Given: $\psi = 15r\sin\theta - 30\ln r - (20/r)\sin\theta$

Polar form of Eq. 14.16:

$v_t = -\partial\psi/\partial r = -15\sin\theta + 30/r - (20/r^2)\sin\theta$; $v_r = \partial\psi/r\partial\theta = 15\cos\theta - (20/r^2)\cos\theta$

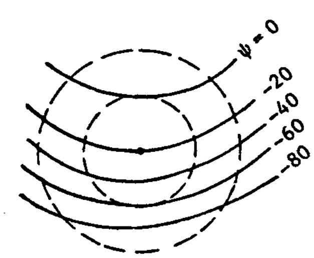
For case where $r = 3$,

$v_t = -15\sin\theta + 10 - 2.22\sin\theta = 10 - 17.22\sin\theta$; $v_r = 15\cos\theta - 2.22\cos\theta = 12.78\cos\theta$

For $r = 3$:

θ	0°	45°	90°	150°	210°	315°	
v_t	10	−2.17	−7.22	1.389	18.61	22.2	◀
v_r	12.78	9.04	0	−11.07	−11.07	9.04	◀

θ	$\psi(r = 2)$	$\psi(r = 4)$
0°	−20.8	−41.6
45°	−6.65	−2.70
90°	−0.794	13.41
135°	−6.65	−2.70
180°	−20.8	−41.6
225°	−34.9	−80.5
270°	−40.8	−96.6
315°	−34.9	−80.5



◀

14.13 *Given is the two-dimensional flow described by $u = x^2 + 2x - 4y$, $v = -2xy - 2y$. (a) Does this satisfy continuity? (b) Compute the vorticity. (c) Plot the velocity vectors for $0 < x < 5$ and $0 < y < 4$ and sketch the general flow pattern. (d) Find the location of all stagnation points in the entire flow field. (e) Find the expression for the stream function.*

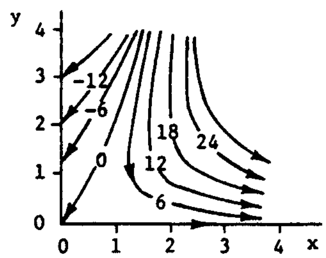
N

Note: This problem is based on Sections 14.1 and 14.3−5.

(*a*) From Eq. 14.3: $\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 2x + 2 - 2x - 2 = 0$; Continuity is satisfied. ◄

(*b*) From Eq. 14.12: $\xi = \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = -2y - 4 \neq 0$ ◄ The flow is rotational

(*c*)



◄

(*d*) Stagnation points occur where both $u = 0$ and $v = 0$.

$v = 0 = -2xy - 2y = -2y(x + 2)$; this is true when $y = 0$ or $x = -1$.

$u = 0 = x^2 + 2x - 4y$; If $x = -1$, $y = -1/4$; If $y = 0$, $x = 0$ or -2.

Thus there are three stagnation points: (0, 0), (−2, 0), (−1, −1/4). ◄

(*e*) Eq. 4.16: $u = \dfrac{\partial \psi}{\partial y}$ $\psi = \int u \partial y = \int (x^2 + 2x - 4y)\partial y = x^2y + 2xy - 2y^2 + f(x)$

$v = -\dfrac{\partial \psi}{\partial x}$ $\psi = -\int v \partial x = +\int (2xy + 2y)\partial x = x^2y + 2xy + f(y)$

So $\psi = x^2y + 2xy - 2y^2$ ◄ Check: $\dfrac{\partial \psi}{\partial y} = u = x^2 + 2x - 4y$; $\dfrac{\partial \psi}{\partial x} = -v = 2xy + 2y$

Sec. 14.6: Velocity Potential – *Exercises (4)*

14.6.1 *If $\phi = y + 2x^2$ is the velocity potential function for a two-dimensional flow, is it irrotational? Does it satisfy the Laplace equation? If not, suggest why.*

N

Eq. 14.23: $\dfrac{\partial \phi}{\partial x} = 4x = -u$; $\dfrac{\partial \phi}{\partial y} = 1 = -v$ does not satisfy Laplace Eq. 14.25 ◄

Eq. 14.12: $\xi = \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 0 - 0 = 0$; Sec.14.6: Yes, it is a potential flow, is irrotational. ◄

Continuity: $\dfrac{\partial v}{\partial x} + \dfrac{\partial v}{\partial y} = 4 + 0 = 4$, non-zero, so continuity (Eq. 14.3) is not satisfied. This is the reason why the Laplace equation is not satisfied. ◄

14.6.2 *Given the stream function $\psi = 10x - 7y$, is this a potential flow? If it is, determine the velocity potential function. Does it satisfy the Laplace equation?*

N

Eq. 14.16: $u = \dfrac{\partial\psi}{\partial y} = -7, \quad v = -\dfrac{\partial\psi}{\partial x} = -10$; Eq. 14.12: $\xi = \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 0$

Yes, this is potential flow, because $\xi = 0$. ◀

From Eq. 14.23: $u = -\dfrac{\partial\phi}{\partial x} = -7, \quad v = -\dfrac{\partial\phi}{\partial y} = -10$

Eq. 14.22: $d\phi = 7dx + 10dy$; integrating: $\phi = 7x + 10y$ ◀

By differentiation: $\dfrac{\partial^2\phi}{\partial x^2} = 0, \quad \dfrac{\partial^2\phi}{\partial y^2} = 0$

So yes, it does satisfy the Laplace equation (Eq. 14.25) in ϕ. ◀

14.6.3 *For the following stream functions, determine if a potential function exists, find the potential function if it does, and determine if the Laplace equation in ϕ is satisfied: (a) $\psi = 3xy + 2x$; (b) $\psi = 3xy + 2x^2$.*

N

(*a*) $\psi = 3xy + 2x$. Eq. 14.16: $u = \dfrac{\partial\psi}{\partial y} = 3x$; $v = -\dfrac{\partial\psi}{\partial x} = -3y - 2$

$\xi = \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 0 - 0 = 0$ $\therefore$ Yes, it is potential flow, a potential function exists ◀

Eq. 14.23: $\dfrac{\partial\phi}{\partial x} = -u = -3x$; $\dfrac{\partial\phi}{\partial y} = -v = 3y + 2$

$\dfrac{\partial^2\phi}{\partial x^2} = -3$; $\dfrac{\partial^2\phi}{\partial y^2} = 3$. $\therefore \dfrac{\partial^2\phi}{\partial x^2} + \dfrac{\partial^2\phi}{\partial y^2} = 0$, the Laplace equation is satisfied. ◀

Integrating $\dfrac{\partial\phi}{\partial x}$ and $\dfrac{\partial\phi}{\partial y}$: $\phi = \dfrac{-3x^2}{2} + f(y)$; $\phi = 2y + \dfrac{3y^2}{2} + f(x)$

Comparing: $f(y) = 2y + \dfrac{3y^2}{2}$; $f(x) = \dfrac{-3x^2}{2}$. $\therefore \phi = 2y + \dfrac{3y^2}{2} - \dfrac{3x^2}{2} + C$ ◀

(*b*) $\psi = 3xy + 2x^2$. Eq. 14.16: $u = \dfrac{\partial\psi}{\partial y} = 3x$; $v = -\dfrac{\partial\psi}{\partial x} = -3y - 4x$

$\xi = \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = -4 + 0 = -4$, no, it is not potential flow, a potential function does not exist ◀

Eq. 14.23: $\dfrac{\partial\phi}{\partial x} = -u = -3x$; $\dfrac{\partial\phi}{\partial y} = -v = 3y + 4x$

$\dfrac{\partial^2\phi}{\partial x^2} = -3$; $\dfrac{\partial^2\phi}{\partial y^2} = 3$, $\therefore$ yes, the Laplace Eq. 14.25 is satisfied. ◀

14.6.4 *For the following stream functions, determine if a potential function exists, find the potential function if it does, and determine if the Laplace equation in ϕ is satisfied: (a) $\psi = 6xy$; (b) $\psi = x\sin y$*

N

(*a*) $\psi = 6xy$. Eq. 14.16: $u = \dfrac{\partial\psi}{\partial y} = 6x$; $v = -\dfrac{\partial\psi}{\partial x} = -6y$

$\xi = \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 0 - 0 = 0$; ∴ Yes, it is potential flow, a potential function exists ◄

Eq. 14.23: $\dfrac{\partial\phi}{\partial x} = -u = -6x$; $\dfrac{\partial\phi}{\partial y} = -v = 6y$

Integrating: $\phi = -3x^2 + f(y)$; $\phi = 3y^2 + f(x)$. Comparing: $\phi = 3(y^2 - x^2)$ ◄

$\dfrac{\partial^2\phi}{\partial x^2} = -6$; $\dfrac{\partial^2\phi}{\partial y^2} = 6$ ∴ the Laplace Eq. 14.25 is satisfied ◄

(*b*) $\psi = x\sin y$. Eq. 14.16: $u = \dfrac{\partial\psi}{\partial y} = x\cos y$; $v = -\dfrac{\partial\psi}{\partial x} = -\sin y$

$\xi = \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y} = 0 - (-x\sin y) \neq 0$ ∴ not potential flow, a potential function does not exist ◄

Eq. 14.23: $\dfrac{\partial\phi}{\partial x} = -u = -x\cos y$; $\dfrac{\partial\phi}{\partial y} = -v = \sin y$

$\dfrac{\partial^2\phi}{\partial x^2} = -\cos y$; $\dfrac{\partial^2\phi}{\partial y^2} = \cos y$ ∴ yes, the Laplace Eq. 14.25 is satisfied ◄

Sec. 14.7: Orthogonality of Streamlines and Equipotential Lines -- *Exercises (2)*

14.7.1 *Which of the flows in Exer. 14.1.1 can be described by a flow net? Write expressions for the stream functions and the potential functions.*

N

By evaluating $\partial u/\partial x + \partial v/\partial y$ (to determine if continuity is satisfied, as in Exer. 14.1.1) and $\partial v/\partial x - \partial u/\partial y$ (to determine if the flow is irrotational, as in Exer. 14.2.1), we find that only flows (*a*) and (*b*) can be represented by a flow net because they satisfy both continuity and irrotationality. ◄

Integrating Eqs. 14.16 and 14.23 we obtain:

(*a*) $\psi = 2y$, $\phi = -2x$ ◄

(*b*) $\psi = 2y - 3x$, $\phi = -3y - 2x$ ◄

14.7.2 *Which of the flows in Exer. 14.1.2 can be described by a flow net? Write expressions for the stream functions and the potential functions.*

N

By evaluating $\partial u/\partial x + \partial v/\partial y$ (to determine if continuity is satisfied, as in Exer. 14.1.1) and $\partial v/\partial x - \partial u/\partial y$ (to determine if the flow is irrotational, as in Exer. 14.2.1), we find that only flows (*c*), (*d*), and (*g*) can be represented by a flow net because they satisfy both continuity and irrotationality. ◄

Integrating Eqs. 14.16 and 14.23 we obtain:

(*c*) $\psi = 3xy$, $\phi = -1.5x^2 + 1.5y^2$ ◄

(*d*) $\psi = 4y + 6x + 2xy$, $\phi = 6y - 4x - x^2 + y^2$ ◄

(*g*) $\psi = x^3/3 - 2x^2y - xy^2 + 2y^3/3$, $\phi = (2/3)x^3 + x^2y - 2xy^2 - (1/3)y^3$ ◄

Sec. 14.7: Orthogonality of Streamlines and Equipotential Lines -- *Problems 14.14–14.27*

14.14 *The flow around the body of Fig. 4.12 may be considered as that due to the sum of two velocity potentials, $\phi_1 = -Ux$, representing an undisturbed flow of velocity U in the x direction, and $\phi_2 = -S \ln r$, representing the radial flow from a source located inside the body behind the stagnation point. To relate U and S, it is observed that the total flow $2\pi S$ from the source (which is hydrodynamically equivalent to the body itself) must be equal to the flow of the main stream which is not passing through the body of width b, or $2\pi S = Ub$. This gives*

$$\phi_2 = -\frac{Ub}{2\pi}\ln r$$

(a) The distance from the stagnation point to the source is determined by setting the radial velocity from the source, $v_r = -\partial\phi/\partial r$, equal and opposite to the undisturbed velocity U. Prove that this establishes the source at a distance $b/2\pi$ behind the stagnation point. The absolute velocity at any point of the field may be determined by the vector sum of the components U and v_r.

There follows an ingenious method of plotting the boundary of such a streamlined body, as shown in Fig. P14.14. Suppose that the streamlines in the undisturbed flow are spaced a distance a apart, where $b/2a = n$, an integer. Next divide the upper half of the source into n radial sectors, each of angle α, that is, $n\alpha = \pi$. Then the undisturbed flow between the x axis and the first streamline is associated with the source flow in the first sector from the stagnation point. Thus the intersection of the first streamline with the first line must be a point on the boundary of the body, through which there can be no flow. Similarly, the intersection of the horizontal line at 2a with the radial line at 2α forms another point, and so on. Further streamlines can be plotted by connecting successive intersections of the original horizontal lines with the radial lines, recognizing that the same flow must exist between any adjacent pair of streamlines. Thus the intersection of a horizontal line ea above the axis with a radial line at $f\alpha$ from the stagnation point must lie on a streamline which is $(e - f)a$ distant from the axis in the undisturbed region, where e and f are integers.

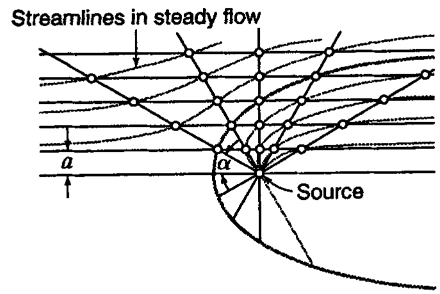


Figure P14.14

(b) Assume a value of U = 20 fps and a two-dimensional flow past a streamlined body for which b = 36 ft. Compute the distance from the source to the stagnation point and to the surface of the body at a radius of 90° to the axis. What is the value of the source velocity at the latter point?
(c) What is the magnitude of the velocity of the fluid along the surface at the 90° point? (Compare with the results of Sample Prob. 4.2.) What is its direction relative to the axis?

BG

(*a*) Given: Source potential, $\phi_2 = (Ub/2\pi)\ln r$; ∴ (Eq. 14.24) $v_r = -\partial\phi/\partial r = Ub/2\pi r$

Setting $v_r = U$ for stagnation condition (on tip of nose) and solving for r, the desired distance behind the nose, $r = b/2\pi$ Q.E.D. ◄

(*b*) For $b = 36$ ft, distance to stagnation point $= -36/2\pi = -5.73$ ft ◄

Since $b/2a = n = 6 = 36/2a$ = the number of radial sectors from half of source,

streamline spacing $= a = 36/12 = 3$ ft

From Fig. P14.14: At $90° = 3\alpha$, $r = 3a = 3(3) = 9$ ft ◄

Source velocity at 90°, $r = 3a$: $v_r = \dfrac{Ub}{2\pi r} = \dfrac{20(36)}{2\pi(9)} = 12.73$ fps ◄

(c) At 90°, $V = \sqrt{U^2 + v_r^2} = \sqrt{20^2 + 12.73^2} = 23.7$ fps ◄

Thus, on the surface at 90°, $V/U = 23.7/20 = 1.185$

In Sample Prob. 4.2 at 90° (≈ 1.6 ft from the stagnation point), $V/U \approx 12/10 = 1.2$

Thus the two agree closely ◄ Direction relative to axis is $\tan^{-1}(12.73/20) = 32.5°$ ◄

14.15 *Solve Prob. 14.14 in SI units; for parts (b) and (c) use U = 5 m/s and b = 12 m.*
See Prob. 14.14 and Fig. P14.14.

SI

(*a*) Given: Source potential, $\phi_2 = (Ub/2\pi)\ln r$; ∴ (Eq. 14.24) $v_r = -\partial\phi/\partial r = Ub/2\pi r$

Setting $v_r = U$ for stagnation condition (on tip of nose) and solving for r, the desired distance behind the nose, $r = b/2\pi$ Q.E.D. ◄

(*b*) For $b = 12$ m, distance to stagnation point $= -12/2\pi = -1.910$ m ◄

Since $b/2a = 12/2a = n = 6$ = the number of radial sectors from half of source,

streamline spacing $= a = 12/12 = 1$ m

Distance (source to surface at 90°) $= r = 3a = 3(1\text{ m}) = 3$ m ◄

Source velocity at 90°, $r = 3a$: $v_r = \dfrac{Ub}{2\pi r} = \dfrac{5(12)}{2\pi(3)} = 3.18$ m/s ◄

(*c*) At 90°, $r = 3a$: $V = \sqrt{U^2 + v_r^2} = \sqrt{5^2 + 3.18^2} = 5.93$ m/s ◄

Thus, on the surface at 90°, $V/U = 5.93/5 = 1.185$

In Sample Prob. 4.2 at 90° (≈ 1.6 ft from the stagnation point), $V/U \approx 12/10 = 1.2$

Thus the two agree closely ◄

Direction relative to axis is $\tan^{-1}(3.18/5) = 32.5°$ ◄

14.16 *Find the distance to the surface of the body and the two velocities called for in Prob. 14.14 for an angle of 30°. (Compare with Sample Prob. 4.2.)*
See Prob. 14.14.

BG

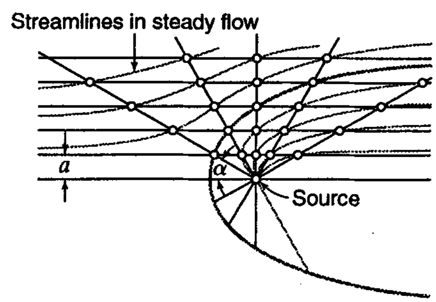


Figure P14.14

At 30°, $r = a/\sin 30° = 3/0.5 = 6.00$ ft ◄

$$v_r = \frac{\partial\phi}{\partial r} = \frac{Ub}{2\pi r} = \frac{20(36)}{2\pi(6.0)} = 19.10 \text{ fps} \quad ◄$$

$$V^2 = U^2 + v_r^2 - 2Uv_r\cos\alpha = 20^2 + 19.10^2 - 2(20)19.10\cos 30°$$

$$= 400 + 365 - 662 = 103$$

$V = 10.16$ fps ◄ (Comparison with Sample Prob. 4.2 is good.)

14.17 *Find the distance to the surface of the body and the two velocities called for in Prob. 14.15 for an angle of 30°. (See Sample Prob. 4.2.)*
Prob. 14.15: u = 5 m/s, b = 12 m. See Prob. 14.14.

SI

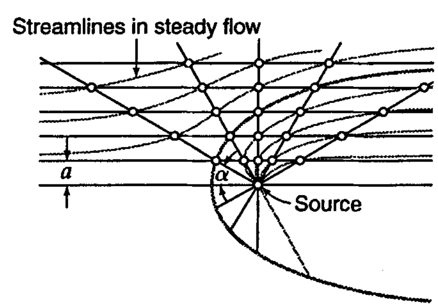


Figure P14.14

At 30°, $r = a/\sin 30° = 1/0.5 = 2$ m ◄

$$v_r = \frac{\partial\phi}{\partial r} = \frac{Ub}{2\pi r} = \frac{5(12)}{2\pi(2)} = 4.77 \text{ m/s} \quad ◄$$

$$V^2 = U^2 + v_r^2 - 2Uv_r\cos\alpha = 5^2 + 4.77^2 - 2(5)4.77\cos 30°$$

$= 6.45$; $V = 2.54$ m/s ◄

14.18 *An ideal fluid flows in a two-dimensional 90° bend (Fig. P14.18). The inner and outer radii of the bend are 0.45 and 1.5 ft. (a) Sketch the flow net and estimate the velocity at the inner and outer walls of the bend if the velocity in the 1.05-ft-wide straight section is 6 fps. (b) Develop an analytic expression for the stream function, in this case noting that* $v_t = -\dfrac{\partial\psi}{\partial r}$ *and* $v_r = \dfrac{\partial\psi}{r\partial\theta}$. *Determine the inner and outer velocities accurately.*

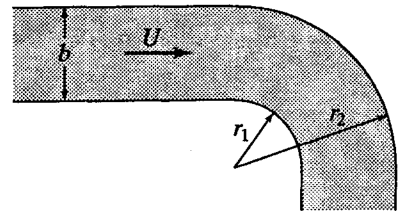


Figure P14.18

BG

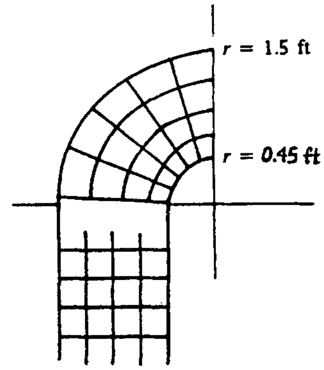
(*a*) Velocity estimates:

Sec. 14.7: $V = -\dfrac{\partial\phi}{\partial s}$

$\therefore\ -\Delta\phi \approx V\Delta s \approx 6\Delta s_0$ so $V \approx 6\Delta s_0/\Delta s$

From sketch: $V_{outside} \approx 6(4\ \text{mm}/7\ \text{mm}) = 3.43$ fps ◀

$V_{inside} \approx 6(4\ \text{mm}/2\ \text{mm}) = 12$ fps ◀



◀

(*b*) Accurate velocity determinations:

$v_t(\Delta r)$ = constant, Δr is proportional to r

$v_t(kr)$ = constant, therefore $v_t = C/r$. Also $v_r = 0$

$\psi = \psi(r, \theta)$, so by the chain rule of differentiation

$$d\psi = \frac{\partial\psi}{r\partial\theta}rd\theta + \frac{\partial\psi}{\partial r}dr = v_r r d\theta - v_t dr = 0 - v_t dr$$

If $\psi_{inside} = 0$, $\psi_{outside} = \psi_{inside} + dq = 0 + (1.05\ \text{ft})(6\ \text{fps}) = 6.3$ cfs/ft

Integrating: $\int_0^{6.3} d\psi = -\int_{0.45}^{1.5} v_t dr = -C\int_{0.45}^{1.5} dr/r$

$\psi = 6.3 = -C[\ln r]_{0.45}^{1.5} = -C\ln(1.5/0.45) = -C\ln 3.33$ and $C = -5.23$

Thus the analytical expression is: $\psi = 5.23 \ln r$ ◀

$v_t = C/r = -5.23/r$; $(v_t)_{r=0.45} = 11.63$ fps ◀

$(v_t)_{r=1.5} = 3.49$ fps ◀

14.19 *An ideal fluid flows in a two-dimensional 90° bend (Fig. P14.18). The inner and outer radii of the bend are 0.15 and 0.50 m. (a) Sketch the flow net and estimate the velocity at the inner and outer walls of the bend if the velocity in the 0.35-m-wide straight section is 2.8 m/s. (b) Develop an analytic expression for the stream function, in this case noting that $v_t = -\dfrac{\partial\psi}{\partial r}$ and $v_r = \dfrac{\partial\psi}{r\partial\theta}$. Determine the inner and outer velocities accurately.*

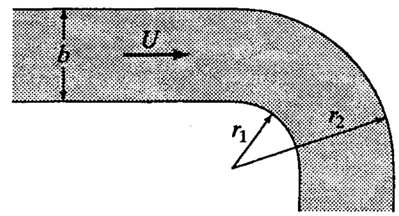


Figure P14.18

SI

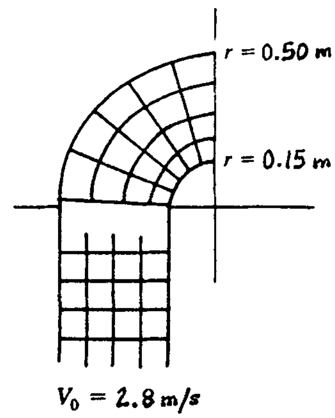
(*a*) Velocity estimates:

Sec 14.7: $V = -\dfrac{\partial\phi}{\partial s}$

$\therefore\ -\Delta\phi \approx V\Delta s \approx 2.8\,\Delta s_0\ ;\qquad V \approx 2.8\,\Delta s_0/\Delta s$

From sketch: $V_{\text{outside}} \approx 2.8(4\text{ mm}/7\text{ mm}) = 1.600\text{ m/s}$ ◄

$V_{\text{inside}} \approx 2.8(4\text{ mm}/2\text{ mm}) = 5.60\text{ m/s}$ ◄



◄

(*b*) Accurate velocity determinations:

$v_t(\Delta r)$ = constant, Δr is proportional to r

$v_t(kr)$ = constant, therefore $v_t = C/r$. Also $v_r = 0$

$\psi = \psi(r, \theta)$, so by chain rule of differentiation

$$d\psi = \frac{\partial\psi}{r\partial\theta}r\,d\theta + \frac{\partial\psi}{\partial r}dr = v_r r\,d\theta - v_t dr = 0 - v_t dr$$

If $\psi_{\text{inside}} = 0$, $\quad \psi_{\text{outside}} = \psi_{\text{inside}} + dq = 0 + (0.35\text{ m})(2.8\text{ m/s}) = 0.980\text{ m}^3\text{/s per m}$

Integrating: $\displaystyle\int_0^{0.980} d\psi = \int_{0.15}^{0.5} v_t\,dr = -C\int_{0.15}^{0.5} dr/r$

$\psi = 0.980 = -C[\ln r]_{0.15}^{0.5} = -C\ln(0.5/0.15) = -C\ln 3.33\ ;\qquad C = -0.814$

Thus the analytical expression is $\psi = 0.814\ln r$ ◄

$v_t = C/r = -0.814/r\ ;\quad (v_t)_{r=0.15} = 5.43\text{ m/s}$ ◄ $\quad (v_t)_{r=0.5} = 1.628\text{ m/s}$ ◄

14.20 *For the two-dimensional flow of a frictionless incompressible fluid against a flat plate normal to the initial velocity, the stream function is given by $\psi = -2axy$, while its conjugate function, the velocity potential, is*

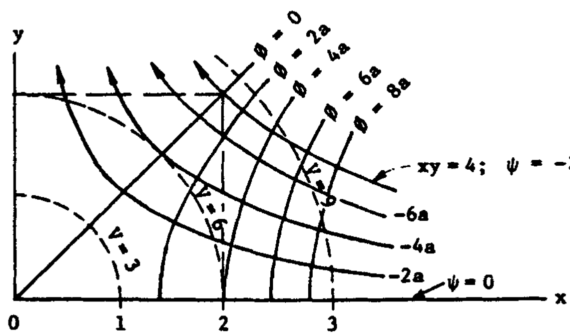
$$\phi = a(x^2 - y^2)$$

where a is a constant and the flow is symmetrical about the yz plane (Fig. 14.1). By direct differentiation, demonstrate that these functions satisfy Eq. (14.25). Using a scale of 1 in = 1 unit of distance, plot the streamlines given by $\psi = \pm 2a, \pm 4a, \pm 6a, \pm 8a$, and the equipotential lines given by $\phi = 0, \pm 2a, \pm 4a, \pm 6a, \pm 8a$. Observe that this flow net also gives the ideal flow around an inside square corner. Compare your results with Sample Prob. 14.5 and note the effect of changing the sign of ψ and ϕ.

N

Differentiating: $\partial\psi/\partial y = -2ax$; $\partial\phi/\partial x = 2ax$; $-\partial\psi/\partial x = 2ay$; $\partial\phi/\partial y = a(-2y)$;

$\partial^2\phi/\partial x^2 = 2a$; $\partial^2\phi/\partial y^2 = -2a$; Thus Eq. 14.25 is satisfied ◄



◄

14.21 *In Prob. 14.20 determine the velocity components u and v, and demonstrate that they satisfy the differential equations for continuity and irrotational flow. In which direction is the flow? Prove that the absolute velocity is given by $V = 2ar$, where r is the radius to the point from the origin. Now assume that the linear scale is 1 in = 1 ft. Determine the constant a such that the flow net of Prob. 14.20 will represent a flow of 3 ft²/sec between any two adjacent streamlines. What are the dimensions of a? Draw curves of equal velocity for values of 3, 6, 9, 12 fps. How does the velocity vary along the surface of the plate?*
See Prob. 14.20.

BG

Given: $\psi = -2axy$, then: $u = \partial\psi/\partial y = -2ax$; $v = -\partial\psi/\partial x = 2ay$

Differentiating further: $(\partial u/\partial x) + (\partial v/\partial y) = -2a + 2a = 0$, so continuity (Sec. 14.4) is satisfied ◄

$(\partial v/\partial x) - (\partial u/\partial y) = 0 - 0 = 0$, so (from Eq. 14.6) flow is irrotational ◄

$V = \sqrt{u^2 + v^2} = \sqrt{4a^2x^2 + 4a^2y^2} = 2a\sqrt{x^2 + y^2} = 2ar$ ◄

Flow between two streamlines in flow net of Prob. 14.20 is given by $\psi_2 - \psi_1 = 2a$

$\therefore\ 2a = 3\ \text{ft}^2/\text{sec}$ (given); $a = 3/2\ \text{sec}^{-1}$ ◄

Dimensions of "a" are most clearly seen from $V = 2ar$; $a = V/2r = (L/T)/L = T^{-1}$ ◄

$V \propto r$, so the velocity varies linearly along the plate. ◄

14.22 *A cylindrical drum with a 2 ft radius is securely held in position in an open channel of rectangular section. The channel is 10 ft wide, and the flow rate is 240 cfs. Water flows beneath the drum as shown in Fig. P14.22. Sketch the flow net, and determine from flow net measurements the pressure at the points indicated along the wetted drum surface. Neglect fluid friction. Sketch the pressure distribution, and by numerical integration determine an approximate value of the horizontal thrust on the cylinder. (Compare with Exer. 6.4.1.)*

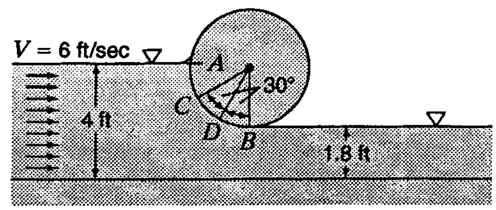
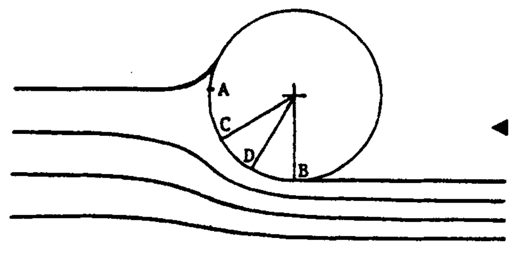


Figure P14.22

BG

Sketch the streamlines by eye and use continuity to determine the velocities. A high degree of accuracy is not expected.



Energy Eq. from upstream to point on drum:

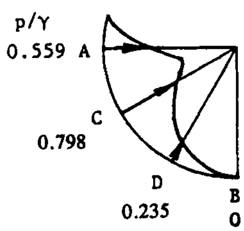
At A, $p/\gamma = p_s/\gamma = V^2/2g = 6^2/2g = 0.559$ ft ◄

At B, in contact with the atmosphere ($p_{atm} = 0$ psig):

$p/\gamma = 0$ ◄

At C, from flow net, $V \approx 7$ fps. $4 + 6^2/2g \approx (4 - 1.0) + p/\gamma + 7^2/2g$; $p/\gamma \approx 0.80$ ft. ◄

At D, from flow net, $V \approx 11.5$ fps. $4 + 6^2/2g \approx (4 - 1.73) + p/\gamma + 11.5^2/2g$; $p/\gamma \approx 0.24$ ft ◄



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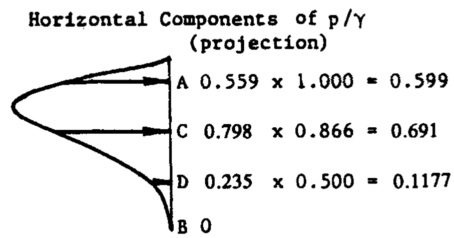


Area under projected p/γ diagram $\approx$ average horizontal component = Area/2.2 = 0.44 ft = 0.19 psi

Horizontal thrust $\approx 0.19(2.2 \times 12)(15 \times 12) = 547$ lb ◄

Note to instructor: See Exer. 6.4.1 for the application of impulse-momentum to a similar problem.

14.23 *Refer to Sample Prob. 6.1. Sketch a flow net. Using the given dimensions in BG units through application of Bernoulli's principle, determine the approximate pressure distribution along the channel bottom and around the curved structure. By numerical integration estimate the magnitude of the horizontal and vertical components of the force of the water on the structure.*

Sample Prob. 6.1: Q = 481 cfs = 48.1 cfs/ft.

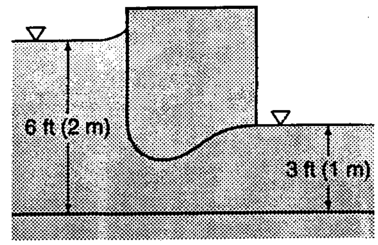
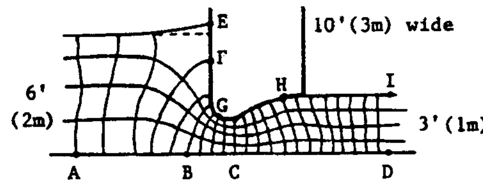


Figure S6.1

BG

$V_A = 48.1/6.0 = 8.02$ ft ; $V_B = 48.1/1 \approx 16$ fps

$V_C = 48.1/2 \approx 24$ fps ; $V_D = 48.1/3.0 = 16.03$ fps



◄

Energy equation between points:

$$6 + \frac{V_A^2}{2g} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} = \frac{p_C}{\gamma} + \frac{V_C^2}{2g} = \frac{p_D}{\gamma} + \frac{V_D^2}{2g}$$

$$6 + 1.00 \approx \frac{p_B}{\gamma} + \frac{16^2}{64.4} \approx \frac{p_C}{\gamma} + \frac{24^2}{64.4} = \frac{p_D}{\gamma} + \frac{16.03^2}{64.4}$$

from which $p_B/\gamma \approx 3.02$ ft, $p_C/\gamma \approx -1.944$ ft, $p_D/\gamma = 3.01$ ft ◄◄

Selecting arbitrary points F, G, and H: $V_E = 0$, $V_F \approx 48.1/6 = 8$ fps

$V_G = 48.1/1.6 \approx 30$ fps, $V_H \approx 48.1/2.8 = 17$ fps, $V_I = 16.03$ fps

Scaling off the sketch: $z_F \approx 4$ ft, $z_G \approx 1.7$ ft, $z_H \approx 2.8$ ft; $z_I = 3.0$ ft (given);

$z_E = 7.0$ ft (from above calculations).

Energy: $$7 = 4 + \frac{p_F}{\gamma} + \frac{V_F^2}{2g} = 1.7 + \frac{p_G}{\gamma} + \frac{V_G^2}{2g} = 2.8 + \frac{p_H}{\gamma} + \frac{V_H^2}{2g} = 3.0 + \frac{p_I}{\gamma} + \frac{V_I^2}{2g}$$

Substituting the approximate values of V gives:

$$\frac{p_E}{\gamma} = 0,\ \frac{p_F}{\gamma} \approx 2\text{ ft},\ \frac{p_G}{\gamma} \approx -8.7\text{ ft},\ \frac{p_H}{\gamma} \approx -0.5\text{ ft},\ \frac{p_I}{\gamma} = 0$$ ◄◄

With careful detail a reasonably accurate estimate of the horizontal and vertical components of the force on the structure can be obtained. ◄

(From Sample Prob. 6.1, by impulse-momentum analysis, $F_x = 936$ lb.)

14.24 *Work Prob. 14.23 using the dimensions as given in SI units in Sample Prob. 6.1.*

Prob. 14.23: Refer to Sample Prob. 6.1. Sketch a flow net. Using the given dimensions in BG units through application of Bernoulli's principle, determine the approximate pressure distribution along the channel bottom and around the curved structure. By numerical integration estimate the magnitude of the horizontal and vertical components of the force of the water on the structure.

Sample Prob. 6.1: $Q = 15.34\ m^3/s = 5.11\ m^3/s$ per meter.

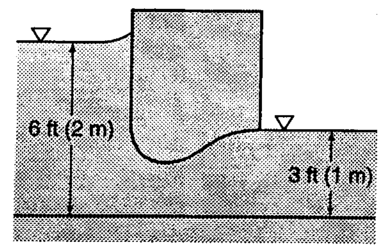
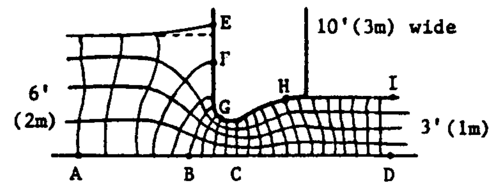


Figure S6.1

SI

From sketch of flowfield:



◄

$$V_A = \frac{5.11}{2.0} = 2.56 \text{ m/s} ; \quad V_B \approx \frac{5.11}{1.0} = 5.11 \text{ m/s}$$

$$V_C \approx \frac{5.11}{0.67} = 7.63 \text{ m/s} ; \quad V_D = \frac{5.11}{1.0} = 5.11 \text{ m/s}$$

Energy equation between points:

$$2 + \frac{V_A^2}{2g} = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} = \frac{p_C}{\gamma} + \frac{V_C^2}{2g} = \frac{p_D}{\gamma} + \frac{V_D^2}{2g}$$

from which $p_B/\gamma \approx 1.0$ m, $p_C/\gamma \approx -0.637$ m, $p_D/\gamma = 1.0$ m ◄◄

Selecting arbitrary points *F*, *G*, and *H*: $V_E = 0$, $V_F \approx 5.11/2 = 2.56$ m/s

$V_G \approx 5.11/0.5 = 10.23$ m/s, $V_H \approx 5.11/0.95 = 5.38$ m/s, $V_I = 5.11$ m/s

Scaling off sketch: $z_F \approx 1.3$ m, $z_G \approx 0.6$ m, $z_H \approx 0.95$ m ; $z_I = 1$ m (given);

$z_E = 2.33$ m (from above calculations).

$$\therefore\ 2.33 = 1.3 + \frac{p_F}{\gamma} + \frac{V_F^2}{2g} = 0.6 + \frac{p_G}{\gamma} + \frac{V_G^2}{2g} = 0.95 + \frac{p_H}{\gamma} + \frac{V_H^2}{2g} = 1.0 + \frac{p_I}{\gamma} + \frac{V_I^2}{2g}$$

Substituting the approximate values of *V* gives:

$$\frac{p_E}{\gamma} = 0, \quad \frac{p_F}{\gamma} \approx 0.7 \text{ m}, \quad \frac{p_G}{\gamma} \approx -3.6 \text{ m}, \quad \frac{p_H}{\gamma} \approx -0.094 \text{ m}, \quad \frac{p_I}{\gamma} = 0$$ ◄◄

With careful detail a reasonably accurate estimate of the horizontal and vertical components of the force on the structure can be obtained. ◄

(From Sample Prob. 6.1, by impulse-momentum analysis, $F_x = 4.91$ kN)

14.25 *The three-dimensional counterpart of the flow in Probs. 14.20 and 14.21 is that of flow along the y axis approaching the plate in the xz plane. As the flow must be symmetrical about the y axis, the traces of stream and equipotential surfaces in the xy plane will be representative of those in all planes containing the y axis. The velocity potential is now given by $\phi = -a(0.5x^2 - y^2)$, and the stream function by $\psi = -ax^2y$. Notice that these functions no longer satisfy Eq. (14.25). Why not? Again plot streamlines and equipotential lines for the values given in Prob. 14.20. The velocities u and v may still be determined by Eq. (14.23). Prove that the absolute velocity for this case is given by $V = a\sqrt{x^2+4y^2}$. With the value of a = $1.5 sec^{-1}$ found in Prob. 14.21, draw curves of equal velocity for values of 3, 6, 9, 12 fps. How does the velocity vary along the surface? What is the total flow between any adjacent stream surfaces?*

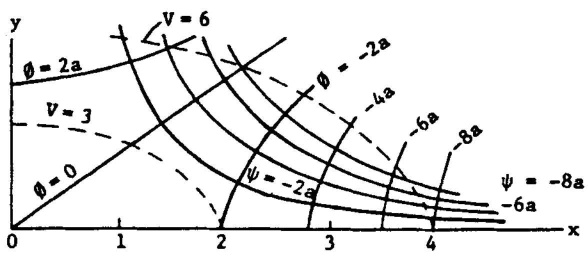
BG

Given: $\phi = -a(0.5x^2 - y^2)$; $\psi = -ax^2y$. Eq. 14.23: $u = -\partial\phi/\partial x = ax$; $v = -\partial\phi/\partial y = -2ay$

Adding vectorially: $V = \sqrt{u^2 + v^2} = a\sqrt{x^2 + 4y^2}$, Q.E.D. ◄

Substituting for $a = 1.5 \text{ sec}^{-1}$ and $V = 3$ fps: $3 = 1.5\sqrt{x^2 + 4y^2}$

or $x^2 + 4y^2 = 2^2 = 4$; from which $x = 2\sqrt{1 - y^2}$



◄

On the surface ($y = 0$), the velocity varies linearly with the distance from the y-axis. ◄

Flow between two stream surfaces is $Q = 2\pi(\Delta\psi) = 2\pi(2a) = 2\pi(2)(3/2) = 6\pi$ cfs ◄

Note: Although Eq. 14.25 is not satisfied as it stands, the Laplace equation is still valid for cylindrical

coordinates: $\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} + \frac{\partial^2\phi}{\partial z^2} = 0$

In this example, "x" is the radial direction while "y" is the z-direction. Also $\partial\phi/\partial\theta = 0$.

14.26 *For the two-dimensional flow around any angle α, the velocity potential and stream function are given in polar coordinates as $\phi = -ar^{\pi/\alpha}\cos(\pi\theta/\alpha)$ and $\psi = -ar^{\pi/\alpha}\sin(\pi\theta/\alpha)$, respectively. (a) Prove that the functions given in Prob. 14.20 are a specialization of these expressions for $\alpha = \pi/2$. (b) Take the case of $\alpha = 3\pi/2$, and plot streamlines and equipotential lines for the values given in Prob. 14.20. Compare the velocity at the corner with that at the corner in Prob. 14.20.*

N

(*a*) Given: $\phi = -ar^{\pi/\alpha}\cos(\pi\theta/\alpha)$; $\psi = -ar^{\pi/\alpha}\sin(\pi\theta/\alpha)$

Substituting for $\alpha = \pi/2$: $\phi = -ar^2\cos(2\pi\theta/\pi)$; $\psi = -ar^2\sin 2\theta$.

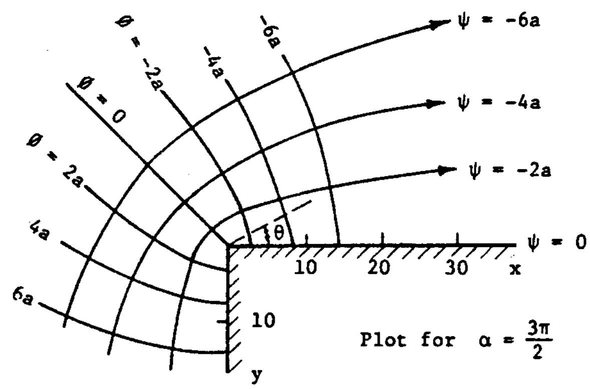
For rectangular coordinates, $x = r\cos\theta$, $y = r\sin\theta$; $r = \sqrt{x^2 + y^2}$

Thus: $\phi = -ar^2\cos 2\theta = -ar^2(2\cos^2\theta - 1) = -ar^2[2(x^2/r^2) - 1]$

$= -ar^2(2x^2 - r^2)/r^2 = -a[2x^2 - (x^2 + y^2)] = -a(x^2 - y^2)$ Q.E.D. (cf Prob. 14.20) ◄

$\psi = -ar^2\sin 2\theta = -ar^2(2\sin\theta\cos\theta) = -ar^2[2(y/r)(x/r)] = -2axy$ Q.E.D. (cf Prob. 14.20) ◄

(*b*)



◄

For $\alpha = 3\pi/2$: At corner, for $\phi = -ar^{2/3}\cos(2\theta/3)$

Eq. 14.24: $v_t = -\frac{1}{r}\frac{\partial\phi}{\partial\theta} = -\frac{1}{r}(-ar^{2/3})\left(-\frac{2}{3}\sin\frac{2}{3}\theta\right) = -\frac{2}{3}\frac{a}{r^{1/3}}\sin\frac{2}{3}\theta$

This reveals that as $r \to 0$, $v_t \to \infty$ ◄ while for an inside corner, as $r \to 0$, $v_t \to 0$ ◄

In either case, $v_r = 0$ along a line bisecting the total angle θ ◄

14.27 *Superimpose a point source ($Q = 100$ cfs) on a rectilinear flow field ($U = 20$ fps). Plot the body contour at $\theta = 30, 60, 90, 120, 150, 180°$ using a scale of 1 in = 1 ft. Compute the velocities along the body contour at these points. Determine the pressures at these points assuming $\rho = 1.94$ slug/ft^3 with zero pressure in the undisturbed rectilinear flow field. What is the velocity and pressure in the combined flow field at the following points? Hint: Refer to Prob. 14.14.*

(a) $\theta = 45°$, $r = 4.0$ ft
(b) $\theta = 90°$, $r = 2.0$ ft
(c) $\theta = 90°$, $r = 4.0$ ft
(d) $\theta = 135°$, $r = 2.0$ ft

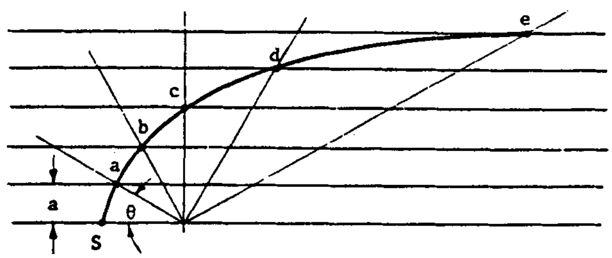
BG

Given: $U = 20$ fps, $v_r = 100/2\pi r$ fps

Distance from source to stagnation point: $U = v_r$; $r = 5/2\pi = 0.796$ ft

From Prob. 14.14: For upper half let $n = 6$, thus $\theta = 30°$

$q = Ua = 100/2n$; thus $a = 100/(2nU) = 0.417$ ft



Scale off radial distances to points a, b, c, d, e and f

$v_x = U - v_r\cos\theta$; $v_y = v_r\sin\alpha$. Vector addition: $v = \sqrt{v_x^2 + v_y^2}$

Point	r (ft)	θ	v_r	v_x	v_y	v (fps)
S	0.80	0°	20.0	0	0	0
a	0.81	30°	19.64	2.99	9.90	10.34
b	0.96	60°	16.57	11.71	14.46	18.71
c	1.24	90°	12.83	20.0	12.83	23.8
d	1.90	120°	8.38	24.2	7.26	25.3
e	4.16	150°	3.83	23.3	1.915	23.4

▲

$p_0 + \rho V_0^2/2 = p + \rho V^2/2$. $0 + 1.940(20^2/2) = 388$ lb/ft^2 $= p + \rho V^2/2$; $\therefore$ $p = 388 - \rho V^2/2$

Hence the pressures at 0°, 30°, etc. are: 388, 284, 48.4, −161, −232, −143 psf, respectively ◄

At other points in the flow field:

(*a*) $v_x = [20 - 100/(2\pi \times 4)]\cos 45°$; $v_y = [100/(2\pi \times 4)]\sin 45°$

$v_x = 11.33$ fps; $v_y = 2.81$ fps; $v = 11.67$ fps ◄

$p = \rho 20^2/2 - \rho V^2/2 = 256$ psf ◄

(*b*) Similarly $v_x = 20.0$ fps, $v_y = 7.95$ fps, $v = 21.5$ fps, $p = -60.3$ psf ◄

(*c*) $v_x = 20.0$ fps, $v_y = 4.98$ fps, $v = 20.6$ fps, $p = -23.6$ psf ◄

(*d*) This point is inside the bounds of the body. Therefore, it is not in the flow field. ◄

Sec. 14.8: Flow Through Porous Media -- *Exercises (4)*

14.8.1 *The estimated average depth of saturated flow in a sloping water table aquifer (porosity 0.25, K = 5 ft/day) is 11.8 ft. The distance between 1-ft water table contours is 34 ft. What is the flow rate through a 1-ft width of aquifer, and what is the average pore velocity?*

BG

Eq. 14.28: $V = -K(dh/ds) = -5(-1/34) = 0.1471$ ft/day

$q = AV = 1(11.8)0.1471 = 1.735$ ft^3/d per ft width ◄

Eq. 14.29: $V_p = V/n = 0.1471/0.25 = 0.588$ ft/day ◄

14.8.2 *The estimated average depth of saturated flow in a sloping water table aquifer (porosity 0.23, K = 1.6 m/d) is 3.8 m. The distance between 1-m water table contours is 37 m. What is the flow rate through a 1-m width of aquifer, and what is the average pore velocity?*

SI

Eq. 14.28: $V = -K(dh/ds) = -1.6(-1/37) = 0.0432$ m/d

$q = AV = 1(3.8)0.0432 = 0.1643$ m^3/d per m width ◄

Eq. 14.29: $V_p = V/n = 0.432/0.23 = 0.1880$ m/d ◄

14.8.3 *From dye tests between two wells, ground water is estimated to travel at 0.034 ft/day. If the corresponding water table slope is 2.25 in per 100 ft, and the average porosity of the aquifer is 0.15, estimate its hydraulic conductivity.*

BG

From Eq. 14.29: $V = nV_p = 0.15(0.034) = 0.00510$ ft/day ; $dh/ds = (2.25/12)/100 = 0.001875$

Eq. 14.28: $0.00510 = K(0.001875)$; $K = 2.72$ ft/day ◄

14.8.4 *From dye tests between two wells, ground water is estimated to travel at 0.0087 m/d. If the corresponding water table slope is 175 mm per 100 m, and the average porosity of the aquifer is 0.19, estimate its hydraulic conductivity.*

SI

From Eq. 14.29: $V = nV_p = 0.19(0.0087) = 0.001653$ m/d ; $dh/ds = 0.175/100 = 0.00175$

Eq. 14.28: $0.001653 = K(0.00175)$; $K = 0.945$ m/d ◄

Sec. 14.8: Flow Through Porous Media -- *Problems 14.28–14.29*

14.28 *Figure 14.28 represents a steady state flow condition through a timber crib dam. The crib is in two sections, each containing different soil: a = 8 ft, b = 14 ft, c = 5 ft, and d = 4 ft. (a) Determine the hydraulic conductivity of the soil in Region 1 if the soil in Region 2 has a hydraulic conductivity of 300 gpd/ft². (b) Determine the seepage rate through the dam in gpd per foot of dam length (i.e., per foot perpendicular to the plane of the figure).*

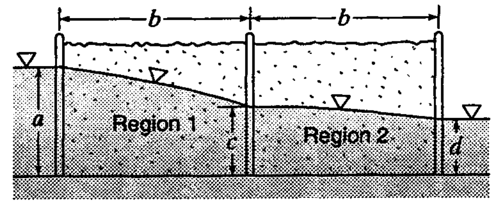


Figure P14.28

BG

(*a*) Using mean areas: $Q = \bar{A}_1 V_1 = \bar{A}_2 V_2$

and using Eq. 14.27 for V: $\frac{8+5}{2} L K_1 \frac{3}{14} = \frac{4+5}{2} L K_2 \frac{1}{14}$

so $K_1 = \frac{9}{3(13)} K_2 = \frac{9}{39}(300 \text{ gpd/ft}^2) = 62.2 \text{ gpd/ft}^2$ ◄

(*b*) $q = q_2 = \bar{A}_2 V_2 = \frac{(4+5) \text{ ft}}{2}(1 \text{ ft})\left(300 \frac{\text{gpd}}{\text{ft}^2}\right)\frac{1}{14} = 96.4$ gpd per ft length ◄

<u>Alternative solution</u> using calculus to avoid using mean areas:

(*a*) Using Eq. 14.27 for unit width: $q = hV = -Kh\frac{dh}{ds}$; $q\,ds = -Kh\,dh$; $q\int_1^2 ds = -K\int_1^2 h\,dh$

$q[s]_{s_1}^{s_2} = -K\left[\frac{1}{2}h^2\right]_{h_1}^{h_2}$; $q[s_2 - s_1] = -\frac{1}{2}K\left[h_2^2 - h_1^2\right]$ so $q(14-0) = -\frac{1}{2}K_A(5^2 - 8^2)$

$14q = \frac{1}{2}(39)K_A$; likewise $14q = -\frac{1}{2}K_B(16-25) = \frac{1}{2}(9)K_B$

$\therefore\ 39K_A = 9K_B$; $K_A = \frac{9}{39}K_B = \frac{9}{39}(300) = 62.2 \text{ gpd/ft}^2$ ◄

14.29 *Figure 14.28 represents a steady state flow condition through a timber crib dam. The crib is in two sections, each containing different soil: a = 2.5 m, b = 4.0 m, c = 1.5 m, and d = 1.2 m. (a) Determine the hydraulic conductivity of the soil in Region 1 if the soil in Region 2 has a hydraulic conductivity of 12 m/d. (b) Determine the seepage rate through the dam in L/h per meter of dam length (i.e., per meter perpendicular to the plane of the figure).*

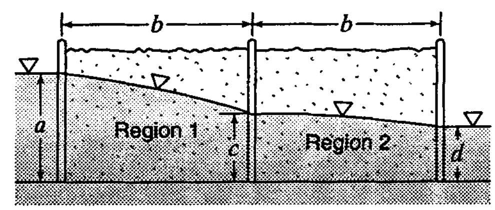


Figure P14.28

BG

(*a*) Using mean areas: $Q = \bar{A}_1 V_1 = \bar{A}_2 V_2$

and using Eq. 14.27 for V: $\frac{2.5+1.5}{2} L K_1 \frac{1.0}{4.0} = \frac{1.5+1.2}{2} L K_2 \frac{0.3}{4.0}$

so $K_1 = \frac{2.7}{4.0}\left(\frac{0.3}{1.0}\right)K_2 = \frac{0.81}{4.0}(12 \text{ m/d}) = 2.43 \text{ m/d}$ ◄

(*b*) $q = q_2 = \bar{A}_2 V_2 = \frac{(1.5+1.2) \text{ m}}{2}(1 \text{ m})(12 \text{ m/d})\left(\frac{0.3}{4.0}\right) = 1.215 \text{ m}^3\text{/d}$ per m length ◄

<u>Alternative solution</u> using calculus to avoid using mean areas:

(*a*) Using Eq. 14.27 for unit width: $q = hV = -Kh\frac{dh}{ds}$; $q\,ds = -Kh\,dh$; $q\int_1^2 ds = -K\int_1^2 h\,dh$

$q[s]_{s_1}^{s_2} = -K\left[\frac{1}{2}h^2\right]_{h_1}^{h_2}$; $q[s_2 - s_1] = -\frac{1}{2}K\left[h_2^2 - h_1^2\right]$ so $q(4.0-0) = -\frac{1}{2}K_A(1.5^2 - 2.5^2)$

$4.0q = \frac{1}{2}(4.0)K_A$; likewise $4.0q = -\frac{1}{2}K_B(1.2^2 - 1.5^2) = \frac{1}{2}(0.81)K_B$

$\therefore\ 4.0K_A = 0.81K_B$; $K_A = \frac{0.81}{4.0}K_B = \frac{0.81}{4.0}(12 \text{ m/d}) = 2.43 \text{ m/d}$ ◄

Chapter 15

Hydraulic Machinery -- Pumps

PROBLEM SELECTION GUIDE

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
15.2	*Head Developed by a Pump*						
	P 15.1	SI	Easy	Short	1		
15.3	*Pump Efficiency*						
	P 15.2	BG	Easy	Short	1		
	15.3	BG	Easy	Short	2		
	15.4	BG	Easy	Short	1		
	15.5	BG	V Easy	V Short	1		
	15.6	SI	V Easy	V Short	1		
15.4	*Similarity Laws for Pumps*						
	X[1]15.4.1	SI	Easy	Short	1	S15.2	
	15.4.2	BG	Easy	Short	2	15.4.3	
	15.4.3	SI	Easy	Short	2	15.4.2	
	15.4.4	BG	Easy	Short	1		
	P 15.7	BG	Easy	Short	2		
	15.8	SI	Easy	Short	1		
	15.9	SI	Easy	Short	1		
	15.10	N	V Easy	V Short	1		
15.6	*Performance Characteristics at Different Speeds and Sizes*						
	X 15.6.1	BG	Easy	Short	1		
	15.6.2	BG	Easy	Short	1		
	15.6.3	BG	Easy	Medium	1		Plot graph
	P 15.11	BG	Easy	Short	1		
	15.12	BG	Easy	Medium	1		Plot graph
	15.13	BG	Easy	Medium	1		Plot graph
15.7	*Operating Point of a Pump*						
	X 15.7.1	BG	Medium	Long	3	S15.4,15.7.2	† Plot curves
	15.7.2	BG	Medium	Long	3	S15.4,15.7.1	† Plot curves
	P 15.14	BG	Medium	Medium	1		
	15.15	BG	Medium	Long	3	S15.4,15.16	† Plot curves
	15.16	BG	Hard	Long	3	S15.4,15.15	† Minor losses. Plot curves
	15.17	BG	Medium	Long	2	15.18	† Plot curve
	15.18	BG	Medium	Long	2	15.17	† Plot curve
	15.19	SI	Medium	Long	1		† Plot graph

/cont...

[1] For all Exercises (identified by "X"), answers are given in Appendix F of the textbook.
† Answers are sensitive to values (ϕ_e, σ_c, etc.) read from a figure or plotted graph.
X = Exercise, P = (end-of-chapter) Problem, S = Sample Problem.

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
15.8	***Specific Speed of Pumps***						
	X 15.8.1	N	Medium	Medium	1		Derivation
	15.8.2	BG	Easy	V Short	1	S15.5	
	15.8.3	BG	Easy	Short	1	S15.5	
	15.8.4	BG	Easy	V Short	1		
	15.8.5	SI	Easy	V Short	1		
	P 15.20	BG	Easy	Short	2		
	15.21	BG	Easy	Medium	2		
	15.22	BG	Easy	Medium	2		
	15.23	BG	Easy	Short	1		
	15.24	SI	Easy	Short	1		
	15.25	SI	Easy	Medium	3		
15.9	***Peripheral-Velocity Factor***						
	X 15.9.1	BG	Easy	Short	1	S15.7	†
	15.9.2	BG	Easy	V Short	1	S15.7	†
	15.9.3	B	Easy	Medium	1		†
	P 15.26	BG	Easy	Short	1	15.27	†
	15.27	B	Easy	Short	1	15.26	†
	15.28	B	Easy	Short	1		†
	15.29	B	Easy	Short	1		†
	15.30	BG	Hard	Long	1		† Trial and Error
	15.31	BG	Medium	Medium	1		†
	15.32	BG	Easy	Short	1		†
	15.33	BG	Easy	Short	1		†
	15.34	BG	Easy	V Short	1		†
	15.35	SI	Medium	V Short	1		
15.10	***Cavitation in Pumps***						
	X 15.10.1	BG	Medium	Short	1	15.10.2	†
	15.10.2	BG	Medium	Short	1	15.10.1	†
	15.10.3	SI	Medium	Short	1	S15.8, P15.37, P15.44	
	P 15.36	SI	Medium	V Short	1		
	15.37	BG	Medium	Short	1	S15.8, X15.10.3, 15.44	
	15.38	BG	Medium	Short	1	15.39	
	15.39	BG	Medium	Medium	1	15.38	
	15.40	BG	Medium	Short	1		
	15.41	SI	Medium	Medium	1		†
	15.42	SI	Hard	Long	1	15.43	†
	15.43	SI	Hard	Long	1	15.42	†
	15.44	SI	Medium	Short	1	S15.8, X15.10.3, 15.37	
	15.45	BG	Medium	Medium	1		†

/cont...

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
15.11	*Viscosity Effect*						
	P 15.46	B	Medium	Short	1		†
15.12	*Selection of Pumps*						
	X 15.12.1	BG	Medium	Medium	2	S15.10	†
	15.12.2	BG	Medium	Medium	2	S15.10	†
	15.12.3	SI	Hard	Long	1		†
	P 15.47	BG	Medium	Long	4	15.48	†
	15.48	BG	Medium	Long	4	15.47	†
	15.49	BG	Medium	Medium	3		†
	15.50	BG	Hard	Medium	1		†
	15.51	BG	Medium	Medium	1	15.52	† Other solutions are possible.
	15.52	BG	Medium	Long	1	15.51	† Other solutions are possible.
	15.53	BG	Medium	Medium	1		Must estimate *f* (Secs. 8.13-14)
	15.54	SI	Hard	Medium	1		Partly unanswerable.
15.13	*Pumps Operating in Series and Parallel*						
	X 15.13.1	BG	Medium	Long	3	S15.13	† Plot curves
	15.13.2	BG	Medium	Medium	2	S15.13	†
	P 15.55	BG	Medium	Medium	1		† Other solutions are possible.
	15.56	BG	Medium	Medium	1		† Other solutions are possible.
	15.57	BG	Hard	Long	2		† Uses Sec. 8.27. Plot curves
	15.58	BG	Medium	Medium	1		
Ch.15	*Miscellaneous*						
	P 15.59	BG	Medium	Long	1		† Plot curves, graph
	15.60	BG	Easy	Short	1		†
	15.61	B	Easy	Medium	1		
	15.62	BG	Easy	Short	1		†

Chapter 15
HYDRAULIC MACHINERY – PUMPS

Sec. 15.2: Head Developed by a Pump -- *Problem 15.1*

15.1 *The diameter of the discharge side of a pump is 100 mm, and that of the intake pipe is 120 mm (Fig. P15.1). The pressure gage at discharge reads 240 kPa and the pressure gage at intake reads 60 kPa. If Q = 45 L/s of water and the pump efficiency is 0.82, find the power delivered to the pump by the drive shaft. The intake of the pump is 350 mm below the discharge side. Neglect the effects of pre-rotation in the entry pipe and assume smooth flow at discharge from the pump.*

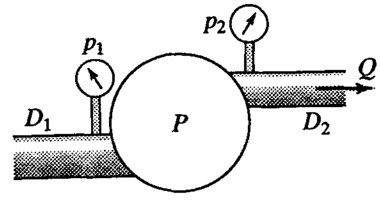


Figure P15.1

SI

$V_s = Q/A_s = (0.045\ \text{m}^3/\text{s})/(\pi 0.120^2/4) = 3.98\ \text{m/s}$

$V_d = V_s(120/100)^2 = 5.73\ \text{m/s}$

Eq. 15.1: $h = (p_d/\gamma + V_d^2/2g + z_d) - (p_s/\gamma + V_s^2/2g + z_s)$

$$h = \left(\frac{240\ \text{kN/m}^2}{9.81\ \text{kN/m}^3} + \frac{5.73^2}{2(9.81)} + 0.35\right) - \left(\frac{60\ \text{kN/m}^2}{9.81\ \text{kN/m}^3} + \frac{3.98^2}{2(9.81)} + 0\right) = 19.56\ \text{m}$$

Eq. 15.2: $P = \gamma Q h/\eta = 9.81(0.045)19.56/0.82 = 10.53\ \text{kW}$ ◄

Sec. 15.3: Pump Efficiency -- *Problems 15.2–15.6*

15.2 *The diameter of the discharge side of a pump is 6 in, and that of the intake pipe is 8 in (Fig. P15.1). The pressure gage at discharge reads 30 psi, and the vacuum gage at intake reads 10 inHg. If Q = 3.0 cfs of water and the brake horsepower is 35.0, find the efficiency of the pump. The intake and discharge are at the same elevation.*

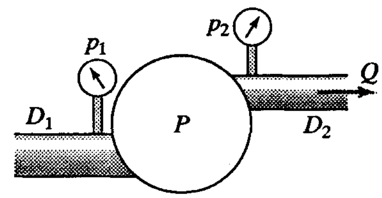


Figure P15.1

BG

$V_d = Q/A_d = 3.0/(\pi 0.5^2/4) = 15.29\ \text{fps};\quad V_s = V_d(6/8)^2 = 8.60\ \text{fps}$

10 inHg vacuum is equivalent to $-33.9(10.0/29.9) = -11.34$ ft of water

$$\text{Eq. 15.1: } h = \left(\frac{p_d}{\gamma} + \frac{V_d^2}{2g}\right) - \left(\frac{p_s}{\gamma} + \frac{V_s^2}{2g}\right) = \left(\frac{30(144)}{62.4} + \frac{15.29^2}{64.4}\right) - \left(-11.34^2 + \frac{8.60^2}{64.4}\right)$$

$$= 69.2 + 3.6 + 11.34 - 1.15 = 83.0\ \text{ft}$$

Eq. 15.2: $\eta = (\gamma Q h/550)/35.0 = 62.4(3.0)83.0/(550 \times 35) = 0.81 = 81\%$ ◄

15.3 *When operating at its BEP, the pump of Fig. 15.6 develops 60 ft of head at a capacity of 10,500 gpm. (a) Calculate the horsepower delivered to the water by the pump. (b) If the brake horsepower is 192 hp, what is the efficiency of the pump at its BEP? (c) Does your answer to (b) agree with Fig. 15.6?*

Fig. 15.6: At Q = 10,500 gpm (the BEP), pump efficiency = 83%.

BG

(*a*) Water Power = $\gamma Q h/550 = 62.4(10{,}500/449)60/550 = 159.3$ hp ◄

(*b*) $\eta = 159.3/192 = 0.83 = 83\%$ ◄ (*c*) Yes, answer (*b*) agrees with Fig. 15.6 ◄

15.4 *The pump of Fig. 15.6 delivers 29 ft of head at a capacity of 15,000 gpm. The pump efficiency under these conditions of operation is 47%. Calculate the water horsepower and the brake horsepower, and check to see if your answers agree with Fig. 15.6.*

Fig. 15.6: At Q = 15,000 gpm, water horsepower = 110, brake horsepower = 235.

BG

Water horsepower = $\gamma Q h/550 = 62.4(15{,}000/449)29/550 = 109.9$ hp ◄

Brake horsepower = Water hp/η = 109.9/0.47 = 234 hp ◄ They check closely with Fig. 15.6 ◄

15.5 *A centrifugal pump delivers 40 gpm of gasoline (s = 0.72) when developing a net head of 15 ft. If the shaft power is 0.126 hp, what is the efficiency of the pump?*

BG

Eq. 15.2: $\eta = \gamma Qh/\text{shaft power} = 0.72(62.4)(40/449)15/(550\times 0.126) = 0.87$ ◄

15.6 *A pump delivers 1000 L/s of water at a head of 8.5 m. If the efficiency of the pump is 68%, what is the shaft power?*

SI

Eq. 15.2: Shaft power $= \gamma Qh/\eta = 9.81(1.0)8.5/0.68 = 122.6$ kW ◄

Sec. 15.4: Similarity Laws for Pumps -- ***Exercises*** *(4)*

15.4.1 *A model centrifugal pump has a scale ratio of 1:15. The model when tested at 3600 rpm, delivered 0.10 m³/s of water at a head of 40 m with an efficiency of 80 percent. Assuming the prototype has an efficiency of 88 percent, what will be its speed, capacity, and power requirement at a head of 50 m?*

SI

Eq. 15.5: $h_m = K_h D_m^2 n_m^2$; $40 = K_h(D_p/15)^2 3600^2$; $K_h = 40(225)/[D_p^2 3600^2] = 0.000\,694/D_p^2$

Eq. 15.5: $h_p = K_h D_p^2 n^2 = 50 = (0.000\,694/D_p^2)D_p^2 n_p^2$; $n_p^2 = 72{,}046$; $n_p = 268$ rpm ◄

From Eq. 15.4: $Q \propto D^3 n$; $Q_p = Q_m(15)^3(268/3600) = 251Q_m$; $Q_p = 251(0.10) = 25.1$ m³/s ◄

From Eq. 15.2: $P_p = \gamma Q_p h_p/\eta = 9.81(25.1)50/0.88 = 14\,000$ kW ◄

15.4.2 *A 19½-in-diameter centrifugal-pump impeller discharges 20 cfs at a head of 100 ft when running at 1200 rpm. (a) If its efficiency is 85 percent, what is the brake horsepower, i.e., what is the horsepower input to the shaft of the pump? (b) If the same pump were run at 1500 rpm, what would be h, Q, and the brake horsepower for homologous conditions?*

BG

(*a*) Eq. 15.2: $bhp = \gamma Qh/(550\eta) = 62.4(20)100/[500(0.85)] = 267$ bhp ◄

(*b*) Eq. 15.5: $h = 100(1500/1200)^2 = 156.3$ ft ◄

Eq. 15.4: $Q = 20(1500/1200) = 25$ cfs ◄

Eq. 15.6: $bhp = 267(1500/1200)^3 = 521$ bhp ◄

15.4.3 *An axial-flow pump delivers 300 L/s at a head of 6.0 m when rotating at 1800 rpm. (a) If its efficiency is 80 percent, how many kilowatts of power must the shaft deliver to the pump? (b) If this same pump were operated at 1500 rpm, what would be h, Q, and the power delivered by the shaft for homologous conditions?*

SI

(*a*) Eq. 15.2: Power $= \gamma Qh/\eta = 9.81(0.3)6/0.8 = 22.1$ kW ◄

(*b*) Eqs. 15.4, 15.5, and 15.6: $Q \propto n,\ \ h \propto n^2,\ \ hp \propto n^3$

$\therefore\ Q = 300(1800/1500) = 360$ L/s ◄

$h = 6(1800/1500)^2 = 8.64$ m ◄

$P = 22.1(1800/1500)^3 = 38.1$ kW ◄

15.4.4 *In Sample Prob. 15.2 the model with a scale ratio of 1:10 has a maximum efficiency of 84%. Using Eq. (15.7), calculate the maximum efficiency of the prototype pump and compare the result with the data given in the sample problem. Note that Eq. (15.7) is empirical, and will provide only approximate values.*
Sample Prob. 15.2: Assume a prototype pump efficiency of 90%.

BG

From Eq. 15.7: $1 - \eta_p = (1 - \eta_m)(D_m/D_p)^{1/5} = (1 - 0.84)(1/10)^{1/5} = 0.1010$

$\eta_p = 1 - 0.1010 = 0.899 = 89.9\%$ ◄

This compares very well with the prototype efficiency of 90% assumed in Sample Prob. 15.2 ◄

Sec. 15.4: Similarity Laws for Pumps -- *Problems* *15.7–15.10*

15.7 *A centrifugal pump with an 18-in-diameter impeller is rated at 25 cfs under a head of 100 ft when running at 1200 rpm. (a) If its efficiency is 85%, what is the brake horsepower? (b) If the same pump is run at 1800 rpm, what would be its rating in terms of h, Q, and the brake horsepower? Assume the efficiency does not change with the change in speed.*

BG

(*a*) Eq. 15.2: Brake hp $= \gamma Qh/(550\eta) = 62.4(25)100/(550 \times 0.85) = 333.7$ ◄

(*b*) Eq. 15.4: $Q \propto n$, $Q = 25(1800/1200) = 37.5$ cfs ◄

Eq. 15.5: $h \propto n^2$, $h = 100(1800/1200)^2 = 225$ ft ◄

Eq. 15.2: Brake hp $= 62.4(37.5)2.25/(550 \times 0.85) = 1126$

or Eq. 15.6: $P \propto n^3$, $P = 333.7(1800/1200)^3 = 1126$ hp ◄

15.8 *A centrifugal pump with a 300-mm-diameter impeller is rated at 40 L/s against a head of 25 m when rotating at 1750 rpm. What would be the rating of a pump of identical geometric shape with a 200-mm-diameter impeller? Assume pump efficiencies and pump speeds are identical.*

SI

Eq. 15.4: $Q \propto D^3$, $Q = 40(200/300)^3 = 11.85$ L/s ◄

Eq. 15.5: $h \propto D^2$, $h = 25(200/300)^2 = 11.11$ m ◄

Pump rating: 11.85 L/s against a head of 11.11 m ◄

15.9 *At its optimum point of operation a given centrifugal pump delivers 3.2 m³/s of water against a head of 25 m when rotating at 1450 rpm. (a) If the efficiency is 82%, what is the brake power of the drive shaft? (b) If a homologous pump with an impeller diameter one half as large is rotating at 1200 rpm, what would be the discharge, head, and shaft power at its point of optimum efficiency? Assume both pumps operate at the same efficiency.*

SI

Eq. 15.4: $Q \propto nD^3$, $Q = 3.2(1200/1450)(1/2)^3 = 0.33$ m³/s ◄

Eq. 15.5: $h \propto n^2D^2$, $h = 25(1200/1450)^2(1/2)^2 = 4.28$ m ◄

Eq. 15.2: Brake power $= \gamma Qh/\eta = 9.81(0.33)4.28/0.82 = 16.9$ kW ◄

15.10 *Using Eq.(15.7), find the approximate efficiency of the smaller pump of Problem 15.9.*
Prob. 15.9: $D_m = 0.5D_p$, $\eta_p = 0.82$

N

Eq. 15.7: $(1 - \eta_p)/(1 - \eta_m) = (D_m/D_p)^{1/5}$; $(1 - 0.82)/(1 - \eta_m) = (1/2)^{1/5} = 0.87$

From which $\eta_m = 0.793 = 79.3\%$ ◄

Sec. 15.6: Performance Characteristics at Different Speeds and Sizes -- *Exercises* *(3)*

15.6.1 *Refer to Fig. 15.8. Transfer the point ($h = 70$ ft and $Q = 7500$ gpm) to a point on Curve 3 if $n_1 = 1450$ rpm and $n_3 = 1160$ rpm. Check your result with Fig. 15.8.*

BG

Eqs. 15.4 and 15.5: $Q \propto n$, $h \propto n^2$

$Q_3/Q_1 = n_3/n_1$; $Q_3 = Q_1(n_3/n_1) = 7500(1160/1450) = 6000$ gpm ◄

$h_3/h_1 = (n_3/n_1)^2$; $h_3 = h_1(n_3/n_1)^2 = 70(1160/1450)^2 = 44.8$ ft ◄

These check with curve 3 of Fig. 15.8. ◄

15.6.2 *Refer to Fig. 15.9. Transfer the point ($h = 70$ ft and $Q = 7500$ gpm) to a point on the D_2 curve if $D_1 = 16.7$ in and $D_2 = 17.5$ in. Check your result with Fig. 15.9.*

BG

Eqs. 15.4 and 15.5: $Q \propto D^3, \quad h \propto D^2$

$Q_2 = Q_1(D_2/D_1)^3 = 7500(17.5/16.7)^3 = 8630$ gpm ◄

$h_2 = h_1(D_2/D_1)^2 = 70(17.5/16.7)^2 = 76.9$ ft ◄ These check with the D_2 curve ◄

15.6.3 *Plot h versus Q for the pump of Fig. 15.7 for the case where $n = 600$ rpm.*
Fig. 15.7: For $Q = 0, 75, 122$ cfs respectively $h = 40, 23, 16$ ft.

BG

Eqs. 15.4 and 15.5: $Q \propto n, \quad h \propto n^2$; $n_2/n_1 = 600/450 = 1.333$; $(n_2/n_1)^2 = 1.778$

Shutoff head $= 40(1.778) = 71.1$ ft, $Q = 0$ cfs

BEP: $h = 16(1.778) = 28.4$ ft ◄ $Q = 122(1.333) = 162.6$ cfs

etc.: 75 cfs at 23 ft becomes $Q = 75(1.333) = 100$ cfs ◄ at $h = 23(1.778) = 40.9$ ft ◄

Sec. 15.6: Performance Characteristics at Different Speeds and Sizes -- ***Problems 15.11−15.13***

15.11 *At its BEP, a given pump develops 30 ft of head when operating at 720 rpm. The flow rate is 4.2 cfs. Find the head and flow rate for a homologous pump operating at 600 rpm if its diameter is 90% that of the given pump.*

BG

Eq. 15.4: $Q \propto nD^3, \quad Q = 4.2(600/720)0.9^3 = 2.55$ cfs ◄

Eq. 15.5: $h \propto n^2D^2, \quad h = 30(600/720)^2 0.9^2 = 16.88$ ft ◄

15.12 *Plot h versus Q for the pump of Fig. 15.6 for the case where $n = 1200$ rpm.*
Fig. 15.6 ($n = 1450$ rpm): For $Q = 0, 6000, 10{,}500, 15{,}000$ gpm respectively, $h = 80, 73, 60, 29$ ft.

BG

Eq. 15.4: $Q \propto n, \quad Q' = Q(1200/1450) = 0.828Q$; Eq. 15.5: $h \propto n^2, \quad h' = h(1200/1450)^2 = 0.685h$

$Q = 0$, $h = 80$ ft becomes 0 gpm, 54.8 ft ◄

$Q = 6000$ gpm, $h = 73$ ft becomes 4970 gpm, 50.0 ft ◄

$Q = 10{,}500$ gpm, $h = 60$ ft becomes 8690 gpm, 41.1 ft ◄

$Q = 15{,}000$ gpm, $h = 29$ ft becomes 12,420 gpm, 19.9 ft ◄

etc.

15.13 *Plot h versus Q for the pump of Fig. 15.7 for the case where $n = 600$ rpm.*
Fig. 15.7: For $Q = 0, 50, 122$ cfs respectively $h = 40, 26, 16$ ft.

BG

Eq. 15.4: $Q \propto n, \quad Q' = Q(600/450) = 1.333Q$; Eq. 15.5: $h \propto n^2, \quad h' = h(600/450)^2 = 1.777h$

0 cfs, 40 ft becomes 0 cfs, 71.1 ft ◄

50 cfs, 26 ft becomes 66.7 cfs, 46.2 ft ◄

122 cfs, 16 ft becomes 162.6 cfs, 28.4 ft ◄ etc.

Sec. 15.7: Operating Point of a Pump – *Exercises (2)*

15.7.1 *Repeat Sample Prob. 15.4 for the case where the length of the 18-inch diameter pipe is 1000 ft.*
Sample Prob. 15.4: $f = 0.032$, $L = 500$ ft, $D = 18$ in, resulting in $h = \Delta z + 0.0530Q^2$.

BG

Refer to the solution to Sample Prob. 15.4 (textbook page 661).

This 1000 ft pipe is twice as long as the 500 ft pipe of Sample Prob. 15.4.

∴ here, $h = \Delta z + 2(0.0530)Q^2 = \Delta z + 0.1061Q^2$ where Q is in cfs. Coordinates of system curves:

Q (cfs)	Q (gpm)	(*a*) $h = h_f$ (ft)	(*b*) $20 + h_f$	(*c*) $65 + h_f$
0	0	0	20	65
10	4490	10.6	30.6	75.6
20	8980	42.4	62.4	107.4
30	13,460	95.6	115.6	160.6

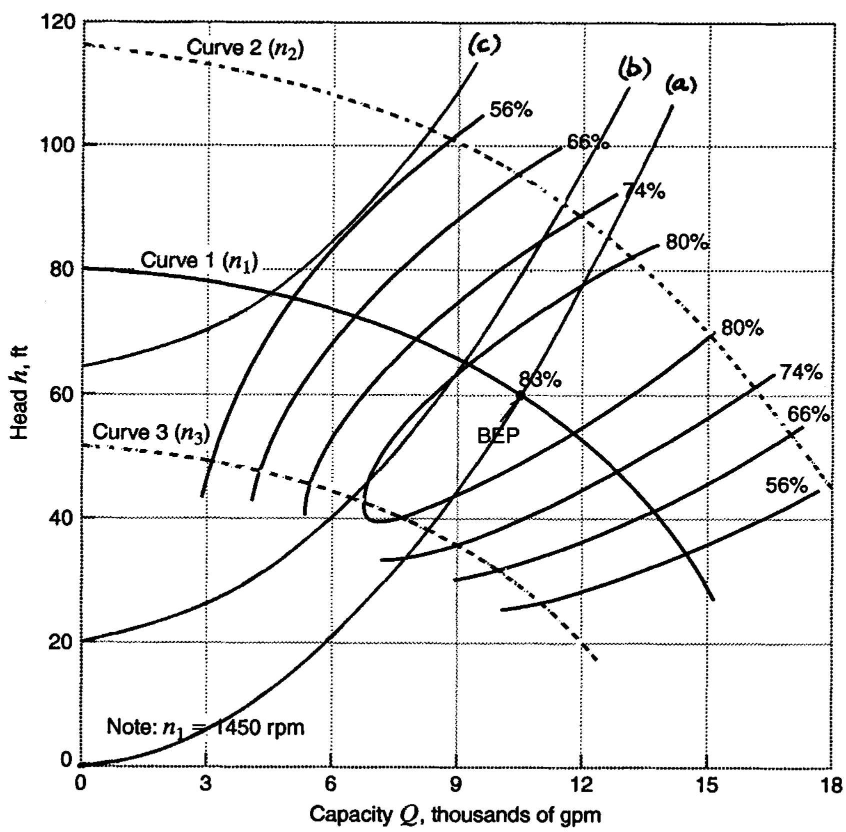


Figure 15.8, modified

By plotting the system curves on Fig. 15.8 we find the points of intersection with Curve 1:

(*a*) $h \approx 60$ ft and $Q \approx 10{,}500$ gpm ◄

(*b*) $h \approx 65$ ft and $Q \approx 9200$ gpm ◄

(*c*) $h \approx 76$ ft and $Q \approx 4500$ gpm ◄

From the same figure, approx. pump efficiencies are: 83%, 79%, and 52% respectively ◄

15.7.2 *Repeat Sample Prob. 15.4 for the case where the pipe diameter is 12 inches rather than 18 inches, all other data remaining the same.*

Sample Prob. 15.4: $f = 0.032$, $L = 500$ ft, $D = 18$ in, resulting in $h = \Delta z + 0.0530Q^2$.

BG

Note: Pipe friction $h_f = f(L/D)V^2/2g$; $V = Q/A = Q/(\pi D^2/4)$; $V \propto 1/D^2$, $V^2 \propto 1/D^4$

$$\therefore \text{ pipe friction } h_f \propto 1/D^5 \ ; \quad \frac{\text{Pipe friction } h_f \text{ for } D = 12 \text{ in}}{\text{Pipe friction } h_f \text{ for } D = 18 \text{ in}} = \left(\frac{18}{12}\right)^5 = 7.59$$

Sample Prob. 15.4 for 18-1n-diameter pipe: $h_f = 0.0530Q^2$ (Q expressed in cfs)

Thus, for the 12-in-diameter pipe, $h = \Delta z + 7.59(0.0530)Q^2 = \Delta z + 0.403Q^2$

Coordinates of system curves:

		(*a*)	(*b*)	(*c*)
Q (cfs)	Q (gpm)	$h = h_f$ (ft)	$20 + h_f$	$65 + h_f$
0	0	0	20	65
5	2240	10.1	30.1	75.1
10	4490	40.3	60.3	105.3
15	6730	90.7	110.7	155.7

By plotting the system curves on Fig. 15.8 we find the points of intersection with Curve 1.

(*a*) $h \approx 73$ ft and $Q \approx 6200$ gpm ◄

(*b*) $h \approx 75$ ft and $Q \approx 5300$ gpm ◄

(*c*) $h \approx 78$ ft and $Q \approx 2500$ gpm ◄

Approx. pump efficiencies are: 63%, 56%, and 33% respectively ◄

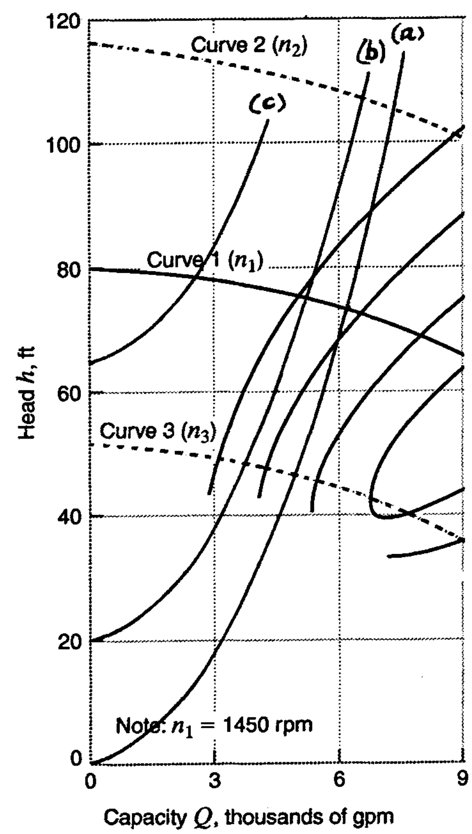


Figure 15.8, modified

Sec. 15.7: Operating Point of a Pump -- *Problems* 15.14–15.19

15.14 *A centrifugal pump is installed to deliver water from a reservoir of water surface elevation zero to another of elevation 300 ft (Fig. P15.14). The 12-in-diameter suction pipe (f = 0.020) is 100 ft long and the 10-in-diameter discharge pipe (f = 0.026) is 5000 ft long. The pump characteristic at 1200 rpm is defined by $h_p = 375 - 24Q^2$, where the pump head h_p is in ft and Q is in cfs. Calculate the rate at which this pump will deliver water, assuming the setting is low enough to avoid cavitation.*

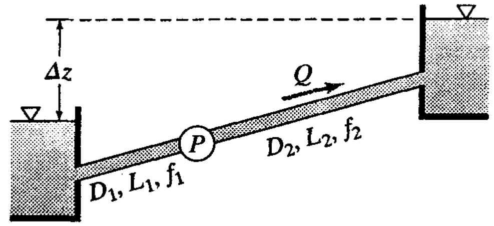


Figure P15.14

BG

Energy equation neglecting minor loss at discharge:

$$0 - 0.020(100/1)V_{12}^2/2g + h_p - 0.026[5000/(10/12)]V_{10}^2/2g = 300$$

$$V_{12} = \frac{Q}{\pi(6/12)^2} = 1.273Q, \quad V_{10} = \frac{Q}{\pi(5/12)^2} = 1.833Q$$

From the above: $h_p = 300 + 8.19Q^2$ (system characteristic)

Given the pump characteristic is: $h_p = 375 - 24Q^2$

Equating the h's: $300 + 8.19Q^2 = 375 - 24Q^2$

or $32.19Q^2 = 75$ from which

$Q = 1.526$ cfs $= 1.526(449$ gpm$) = 685$ gpm ◄

15.15 *A pump whose characteristics are shown on Fig. 15.7 when operating at 450 rpm is used to pump water from reservoir A to reservoir B through a 24-in-diameter pipe (f = 0.028) of length 170 ft. Neglecting minor losses, find the flow rate and pump efficiency for the following conditions; (a) reservoir water-surface elevations identical; (b) water surface in reservoir B is 10 ft higher than that in reservoir A; (c) water-surface elevation in reservoir B is 10 ft lower that in reservoir A.*

BG

Pump characteristic curve is given in Fig. 15.7 (see below).

Friction head loss: $h_f = f(L/D)(V^2/2g) = f(L/D)Q^2/[(\pi r^2)^2 2g] = 0.028(170/2.0)Q^2/[(\pi 1^2)^2(2)32.2]$

$h_f = 0.00375Q^2$ (Q in cfs)

System characteristic curves: $z_A + h_p - h_f = z_B$; $h_p = h_f + (z_B - z_A)$

(*a*) $h_p = h_f$; (*b*) $h_p = h_f + 10$; (*c*) $h_p = h_f - 10$

Coordinates of system curves:

Q (cfs)	(*a*) h_f (ft)	(*b*) $h_f + 10$	(*c*) $h_f - 10$
50	9.4	19.4	−0.6
70	18.4	28.4	8.4
90	30.4	40.4	20.4
100	37.5	47.5*	27.5

*For plotting purposes only

By plotting the curves we find h and Q, the points of intersection:

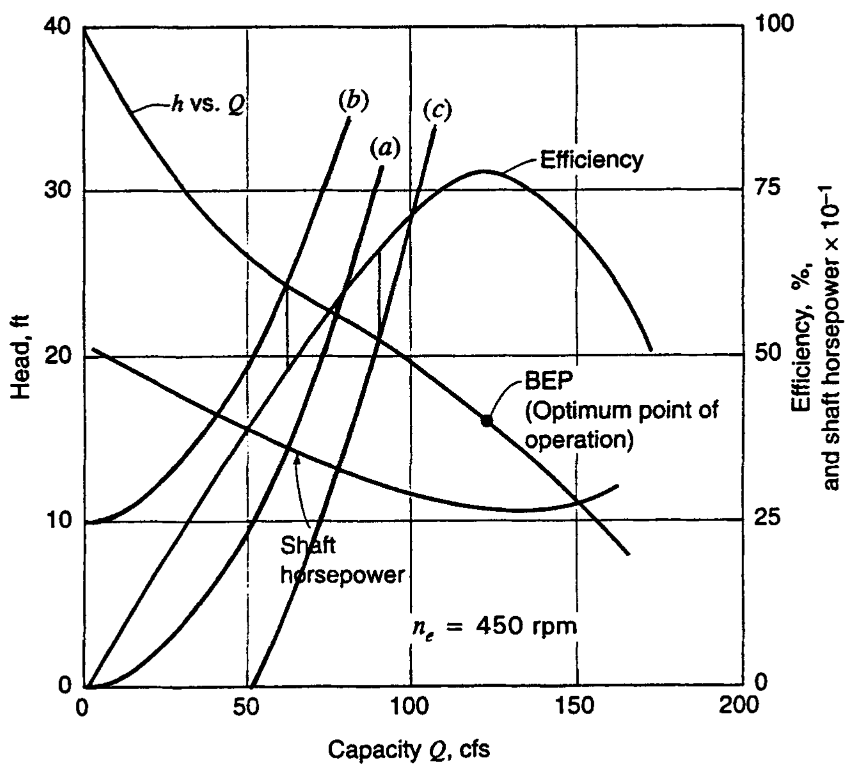


Figure 15.7, modified

(*a*) $h \approx 22$ ft and $Q \approx 76$ cfs, $\eta \approx 60\%$ ◄

(*b*) $h \approx 24$ ft and $Q \approx 62$ cfs, $\eta \approx 48\%$ ◄

(*c*) $h \approx 21$ ft and $Q \approx 92$ cfs, $\eta \approx 66\%$ ◄

15.16 *Repeat Prob. 15.15 using the same data except consider minor losses: square entrance and submerged discharge. Neglect bend losses.*

Prob. 15.15: A pump with the characteristics of Fig. 15.7 (see with Solution 15.15) pumps water from reservoir A to reservoir B through a 170-ft-long pipeline (f = 0.028, D = 24 in). Find Q and the pump η for (a) reservoir water surface elevations identical, (b) water surface B 10 ft higher than A; (c) water surface B 10 ft lower than A.

BG

The pump characteristic curve is given in Fig. 15.7 (see previous problem).

Friction head loss: $h_f = f(L/D)(V^2/2g) = f(L/D)Q^2/[(\pi r^2)^2 2g] = 0.028(170/2.0)Q^2/[(\pi 1^2)^2(2)32.2]$

$h_f = 0.00375Q^2$ (Q in cfs). Consider minor losses: $0.5(V^2/2g) + 1.0(V^2/2g) = 1.5(V^2/2g)$

$V = Q/A = Q/(\pi 1^2) = Q/\pi$; $1.5(V^2/2g) = 1.5Q^2/[(\pi)^2(2)32.2] = 0.002\,36Q^2$

Total head loss $h_L = h_f + h' = 0.00375Q^2 + 0.00236Q^2 = 0.00611Q^2$

System charateristic curves: (*a*) $h = h_L$; (*b*) $h = h_L + 10$; (*c*) $h = h_L - 10$

Coordinates of system curves:

Q (cfs)	(*a*) h_L (ft)	(*b*) $h_L + 10$	(*c*) $h_L - 10$
30	5.5	15.5	−4.5
50	15.3	25.3	5.3
70	29.9	39.9	19.9
90	49.5	59.5	39.5

By plotting the curves we find h and Q, the points of intersection:

(*a*) $h \approx 25$ ft and $Q \approx 60$ cfs, $\eta \approx 48\%$ ◄

(*b*) $h \approx 26$ ft and $Q \approx 52$ cfs, $\eta \approx 39\%$ ◄

(*c*) $h \approx 23$ ft and $Q \approx 74$ cfs, $\eta \approx 56\%$ ◄

15.17 *A pump whose characteristics are shown in Fig. 15.8, operating at 1450 rpm, is placed in a 24-in-diameter pipe (f = 0.032) of length 5000 ft (Fig. P15.17). The discharge end of the pipe is 30.0 ft lower than the water-surface elevation in the reservoir. The water discharges freely into the atmosphere. (a) Find the flow rate. (b) What would be the flow rate if the pump had not been installed in the pipeline?*

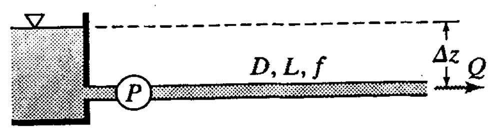


Figure P15.17

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(*a*) Energy equation: $0 + 30 + 0 + h_p - 0.032(5000/2)V^2/2g = 0 + 0 + V^2/2g$ (1)

$V = Q/A = Q/(\pi 1^2) = Q/\pi$ (2)

Substitute (2) into (1) to obtain system characteristic: $h_p = 0.1259Q^2 - 30$

Q (gpm)	Q (cfs)	h_p (ft)
3000	6.68	−24.4
6000	13.37	−7.5
9000	20.1	20.6
12,000	26.7	60.0
15,000	33.4	110.6

Plot the system curve on Fig. 15.8 (see Fig. with Solution 15.18) and find the point of intersection with

Curve 1: $h \approx 55$ ft, $Q \approx 11{,}800$ gpm, $\eta \approx 81\%$ ◄

(*b*) If there was no pump: $30 - 0.1259Q^2 = 0$ or $Q = \sqrt{30/0.1259} = 15.44$ cfs $= 6930$ gpm ◄

15.18 *Repeat Problem 15.17 with all data the same except operate the pump at 1160 rpm rather than 1450 rpm. Note the pump efficiency under this mode of operation.*

Prob. 15.17: A pump with the characteristics of Fig. 15.8 ($n = n_1 = 1450$ rpm) pumps water from a reservoir through a 5000-ft-long pipeline ($f = 0.032$, $D = 24$ in) to discharge freely into the atmosphere 30 ft lower than the reservoir surface (Fig. P15.17). (a) Find Q. (b) Find Q if the pump had not been installed.

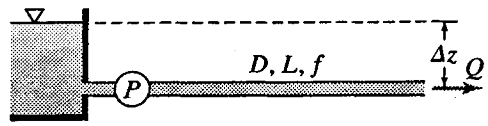


Figure P15.17

BG

Sec. 15.6: Characteristic performance curves for the same pump operating at different speeds are related approximately by Eqs. 15.4 and 15.5, as shown in Fig. 15.8.

At shutoff ($Q = 0$), $h = 80$ ft at 1450 rpm

Eq. 15.5: $h \propto n^2$

With $n = 1200$ rpm, $h = 80(1160/1450)^2 = 51.2$ ft

In Fig. 15.8 when $Q = 0$ we see the n_3 curve has an h of approx 51.5 ft. Hence the n_3 curve may be taken to represent the pump characteristic for operation at 1160 rpm.

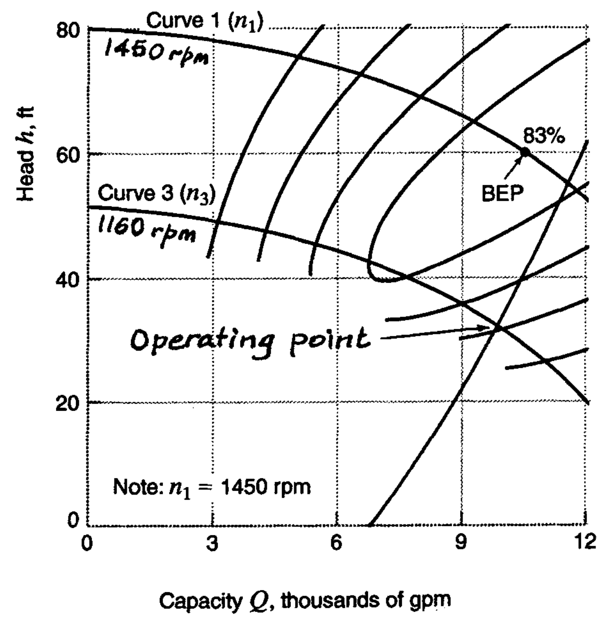


Figure 15.8, modified

Energy equation:

(*a*) $30 + h_p - 0.032(5000/2)V^2/2g = V^2/2g$ (1)

$V = Q/A = Q/(\pi 1^2) = Q/\pi$ (2)

Substitute (2) into (1) to obtain system characteristic: $h_p = 0.1259Q^2 - 30$

Q (gpm)	Q (cfs)	h_p (ft)
3000	6.68	−24.4
6000	13.37	−7.5
9000	20.1	20.6
12,000	26.7	60.0
15,000	33.4	110.6

The n_3 curve and plotted system curve (see figure) intersect at $h \approx 32$ ft and $Q \approx 9900$ gpm at which point $\eta \approx 67\%$ ◀

(*b*) If there was no pump: $30 - 0.1259Q^2 = 0$ or $Q^2 = 30/0.1259 = 238$

$Q = \sqrt{238} = 15.44$ cfs $= 6930$ gpm ◀

15.19 *The characteristic curve for a pump operating at 1200 rpm is given by the following:*

Head, m	*Q, L/s*
20	*0*
15	*165*
10	*250*

This pump is installed in a pipeline to augment the flow from reservoir A to reservoir B (Fig. 15.19). When the water-surface elevation in reservoir A exceeds that in reservoir B by 15 m, the flow rate is 150 L/s. What will the flow rate be when the elevation difference is 3 m? Assume the system curve is parabolic.

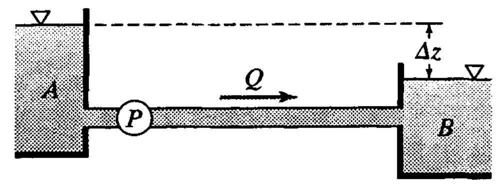
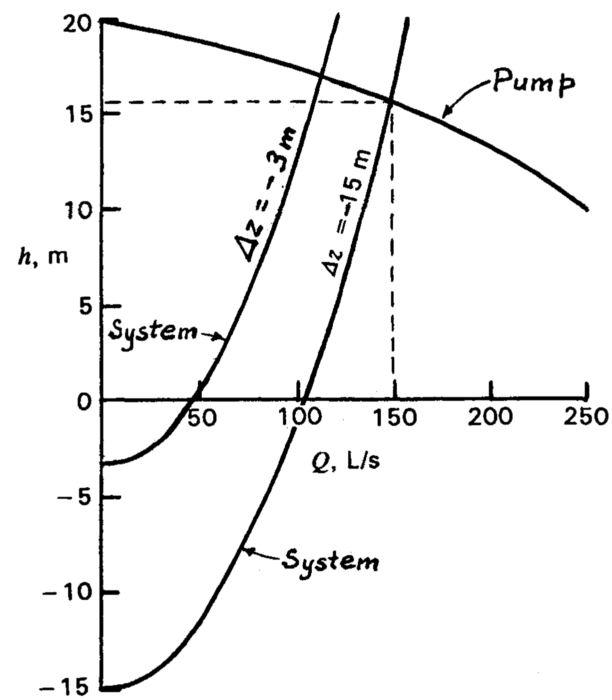


Figure P15.19

SI

Plot the pump characteristic curve from the given data (see figure).



Note (from the pump characteristic curve) that when $Q = 150$ L/s the head developed by the pump is 15.5 m.

Energy equation when $z_B - z_A = \Delta z = -15$ m:

$$15 + h_p - h_L = 0, \quad \text{where} \quad h_L = kQ^2$$

i.e., $15 + 15.5 - k(150)^2 = 0$ (Q in L/s)

$\therefore\ k = 30.5/150^2 = 0.001356$ and

$$\Delta z + h_p - 0.001356Q^2 = 0$$

When $\Delta z = -3$ m the system curve is:

$$3 + h_p - kQ^2 = 0 \quad \text{or} \quad h_p = 0.001356Q^2 - 3$$

Assume values of Q, calculate values of h_p, and plot on the diagram to find the point of intersection.

System curve for $\Delta z = -3$ m:

Q (L/s)	$0.001356Q^2$	h_p (m)
0	0	−3.0
50	3.39	0.4
100	13.55	10.6
120	19.5	16.5
130	22.9	19.9

Point of intersection is at $h \approx 17.5$ m, $Q \approx 105$ L/s ◄

Sec. 15.8: Specific Speed of Pumps – *Exercises (5)*

15.8.1 *Derive the expression for the specific speed of a pump by eliminating D from Eqs. (15.4) and (15.5) in such a way that n becomes a term in the numerator.*

N

Eq. 15.4: $Q = K_Q n D^3$; Eq. 15.5: $h = K_h n^2 D^2$

To obtain an expression in the form of Eq. 15.8 we must eliminate the D from these two equations.

Raise Eq. 15.4 to the 1/2 power: $Q^{1/2} = K_Q^{1/2} n^{1/2} D^{3/2}$ (a)

Raise Eq. 15.5 to the 3/4 power: $h^{3/4} = K_h^{3/4} n^{3/2} D^{3/2}$ (b)

Divide Eq. (a) by Eq. (b): $\dfrac{Q^{1/2}}{h^{3/4}} = \dfrac{K_Q^{1/2} n^{1/2} D^{3/2}}{K_h^{3/2} D^{3/2}} = \dfrac{K_Q^{1/2} n^{1/2}}{K_h^{3/4} n^{3/2}}$; $\dfrac{K_Q^{1/2}}{K_h^{3/4}} = \dfrac{nQ^{1/2}}{h^{3/4}} = N_s$ ◄

15.8.2 *What is the specific speed of the pump whose performance characteristics are given in Fig. 15.7? What type of pump is this?*

Fig. 15.7: BEP is at Q = 122 cfs and h = 16 ft (n_e = 450 rpm).

BG

Eq. 15.8a: $N_s = n_e\sqrt{\text{gpm}}/h^{3/4} = 450\sqrt{122(449)}/16^{3/4} = 13{,}163$ ◄

Fig. 15.11 for N_s = 13,163: This is an axial-flow pump. ◄

15.8.3 *What are the specific speeds of the pumps on the Colorado River Aqueduct at the Hayfield, Gene and Iron Mountain Plants? (See Sec. 15.14). Find also the corresponding specific speeds $(N_s)_{cfs}$.*

Sec. 15.14: The Hayfield pumps (n_e = 450 rpm) each deliver 200 cfs at h = 444 ft; the Gene pumps (n_e = 400 rpm) each deliver 200 cfs at h = 310 ft; the Iron Mountain pumps (n_e = 300 rpm) each deliver 200 cfs at h = 146 ft.

BG

Eq. 15.8a: $N_s = n_e\sqrt{\text{gpm}}/h^{3/4}$

Hayfield: $N_s = 450\sqrt{200(449)}/444^{3/4} = 1395$ ◄

Gene: $N_s = 400\sqrt{200(449)}/310^{3/4} = 1622$ ◄

Iron Mountain: $N_s = 300\sqrt{200(449)}/146^{3/4} = 2140$ ◄

Sec. 15.8: These are all radial-flow pumps. Sec. 15.8: $(N_s)_{\text{cfs}} = 0.0472(N_s)_{\text{gpm}}$. So the corresponding values of $(N_s)_{\text{cfs}}$ are 65.8, 76.6 and 101.0 respectively. ◄◄

15.8.4 *Find the specific speed of a 10-stage pump that develops a total head of 600 ft at a capacity of 1600 gpm when operating at maximum efficiency at a rotative speed of 900 rpm.*

BG

h = 600/10 = 60 ft per stage ; Eq. 15.8a: $N_s = n_e\sqrt{\text{gpm}}/h^{3/4} = 900\sqrt{1600}/60^{3/4} = 1670$ ◄

Fig. 15.11 for N_s = 1670: This is a radial-flow pump.

15.8.5 *A pump is to discharge 0.8 m^3/s at a head of 40 m when running at 300 rpm. What type of pump will be required?*

SI

Eq. 15.8b: $(N_s)_{\text{SI}} = \omega\sqrt{Q}/(gh)^{3/4}$ where $\omega = 2\pi n/60$ (ω rad/s, n rev/min)

$(N_s)_{\text{SI}} = 2\pi(300/60)\sqrt{0.8}/(9.81 \times 40)^{3/4} = 0.318$

Sec. 15.8 and Fig. 15.11: This is a radial-flow centrifugal pump. ◄

Sec. 15.8: Specific Speed of Pumps – ***Problems*** *15.20–15.25*

15.20 *All dimensions of a model pump are one-third as large as the corresponding dimensions of a prototype pump. The model pump operating at 1800 rpm delivers 3.0 cfs at a head of 35 ft. (a) Find the speed and capacity of the prototype if it is to develop a head of 35 ft. (b) Find the specific speed of the model pump and the prototype. They should be the same. Assume the efficiency of the model and prototype are the same.*

BG

(*a*) Use Eqs. 15.4 and 15.5: $Q \propto nD^3$ and $h \propto n^2D^2$

Eq. 15.5: $h_p/h_m = 1.0 = n_p^2D_p^2/(n_m^2D_m^2) = n_p^2/(1800)^2(3D_m/D_m)^2$; $n_p = 600$ rpm ◄

Eq. 15.4: $Q_p/Q_m = n_pD_p^3/(n_mD_m^3) = (600/1800)(3D_m/D_m)^3 = 9$; $Q_p = 9Q_m = 27.0$ cfs ◄

(*b*) Eq. 15.8*a*: $(N_s)_m = 1800\sqrt{3(449)}/(35)^{3/4} = 4590$ ◄

$(N_s)_p = 600\sqrt{27(449)}(35)^{3/4} = 4590$ ◄

15.21 *All dimensions of a model pump are one-third as large as the corresponding dimensions of a prototype pump. If the model pump operating at 1800 rpm delivers 3.0 cfs at a head of 35 ft, find (a) the speed and head of the prototype when it delivers 3.0 cfs. Find also (b) the specific speed of the model pump and the prototype. They should be the same. Assume the efficiency of the model and prototype are the same.*

BG

(*a*) Use Eqs. 15.4 and 15.5: $Q \propto nD^3$ and $h \propto n^2D^2$

Eq. 15.4: $Q_p/Q_m = n_pD_p^3/(n_mD_m^3) = 1 = (n_p/n_m)(3D_m/D_m)^3$; $n_p = n_m/27 = 1800/27 = 66.7$ rpm ◄

Eq. 15.5: $h_p/h_m = n_p^2D_p^2/(n_m^2D_m^2) = (66.7/1800)^2(3D_m/D_m)^2$

$h_p = h_m(1/27)^2(9/1) = h_m/81 = 35/81 = 0.432$ ft ◄

(*b*) Eq. 15.8*a*: $(N_s)_m = 1800\sqrt{3(449)}/(35)^{3/4} = 4590$ ◄

$(N_s)_p = 66.7\sqrt{3(449)}/(0.432)^{3/4} = 4590$ ◄

15.22 *All dimensions of a model pump are one-third as large as the corresponding dimensions of the prototype pump. If the model operating at 1800 rpm delivers 3.0 cfs at a head of 35 ft, find (a) the speed and capacity of the prototype if it is to develop a head of 140 ft. Assume the efficiency of the prototype and the model are the same. Find also (b) the specific speed of the model and the prototype.*

BG

(*a*) Use Eqs. 15.4 and 15.5: $Q \propto nD^3$ and $h \propto n^2D^2$

Eq. 15.5: $h_p/h_m = n_p^2D_p^2/(n_m^2D_m^2) = (140/35) = (n_p^2/n_m^2)(3D_m/D_m)^2$

$n_p = n_m(4/9)^{1/2} = 1800(2/3) = 1200$ rpm ◄

Eq. 15.4: $\dfrac{Q_p}{Q_m} = \dfrac{n_pD_p^3}{n_mD_m^3} = \dfrac{1200}{1800}\left(\dfrac{3D_m}{D_m}\right)^3$; $Q_p = Q_m\dfrac{2}{3}\left(\dfrac{27}{1}\right) = 18Q_m = 54.0$ cfs ◄

(*b*) Eq. 15.8*a*: $(N_s)_m = 1800\sqrt{3(449)}/(35)^{3/4} = 4590$ ◄

$(N_s)_p = 1200\sqrt{27(449)}/(140)^{3/4} = 4590$ ◄

15.23 *A centrifugal pump is required to deliver 1600 gpm against a head of 350 ft at a rotative speed 1150 rpm. Would a two-stage pump be more efficient than a single-stage?*

BG

One-stage pump, $h = 350$ ft ; $N_s = n_e\sqrt{\text{gpm}}/h^{3/4} = 1150\sqrt{1600}/(350)^{3/4} = 569$

Two-stage pump, $h = 350/2 = 175$ ft ; $N_s = 1150\sqrt{1600}/(175)^{3/4} = 956$

Three-stage pump, $h = 350/3 = 116.7$; $N_s = 1150\sqrt{1600}/(116.7)^{3/4} = 1295$

Fig. 15.11 indicates that pumps with $N_s = 569$ have a rather low efficiency. Therefore yes, it is better to use a two-stage (or three-stage) pump. ◄

15.24 *A pump is to deliver 2.0 m^3/s against a head of 160 m when operating at 300 rpm. What type of pump will be required?*

SI

300 rpm = $2\pi(300)/60$ = 31.4 rad/s = ω_e

Eq. 15.9*b*: $(N_s)_{SI} = \omega_e\sqrt{Q}/(gh)^{3/4} = 31.4\sqrt{2}/(9.81 \times 160)^{3/4} = 0.178$

Fig. 15.11 indicates that this is a radial-flow pump with a low specific speed (outside the range of most radial-flow centrifugal pumps). It is not very desirable because of low efficiency. ◄

15.25 *At its optimum point of operation, a given centrifugal pump with an impeller diameter of 500 mm delivers 3.2 m^3/s of water against a head of 25 m when rotating at 1450 rpm. (a) If its efficiency is 82%, what is the brake power of the drive shaft? (b) If a homologous pump with a diameter of 800 mm is rotating at 1200 rpm, what would be the discharge, head, and shaft power? Assume both pumps operate at the same efficiency. (c) Compute the specific speed of both pumps.*

SI

(*a*) Brake power = $\gamma Qh/\eta$ = 9.81(3.2)25/0.82 = 957 kW ◄

(*b*) Eq. 15.4: $Q \propto nD^3$, $Q = 3.2(1200/1450)(800/500)^3 = 10.85$ m^3/s ◄

Eq. 15.5: $h \propto n^2D^2$, $h = 25(1200/1450)^2(800/500)^2 = 43.8$ m ◄

Eq. 15.6: $P \propto n^3D^5$, $P = 957(1200/1450)^3(800/500)^5 = 5687$ kW ◄

(*c*) For n = 1450 rpm, $\omega = 2\pi n/60$ = 151.8 rad/s ; $(N_s)_{SI} = 151.8(3.2)^{1/2}/(9.81 \times 25)^{3/4} = 4.38$ ◄

For n = 1200 rpm, $\omega = 2\pi(1200)/60$ = 125.6 rad/s

$(N_s)_{SI} = 125.6(10.85)^{1/2}/(9.81 \times 43.8)^{3/4} = 4.38$ ◄ This is an axial-flow pump.

Sec. 15.9: Peripheral-Velocity Factor – *Exercises (3)*

15.9.1 *Estimate the diameter of the impeller of the pump whose operating characteristics are shown in Fig. 15.7.*

BG

BEP is Q = 122 cfs at h = 16 ft ; Eq. 15.8*a*: $N_s = 450\sqrt{122(449)}/16^{3/4} = 13{,}163$

Fig. 15.11 for N_s = 13,163: $\phi_e \approx 2.3$

Eq. 15.10: $D = 153.3\phi_e\sqrt{h}/n_e \approx 153.3(2.3)\sqrt{16}/450 = 3.1$ ft = 38 in ◄

15.9.2 *Estimate the diameter of the impeller of a pump whose BEP is defined by Q = 5000 gpm at a head of 82 ft when operating at 3000 rpm.*

BG

Eq. 15.8*a*: $N_s = 3000\sqrt{5000}/82^{3/4} = 7785$; Fig. 15.11 for N_s = 7785: $\phi_e \approx 1.8$

Eq. 15.10: $D \approx 153.3(1.8)\sqrt{82}/3000 = 0.83$ ft = 10 in ◄

15.9.3 *At maximum efficiency, a four-stage pump delivers 400 L/s against a head of 300 m at a rotative speed ω of 125.6 rad/sec. Estimate the diameter of the impellers if all four impellers are identical. Calculate $(N_s)_{SI}$, convert to N_s, and then obtain an estimated value of D.*

B

Four-stage pump: h = 320/4 = 80 m

Eq. 15.8*b*: $(N_s)_{SI} = \omega_e\sqrt{Q}/(gh)^{3/4} = 125.6\sqrt{0.4}/(9.81 \times 80)^{3/4} = 0.54$

From Sec. 15.8: N_s = 0.54/0.000 368 = 1467. Fig. 15.11 for N_s = 1467: $\phi_e \approx 1.0$; $n_e = \omega_e(60/2\pi)$

Eq. 15.10: $D \approx 153.3(1.0)\sqrt{80(3.28)}/[125.6(60/2\pi)] = 2.07$ ft = 24.8 in ◄

$D \approx 2.07(0.3048) = 0.630$ m = 630 mm ◄

Sec. 15.9: Peripheral-Velocity Factor – ***Problems** 15.26–15.35*

15.26 *A three-stage pump is rated at 7500 gpm against a head of 600 ft at a speed of 1200 rpm. The three impellers are identical. Find the N_s of this pump and the approximate diameter of the impellers.*

BG

h = 600/3 = 200 ft/stage. Eq. 15.8*a*: $N_s = n_e\sqrt{\text{gpm}}/h^{3/4} = 1200\sqrt{7500}/200^{3/4} = 1954$ ◀

From Fig. 15.11 for N_s = 1954: $\phi_e \approx 1.05$

Eq. 15.10: $D = 153.3\phi_e\sqrt{h}/n \approx 153.3(1.05)\sqrt{200}/1200 = 1.90$ ft = 22.8 in ◀

15.27 *A four-stage pump is designed to deliver 65 L/s against a head of 120 m when operating at 1450 rpm. The four impellers are identical. Find the specific speed of this pump. What type of pump is it? Estimate the diameter of the impellers.*

B

h = 120/4 = 30 m/stage

Eq. 15.8*b*: $(N_s)_{SI} = \omega_e\sqrt{Q}/(gh)^{3/4} = [2\pi(1450)/60]\sqrt{0.065}/(9.81\times 30)^{3/4} = 0.545$

From Sec. 15.8: $N_s = \dfrac{(N_s)_{SI}}{0.000368} = \dfrac{0.545}{0.000368} = 1481$; Fig. 15.11 for N_s = 1481: $\phi_e \approx 1.02$

Eq. 15.10: $D = 153.3P_e\sqrt{h}/n \approx 153.3(1.02)\sqrt{30(3.28)}/1450 = 1.07$ ft = 12.8 in = 326 mm ◀

This is a radial flow pump ◀

15.28 *Estimate the diameter of the impeller of the model pump described in Exer. 15.4.1.*
Exer. 15.4.1: The pump (n_e = 3600 rpm) delivers 0.10 m^3/s at h = 40 m.

B

Eq. 15.8*b*: $(N_s)_{SI} = \omega_e\sqrt{Q}/(gh)^{3/4} = 2\pi(3600/60)\sqrt{0.10}/(9.81\times 40)^{3/4} = 1.35$

Fig. 15.11 for $(N_s)_{SI}$ = 1.35: $\phi_e \approx 1.3$

Eq. 15.10: $D \approx 153.3(1.3)\sqrt{40(3.28)}/3600 = 0.64$ ft = 8 in = 190 mm ◀

15.29 *Approximately what would be the diameter of the impeller of a pump that at its BEP delivers 150 L/s of water at a head of 45 m when operating at 1750 rpm.*

B

Eq. 15.8*b*: $(N_s)_{SI} = \omega_e\sqrt{Q}/(gh)^{3/4} = 2\pi(1750/60)\sqrt{0.150}/(9.81\times 45)^{3/4} = 0.737$

From Sec. 15.8: N_s = 0.737/0.000 368 = 2000 ; Fig. 15.11 for N_s = 2000: $\phi_e \approx 1.10$

Eq. 15.10: $D \approx 153.3(1.10)\sqrt{45(3.28)}/1450 = 1.41$ ft = 17.0 in = 430 mm ◀

Sec. 15.8 and Fig. 15.11 for N_s = 2000: This is a radial-flow centrifugal pump. ◀

15.30 *Approximately how much head will be developed by a pump when operating at its point of optimum efficiency if it delivers 6500 gpm when running at 1200 rpm? The diameter of the impeller is 15.0 in.*

BG

This problem must be solved by trial using Eq. 15.8*a*, Eq. 15.10 and Fig. 15.11.

Eq. 15.10 assuming ϕ_e = 1.0: $D = 153.3(1.0)\sqrt{h}/1200 = 15/12$ so $\sqrt{h} = 9.78$ ft, h = 95.7 ft

Eq. 15.8*a*: $N_s = 1200\sqrt{6500}/95.7^{3/4} = 3160$; Fig. 15.11 for N_s = 3160: $\phi_e \approx 1.25$

Try ϕ_e = 1.2: $D = 153.3(1.2)\sqrt{h}/1200 = 15/12$, so $\sqrt{h} = 8.15$, h = 66.5 ft

Eq. 15.8*a*: $N_s = 1200\sqrt{6500}/66.5^{3/4} = 4150$; Fig. 15.11 for N_s = 4150: $\phi_e \approx 1.4$

Try ϕ_e = 1.5: $D = 153.3(1.5)\sqrt{h}/1200 = 15/12$ so $\sqrt{h} = 6.52$, h = 42.6 ft

Eq. 15.8*a*: $N_s = 1200\sqrt{6500}/42.6^{3/4} = 5790$; Fig. 15.11 for N_s = 5790: $\phi_e \approx 1.52$ (close!)

Hence $h \approx 43$ ft ◀

15.31 *Calculate the values of ϕ_e for the pumps of Sample Prob. 15.2 and compare these values with those given in Fig. 15.11. Perform these calculations for both the model and the prototype.*

Sample Prob. 15.2: Model pump, diameter 7.4 in, delivers 3.0 cfs at 3600 rpm when h = 125 ft; prototype delivers 300 cfs at 360 rpm when h = 125 ft. The scale ratio is 1:10.

BG

Model: Eq. 15.8*a*: $N_s = 3600\sqrt{3.0(449)}/125^{3/4} = 3535$

Fig. 15.11 for $N_s = 3535$: $\phi_e \approx 1.3$

Eq. 15.10: $D \approx 7.4/12 \approx 153.3\phi_e\sqrt{125}/3600$ so $\phi_e \approx 1.29$, good check ◀

Prototype: Eq. 15.8*a*: $N_s = 360\sqrt{300(449)}/125^{3/4} = 3535$

Fig. 15.11 for $N_s = 3535$: $\phi_e \approx 1.3$

Eq. 15.10: $D \approx 10(7.4/12) \approx 153.3\phi_e\sqrt{125}/360$ so $\phi_e \approx 1.29$, good check ◀

15.32 *At maximum efficiency, the pump at Rocky River delivers 280 cfs at a head of 238 ft when operating at 327 rpm. Calculate the specific speed of this pump and provide an estimate of the diameter of the impeller.*

BG

Eq. 15.8*a*: $N_s = 327\sqrt{280(449)}/238^{3/4} = 1913$

Fig. 15.11, for $N_s = 1913$: $\phi_e \approx 1.04$

Eq. 15.10: $D \approx 153.3(1.04)\sqrt{238}/327 = 7.5$ ft ◀

15.33 *At the Grand Coulee project on the Columbia River there are several identical pumps, each with an impeller diameter of 167⅜ inches. The rotative speed of these pumps is 200 rpm and the maximum efficiency is 90.8% when discharging 1250 cfs at a head of 344 ft. Find the specific speed and calculate ϕ_e. How does the calculated value of ϕ_e compare with the value shown in Fig. 15.11?*

BG

Eq. 15.8*a*: $N_s = 200\sqrt{1250(449)}/344^{3/4} = 1877$

Eq. 15.10: $D = 153.3\phi_e\sqrt{344}/200 = 167.4/12$ so $\phi_e = 0.98$ ◀

Fig. 15.11 for $N_s = 1877$: $\phi_e \approx 1.0$ Good check

15.34 *Approximately what head would you expect a pump (impeller diameter = 10.0 in) to develop at 1200 rpm if its specific speed is 3500?*

BG

Fig. 15.11 for $N_s = 3500$: $\phi_e \approx 1.3$

Eq. 15.10: $D = 153.3\phi_e\sqrt{h}/n$; $10/12 \approx 153.3(1.3)\sqrt{h}/1200$, so $h \approx 25$ ft ◀

15.35 *A given axial-flow pump delivers 300 L/s at a head of 6 m when operating at 1800 rpm. If this pump were operating on the moon at the same speed, what head would it develop when delivering water at 300 L/s? Assume the gravitational constant of the moon is one-sixth that of the earth. Assume cavitation does not occur.*

SI

Eq. 15.3 indicates that if n and D are constant, gh = constant. Hence, since g of moon is $(1/6)g$ of earth, h on moon = $6h$ on earth.

So $h_{moon} = 6h_{earth} = 6 \times 6 = 36$ m ◀

Sec. 15.10: Cavitation in Pumps -- ***Exercises (3)***

15.10.1 *A mixed-flow pump, located at sea level, with a specific speed of 6000 is to be used to pump 80°F water from a reservoir. The head to be developed is 40 ft. What is the greatest elevation above the reservoir water surface that the pump can be placed such that cavitation will not occur in the pump? Assume the head loss from the reservoir to the pump is 1.5 ft.*

BG

Fig. 15.12 for $N_s = 6000$: $\sigma_c \approx 0.63$; Table A.1 at 80°F: $p_v/\gamma = 1.17$ ft and $\gamma = 62.22$ lb/ft^3

Eq. 15.14: $(z_s)_{max} \approx (p_0)_{abs}/\gamma - p_v/\gamma - \sigma_c h - h_L = 14.7(144)/62.22 - 1.17 - 0.63(40) - 1.5$

$(z_s)_{max} \approx 34.02 - 1.17 - 25.20 - 1.50 = 6.2$ ft above the reservoir water surface ◄

An approximate answer (10 ft) can be found through use of Fig. 15.13.

15.10.2 *Solve Exer. 15.10.1 for the case where the pump is located at elevation 10,000 ft with a water temperature of 50°F. Assume all other data remain the same.*

Exer. 15.10.1: The pump ($N_s = 6000$) delivering water from a reservoir is to develop $h = 40$ ft, when the friction $h_L = 1.50$ ft. Find $(z_s)_{max}$ above the reservoir without cavitation in the pump.

BG

Fig. 15.12 for $N_s = 6000$: $\sigma_c \approx 0.63$; Table A.1 at 50°F: $p_v/\gamma = 0.41$ ft and $\gamma = 62.41$ lb/ft

Table A.3 at elevation 10,000 ft: $p_{atm} = 10.11$ psia

Eq. 15.14: $(z_s)_{max} \approx 10.11(144)/62.41 - 0.41 - 0.63(40) - 1.5 = 23.33 - 0.41 - 25.20 - 1.50$

$(z_s)_{max} \approx -3.8$ ft

To safeguard against cavitation this pump must be placed about 3.8 ft or more below the elevation of the reservoir water surface. ◄

15.10.3 *A pump with a critical value of $\sigma = 0.20$ is to pump against a head of 20 m. The barometric pressure is 98.5 kPa abs, and the vapor pressure is 5.2 kPa abs. Friction losses from the reservoir to the pump are 0.5 m. Find the maximum allowable height of the pump relative to the water surface in the reservoir to ensure that cavitation will not occur.*

SI

Eq. 15.14: $(z_s)_{max} = (p_0)_{abs}/\gamma - p_v/\gamma - \sigma_c h - h_L = (98.5 - 5.2)/9.81 - 0.2(20) - 0.5$

$(z_s)_{max} = 9.51 - 4.00 - 0.50 = 5.01$ m

To prevent cavitation, the pump should be set less than 5.01 m above the reservoir water surface. ◄

Sec. 15.10: Cavitation in Pumps -- ***Problems*** *15.36–15.45*

15.36 *Is the pump in Prob. 15.35 likely to cavitate? Why?*

Prob. 15.35: An axial-flow pump ($n = 1800$ rpm) which on earth delivers 300 L/s at $h = 6$ m is to deliver the same flow on the moon.

BG

Yes, the pump is quite likely to cavitate because the atmospheric pressure on the moon is very low. ◄

15.37 *A pump with a critical value of σ_c of 0.20 is to pump against a head of 200 ft. The barometric pressure is 14.3 psia, and the vapor pressure of the water is 0.8 psia. Assume the friction losses in the intake piping are 4 ft. Find the maximum allowable height of the pump relative to the water surface at intake to assure that cavitation will not occur.*

BG

Eq. 15.14: $(z_s)_{max} = (p_0)_{abs}/\gamma - p_v/\gamma - \sigma_c h - h_L$

$(z_s)_{max} = (14.3 - 0.8)144/62.4 - 0.2(200) - 4 = 31.15 - 40 - 4 = -12.85$ ft

The pump must be placed at least 12.85 ft below the reservoir water surface. ◄

15.38 *A boiler feed pump delivers water at 200°F, which it draws from an open hot well with a friction loss of 2 ft in the intake pipe (Fig. P15.38). The barometric pressure is 29 inHg, and the value of σ_c for the pump is 0.16. What must be the elevation of the water surface in the hot well relative to that of the pump intake? The total pumping head is 140 ft.*

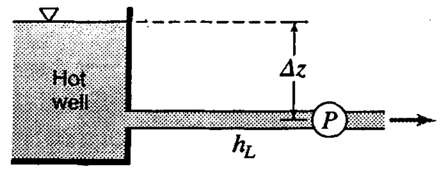


Figure P15.38

BG

29 inHg = (29/12)13.55 = 32.75 ft of water = p_{atm}/γ

Table A.1 at 200°F: p_v/γ = 27.59 ft

Eq. 15.14: $(z_s)_{max}$ = 32.75 - 27.59 - 0.16(140) - 2 = 3.16 - 22.4 = -19.24 ft

The pump must be placed at least 19.24 ft below the hot well water surface to prevent cavitation. ◄

15.39 *Repeat Prob. 15.38 with all data identical except for the total pumping head. Consider three cases as follows: total pumping heads of 40 ft, 100 ft, and 180 ft.*

Prob. 15.38: A pump (σ_c = 0.16) draws 200°F water through intake piping with friction h_f = 2 ft (Fig. P15.38). If p_0 = 29 inHg, find $(z_s)_{max}$ for no cavitation in the pump.

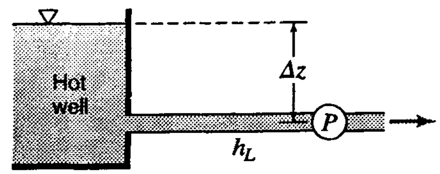


Figure P15.38

BG

29 inHg = (29/12)13.55 = 32.75 ft of water = p_{atm}/γ

Table A.1 at 200°F: p_v/γ = 27.59 ft

Eq. 15.14: $(z_s)_{max}$ = 32.75 - 27.59 - 0.16h - 2 = 3.16 - 0.16h ft

h (ft)	Required distance below water surface (ft)
40	3.24
100	12.84
180	25.6

▲

15.40 *Suppose a pump was pumping water at a head of 30 ft with a water temperature of 100°F and a barometric pressure of 14.2 psia. At intake to the pump the pressure is 17 inHg vacuum and the pipeline velocity is 16 ft/sec. What are the values of NPSH and σ?*

BG

Table A.1 at 100°F: γ = 62.00 lb/ft^3 and p_v/γ = 2.19 ft

$(p_s)_{abs}/\gamma = p_{atm}/\gamma + p_{gage}/\gamma$ = 14.2(144)/62.00 + (-17 inHg)33.9/29.9 = 33.0 - 19.3 = 13.7 ft

Eq. 15.11: $NPSH = (p_s)_{abs}/\gamma + V_s^2/2g - p_v/\gamma$ = 13.7 + 16^2/(2 × 32.2) - 2.19 = 15.49 ft ◄

Eq. 15.11: $\sigma = NPSH/h$ = 15.49/30 = 0.516 ◄

15.41 *A centrifugal pump with $(N_s)_{SI}$ = 1.48 is to pump 30 L/s of gasoline (s = 0.82, vapor pressure = 32 kPa) from an open tank (Fig. P15.41). The pump impeller rotates at 157 rad/s. Assuming the free surface in the tank is at elevation 100.0 m and the barometric pressure is 101 kPa abs, what is the highest elevation at which the pump centerline may be placed without cavitation problems? The suction pipe (f = 0.022) is 40 m long and has a diameter of 120 mm. Assume square entrance conditions where the pipe takes off from the tank. Express the answer in terms of h.*

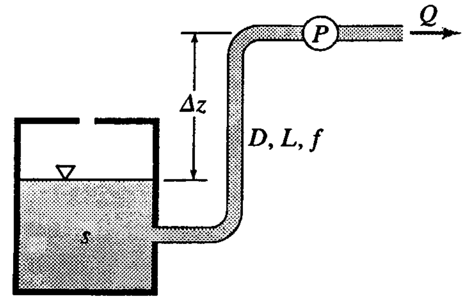


Figure P15.41

SI

Fig. 15.12 for $(N_s)_{SI}$ = 1.48: $\sigma_c \approx 0.40$

$h_L = 0.5V^2/2g + 0.022(40/0.120)V^2/2g = 7.83V^2/2g$; $V = Q/A = 0.030/[\pi(0.120)^2/4] = 2.65$ m/s

$\therefore\ h_L = 7.83(2.65)^2/(2\times 9.81) = 2.81$ m

Eq. 15.14: $(z_s)_{max} = (p_s)_{abs}/\gamma - p_v/\gamma - \sigma_c h - h_L \approx 101/(0.82\times 9.81) - 32/(0.82\times 9.81) - 0.40h - 2.81$

$(z_s)_{max} \approx 12.56 - 3.98 - 0.40h - 2.81 = 5.77 - 0.40h$

Max elev. (meters) = $100.0 + (z_s)_{max} \approx 105.77 - 0.40h$ ◀

15.42 *At rated capacity a four-stage centrifugal pump rotates at 1200 rpm and delivers 360 L/s of water against a head of 180 m. What diameter of impellers should be used? The suction and discharge flanges have inside diameters of 300 mm and 320 mm respectively, and both are located 0.7 m below the pump centerline. At what gage pressure at the suction flange may cavitation be expected? Water temperature is 34°C and barometric pressure is 98 kPa abs.*

SI

$h = 180/4 = 45$ m per stage

Eq. 15.8*b*: $(N_s)_{SI} = \omega_e\sqrt{Q}/(gh)^{3/4} = 2\pi(1200/60)\sqrt{0.360}/(9.81\times 45)^{3/4} = 0.782$

Fig. 15.11 for $(N_s)_{SI}$ = 0.782: $\phi_e \approx 1.1$

Eq. 15.10: $D = 153.3\phi_e\sqrt{h}/n_e \approx 153.3(1.1)\sqrt{45(3.28)}/1200 = 1.7$ ft = 21 in = 520 mm ◀

Eq. 15.14: $(z_s)_{max} = (p_0)_{abs}/\gamma - p_v/\gamma - \sigma_c h - h_L$

where we let $(z_s)_{max} = -0.7$ m for the condition of incipient cavitation

Table A.1 at 34°C: $\gamma = 9.75$ kN/m^3 and $p_v/\gamma = 0.56$ m

$(p_0)_{abs}/\gamma = p_{atm}/\gamma = 98/9.75 = 10.05$ m ; Fig. 15.12 for $(N_s)_{SI}$ = 0.782: $\sigma_c \approx 0.16$

Hence Eq. 15.14 can be expressed as: $-0.7 \approx 10.05 - 0.56 - 0.16(180/4) - h_L$; $h_L \approx 3$ m

Energy equation from surface of reservoir to suction side of pump, with reservoir water surface as datum:

$0 - h_L = +z_s + p_s/\gamma + V_s^2/2g$ where $z_s = -0.7$ m and $V_s = Q/A_s$

$V_s = 0.36/(\pi 0.3^2/4) = 5.10$ m/s; $V_s^2/2g = 5.10^2/(2\times 9.81) = 1.326$ m

Hence, energy equation may be written as: $0 - 3 \approx -0.7 + p_s/\gamma + 1.33$

from which $p_s/\gamma \approx -3 - 1.33 + 0.7 = -3.63$ m ; $p_s \approx -3.63(9.75) = -35.4$ kN/m^2

Inside cover: 760 mmHg is equivalent to 101.32 kN/m^2.

Hence the pressure at suction ≈ −35.4(760/101.32) = Vacuum of 210 mmHg ◀

15.43 *Rework Prob. 15.42 for the case of a three-stage pump. Assume speed, flow rate, and total head are unchanged.*

Prob. 15.42: At rated capacity a four-stage centrifugal pump rotates at 1200 rpm and delivers 360 L/s of water against a head of 180 m. What diameter of impellers should be used? The suction and discharge flanges have inside diameters of 300 mm and 320 mm respectively, and both are located 0.7 m below the pump centerline. At what gage pressure at the suction flange may cavitation be expected? Water temperature is 34°C and barometric pressure is 98 kPa abs.

SI

Eq. 15.8*b*: $(N_s)_{SI} = \omega_e\sqrt{Q}/(gh)^{3/4} = 2\pi(1200/60)\sqrt{0.360}/(9.81 \times 180/3)^{3/4} = 0.631$

Fig. 15.11 for $(N_s)_{SI} = 0.631$: $\phi_e \approx 1.02$; $h = 180/3 = 60$ m

Eq. 15.10: $D = 153.3\phi_e\sqrt{h}/n_e \approx 153.3(1.02)\sqrt{60(3.28)}/1200 = 1.83$ ft $= 21.9$ in $= 560$ mm ◄

Fig. 15.12 for $(N_s)_{SI} = 0.631$: $\sigma_c \approx 0.10$. Table A.1 at 34°C: $\gamma = 9.75$ kN/m^3 and $p_v/\gamma = 0.56$ m

Eq. 15.14 with $(z_s)_{max} = -0.7$ m for the condition of incipient cavitation:

$-0.7 \approx 10.05 - 0.56 - 0.10(180/3) - h_L$ from which $h_L \approx 10.05 - 0.56 - 6.0 + 0.7 = 4.2$ m

Energy equation from surface of reservoir to suction side of pump, with reservoir water surface as datum:

$0 - h_L = +z_s + p_s/\gamma + V_s^2/2g$ where $z_s = -0.7$ and $V_x = Q/A_s$

$V_s = 0.36/(\pi 0.3^2/4) = 5.10$ m/s ; $V_s^2/2g = 5.10^2/(2 \times 9.81) = 1.326$ m

$p_s/\gamma = -z_s - V_s^2/2g - h_L \approx -(-0.7) - 1.3 - 4.2 = -4.8$ m ; $p_s \approx -4.8(9.75) = -47$ kN/m^2

Using the inside cover: The pressure at suction $\approx -47(760/101.32) =$ Vacuum of 350 mmHg ◄

15.44 *A pump with a critical value of $\sigma_c = 0.18$ is to pump against a head of 80 m. The barometric pressure is 98.5 kPa abs, and the vapor pressure is 5.4 kPa abs. Assume the friction losses in the intake are 1.2 m. Find the maximum allowable elevation of the pump relative to the water surface at intake.*

SI

Eq. 15.14: $(z_s)_{max} = (p_s)_{abs}/\gamma - p_v/\gamma - \sigma_c h - h_L = (98.5 - 5.4)/9.81 - 0.18(80) - 1.2$

$(z_s)_{max} = 9.49 - 14.40 - 1.20 = -6.11$ m

The pump must be set at least 6.11 m below the elevation of the reservior water surface. ◄

15.45 *Determine the specific speed of a centrifugal pump that is rated at 3500 gpm under a head of 70 ft at 1750 rpm? What would be the head and capacity of this pump if operated at 1160 rpm? For each rotative speed note the maximum tolerable suction lift as recommended in Fig. 15.13.*

BG

Eq. 15.8*a*: $N_s = 1750\sqrt{3500}/70^{3/4} = 4280$ ◄

Fig. 15.13 for $N_s = 4280$ (1750 rpm): the maximum tolerable suction lift is about 3.0 ft. ◄

Changing the rotative speed will not change N_s as N_s is defined at the (rated) BEP.

From Eq. 15.5 with D unchanged: $h_2 = h_1(n_2D_2/n_1D_1)^2 = 70(1160/1750)^2 = 30.8$ ft ◄

From Eq. 15.4 with D unchanged: $Q_2 = Q_1(n_2D_2^3/n_1D_1^3) = 3500(1160/1750) = 2320$ gpm ◄

Fig. 15.13 for $N_s = 4280$ and $h = 30.8$ ft (1160 rpm): the tolerable suction lift is about 21 ft. ◄

Note: The lower speed results in a lower flow rate which permits a higher tolerable lift.

Sec. 15.11: Viscosity Effect – *Problem 15.46*

15.46 *By scaling the necessary data off Fig. 15.14, find the approximate specific gravity of the liquid whose kinematic viscosity is 396 times that of water. Note that the maximum efficiency when pumping this liquid is 18%, and when pumping water it is 85%.*

B

By measuring Fig. 15.14 we find the following ratios at maximum efficiency:

bhp for water/bhp for given liquid ≈ 0.52

Q for water/Q for given liquid ≈ 1.48

h for water/h for given liquid ≈ 1.30

Eq. 15.2: $\eta = \dfrac{\gamma Q h}{\text{bhp}}$; so for the ratio of water to liquid, we can write:

$$\frac{0.85}{0.18} \approx \frac{\gamma}{s\gamma}(1.48)1.30\left(\frac{1}{0.52}\right)$$

from which $s \approx 0.78$ ◄

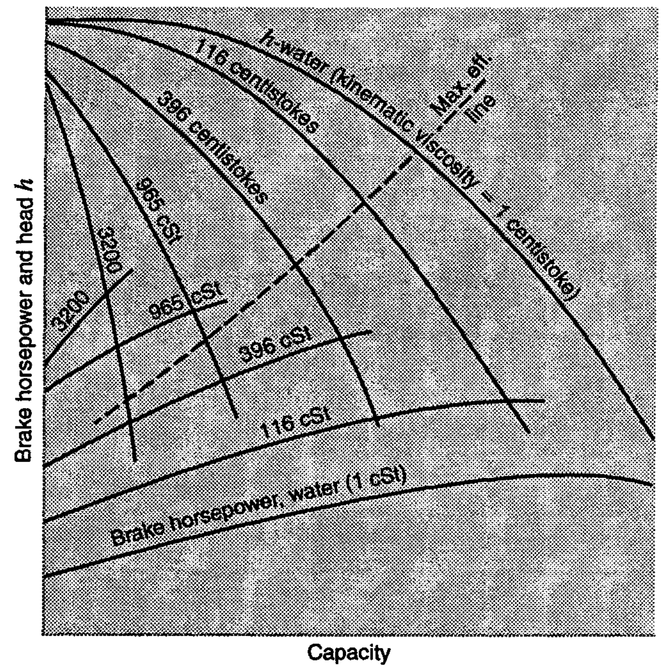


Figure 15.14

Sec. 15.12: Selection of Pumps – *Exercises (3)*

15.12.1 *Repeat Sample Prob. 15.10 for the case where the reservoir water surface elevations are 120 ft and 160 ft respectively, with all other data remaining the same.*

Sample Prob. 15.10: Water is pumped at 20 cfs from reservoir A to reservoir B (total friction h_L = 18.88 ft). The intake piping friction h_L = 2.83 ft, and p_v/γ = 0.41 ft. Find (a) N_s of the most suitable pump with n = 600 rpm, and (b) whether the pump is safe from cavitation at the surface elevation of reservoir A.

BG

(*a*) $h = (160 - 120) + 18.88 = 58.9$ ft ; Eq. 15.8*a*: $N_s = 600\sqrt{20(449)}/58.9^{3/4} = 2670$ ◄

(*b*) Eq. 15.14: $(z_s)_{\max} = (p_0)_{\text{abs}}/\gamma - p_v/\gamma - \sigma_h - h_L$

From Table A.3 at elevation 140 ft: $(p_0)_{\text{abs}}/\gamma = 14.63(144)/62.4 = 33.8$ ft

Fig. 15.12 for $N_s = 2670$: $\sigma_c \approx 0.21$; ∴ $\sigma_c h \approx 0.21(58.9) = 12$ ft

$(z_s)_{\max} \approx 33.8 - 0.41 - 12 - 2.83 = 18$ ft

The pump can be placed as much as about 18 ft above the water surface elevation in reservoir A.

∴ with $z_s = 0$, yes, the pump will be safe against cavitation. ◄

15.12.2 *Repeat Sample Prob. 15.10 for the case where the flow rate is 50 cfs rather than 20 cfs, with all other data remaining the same.*

Sample Prob. 15.10: Water is pumped from reservoir A (surface elevation 5910 ft) to reservoir B (6060 ft) through a 24-in-diameter pipeline (f = 0.03) 2000 ft long. The pump is 300 ft from reservoir A, at the same elevation as its surface. $(p_0)_{abs}/\gamma = 27.3$ ft and $p_v/\gamma = 0.41$ ft. Find (a) N_s of the most suitable pump with n = 600 rpm, and (b) whether the pump is safe from cavitation.

BG

(*a*) $V = Q/A = 50/\pi(1)^2 = 15.92$ ft/sec

$h = (6060 - 5910) + 0.03(2000/2)15.92^2/(2 \times 32.2) = 268$ ft

Eq. 15.8*a*: $N_s = 600\sqrt{50(449)}/268^{3/4} = 1357$ ◄

(*b*) Eq. 15.14: $(z_s)_{max} = (p_0)_{abs}/\gamma - p_v/\gamma - \sigma_c h - h_L$

Fig. 15.12 for $N_S = 1357$: $\sigma_c \approx 0.08$, $\therefore\ \sigma_c h \approx 0.08(268) \approx 21$ ft

Eq. 8.10: $h_L = 0.03(300/2)15.92^2/(2 \times 32.2) = 17.7$ ft

$(z_s)_{max} = 27.3 - 0.41 - 21 - 17.7 = -12$ ft

This pump must be placed at least approx. 12 ft below the water surface elevation in reservoir A.

$\therefore$ with $z_s = 0$, no, the pump will not be safe against cavitation. ◄

15.12.3 *Water flows at 80°C through a 100-mm-diameter pipe (f = 0.024) of length 300 m from a large pressurized tank (p = 200 kPa, water surface elevation 60.0 m) to discharge into the atmosphere at elevation 68.0 m. If the flow rate is to be 20 L/s, find the head that a pump, rotating at 1200 rpm placed in the pipeline, must develop and determine its specific speed. Assume the pressure in the tank remains constant and neglect minor losses. This pump is placed 6.0 m above the water level in the tank. Will cavitation occur if the head loss to the pump is 0.8 m?*

SI

Table A.1 at 80°C: $\gamma = 9.53$ kN/m³ ; $V = Q/A = (20 \times 10^{-3}\text{ m}^3/\text{s})/(\pi 0.05^2\text{ m}^2) = 2.55$ m/s

Energy, from tank atmosphere to outlet: $z_1 + p_1/\gamma - f(L/D)V^2/2g + h_p = z_2$

$60.0\text{ m} + 200\text{ kN/m}^2/(9.53\text{ kN/m}^3) - 0.024(300/0.10)2.55^2/[2(9.81)] + h_p = 68.0\text{ m}$

$h_p = 8.0 - 21.0 + 23.9 = 10.9$ m ◄

Eq. 15.8*b*: $(N_s)_{SI} = \omega\sqrt{Q}/(gh)^{3/4}$ where $\omega = 2\pi n/60 = 6.28(1200)/60 = 125.6$ rad/sec

$(N_s)_{SI} = 125.6\sqrt{20 \times 10^{-3}}/(9.81 \times 10.9)^{3/4} = 0.534$

Eq. 15.14: $(z_s)_{max} = (p_0)_{abs}/\gamma - p_v/\gamma - \sigma_c h - h_L$

Table A.3: $(p_0)_{abs} = 101.33$ kN/m² ; Table A.1: $\gamma = 9.53$ kN/m³, $p_v/\gamma = 4.96$ m

Fig. 15.12 for $(N_s)_{SI} = 0.534$: $\sigma_c \approx 0.09$

Thus: $(z_s)_{max} \approx (101.33/9.53) - 4.96 - 0.09(10.9) - 0.8 = 10.63 - 4.96 - 0.98 - 0.8 = 3.9$ m

$z_s = 6.0$ m is above $(z_s)_{max} \approx 3.9$ m, so yes, the pump will cavitate. ◄

Sec. 15.12: Selection of Pumps -- ***Problems*** *15.47–15.54*

15.47 *Water at 60°F flows by gravity between two reservoirs. The water surface in Reservoir A is at elevation 5010 ft and that in Reservoir B is at elevation 4980 ft. The water flows through a 24-in-diameter pipe (f = 0.026) of length 8000 ft (Fig. P15.47). (a) What is the flow rate? Neglect minor losses. (b) If one wishes to double the flow rate using a pump operating at 1800 rpm, what specific speed pump would you recommend? (c) Approximately what would be the diameter of the impeller of the pump? (d) If the pump were set at the upper end of the pipeline, at what elevation must the pump be set to safeguard against cavitation if the head loss from Reservoir A to the pump is 1.5 ft?*

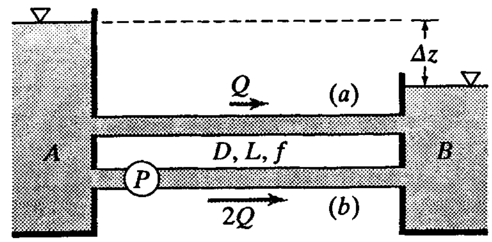


Figure P15.47

BG

(*a*) Energy equation from water surface A to water surface B: $5010 - 0.026(8000/2)V^2/2g = 4980$

$104V^2/2g = 5010 - 4980 = 30;\ V^2/2g = 30/104 = 0.2885$

$V^2 = 2(32.2)0.288 = 18.58;\ V = 4.31$ ft/sec, $Q = \pi(2^2/4)4.31 = 13.53$ cfs ◀

(*b*) Double the flow rate: $5010 - 0.026(8000/2)(2 \times 4.31)^2/2g + h_p = 4980$

$30 - 104(8.62)^2/(2 \times 32.2) + h_p = 0;\ h_p = 120.0 - 30 = 90.0$ ft

$Q = AV = \pi(2^2/4)8.62 = 27.1$ cfs ; Eq. 15.8*a*: $N_s = 1800\sqrt{27.1(449)}/90.0^{3/4} = 6795$

Recommend specific speed of 6800 ◀

(*c*) Fig. 15.11 for $N_s = 6800$: $\phi_e \approx 1.75$

Eq. 15.10: $D \approx 153.3(1.75)\sqrt{90.0}/1800 = 1.41$ ft $= 16.9$ in ◀

(*d*) Eq. 15.14: $(z_s)_{max} = (p_0)_{abs}/\gamma - p_v/\gamma - \sigma_c h - h_L$

where (Table A.3 at 4995 ft) $(p_0)_{abs}/\gamma = p_{atm}/\gamma = 12.23(144)/62.4 = 28.2$ ft

Table A.1 at 60°F: $p_v/\gamma = 0.59$ ft ; Fig. 15.12 for $N_s = 6800$: $\sigma_c \approx 0.74$

$(z_s)_{max} \approx 28.2 - 0.6 - 0.74(90.0) - 1.5 = -40$ ft

Must set pump at least about 40 ft below the elevation of the reservoir water surface. ◀

15.48 *Repeat Prob. 15.47 for the case where the pump is to operate at 600 rpm rather than 1800 rpm, with all other data remaining the same.*

Prob. 15.47: Water at 60°F flows by gravity from reservoir A (surface elevation 5010 ft) to reservoir B (4980 ft) through a pipeline (f = 0.026, L = 8000 ft, D = 24 in) (Fig. P15.47). (a) Find Q, neglecting minor losses. (b) What N_s would you use for a pump (n = 1800 rpm) to double the Q? (c) Find the approximate pump impeller diameter. (d) With the pump at reservoir A, and a 1.5-ft intake h_L, find $(z_s)_{max}$ above the surface of reservoir A for no cavitation in the pump.

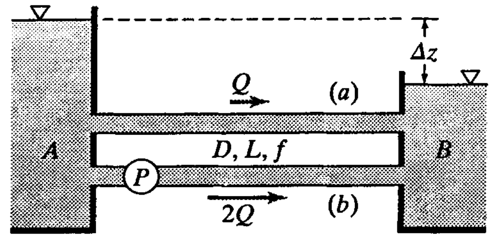


Figure P15.47

BG

(*a*) Energy equation from water surface A to water surface B: 5010 − 0.026(8000/2)$V^2/2g$ = 4980

$104V^2/2g = 5010 - 4980 = 30;\ \ V^2/2g = 30/104 = 0.2885$

$V^2 = 2(32.2)0.288 = 18.58;\ \ V = 4.31$ ft/sec, $Q = \pi(2^2/4)4.31 = 13.53$ cfs ◀

(*b*) Double the flow rate: $5010 - 0.026(8000/2)(2\times 4.31)^2/2g + h_p = 4980$

$30 - 104(8.62)^2/(2\times 32.2) + h_p = 0;\ \ h_p = 120.0 - 30 = 90.0$ ft

$Q = AV = \pi((2^2/4)8.62 = 27.1$ cfs ; Eq. 15.8*a*: $N_s = 600\sqrt{27.1(449)}/90.0^{3/4} = 2264$

Recommend specific speed of 2260 ◀

(*c*) Fig. 15.11 for N_s = 2260: $\phi_e \approx 1.1$

Eq. 15.10: $D \approx 153.3(1.1)\sqrt{90.0}/600 = 2.7$ ft = 32 in ◀

(*d*) Eq. 15.14: $(z_s)_{max} = (p_0)_{abs}/\gamma - p_v/\gamma - \sigma_c h - h_L$

where (Table A.3 at 4995 ft) $(p_0)_{abs}/\gamma = p_{atm}/\gamma = 12.23(144)/62.4 = 28.2$ ft

Table A.1 at 60°F: $p_v/\gamma = 0.59$ ft ; Fig. 15.12 for N_s = 2260: $\sigma_c \approx 0.185$

Eq. 15.14: $(z_s)_{max} \approx 28.2 - 0.6 - 0.185(90.0) - 1.5 = 9.5$ ft

The pump can be set as much as about 9.5 ft above the reservoir water surface. ◀

15.49 *Water at 170°F is pumped through a 15-in-diameter pipe (f = 0.028) 2000 ft long from a large pressurized tank (p = 30 psi) whose water surface is at elevation 200 ft to a point of discharge in the atmosphere at elevation 242 ft (Fig. P15.49). (a) If the flow rate is 5800 gpm, find the head that a pump, placed in the pipeline and rotating at 1200 rpm, must develop and determine the specific speed of the pump. Assume the pressure in the tank remains constant and neglect minor losses. (b) Approximately what will be the diameter of the impeller? (c) At what elevation must the pump be set to safeguard against cavitation if the head loss from the tank to the pump is 1.6 ft?*

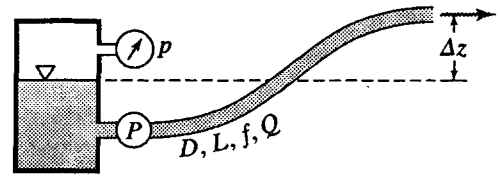


Figure P15.49

BG

Table A.1 at 170°F: γ = 60.8 lb/ft^3, p_v = 5.99 psia

(*a*) $V = Q/A = (5800/449)/[\pi(15/12)^2/4] = 10.53$ fps ; $V^2/2g = (10.53)^2/(2 \times 32.2) = 1.721$ ft

Energy eq. from tank atmos. to outlet: $200 + 30(144)/60.8 - 0.028[(2000)/(15/12)]1.721 + h_p = 242$

$200 + 71.1 - 77.1 + h_p = 242$; $h_p = 48.0$ ft ◄

Eq. 15.8*a*: $N_s = n_e\sqrt{\text{gpm}}/h^{3/4} = 1200\sqrt{5800}/(48.0)^{3/4} = 5010$ ◄

(*b*) Fig. 15.11 for $N_s = 5010$: $\phi_e \approx 1.5$

Eq. 15.10: $D = 153.3\phi_e\sqrt{h}/n \approx 153.3(1.5)\sqrt{48.0}/1200 = 1.3$ ft = 16 in ◄

(*c*) Eq. 15.14: $(z_s)_{max} = p_0/\gamma - p_v/\gamma - \sigma_c h - h_L$ where $p_0 = p_{atm} + p_{tank}$

Fig. 15.14 for $N_s = 5010$: $\sigma_c \approx 0.5$

$(z_s)_{max} \approx (14.7 + 30.0)144/60.8 - 5.99(144)/60.8 - 0.5(48.0) - 1.6 = 66$ ft

The pump could be placed as much as about 66 ft above the water level in the lower tank. ◄

15.50 *A pump manufacturer is asked to provide a pump that will deliver 84,500 gpm against a head of 225 ft. Several speeds of operation are considered, namely 225 rpm, 600 rpm, 1200 rpm, and 1800 rpm. Determine the specific speeds of pumps that will operate at these speeds and estimate the diameter of each of their impellers. What factors might one consider in making a choice among them?*

BG

Eq. 15.8*a*: $N_s = n_e\sqrt{\text{gpm}}/h^{3/4} = n_e\sqrt{84{,}500}/225^{3/4} = 4.3n_e$

Eq. 15.10: $D = 153.3\phi_e\sqrt{h}/n_e = 153.8\phi_e\sqrt{225}/n_e = 2300\phi_e/n_e$ where ϕ_e depends on N_s (Fig. 15.11)

n_e (rpm)	N_s	ϕ_e	D (ft)
225	968	≈0.95	≈9.7
600	2580	≈1.15	≈4.41
1200	5160	≈1.55	≈2.97
1800	7740	≈1.80	≈2.30
	▲		▲

Low speed pumps tend to have large inpellers. High speed pumps are more likely to have problems with cavitation.

15.51 *You are asked to select a single-stage or multi-stage pump to deliver 10 cfs against a head of 300 ft when operating at 450 rpm. Specify the number of stages, the specific speed, and the approximate diameter of the impeller.*

BG

Assume a single-stage pump. Eq. 15.8*a*: $N_s = 450\sqrt{10(449)}/300^{3/4} = 418$

N_s is too low, so select a multi-stage pump.

Stages	h (ft)	N_s	ϕ_e*	D**
2	150	703	≈0.90	≈3.8 ft
3	100	953	≈0.95	≈3.2 ft
4	75	1183	≈0.98	≈2.9 ft
6	50	1603	≈1.01	≈2.43 ft

*From Fig. 15.11; **From Eq. 15.10

Select a multi-stage pump, say 3-stage with $N_s = 953$ having an impeller diameter of about 3.2 ft. ◄

A 4-stage pump ($N_s = 1183$, $D \approx 2.9$ ft) has higher efficiency (Fig. 15.11) but costs more than 3 stages.

15.52 *You are asked to select a single-stage or multi-stage pump to deliver 300 cfs against a head of 10 ft when operating at 450 rpm. Specify the number of stages, the specific speed and the approximate diameter of the impeller.*

BG

Assume a single-stage pump. Eq. 15.8*a*: $N_s = 450\sqrt{300(449)}/10^{3/4} = 29{,}370$

Impossible, there is no pump with $N_s = 29{,}370$ (Fig. 15.11).

Select for example, 10 pumps acting in parallel. Each pump must deliver 300/10 = 30 cfs

Eq. 15.8*a*: $N_s = 450\sqrt{30(449)}/10^{3/4} = 9280$, a mixed flow pump (Fig. 15.11).

Fig. 15.11 for $N_s = 9280$: $\phi_e \approx 2.0$; Eq. 15.10: $D = 153.2\phi_e\sqrt{10}/450 \approx 153.3(2.0)\sqrt{10}/450 = 2.1$ ft

Hence, 10 mixed-flow pumps ($N_s = 9280$, impeller dia ≈ 2.1 ft) in parallel is a possible solution. ◄
As an alternative use 6 pumps operating in parallel, each pump to deliver 300/6 = 50 cfs.

$N_s = 450\sqrt{50(449)}/10^{3/4} = 11{,}990$, an axial-flow pump (Fig. 15.11).

Fig. 15.11 for $N_s = 11{,}990$: $\phi_e \approx 2.2$; $D \approx 153.3(2.2)\sqrt{50}/450 = 5.3$ ft

An alternative would be to use 6 axial-flow pumps ($N_s = 11{,}990$) with impeller diameter ≈ 5.3 ft. ◄
Many alternatives are possible. A change in speed of operation adds alternatives. To decide which is the better choice an economy study is necessary. Such a study would include investment, maintenance and operating costs, etc. Also, the selected pumps must be determined to be free from cavitation.

15.53 *A submersible centrifugal pump in a water well is to be set 120 ft below the land surface. The steel discharge pipe from the pump to the surface has a diameter of 6.0 in. The water surface in the well at maximum drawdown is 106 ft below the land surface. How many stages (N_s = 2650) would you recommend for the pump if the discharge from the well at maximum drawdown is 800 gpm and the pump operates at 1450 rpm?*

BG

Eq. 15.8*a*: $N_s = n_e\sqrt{\text{gpm}}/h^{3/4}$ or $h = \left(n_e\sqrt{\text{gpm}}/N_s\right)^{4/3} = [1450(800)^{1/2}/2650]^{4/3} = 38.5$ ft per stage

But $h = 120 + f(L/D)V^2/2g$ where $V = Q/A = (800/449)/(\pi 0.25^2) = 9.08$ ft/sec

f will have a value of about 0.015 (Table 8.1 and Fig. 8.11). Let $f = 0.02$ to be on safe side

$\therefore\ h = 106 + 0.02(120/0.5)9.08^2/(2 \times 32.2) = 112.1$ ft

112.1/38.5 = 2.91 stages. Use a three-stage pump. ◄

15.54 *A pump is required to deliver 830 L/s against a head of 230 m. If the minimum desirable specific speed from an efficiency standpoint is $(N_s)_{SI}$ = 0.35 and the motor speed in 1500 rpm, how many stages would you recommend? Also, what is the maximum permissible suction lift?*

SI

Eq. 15.8*b*: $(N_s)_{SI} = \omega_e\sqrt{Q}/(gh)^{3/4}$; $0.35 = 2\pi(1500/60)\sqrt{0.830}/(9.81h)^{3/4} = 25.8/h^{3/4}$; $h = 308.6$ m

308.6 m > 230 m ; Hence one stage is adequate.

$(N_s)_{SI} = \omega_e\sqrt{Q}/(gh)^{3/4} = 2\pi(1500/60)\sqrt{0.830}/(9.81\times 230)^{3/4} = 0.437$

Use a single-stage pump having $(N_s)_{SI} = 0.437$ ◄

The maximum permissible suction lift cannot be calculated because the following data are missing: elevation where the pump is to be used, temperature of the water to be pumped, and head loss from the source of the water to the pump (see Equation 15.14). ◄

Sec. 15.13: Pumps Operating in Series and Parallel -- *Exercises (2)*

15.13.1 *Suppose the pumps of Sample Prob. 15.13 operate at 1500 rpm. What then would be the flow rates for (a) a single pump; (b) two pumps in series; (c) two pumps in parallel. All other data to remain the same.*

Prob. 15.13: Water is pumped from reservoir A to reservoir B, which has a 35-ft higher water surface. The loss in the pipeline is given by $h_L = 20Q^2$ (h_L in ft, Q in 100's of gpm). Two available pumps (n = 1800 rpm) each have the following characteristics:

h, ft:	*100*	*90*	*80*	*60*	*40*	*20*
Q, gpm:	*0*	*110*	*180*	*250*	*300*	*340*

BG

With changes in speed the pump characteristics will change in accordance with Eqs. 15.4 and 15.5:

$Q \propto n$ and $h \propto n^2$. Hence $Q' = Q(1500/1800) = 0.833Q$ and $h' = h(1500/1800)^2 = 0.694h$

The pump characteristics shown in Sample Problem 15.13 thus become for 1500 rpm:

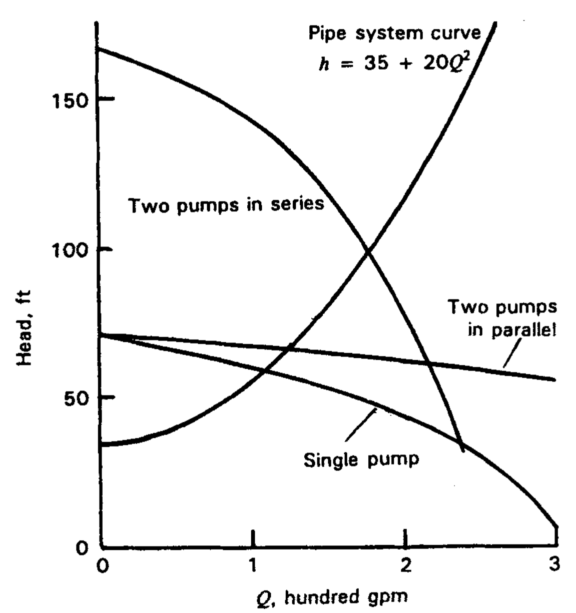
Head (ft)	Flow (gpm)
69.4	0
62.5	91.7
55.6	150.0
41.7	208.3
27.8	250.0
13.9	283.3

From the plot, the points of intersection are:

(*a*) Single pump: $Q \approx 112$ gpm ◄

(*b*) Two pumps in series: $Q \approx 179$ gpm ◄

(*c*) Two pumps in parallel: $Q \approx 124$ gpm ◄



15.13.2 *Two pumps whose characteristics are given in Sample Prob. 15.13 are to be used in parallel. They must develop a head $h = 35 + 20Q^2$ as in the sample problem. (a) What is the flow rate with both pumps operating at 1800 rpm? (b) The speed of one of the pumps is reduced until it no longer delivers water. At approximately what speed will this happen?*

Sample Prob. 15.13: The two pumps (n = 1800 rpm) each have the following characteristics:

h, ft:	*100*	*90*	*80*	*60*	*40*	*20*
Q, gpm:	*0*	*110*	*180*	*250*	*300*	*340*

BG

(*a*) From Fig. S15.13 (intersection of curves for system and two pumps in parallel):

$Q \approx 170$ gpm ◄

(*b*) Slow down one pump until its shutoff head drops to the point where the single pump curve intersects the pipe system curve. This point of intersection (Fig. S15.13) is

$h \approx 84$ ft, $Q \approx 156$ gpm.

The reduced speed to give a shutoff head of 84 ft is given by Eq. 15.5:

$h \propto n^2$, so $84/100 \approx (n/1800)^2$

$n \approx 1650$ rpm ◄

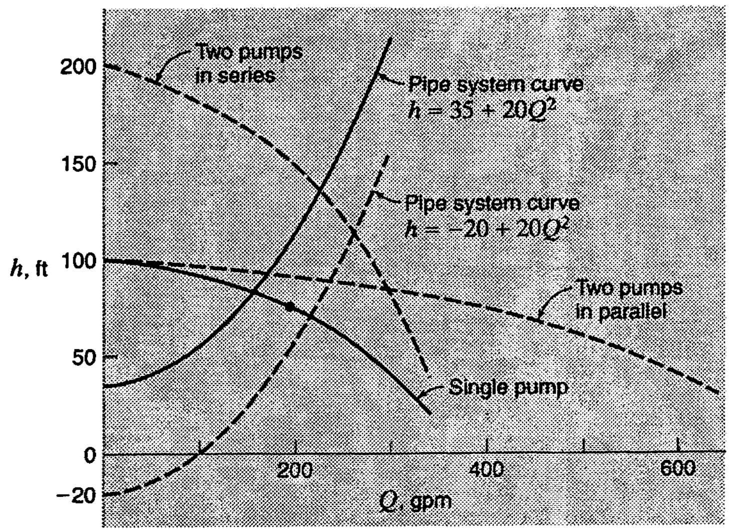


Figure S15.13

Sec. 15.13: Pumps Operating in Series and Parallel -- *Problems 15.55–15.58*

15.55 *In planning a pumping station an engineer decided to use two identical pumps operating in parallel. These pumps, when both are operating, must deliver a total of 200 cfs against a head of 40 ft. Calculate the specific speed of each of these pumps and estimate the diameter of the impellers.*

BG

Pumps in parallel -- each pump must deliver 100 cfs against a head of 40 ft.

Eq. 15.8*a*: $N_s = n_e\sqrt{\text{gpm}}/h^{3/4} = n_e\sqrt{100(449)}/40^{3/4} \approx 13.32n_e$

Analyze for different speeds of operation:

n_e (rpm)	N_s	ϕ_e*	D**
450	5994	≈1.65	≈3.55 ft
600	7992	≈1.85	≈2.99 ft
900	11,988	≈2.20	≈2.37 ft
*From Fig. 15.11 **From Eq. 15.10	▲		▲

The above represent possible alternatives. Lower rotation speeds will result in lower values of N_s, but larger impellers will be required.

15.56 *In planning a pumping station, an engineer decided to use two identical pumps operating in series. These pumps, when both are operating, must deliver 200 cfs against a total head of 40 ft. Calculate the specific speed of each of these pumps and estimate the diameter of the impellers.*

BG

Pumps in series -- each pump must deliver 200 cfs against a head of 20 ft.

Eq. 15.8*a*: $N_s = n_e\sqrt{200(449)}/20^{3/4} = 31.7n_e$

n_e (rpm)	N_s	ϕ_e*	D**
300	9550	≈2.00	≈4.57 ft
450	14,265	≈2.35	≈3.58 ft
*From Fig. 15.11 **From Eq. 15.10	▲		▲

The above represents possible alternatives. Lower rotation speeds will result in lower values of N_s, but larger impellers will be required.

15.57 *A very long pipe connects two reservoirs whose water levels are the same. Identical pumps A and B are connected to the pipe in parallel. The operating characteristics of each of the pumps are shown in Fig. 15.6. The pumps are operated at 1450 rpm. When only one pump is operating the flow rate was found to be 13,200 gpm. (a) How much flow will there be when both pumps operate? (b) Find also the rate of delivery (with one and two pumps) if the pump speeds are reduced to 1200 rpm.*

BG

Sec. 8.27: The pipe is very long, so minor losses are negligible.

Fig. 15.6: Single pump (n = 1450 rpm) at Q = 13,200 gpm: h = 44 ft.

Since the two water levels are the same $h = h_L$, and the system curve is $h = kQ^2$.

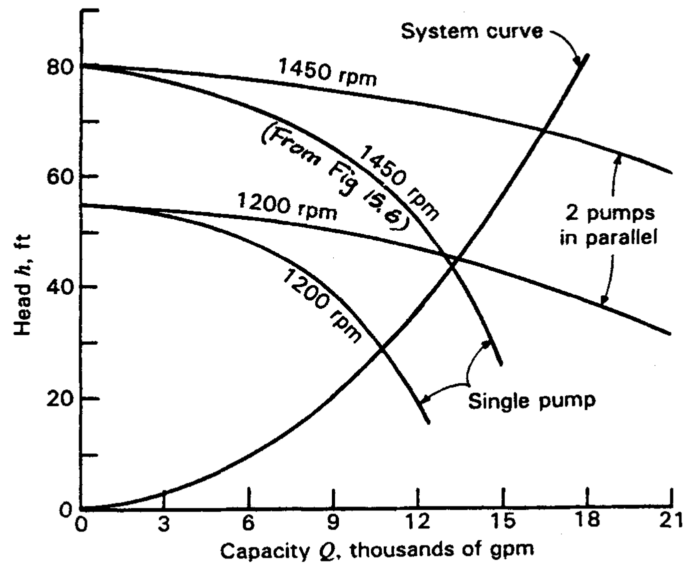
Thus (with Q in thousands of gpm):

$$44 = k(13.2)^2$$

from which $k = 44/(13.2)^2 = 0.253$

Hence $h = 0.253Q^2$

Q	h
6	9.0
9	20.2
12	36.0
15	56.2
18	81.0



(*a*) From graph for two pumps in parallel (n = 1450 rpm): Curves intersect at $h \approx 68$ ft,

$Q \approx 16{,}500$ gpm ◀

(*b*) Eq. 15.4: $Q \propto nD^3$. So for same pump (D), $Q' = (n'/n)Q = (1200/1450)Q = 0.828Q$

Eq. 15.5: $h \propto n^2D^2$. So for same pump (D), $h' = (n'/n)^2h = 0.828^2h = 0.685h$

So corresponding points are (plotted on graph above):

1450 rpm		1200 rpm	
Q (1000 gpm)	h	Q (1000 gpm)	h
0	80 ft	0	54.8 ft
6	74 ft	4.96	50.7 ft
10.5	60 ft	8.69	41.1 ft
13.2	43 ft	10.92	29.5 ft
15.0	25 ft	12.41	17.1 ft

One pump (n = 1200 rpm): Curves intersect at Q = 10,700 gpm, h = 28.5 ft ◀

Two pumps in parallel (n = 1200 rpm): Curves intersect at Q = 13,500 gpm, h = 45 ft ◀

15.58 *Select the specific speed of the pump or pumps required to convey 1.0 cfs of water against a head of 115 ft. The pump rotative speed is 1750 rpm. Consider the following cases: a single pump; two pumps in parallel; three pumps in parallel; two pumps in series; three pumps in series.*

BG

Inside cover: $Q = 1$ cfs $= 448.8$ gpm. Eq. 15.8 for single pump: $N_s = 1750\sqrt{448.8}/115^{3/4} = 1055$ ◄

Pumps in parallel: $h = 115$ ft, $Q = 448.8/n_p$ where n_p = number of pumps in parallel

2 pumps in parallel:. $N_s = 1750\sqrt{448.8/2}/115^{3/4} = 740$ ◄

3 pumps in parallel: $N_s = 1750\sqrt{448.8/3}/115^{3/4} = 610$ ◄

This value of N_s is very low, having a low efficiency (Fig. 15.11).

Pumps in series: $Q = 448.8$ gpm, $h = 115/n_p$ where n_p = number of pumps in series

2 pumps in series: $N_s = 1750\sqrt{448.8}/(115/2)^{3/4} = 1781$ ◄

3 pumps in series: $N_s = 1750\sqrt{448.8}/(115/3)^{3/4} = 2399$ ◄

Chapter 15: Miscellaneous -- *Problems 15.59–15.62*

15.59 *The pump of Fig. 15.6 is placed in an 8-in-diameter pipe (f = 0.030) 150 ft long that is used to lift water from one pond to another (Fig. P15.59). The difference in water surface elevation between the two ponds varies from 20 ft to 50 ft. Plot a curve showing delivery rate versus water-surface elevation difference.*

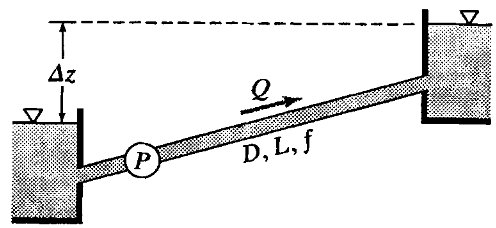


Figure P15.59

BG

$h = \Delta z + f(L/D)V^2/2g$ where

$V = Q/A = Q/[(\pi/4)(8/12)^2] = 2.865Q$

$h_L = 0.030[150/(8/12)](2.865Q)^2/(2 \times 32.2)$

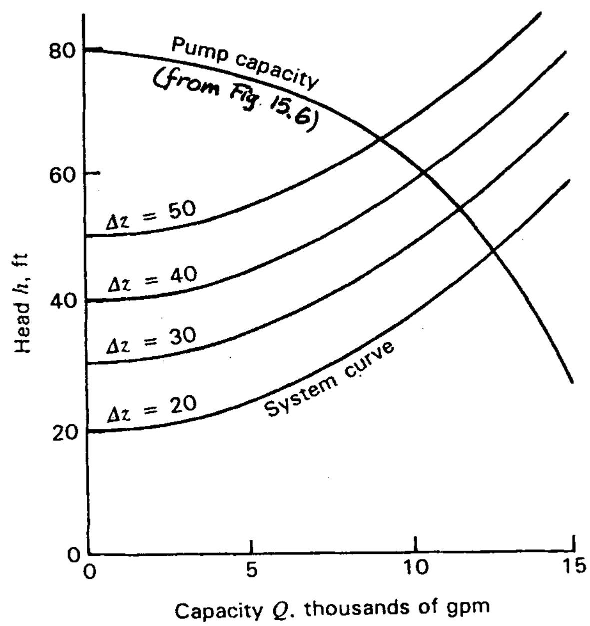
$h_L = 0.860Q^2$ where Q is expressed in cfs

$h_L = 0.860(449Q/1000)^2 = 0.1734Q^2$

where Q is expressed in thousands of gpm

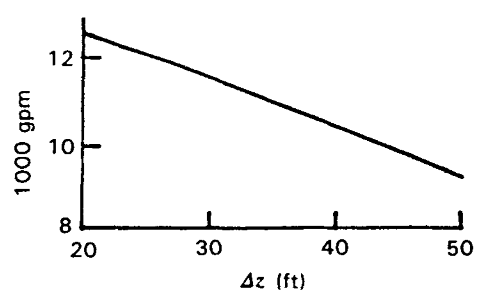
Q (1000 gpm)	h_L (ft)
1	0.173
5	4.34
10	17.3
15	38.9

Plot system curves, $h = \Delta z + h_L$, for $\Delta z = 20$, 30, 40, 50 ft (see graph). From graph, these curves intersect with the pump's head-capacity curve at:



Δz (ft)	Q (gpm)
20	≈12,600
30	≈11,600
40	≈10,500
50	≈9,200

The required curve is:



15.60 *Under normal operating conditions a centrifugal pump with an impeller diameter of 20.0 in delivers water at a head of 170 ft with an efficiency of 70% at 1800 rpm. Compute the peripheral velocity of the impeller, the specific speed, and the flow rate.*

BG

Peripheral velocity $= \omega r = 2\pi(n/60)10/12 = 2\pi(1800/60)10/12 = 157.1$ ft/sec ◄

Eq. 15.10: $D = 20/12 = 153.3\phi_e\sqrt{h}/n_e = 153.3\phi_e\sqrt{170}/1800$; $\phi_e = 1.501$

Fig. 15.11 for $\phi_e = 1.5$: $N_s \approx 4700$ ◄

Eq. 15.8*a*: $N_s \approx 4700 \approx n_e\sqrt{\text{gpm}}/h^{3/4}$; $\sqrt{\text{gpm}} \approx 4700(170)^{3/4}/1800 = 123$; gpm $\approx$ 15,000 gpm ◄

15.61 *A centrifugal pump driven by an electric motor lifts water a total height of 200 ft. The pump efficiency is 78% and the motor efficiency is 88%. The lift is through 1220 ft of 6-in-diameter pipe and the pumping rate is 350 gpm. If $f = 0.022$ and power costs 7¢/kWh, what is the cost of pumping a million gallons of water?*

B

$h = \Delta z + f(L/D)V^2/2g$ where $V = Q/A = (350/448.8)/(\pi 0.25^2) = 3.97$ ft/sec

$\therefore\ h = 200 + 0.022[1220/(6/12)](3.97)^2/(2 \times 32.2) = 213.1$ ft

Power used $= \gamma Qh/[0.78(0.88)550] = 27.48$ hp $= 0.746(27.48)$ kW $= 20.49$ kW

$$\text{Time to pump} = \frac{1{,}000{,}000\ \text{gal}}{350\ \text{gal/min}}\left(\frac{\text{hr}}{60\ \text{min}}\right) = 47.6\ \text{hr}$$

Cost = (\$0.070/kWh)(20.49 kW)(47.6 hr) = \$68.29 ◄

15.62 *What is the specific speed of the four-stage pumps mentioned in the last-but-one paragraph of Sec. 15.14? Approximately what is the diameter of the impellers?*

Sec. 15.14: The pumps ($n = 600$ rpm) each deliver 315 cfs against a total head of 1970 ft.

BG

Eq. 15.8*a*: $N_s = n_e\sqrt{\text{gpm}}/h^{3/4}$ where $h = 1970/4 = 492.5$ ft per stage

$N_s = 600\sqrt{315(448.8)}/492.5^{3/4} = 2158$ ◄

Fig. 15.11 for $N_s = 2158$: $\phi_e \approx 1.13$

Eq. 15.10: $D \approx 153.3(1.13)\sqrt{492.5}/600 = 6.4$ ft $= 77$ in ◄

Chapter 16

Hydraulic Machinery -- Turbines

PROBLEM SELECTION GUIDE

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
16.3	***Action of the Impulse Turbine***						
	X[1]16.3.1	BG	Medium	Short	1		Uses Secs. 5.9 and 6.7
	16.3.2	BG	Easy	Short	3	S16.1	Uses Secs. 5.9 and 6.8
	16.3.3	SI	Medium	Short	3		Uses Secs. 5.9 and 6.8
	P 16.1	BG	Medium	Long	3	S16.1	Uses Sec. 5.9
	16.2	BG	Medium	Long	3	S16.1	Uses Sec. 5.9
	16.3	SI	Medium	Medium	3		Uses Secs. 5.9 and 6.8
	16.4	BG	Medium	Medium	4	16.5	Uses Secs. 5.9 and 6.8
	16.5	BG	Medium	Medium	4	16.4	Uses Secs. 5.9 and 6.8
	16.6	BG	Medium	Long	3	S16.1	Uses Secs. 5.9 and 6.8
16.4	***Head on an Impulse Turbine and Efficiency***						
	X 16.4.1	BG	Easy	Medium	3	S16.4	Uses Sec. 5.9
	16.4.2	BG	Medium	Long	1	S16.4	Uses Sec. 5.9
	16.4.3	BG	Easy	Short	1		Uses Sec. 5.9
	P 16.7	BG	Easy	Long	1	S16.4	Uses Sec. 5.9
	16.8	BG	Easy	V Short	1		Uses Sec. 5.9
	16.9	BG	Easy	Short	2		Uses Sec. 6.8
	16.10	SI	Hard	Long	1		Uses Sec. 5.9
	16.11	BG	Medium	Long	1		Uses Secs. 5.9 and 6.8
	16.12	SI	Medium	Medium	1		☐ T & E for *f* using Sec. 8.13
	16.13	BG	Medium	Long	3		
16.5	***Nozzles for Impulse Turbines***						
	X 16.5.1	BG	Medium	Medium	1	S16.5, 16.5.2	Trials for D_j
	16.5.2	BG	Medium	Medium	1	S16.5, 16.5.1	Trials for D_j
	P 16.14	BG	Medium	Medium	2		
	16.15	BG	Hard	Medium	1		Uses calculus
16.7	***Action of the Reaction Turbine***						
	X 16.7.1	SI	Easy	V Short	1		
	16.7.2	BG	Easy	V Short	1		

/cont...

[1] For all Exercises (identified by "X"), answers are given in Appendix F of the textbook.
† Answers are sensitive to values (ϕ_e, σ_c, η, h_{max}) read from a figure.
X = Exercise, P = (end-of-chapter) Problem, S = Sample Problem. ☐ = could use computing aids.

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
16.8	***Draft Tubes and Effective Head on Reaction Turbines***						
	X 16.8.1	BG	Medium	Medium	1	S16.7, P16.18	Uses Sec. 8.24
	16.8.2	BG	V Easy	V Short	1	16.8.3	
	16.8.3	SI	V Easy	V Short	1	16.8.2	
	P 16.16	SI	V Easy	V Short	1		
	16.17	SI	Medium	Medium	1		☐ Trial and Error for f using Sec. 8.13
	16.18	SI	Medium	Medium	1	S16.7, X16.8.1	Uses Sec. 8.24
	16.19	BG	Medium	Medium	3		
16.9	***Efficiency of Turbines***						
	P 16.20	N	V Easy	V Short	1	16.24	
16.10	***Similarity Laws for Reaction Turbines***						
	X 16.10.1	SI	Medium	Medium	3	P16.21	
	16.10.2	BG	Easy	V Short	2		
	16.10.3	BG	Medium	Medium	1		
	P 16.21	BG	Easy	Medium	2	X16.10.1	
	16.22	BG	Easy	Short	1		
	16.23	SI	Easy	V Short	1	16.25	
	16.24	BG	V Easy	V Short	1	16.20	
	16.25	BG	Easy	V Short	1	16.23	
16.11	***Peripheral Velocity Factor and Specific Speed of Turbines***						
	X 16.11.1	BG	Easy	Short	1	16.11.2-3	†
	16.11.2	BG	Easy	Short	1	16.11.1-3	†
	16.11.3	BG	Easy	Short	1	16.11.1-2	†
	P 16.26	BG	Medium	Medium	4		†
	16.27	BG	Easy	Short	1		† Partly unsolvable
	16.28	B	Easy	Short	1	16.29	† Unit conversions
	16.29	BG	Easy	Short	1	16.28	†
	16.30	B	Easy	Short	1	16.31	† Assume η from Fig 16.13; unit conv's
	16.31	BG	Easy	Short	1	16.30	†
	16.32	BG	Easy	Short	2	16.33-34	†
	16.33	BG	Easy	Short	2	16.32-34	†
	16.34	BG	Easy	Short	2	16.32-33	†

/cont...

Sec	Exer/Prob	Units	Difficulty	Length	Parts	Similar	Special features
16.12	***Cavitation in Turbines***						
	X 16.12.1	BG	Easy	Short	1		†
	16.12.2	BG	Easy	Medium	2		† Interpolation
	16.12.3	BG	Easy	Short	1		†
	P 16.35	BG	Easy	Short	1	16.36	†
	16.36	BG	Easy	Short	1	16.35	†
	16.37	BG	Easy	Medium	4	16.38	†
	16.38	BG	Easy	Medium	4	16.37	† Interpolation.
	16.39	BG	Easy	Short	1	16.38	†
	16.40	BG	Easy	Short	2		†
16.13	***Selection of Turbines***						
	X 16.13.1	BG	Medium	Medium	2	16.13.2	†
	16.13.2	BG	Medium	Medium	2	16.13.1	†
	16.13.3	BG	Easy	Short	1		†
	16.13.4	BG	Easy	Short	2		
	P 16.41	BG	Hard	Long	2	16.52-53	†
	16.42	BG	Medium	Medium	1		†
	16.43	BG	Medium	Medium	1		
	16.44	BG	Easy	Medium	2		
	16.45	BG	Easy	Medium	1		†
	16.46	BG	Medium	Medium	1		†
	16.47	BG	Medium	Long	2		†
	16.48	BG	Easy	Medium	1		†
	16.49	B	Medium	Long	1		† Unit conversions
	16.50	BG	Medium	Medium	2	16.51	
	16.51	BG	Medium	Long	1	16.50	†
	16.52	B	Medium	Medium	1	16.41*a*,53	† Unit conversions
	16.53	BG	Medium	Medium	1	16.41*a*,52	†

Chapter 16
HYDRAULIC MACHINERY – TURBINES

Sec. 16.3: Action of the Impulse Turbine -- *Exercises (3)*

16.3.1 *In Sample Prob. 16.1 subtract the horsepower of the absolute velocity of the water at bucket exit from the horsepower of the jet at bucket entrance. What does the result represent? Why?*

Sample Prob. 16.1: V_1 = 200 fps, u = 125.6 fps, $v_2 = v_1$ = 74.4 fps, Q = 4.36 cfs.

BG

Eq. 6.17: $V_{2x} = u + v_2\cos\beta_2 = 125.6 + 74.4\cos 160° = 55.7$ fps

Eq. 6.18: $V_{2y} = v_2\sin\beta_2 = 74.4\sin 160° = 25.4$ fps. $V_2 = \sqrt{55.7^2 + 25.4^2} = 61.2$ fps

Per Sec. 5.9: Power at entrance, $P = \dfrac{\gamma Q(V_1^2/2g)}{550} = \dfrac{62.4(4.36)}{550}\left(\dfrac{200^2}{2 \times 32.2}\right) = 307$ hp

Power at exit, $P = \dfrac{\gamma Q(V_2^2/2g)}{550} = \dfrac{62.4(4.36)}{550}\left(\dfrac{61.2^2}{2 \times 32.2}\right) = 28.8$ hp; $\Delta P = 307 - 28.8 = 278$ hp ◄

This represents the hp transferred to the turbine runner. ◄

This is so because we assumed $v_2 = v_1$ ◄

16.3.2 *Repeat Sample Prob. 16.1 for the case where $v_2 = 0.8v_1$ and calculate the percent error in the horsepower made by assuming $v_2 = v_1$ in Sample Prob. 16.1.*

Sample Prob. 16.1: u = 125.6 fps, $v_2 = v_1$ = 74.4 fps, Q = 4.36 cfs, D = 10.0 ft, HP to runner = 278.6. Find (a) F on buckets, (b) T on runner, (c) hp transferred to runner.

BG

$v_2 = 0.8v_1 = 0.8(74.4) = 59.5$ fps ; $\Delta v_x = v_{2x} - v_{1x} = 59.5\cos 160° - 74.4 = -130.3$ fps

(*a*) Eq. 6.24: $F = 1.940(4.36)130.3 = 1102$ lb ◄

(*b*) $T = Fr = 1102(5) = 5510$ ft·lb ◄

(*c*) Eqs. 5.38 and 5.40: Power transferred to runner = $Fu/550 = 1102(125.6)/550 = 251.7$ hp ◄

(*d*) Error = (278.6 - 251.7)/251.7 = 0.1069 = 10.69% ◄

16.3.3 *Consider an impulse turbine with a pitch diameter of 2.0 m and a bucket angle β_2 of 165°. If the jet velocity is 86 m/s, the jet diameter 108 mm, and the rotative speed 375 rpm, find (a) the force on the buckets, (b) the torque on the runner, and (c) the horsepower transferred to the runner. Assume that the velocity relative to the bucket does not change (i.e. assume $v_2 = v_1$).*

SI

$\omega = 375(2\pi/60) = 39.3$ rad/s ; $u = \omega r = 39.3(1\text{ m}) = 39.3$ m/s

$Q = A_1V_1 = \pi(0.108/2)^2 86 = 0.788$ m³/s ; $V_1 = 86 = v_1 + u = v_1 + 39.3$; $v_1 = 46.7$ m/s

$\Delta v_x = v_{2x} - v_{1x} = 46.7\cos 165° - 46.7 = -91.9$ m/s

(*a*) Eq. 6.24: $F = 1000(\text{kg/m}^3)0.788(\text{m}^3/\text{s})91.9\text{ m/s} = 72\,400\text{ kg·m/s}^2 = 72\,400$ N ◄

(*b*) $T = Fr = 72400(1) = 72400$ N·m ◄

(*c*) Eq. 5.38: $P = T\omega = 72400(39.3) = 2840\,000$ N·m/s = 2840 kW ◄

Sec. 16.3: Action of the Impulse Turbine – *Problems 16.1–16.6*

16.1 *Repeat Sample Prob. 16.1 for rotative speeds of 80, 160, 300, and 360 rpm, with all other data remaining the same. Also find the hydraulic efficiency of the flow through the buckets for each speed. Plot horsepower as ordinate versus rotative speed as abscissa to find the approximate speed at which maximum power will be developed. At maximum power, approximately what is the value of u/V_1?*

Sample Prob. 16.1: Impulse turbine, pitch D = 10 ft, β_2 = 160°, $V_i = V_j$ = 200 fps, Q = 4.36 cfs, $v_2 = v_1$. Find (a) F on buckets, (b) T on runner, (c) HP transferred to runner.

BG

Horsepower of jet (= hydraulic horsepower input to turbine buckets)

$$= \gamma Q(V_j^2/2g)/550 = 62.4(4.36)(200^2/64.4)/550 = 307.2 \text{ hp}$$

$u = \omega r = n(2\pi/60)5 = 0.524n$; $v_2 = v_1 = V_1 - u = 200 - u$; $T = Fr = 5F$

n (rpm)	u (fps) $0.524n$	$v_2 = v_1$ $200 - u$	$v_2\cos 160°$	Δv_x (fps) $v_2\cos 160° - v_1$	F (lb) $\rho Q(\Delta v_x)$	T (ft·lb) $5F$	P (hp) $Fu/550$	η' (%) hp/307.2
80	41.9	158.1	−148.5	−306.6	2593	12,965	197.5	64.3
160	83.8	116.2	−109.2	−225.4	1906	9,530	290.0	94.4
240	125.6	74.4	−69.9	−144.3	1220	6,100	278.6	90.7
300	157.0	43.0	−40.4	−83.4	705	3,525	201.2	65.5
360	188.4	11.6	−10.9	−22.5	190	950	65.0	21.1
▲					▲	▲	▲	▲

Maximum power and maximum efficiency of flow through the turbine occur at about 190 rpm. ◄

$u = \omega r \approx 190(2\pi/60)5 = 99$ ft/sec ; $u/V_1 \approx 99/200 = 0.5$ ◄

16.2 *Same as Prob. 16.1, except also consider hydraulic friction in the buckets by letting $v_2 = 0.85\, v_1$.*

Prob. 16.1: Impulse turbine, pitch D = 10 ft, β_2 = 160°, $V_i = V_j$ = 200 fps, Q = 4.36 cfs. Find (a) F on buckets, (b) T on runner, (c) HP transferred to runner, each for rotative speeds of 80, 160, 300, and 360 rpm. Also find the hydraulic efficiency of the flow through the buckets for each speed. Plot horsepower as ordinate versus rotative speed as abscissa to find the approximate speed at which maximum power will be developed. At maximum power, approximately what is the value of u/V_1?

BG

Horsepower of jet (= hydraulic horsepower input to turbine buckets)

$$= \gamma Q(V_j^2/2g)/550 = 62.4(4.36)(200^2/64.4)/550 = 307.2 \text{ hp}$$

$u = \omega r = n(2\pi/60)5 = 0.524n$; $v_1 = V_1 - u = 200 - u$; $v_2 = 0.85 v_1$; $v_{2x} = v_2\cos 160°$

$\Delta v_x = v_2\cos 160° - v_1$; $F = \rho Q \Delta v_x$; $T = Fr = 5F$

n rpm	u (fps) $0.524n$	v_1 (fps) $200 - u$	v_2 (fps) $0.85v_1$	Δv_x $v_2\cos 160° - v_1$	F (lb) $\rho Q(\Delta v_x)$	T (ft·lb) $5F$	HP $Fu/550$	η' HP/307.2
80	41.9	158.1	126.2	−284.3	2405	12,025	183.2	59.6%
160	83.8	116.2	92.8	−209.0	1768	8,840	269.4	87.7%
240	125.6	74.4	59.4	−133.8	1132	5,660	258.5	84.2%
300	157.0	43.0	34.3	−77.3	654	3,270	186.7	60.8%
360	188.4	11.6	9.3	−20.9	177	885	60.6	19.7%
▲					▲	▲	▲	▲

Maximum power and maximum efficiency of flow through the turbine occur at about 180 rpm. ◄

$u = \omega r \approx 180(2\pi/60)5 = 94.2$ ft/sec ; $u/V_1 \approx 94.2/200 = 0.47$ ◄

16.3 *Consider an impulse wheel with a pitch diameter of 2.65 m and a bucket angle of 168°. If the jet velocity is 55.0 m/s, the jet diameter is 100 mm, and the rotative speed is 300 rpm, find (a) the force on the buckets, (b) the torque on the runner, and (c) the power transferred to the runner. Assume $v_2 = 0.85v_1$.*

SI

$u = \omega r = 300(2\pi/60)2.65/2 = 41.6$ m/s ; $V_1 = 55$ m/s $= v_1 + u = v_1 + 41.6$; $v_1 = 13.4$ m/s

$v_2 = 0.85v_1 = 11.4$ m/s, $v_{2x} = v_2 \cos 168° = -11.15$ m/s

Hence $\Delta v_x = v_{2x} - v_{1x} = -11.15 - 13.4 = -24.55$ m/s

Note also, $V_{2x} = u + v_2 \cos 168° = 41.6 - 11.15 = 30.45$ m/s

Thus $\Delta V_x = V_{2x} - V_{1x} = 30.45 - 55 = -24.55$ m/s. Hence $\Delta V_x = \Delta v_x = -24.55$ m/s

(*a*) Eq. 6.24: $F = \rho Q(\Delta v_x) = \rho Q(\Delta V_x)$ where $Q = A_j V_j$

$F = 1000\ \text{kg/m}^3[\pi(0.1/2)^2]55\ \text{m}^3/\text{s}\ (24.55\ \text{m/s}) = 10\,580$ N ◀

(*b*) $T = Fr = 10\,580\ \text{N}(2.65/2)\ \text{m} = 14\,020\ \text{N·m}$ ◀

(*c*) $P = Fu = 10\,580\ \text{N}(41.6\ \text{m/s}) = 440$ kW ◀

or $P = T\omega = 14\,020(300)(2\pi/60)\ \text{N·m/s} = 440$ kW ◀

16.4 *An impulse turbine with a pitch diameter of 4.0 ft and a bucket angle of 160° is rotating at 514.7 rpm. The nozzle produces a jet with a velocity of 240 ft/sec and diameter of 4 in. Find (a) the force on the buckets, (b) the torque on the runner, (c) the horsepower transferred to the runner, and (d) the efficiency η of the turbine under these conditions of operation. Assume the head loss through the nozzle can be expressed as $0.05V_j^2/2g$ and assume the velocity relative to the bucket does not change (i.e., assume $v_2 = v_1$). Neglect bearing friction and windage.*

BG

$Q = A_j V_j = \pi(2/12)^2 240 = 20.93$ cfs ; $h = 1.05V_j^2/2g = 1.05(240^2/64.4) = 939$ ft

$u = \omega r = 514.7(2\pi/60)2 = 107.7$ fps

$V_1 = V_j = 240$ fps $= v_1 + u = v_1 + 107.7$; thus $v_1 = 240 - 107.7 = 132.3$ fps $= v_2$

$\Delta v_x = v_{2x} - v_{1x} = v_1 \cos 160° - v_1 = -124.3 - 132.3 = -256.6$ fps

Note also $V_{2x} = u + v_2 \cos 160° = 107.7 - 124.3 = -16.6$ fps

$\Delta V_x = V_{2x} - V_{1x} = -16.6 - 240 = -256.6$ fps. So $\Delta v_x = \Delta V_x$

(*a*) Eq. 6.24: $F = \rho Q(\Delta V_x) = 1.940(20.93)256.6 = 10{,}420$ lb ◀

(*b*) $T = Fr = 10{,}420(2.0) = 20{,}820$ ft·lb ◀

(*c*) Eqs. 5.38 and 5.40: Shaft horsepower $= T\omega/550 = 20{,}820(2\pi/60)514.7/550 = 2039$ hp ◀

(*d*) Eq. 5.42: $$\eta = \frac{\text{shaft horsepower}}{\gamma Qh/550} = \frac{2039}{62.4(20.93)939/550} = 0.915 = 91.5\%$$ ◀

16.5 *Work Prob. 16.4 where the speed of rotation is 300 rpm, all other data remaining the same.*

Prob. 16.4: Impulse turbine, pitch D = 4.0 ft, β_2 = 160°, n = 514.7 rpm, D_j = 4 in, V_j = 240 fps. Find (a) F on buckets, (b) T on runner, (c) HP transferred to runner, (d) η of turbine. Assume nozzle $h_L = 0.05V_j^2/2g$, $v_2 = v_1$, and neglect bearing friction and windage.

BG

$Q = A_jV_j = \pi(2/12)^2 240 = 20.93$ cfs ; $h = 1.05V_j^2/2g = 1.05(240^2/64.4) = 939$ ft

$u = \omega r = 300(2\pi/60)2 = 62.8$ fps

$V_1 = V_j = 240$ fps $= v_1 + u = v_1 + 62.8$; thus $v_1 = 240 - 62.8 = 177.2$ fps $= v_2$

$\Delta v_x = v_{2x} - v_{1x} = v_2 \cos 160° - v_1 = -166.5 - 177.2 = -343.7$ fps

(*a*) Eq. 6.24: $F = \rho Q(\Delta v_x) = 1.940(20.93)343.7 = 13{,}956$ lb ◀

(*b*) $T = Fr = 13{,}956(2) = 27{,}910$ ft·lb ◀

(*c*) Eqs. 5.38 and 5.40: Shaft horsepower $= T\omega/550 = 27{,}910(2\pi/60)300/550 = 1593$ hp ◀

(*d*) Eq. 5.42: $\eta = \dfrac{\text{shaft horsepower}}{\gamma Qh/550} = \dfrac{1593}{62.4(20.93)939/550} = \dfrac{1593}{2230} = 0.714 = 71.4\%$ ◀

16.6 *(a) Solve Sample Prob. 16.1 for blade angles of 160°, 165°, and 170° for the case of an impulse turbine whose pitch diameter is 8 ft rather than 10 ft. All other data are to remain the same. (b) Repeat the above for the case where $v_2 = 0.80v_1$.*

Sample Prob. 16.1: Impulse turbine, $V_1 = V_j$ = 200 fps, Q = 4.36 cfs, $v_2 = v_1$, n = 240 rpm. Find (a) F on buckets, (b) T on runner, (c) HP transferred to runner.

BG

(*a*) $u = \omega r = 240(2\pi/60)4 = 100.5$ ft/sec

$V_1 = 200$ ft/sec $= v_1 + u = v_1 + 100.5$ ft/sec, $v_1 = 99.5$ ft/sec $= v_2$

$\Delta v_x = v_2 \cos\beta_2 - v_1 = 99.5\cos\beta_2 - 99.5 = 99.5(\cos\beta_2 - 1)$

Eq. 6.24: $F = \rho Q(\Delta v_x) = 1.940(4.36)\Delta v_x = 8.46\Delta v_x$

$T = Fr = 4F$. Eqs. 5.38 and 5.40: $P = Fu/550 = F(100.5)/550 = 0.1828F$ hp

β_2	$\cos\beta_2$	Δv_x fps	F lb	T lb·ft	HP
160°	−0.940	−193.0	1632	6528	298
165°	−0.966	−195.6	1654	6616	302
170°	−0.985	−197.5	1670	6680	305
			▲	▲	▲

Note: As β_2 gets larger the HP gets larger. However as β_2 approaches 170° the discharge from a bucket may interfere with the following bucket. Hence $\beta_2 \approx 165°$ is a good design.

(*b*) $v_2 = 0.80v_1 = 0.80(99.5) = 79.6$ ft/sec ; $\Delta v_x = v_2\cos\beta_2 - v_1 = 79.6\cos\beta_2 - 99.5$

β_2	$\cos\beta_2$	$v_2\cos\beta_2$ fps	Δv_x fps	F lb	T lb·ft	HP
160°	−0.940	74.8	174.3	1474	5896	269
165°	−0.966	76.9	176.4	1492	5968	273
170°	−0.985	78.4	177.9	1505	6020	275
				▲	▲	▲

Sec. 16.4: Head on an Impulse Turbine and Efficiency -- *Exercises (3)*

16.4.1 *Refer to Sample Prob. 16.4. (a) Find the head h" delivered to the buckets of the turbine if the absolute velocity at discharge from the buckets is 60 ft/sec. Assume a bucket loss coefficient of 0.30. (b) Calculate the hydraulic efficiency of the flow through the turbine buckets. (c) What is the overall efficiency of the turbine?*

Sample Prob. 16.4: Impulse turbine, $V_j = 377$ *fps, bucket* $u = 166.5$ *fps, net* $h = 2314$ *ft,* $k_n = 0.05$.

BG

(*a*) $V_1 = 377 = u + v_1 = 166.5 + v_1\ ;\quad v_1 = 211$ fps

Eq. 16.6 with $\left(\dfrac{1}{C_v^2} - 1\right)\left[1 - \left(\dfrac{A_j}{A_B}\right)^2\right]$ replaced by k_n, the nozzle loss coefficient:

$$2314 = 0.05(377^2)/(2\times 32.2) + 0.30(211^2)/(2\times 32.2) + 60^2/(2\times 32.2) + h'' = 110 + 206 + 56 + h''$$

$h'' = 1942$ ft ◄

(*b*) Hydraulic effeciency of flow through the buckets =

$$\frac{\text{energy head transfered to buckets}}{\text{energy head entering buckets}} = \frac{h''}{V_j^2/2g} = \frac{1942(2)32.2}{377^2} = 0.88 = 88\% \quad ◄$$

(*c*) Eq. 5.42: Overall turbine $\eta = \dfrac{\text{Shaft power}}{\text{Input power}} = \dfrac{\gamma Q h''}{\gamma Q h} = \dfrac{h''}{h} = \dfrac{1942}{2314} = 0.84 = 84\%$ ◄

16.4.2 *Repeat Sample Prob. 16.4 for the case of a 4-in-diameter jet with all other data remaining the same.*

Sample Prob. 16.4: Delivery pipe, $L = 10{,}000$ *ft,* $f = 0.025$, $D = 4.0$ *ft, delivers water to a nozzle at an elevation 2420 ft below the reservoir water surface, nozzle loss* $= 0.05V_j^2/2g$. *The jet drives an impulse wheel,* $D_w = 106$ *in,* $\beta_2 = 165°$, *connected to a 20-pole generator in a 60-cycle system. Find the pipe V,* V_j, *Q, u,* u/V_1, *net head h, F on bucket, T on wheel, shaft HP, HP at base of nozzle, and overall turbine* η. *Assume* $v_2 = v_1$, *neglect bearing friction and windage.*

BG

$L/D = 10{,}000/4 = 2500 > 1000$ so neglect minor loss at pipe entrance.

Energy equation from reservoir surface to jet: $2420 - 0.025(10{,}000/4)V^2/2g - 0.05V_j/2g = V_j^2/2g$

Continuity: $V_j = (48/4)^2V = 144V\ ;\quad \therefore\ 2420 - 62.5V^2/2g - 0.05(144V)^2/2g = (144V)^2/2g$

i.e., $2420 = (62.5 + 1037 + 20{,}736)V^2/2g = 21{,}835V^2/2g$

$V = 2.67$ fps ◄ $V_j = 144(2.67) = 385$ fps ◄ $Q = AV = \pi 2^2(2.67) = 33.5$ cfs ◄

Sec. 16.1: $N = 7200/n$ so $n = 7200/N = 7200/20 = 360$ rpm

$\omega = 2\pi n/60 = 2\pi(360)/60 = 37.7$ rad/sec ; $u = \omega r = 37.7(106/2)/12 = 166.5$ fps ◄

$u/V_1 = 166.5/385 = 0.432$ ◄

Eq. 16.5: Net head, $h = z_1 - h_L = 2420 - 62.5(2.67)^2/(2\times 32.2) = 2413$ ft ◄

Eq. 16.2: $F = 1.940(33.5)(385 - 166.5)(1 - \cos 165°) = 27{,}900$ lb ◄

$T = Fr = 27{,}900(106/2)/12 = 123{,}300$ ft·lb ◄

Eqs. 5.38 and 5.40: $P = T\omega/550 = 123{,}300(37.7)/550 = 8450$ hp ◄

Power at head (inlet, base) of nozzle = power input to turbine (including nozzle) =

$P = \gamma Q h/550 = 62.4(33.5)2413/550 = 9170$ hp ◄

Eq. 5.42: Overall turbine $\eta = \dfrac{\text{Shaft power}}{\text{Input power}} = \dfrac{8450}{9170} = 0.921 = 92.1\%$ ◄

16.4.3 *Refer to Sample Prob. 16.1. Calculate the hydraulic efficiency of the flow through the turbine buckets. Note, this is different than the hydraulic efficiency η' of the turbine because the nozzle is considered an integral part of the turbine.*

Sample Prob. 16.1: $V_j = V_1 = 200$ fps, $Q = 4.36$ cfs, P transferred to runner = 278.6 hp.

BG

Power extracted by runner = power transferred to runner = 278.6 hp (given).

$P_{jet} = \gamma Q(V_j^2/2g)/550 = 62.4(4.36)[200^2/(2 \times 32.2)]/550 = 307.2$ hp

Hydraulic efficiency of flow through the turbine buckets =

$$\eta_b = \frac{P \text{ extracted by runner}}{P \text{ of jet}} = \frac{278.6}{307.2} = 0.906 = 90.6\% \quad ◄$$

Note: Here, with $v_2 = v_1$ as the water flows through the buckets, there is no head loss due to fluid friction. However, the bucket hydraulic efficiency is less than 100% because the velocity head at discharge, $V_2^2/2g$ (i.e., the residual velocity head), does not act on the buckets and is effectively lost.

Sec 16.4: Head on an Impulse Turbine and Efficiency -- ***Problems** 16.7–16.13*

16.7 *Repeat Sample Prob. 16.4 for the case of a 6-in-diameter jet, with all other data remaining the same.*

Sample Prob. 16.4: Delivery pipe, L = 10,000 ft, f = 0.025, D = 4.0 ft, delivers water to a nozzle at an elevation 2420 ft below the reservoir water surface, nozzle loss = $0.05V_j^2/2g$. The jet drives an impulse wheel, $D_w = 106$ in, $\beta_2 = 165°$, connected to a 20-pole generator in a 60-cycle system. Find the pipe V, V_j, Q, u, u/V_1, net head h, F on bucket, T on wheel, shaft HP, HP at base of nozzle, overall turbine η. Assume $v_2 = v_1$, neglect bearing friction and windage.

BG

$L/D = 10{,}000/4 = 2500 > 1000$ so neglect minor loss at pipe entrance.

Energy equation from surface of reservoir to jet: $2420 - 0.025(10{,}000/4)V^2/2g - 0.05V^2/2g = V_j^2/2g$

Continuity: $V_j = V(48/6)^2 = 64V$. $\therefore$ $2420 - 62.5V^2/2g - 0.05(64V)^2/2g = (64V)^2/2g$

$2420 = (62.5 + 204.8 + 4096)V^2/2g = 4363V^2/2g$; $V = \sqrt{2(32.2)2420/4363} = 5.98$ fps ◄

$V_j = 64V = 64(5.98) = 383$ fps ◄ $Q = AV = \pi 2^2(5.98) = 75.1$ cfs ◄

Sec. 16.1: $N = 7200/n$ so $n = 7200/N = 7200/20 = 3600$ rpm

$\omega = 2\pi n/60 = 2\pi 360/60 = 37.7$ rad/sec

$u = \omega r = 37.7(106/2)/12 = 166.5$ fps ◄ $u/V_1 = 166.5/383 = 0.435$ ◄

Eq. 16.5: Net head, $h = z_1 - h_L = 2420 - f(L/D)V^2/2g = 2420 - 0.025(10{,}000/4)(5.98)^2/64.4$

$= 2385$ ft ◄

Eq. 16.2: $F = 1.940(75.1)(383 - 166.5)(1 - \cos 165°) = 62{,}000$ lb ◄

Torque on wheel $T = Fr = 62{,}000(106/2)/12 = 273{,}800$ ft·lb ◄

Eqs. 5.38 and 5.40: Shaft power $P = T\omega/550 = 273{,}800(37.7)/550 = 18{,}770$ hp ◄

Power at base (entrance) of nozzle = power input to turbine (including nozzle) =

$P = \gamma Qh/550 = 62.4(75.1)2385/550 = 20{,}320$ hp ◄

Eq. 5.42: Overall turbine efficiency $\eta = \dfrac{\text{Shaft power}}{\text{Input power}} = \dfrac{18{,}770}{20{,}320} = 0.924 = 92.4\%$ ◄

Note: The turbine efficiency is still high. However, the power developed (18,770 hp) using this nozzle with a 6-in-diameter jet is much less than the power (31,960 hp, Sample Prob. 16.4) with the nozzle that produces the 8-in-diameter jet. See Sample Problem 16.5 for an explanation.

16.8 *A torque of 157,600 ft·lb is transmitted to a turbine that is developing 6000 bhp. How many poles does the generator have if it is producing 50-cycle electricity?*

BG

Eqs. 5.38 and 5.40: bhp = 6000 = $T\omega/550$ = $157{,}600(2\pi/60)n/550$; n = 200 rpm

Sec. 16.1: Number of poles = $6000/n$ = 6000/200 = 30 ◄

16.9 *A 1.5-in-diameter water jet with a velocity of 120 ft/sec impinges on a bucket of a stationary impulse wheel with a pitch diameter of 8.0 ft. If the bucket blade angle is 165°, what torque must be applied to the turbine shaft to prevent it from rotating? Consider two cases: (a) $v_2 = v_1$; (b) $v_2 = 0.9v_1$.*

BG

Eq. 6.24: $F = \rho Q(\Delta v_x)$; $Q = A_j V_j = [\pi(1.5/12)^2/4]120 = 1.472$ cfs

(*a*) $\Delta v_x = 120\cos 165° - 120 = -115.9 - 120 = -235.9$ fps

$T = Fr = \rho Q(\Delta v_x)r = 1.940(1.472)235.9(4) = 2695$ ft·lb ◄

(*b*) $\Delta v_x = 0.9(120)\cos 165° - 120 = -104.3 - 120 = -224.3$ fps

$T = \rho Q(\Delta v_x)r = 1.940(1.472)(224.3)4 = 2562$ ft·lb ◄

16.10 *Water is delivered from a reservoir through a 1200-mm-diameter pipe (f = 0.022) 3200 m long to a nozzle that emits a 200-mm-diameter jet (Fig. P16.10). The jet impinges on the buckets of an impulse turbine, pitch diameter 2.5 m, which drives a 16-pole generator that produces 50-Hz electricity. The surface of the reservoir providing the water is at an elevation 800 m higher than the nozzle. Assume that the head loss through the nozzle can be expressed as $0.05V_j^2/2g$ where V_j is the jet velocity. Find the following: velocity of flow in the pipe, jet velocity, flow rate, speed of the buckets, value of u/V_1, net head, force F on the bucket, torque on the wheel, shaft power, power at the base of the nozzle, and overall turbine efficiency. Assume β_2 = 165° and $v_2 = v_1$; neglect bearing friction and windage.*

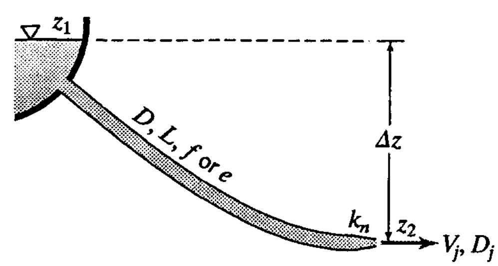


Figure P16.10

SI

Energy equation from reservoir surface to jet: $800 - (0.022)(3200/1.2)V^2/2g - 0.05V_j^2/2g = V_j^2/2g$

Continuity: $V_j = (1200/200)^2V = 36V$. ∴ $800 - 58.7V^2/2g = 1.05(36V)^2/2g = 1360V^2/2g$

$800 = 1420V^2/2g$; V = 3.325 m/s ◄ $V_j = 36V = 36(3.325) = 119.7$ m/s ◄

$Q = AV = \pi(0.200/2)^2 119.7 = 3.76$ m³/s ◄ $N = 6000/n$, $n = 6000/16 = 375$ rpm ◄

$\omega = 2\pi n/60 = 2\pi(375/60) = 39.25$ rad/sec ; $u = \omega r = 39.25(2.500/2) = 49.0$ m/s ◄

Hence $u/V_1 = 49.0/119.7 = 0.41$ ◄

Net head (Fig. 16.1): $h = z_1 - h_L = 800 - 0.022(3200/1.2)3.325^2/[2(9.81)] = 767$ m

Eq. 16.2: $F = \rho Q(V_1 - u)(1 - \cos\beta_2) = 1000(3.76)(119.7 - 49.0)(1 + 0.966) = 523\,000$ N ◄

Torque on wheel: $T = Fr = 523{,}000(2.5/2) = 654\,000$ N·m ◄

Eqs. 5.38 and 5.41: Shaft power = $T\omega$ = $654\,000(39.25)/1000 = 25\,500$ kW ◄

Power at base (entrance) of nozzle = power input to turbine (including nozzle) =

$P = \gamma Q h = 9.81(3.76)767 = 28\,300$ kW ◄

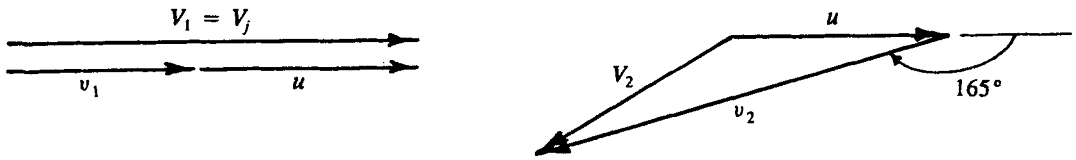
Eq. 5.42: Overall efficiency of turbine: $\eta = T\omega/\gamma Q h = 25\,500/28\,300 = 0.90$ ◄

16.11 *The diameter of a Pelton wheel is 6 ft. The velocity of the 4-in-diameter jet is 400 ft/sec and the bucket angle is 165°. Assuming a k value of 0.21, for bucket speeds of 100, 200, 300 and 400 ft/sec find the torque in ft·lb, the shaft horsepower, and the hydraulic efficiency of the buckets. Neglect bearing friction and windage.*

BG

Flow through buckets: $v_1^2/2g - h_L = v_2^2/2g$ where $h_L = k(v_2^2/2g)$; $v_1^2/2g - 0.21 v_2^2/2g = v_2^2/2g$

$v_1^2 = (0.21 + 1)v_2^2$ from which $v_2 = 0.909 v_1$



$\Delta v = v_2 \cos 165° - v_1 = 0.909 v_1 \cos 165° - v_1 = -1.879 v_1$; $\Delta V = (u + v_2 \cos 165°) - V_1$

Sec. 6.8 shows that $\Delta v = \Delta V$. $Q = AV = (\pi/4)(4/12)^2 400 = 34.9$ cfs

Eq. 6.24: $F = \rho Q(\Delta V) = 1.940(34.9)\Delta V = 67.7\Delta V = 67.7\Delta v$; $T = Fr = 3F = 3(67.7)\Delta v = 203\Delta v$

Shaft hp $= Fu/550 = 67.7(\Delta v)u/550$; Jet hp $= \gamma Q(V_j^2/2g)/550 = 62.4(34.9)[400^2/(2 \times 32.2)]/550 = 9840$

Bucket η = Shaft hp/Jet hp = $[67.7(\Delta v)u/550]/9840$

μ (ft/sec)	v_1 (ft/sec)	Δv (ft/sec)	T (ft·lb)	shaft hp	bucket η
100	300	563.7	114,300	6940	0.71
200	200	375.8	76,300	9250	0.94
300	100	187.9	38,100	6940	0.71
400	0	0	0	0	0

Note: This assumes that all water issuing from the jet acts on the wheel.

16.12 *An impulse turbine is supplied by a 1200-mm-diameter steel penstock (e = 0.10 mm) 500 m long (Fig. P16.10). The jet diameter of the single nozzle is 400 mm and its loss coefficient is 0.05. When the water level in the reservoir is 150 m above the nozzle of the turbine, the brake power is measured to be 7900 kW. What is the turbine efficiency?*

SI

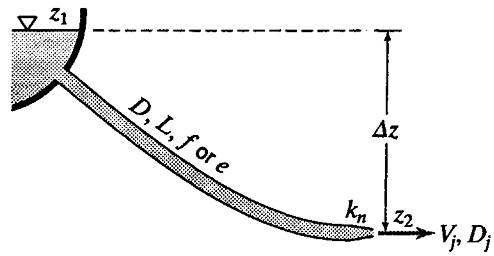


Figure P16.10

Energy equation: $150 - f(L/D)V^2/2g - 0.05V_j^2/2g = V_j^2/2g$

$e/D = 0.10/1200 = 0.000\,083$

Fig. 8.11: Assume $f = 0.013$ (depends on Reynolds number)

Continuity: $V_j(400)^2 = V(1200)^2$; $V_j = (1200/400)^2 V = 9V$

$\therefore\ 150 - 0.013(500/1.2)V^2/2g = 1.05(9V)^2/2g = 85.1V^2/2g$; $V = 5.71$ m/s

Check f: $\mathbf{R} = DV/\nu = 1.2(5.71)/(1 \times 10^{-6}) = 6.85 \times 10^6$; Fig. 8.11 shows $f = 0.012$ is closer

This changes V very little, $V = 5.72$ m/s

Net head at base of nozzle: $h = 150 - 0.012(500/1.2)5.72^2/(2 \times 9.81) = 141.7$ m

Eq. 5.39: Power at base (entrance) of nozzle = power input to turbine (including nozzle) =

$P = \gamma Q h = 9.81(6.47)141.7 = 8990$ kW

Turbine η = brake P/delivered P = 7900/8990 = 0.879 ◄

Alternative solution (*i*): Set up the 7 governing simultaneous equations and solve them automatically in computer software (Secs. 8.27 and 8.17). The 7 governing equations are: an energy equation which includes h_f and h_n, 2 continuity Eqs. 4.7, for V and V_j, Eq. 8.1 for **R**, Eq. 8.13 for h_f, an equation for nozzle loss h_n (Sec. 8.23), and Colebrook Eq. 8.51 for f.

Alternative solution (*ii*): Elimininate many unknowns (Q, V_j, h_f, h_n) and solve for the remainder (f, V, **R**) by trials (Secs. 8.27 and 8.15), as in the solution presented above but using Eq. 8.51 or 8.52 instead of Fig. 8.11 for f.

16.13 *Using the data of Prob. 16.11, find values of (a) head loss in bucket friction, (b) velocity head at discharge, and (c) total head loss for bucket speeds of 180, 190, 200, and 210 fps. At what bucket speed is the discharge loss a minimum? At what bucket speed is the total head loss a minimum?*

Prob. 16.11: Pelton wheel, $V_1 = 400$ fps, $\beta_2 = 165°$, $k = 0.21$.

BG

For $u = 180$ fps:

(*a*) $v_1 = V_1 - u = 400 - 180 = 220$ fps

Sec. 16.4: $v_1^2/2g - kv_2^2/2g = v_2^2/2g$, $\therefore$ $v_2 = v_1/\sqrt{k+1} = 220/\sqrt{0.21+1} = 200$ fps

Bucket friction $h_L = kv_2^2/2g = 0.21(200^2/64.4) = 130.4$ ft ◄

(*b*) $V_{2x} = u + v_2 \cos 165° = 180 + 200(-0.966) = -13.19$ fps ; $V_{2y} = 200 \sin 165° = 51.8$ fps

$V_2 = \sqrt{V_{2x}^2 + V_{2y}^2} = \sqrt{(-13.19)^2 + (51.8)^2} = 53.4$ fps ; $V_2^2/2g = (53.4)^2/(2 \times 32.2) = 44.3$ ft ◄

(*c*) Total head loss = 130.4 + 44.3 = 174.7 ft ◄

u fps	v_2 fps	(*a*) bucket h_L ft	V_{2x} fps	V_{2y} fps	V_2 fps	(*b*) $V_2^2/2g$ ft	(*c*) Total loss ft
180	200.0	130.4	−13.2	51.8	53.4	44.3	174.7
190	191.1	119.1	5.4	49.5	49.8	38.5	157.6
200	182.0	108.0	24.2	47.1	53.7	44.8	152.8
210	172.9	97.5	43.0	44.8	62.0	59.7	157.2
		▲				▲	▲

The discharge loss (*b*) is a minimum at a bucket speed of slightly less than 190 fps. ◄

The total head loss (*c*) is a minimum at a bucket speed of slightly more than 200 fps. ◄

Sec. 16.5: Nozzles for Impulse Turbines – *Exercises (2)*

16.5.1 *Repeat Sample Prob. 16.5 for the case where the pipe length is 10,000 ft. All other data are to remain the same.*

Sample Prob. 16.5: Impulse turbine, pipe $D = 6$ in, $h_e' = 0$, $f = 0.020$, nozzle $h_L = 0.04V_j^2/2g$. Reservoir surface elev. = 500 ft, nozzle elev. = 300 ft. Find D_j for maximum P_j.

BG

Energy Eq: $500 - 0.020(10{,}000/0.5)V^2/2g - 0.04V_j^2/2g = 300 + V_j^2/2g$ (1)

Continuity: $D^2V = D_j^2V_j$; $V = (D_j/0.5)^2V_j = 4D_j^2V_j$ (2)

Combining (1) and (2) gives $200 = 400(4D_j^2V_j)^2/2g + 1.04V_j^2/2g = (6400D_j^4 + 1.04)V_j^2/2g$

Assume values of D_j (as in Sample Prob. 16.5) and find corresponding values for V_j.

Then find corresponding $Q = A_jV_j = (\pi/4)D_j^2V_j$ and from Eq. 16.9: $P_j = \gamma Q(V_j^2/2g)/550$

D_j in	D_j ft	V_j fps	A_j ft²	Q cfs	P_j hp	
0.5	0.0416	110.3	0.001 36	0.150	3.21	
1.0	0.0833	97.7	0.005 45	0.53	8.95	(max)
2.0	0.167	46.3	0.021 9	1.014	3.83	

Thus the maximum power (about 8.95 hp) occurs with a jet diameter of about 1.0 inch. ◄

<u>Alternative solution</u>: This could use differential calculus, see Prob. 16.15.

16.5.2 *Repeat Sample Prob. 16.5 for the case where the pipe diameter is 12 in. All other data are to remain the same.*

Sample Prob. 16.5: Impulse turbine, pipe L = 1000 ft, $h_e' = 0$, $f = 0.020$, nozzle $h_L = 0.04V_j^2/2g$. Reservoir surface elev. = 500 ft, nozzle elev. = 300 ft. Find D_j for maximum P_j.

BG

Energy Eq: $500 - 0.020(1000/1.0)V^2/2g - 0.04V_j^2/2g = 300 + V_j^2/2g$ (1)

Continuity: $D^2V = D_j^2V_j$; $V = (D_j/1)^2V_j = D_j^2V_j$ (2)

Combining (1) and (2) gives $200 = 20(D_j^2V_j)^2/2g + 1.04V_j^2/2g = (20D_j^4 + 1.04)V_j^2/2g$

Assume values of D_j (as in Sample Prob. 16.5) and find corresponding values for V_j.

Then find corresponding $Q = A_jV_j = (\pi/4)D_j^2V_j$ and from Eq. 16.9: $P_j = \gamma Q(V_j^2/2g)/550$

D_j in	D_j ft	V_j fps	A_j ft²	Q cfs	P_j hp	
3.0	0.250	107.3	0.0491	5.27	106.9	
4.0	0.333	100.1	0.0855	8.58	152.4	
5.0	0.417	88.5	0.1365	12.08	166.7	(max)
6.0	0.500	75.0	0.1963	14.72	145.9	
8.0	0.667	50.7	0.352	17.74	79.4	

Thus the maximum power (about 167 hp) occurs with a jet diameter of almost 5.0 in. ◄

Alternative solution: This could use differential calculus, see Prob. 16.15.

Sec 16.5: Nozzles for Impulse Turbines -- *Problems 16.14–16.15*

16.14 *A 36-in pipeline (f = 0.020) 10,000 ft long connects a reservoir whose water surface elevation is 1800 ft to a nozzle at elevation 1000 ft (Fig. P16.10). The jet from the nozzle is used to drive an impulse wheel. (a) If the head loss through the nozzle is expressible as $0.04V_j^2/2g$, determine the horsepower that will be developed by jets having diameters of 8 in, 10 in, and 12 in. (b) On the basis of the answers to (a), approximately what would be the horsepower and diameter of the jet that gives the maximum horsepower?*

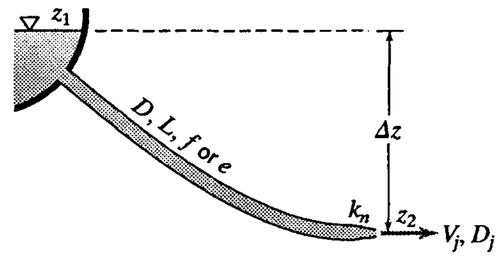


Figure P16.10

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Energy: $1800 - 0.020(10{,}000/3)V^2/2g - 0.04V_j^2/2g = 1000 + V_j^2/2g$ (1)

Continuity: $D^2V = D_j^2V_j;\ \ V = (D_j/3)^2V_j = D_j^2V_j/9$ (2)

Combining (1) and (2) gives $800 = 66.7(D_j^2/9)^2V_j^2/2g + 1.04V_j^2/2g = (0.823D_j^4 + 1.04)V_j^2/2g$

For the given values of D_j find corresponding values of V_j.

Next find corresponding $Q = A_jV_j = (\pi/4)D_j^2V_j$. Then from Eq. 16.9: $P_j = \gamma Q(V_j^2/2g)/550$

D_j in	D_j ft	V_j fps	A_j ft^2	Q cfs	$V_j^2/2g$ ft	P_j hp	
8	0.667	207	0.349	72.2	665	5450	
10	0.833	189.3	0.545	103.2	556	6510	(max)
12	1.000	166.2	0.785	130.5	429	6350	

So the maximum jet power of about 6510 hp occurs with a jet diameter slightly larger than 10 inches. ◄

Alternative solution: This could use differential calculus, see Prob. 16.15.

16.15 *Using the data of Prob. 16.14, express the jet power analytically in terms of D_j and V_j, differentiate, and equate to zero to find the value of D_j for which the jet power is a maximum. Evaluate the value of the maximum jet power. Hint: Express power as $P = f(x)$, where $x = D_j^2$.*

Prob. 16.14: Impulse turbine, pipe D = 3 ft, L = 10,000 ft, f = 0.020, nozzle $h_L = 0.04V_j^2/2g$. Reservoir surface elev. = 1800 ft, nozzle elev. = 1000 ft.

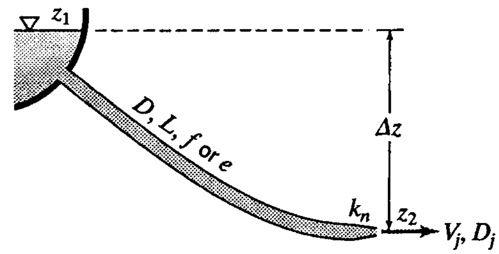

Figure P16.10

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Energy: $$1800 - 0.020\frac{10{,}000}{3}\frac{V^2}{2g} - 0.04\frac{V_j^2}{2g} = 1000 + \frac{V_j^2}{2g} \quad (1)$$

Continuity: $$D^2V = D_j^2V_j \;; \quad V = \left(\frac{D_j}{3}\right)^2 V_j = \frac{D_j^2V_j}{9} \quad (2)$$

Combining (1) and (2) and rearranging gives: $V_j^2 = 2g(800)/(1.04 + 0.823D_j^4)$

Power of jet: $P = \gamma Qh = \gamma A_jV_j(V_j^2/2g) = 62.4(\pi D_j^2/4)V_j^3/2g = 0.761D_j^2V_j^3$ ft·lb/sec

Let $x = D_j^2$ and $y = V_j^2 = \dfrac{2g(800)}{1.04 + 0.823x^2}$

Then $P = 0.761xy^{3/2}$ and $\dfrac{dy}{dx} = \dfrac{-2g(800)2(0.823)x}{(1.04 + 0.823x^2)^2} = \dfrac{-2(0.823)xy}{1.04 + 0.823x^2}$

$$\frac{dP}{dx} = 0.761y^{3/2} + 0.761x\left(\frac{3}{2}\right)y^{1/2}\left[\frac{-2(0.823)xy}{1.04 + 0.823x^2}\right] = 0$$

i.e. $1 = 3(0.823)x^2/(1.04 + 0.823x^2)$; $1.04 + 0.823x^2 = 3(0.823)x^2$

$1.04 = 2(0.823)x^2$; $x^2 = 0.632$; $D_j = \sqrt{x} = 0.632^{1/4} = 0.892$ ft $= 10.70$ in ◀

Then $y = 2g(800)/[1.04 + 0.823(0.632)] = 33{,}000$; $V_j = \sqrt{y} = 181.7$ fps

$P = 0.761xy^{3/2} = 0.761\sqrt{0.632}(33{,}000)^{3/2} = 3.63 \times 10^6$ ft·lb/sec $= 6600$ hp ◀

Sec. 16.7: Action of the Reaction Turbine -- *Exercises* (2)

16.7.1 *A Kaplan turbine operating at 327.3 rpm develops a shaft power of 300 kW when the flow is 5.5 m³/s. Find the torque transmitted from the flowing water to the shaft.*

SI

Eq. 16.10 (and 5.38): Shaft power $P = T\omega = 300$ kW $= 300\,000$ N·m/s

$\omega = 327.3(2\pi/60) = 34.3$ rad/sec ; $T = 300\,000/\omega = 300\,000/34.3 = 8750$ N·m ◀

16.7.2 *A torque of 7500 ft·lb is transferred from the flowing water to a reaction turbine whose shaft power is 285.5 horsepower. How many poles must the generator have to develop 50-cycle electricity?*

BG

Eq. 16.10 and (5.38): Shaft power $= T\omega$; $285.5(550)$ ft·lb/sec $= 7500(2\pi n/60)$; $n = 200$ rpm

Sec. 16.1: Number of poles, $N = 6000/n = 6000/200 = 30$ ◀

Sec. 16.8: Draft Tubes and Effective Head on Reaction Turbines -- ***Exercises*** *(3)*

16.8.1 *Repeat Sample Prob. 16.7 for the case where the flow rate is 90 cfs with all other data remaining the same.*
Sample Prob. 16.7: Original draft tube = pipe, L = 40 ft, f = 0.03, constant D = 3 ft. If replaced by a diverging tube, D increasing uniformly from 3 ft to 8 ft over 40 ft length, how much additional effective head will be developed? Neglect effects of the bend and the tailrace velocity.

BG

Original (straight) draft tube: $V = Q/A = 90/(\pi 1.5^2) = 12.73$ ft/sec

Head loss due to draft tube: $h_L = f(L/D)V^2/2g = 0.03(40/3)12.73^2/(2\times 32.2) = 1.007$ ft

Head loss at discharge: $h_{Lx} = V_2^2/2g - V_C^2/2g = 12.73^2/(2\times 32.2) - 0 = 2.52$ ft

Thus original total head loss = 1.007 + 2.52 = 3.52 ft

Replacement (diverging) draft tube:

Head loss due to draft tube: Eq. 8.78: $h_L = k'(V_1 - V_2)^2/2g$ where $V_1 = 12.73$ ft/sec

Continuity: $V_2 = (D_1/D_2)^2 V_1 = (3/8)^2 12.73 = 1.790$ ft/sec

Cone angle (Fig. 8.19): $\tan(\alpha/2) = (8 - 3)/(2\times 40) = 0.0625$, $\alpha = 7.15°$

Fig. 8.20*a* (solid curve) for $\alpha = 7.15°$: $k' = 0.133$

Thus $h_L = 0.133(12.73 - 1.790)^2/(2\times 32.2) = 0.247$ ft

Head loss at discharge: $h_{Lx} = 1.790^2/(2\times 32.2) - 0 = 0.0498$ ft

New total head loss = 0.247 + 0.0498 = 0.297 ft

Reduction in total head loss = original − new = 3.52 − 0.297 = 3.23 ft

∴ new draft tube will provide 3.23 ft of additional net head. ◄

16.8.2 *A Francis turbine operating at 300 rpm develops a shaft horsepower of 1125 hp when the flow is 120 cfs. What is the net head on the runner if the turbine efficiency is 85%?*

BG

Eq. 16.8: Shaft power = $\gamma Q h \eta$; $1125 = \gamma Q h\eta/550 = 62.4(120)h(0.85)/550$; $h = 97.2$ ft ◄

16.8.3 *A Kaplan turbine operating at 327.3 rpm develops a shaft power of 300 kW when the flow is 5.5 m³/s. What is the net head on the runner if the turbine efficiency is 92%?*

SI

Eq. 16.8: Shaft power = $\gamma Q h \eta$; $300 = 9810(5.5)h(0.92)/1000$; $h = 6.04$ m ◄

Sec. 16.8: Draft Tubes and Effective Head on Reaction Turbines -- ***Problems*** *16.16–16.19*

16.16 *In the test of a Francis turbine, a pressure of 140 kPa was measured at the flange at the entrance to a spiral turbine-case where the diameter is 600 mm. Neglecting the small velocity head in the tailrace, find the net head on the turbine if the flow rate was 2.5 m³/s and the flange was 3.0 m above the tailrace.*

SI

At entrance: $V = 2.5/(\pi 0.3^2) = 8.84$ m/s

Eq. 16.12: $h = z + p/\gamma + V^2/2g - V_C^2/2g = 3.0 + 140(1000)/9810 + 8.84^2/(2\times 9.81) - 0$

$= 3.0 + 14.27 + 3.98 = 21.3$ m ◄

16.17 *A reaction turbine is supplied with water through a 1500-mm-diameter pipe (e = 0.10 mm) that is 50 m long. The water surface in the reservoir is 27 m above the draft-tube inlet that is 4.1 m above the water level in the tailrace (Fig. P16.17). If the turbine efficiency is 92% and the discharge is 12 m³/s, what is the power output of the turbine in kilowatts? By how much must the discharge be increased to increase power production by 500 kW? Assume the velocity in the tailrace can be neglected.*

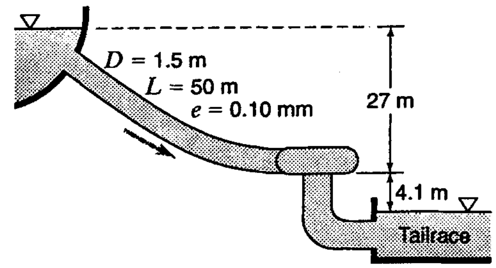

Figure P16.17

SI

$V = Q/A = 12/(\pi 1.5^2/4) = 6.79$ m/s

Table A.1 for water at 20°C: $\nu = 1.00 \times 10^{-6}$ m²/s

$\mathbf{R} = 6.79(1.5)/(1 \times 10^{-6}) = 1.019 \times 10^7$; $e/D = 0.10/1500 = 0.000\,0667$, $f = 0.0114$ (Fig. 8.11)

Net $h = 27 + 4.1 - 0.0114(50/1.5)6.79^2/(2 \times 9.81) = 30.2$ m

$P = \gamma Q h \eta = 9810(12)30.2(0.92) = 3\,270\,000$ W $= 3270$ kW ◄

$(3270 + 500)$ kW $= 9810(\pi 1.5^2/4)V[27 + 4.1 - f(50/1.5)V^2/(2 \times 9.81)]0.92/1000$

By trials or polynomial or equation solver: $V = 7.36$ m/s, $\mathbf{R} = 1.104 \times 10^7$, $f = 0.0113$

$Q = AV = (\pi 1.5^2/4)7.36 = 13.00$ m³/s ; $\Delta Q = 13.00 - 12.0 = 1.001$ m³/s ◄

16.18 *A draft tube leading from the discharge side of a turbine to the submerged discharge in the tailrace as in Fig. 16.11 consists of a pipe (f = 0.025) of constant diameter 0.8 m and length 10.0 m. The flow rate is 9.2 m³/s. If this draft tube were to be replaced by a diverging tube 12.0 m long whose diameter changed uniformly from 0.8 m to 2.5 m over the 12.0 m length (Fig. P16.18), how much additional head would be developed by the replacement draft tube? Neglect head loss due to curvature in the bend.*

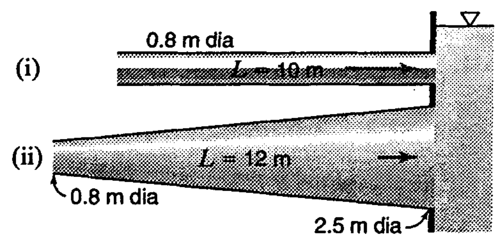

Figure P16.18

SI

Original (straight) draft tube, $L = 10$ m: $V = Q/A = 9.2/(\pi 0.4^2) = 18.30$ m/s

Assume the tailrace velocity V_C is negligible.

$h_L = h_f + h_d' = 0.025(10/0.8)V^2/2g + V^2/2g = (0.313 + 1)(18.30)^2/(2 \times 9.81) = 22.4$ m

Replacement (diverging) draft tube, $L = 12$ m: $V_1 = 18.30$ m/s; $V_2 = Q/A_2 = 9.2/(\pi 1.25^2) = 1.874$ m/s

Cone angle (Fig. 8.19): $\tan(\alpha/2) = (2.5 - 0.8)/(2 \times 12) = 0.0708$, $\alpha = 8.10°$

Fig. 8.20*a* (solid curve) for $\alpha = 8.10°$: $k' = 0.144$

Eq. 8.78: $h' = k'(V_1 - V_2)^2/2g = 0.144(18.30 - 1.874)^2/[2(9.81)] = 1.981$ m

$h_d' = V_2^2/2g = 1.874^2/[2(9.81)] = 0.1790$ m. New total head loss $= 1.981 + 0.1790 = 2.16$ m

Additional head that would be developed = original − new = $22.4 - 2.16 = 20.2$ m ◄

16.19 *At its upper end a draft tube has a diameter of 24.5 in where it joins the discharge side of the turbine at a point 11.0 ft above the surface of the water in the tailrace. The discharge end of the draft tube has a diameter of 42 in and the velocity in the tailrace is negligible. The total head loss in the draft tube is $0.15 V_2^2/2g$ plus the submerged discharge loss of $V_3^2/2g$, where subscripts 2 and 3 refer to the upper and lower ends of the draft tube, respectively. (a) When the flow is 38.8 cfs, what is the pressure at the upper end of the draft tube? (b) Suppose the draft tube were of uniform diameter and the flow rate does not change; what then would be the pressure at the upper end of the tube? (c) How much head is saved by the diverging tube? Assume the draft tube has a length of 18 ft and f = 0.020.*

BG

(*a*) Upper end: $V_2 = Q/A_2 = 38.8/[\pi(24.5/12)^2/4] = 11.85$ fps ; $V_2^2/2g = 11.85^2/(2 \times 32.2) = 2.18$ ft

Lower end: $V_3 = Q/A_3 = 38.8(4)/[\pi(42/12)^2] = 4.03$ fps ; $V_3^2/2g = 4.03^2/(2 \times 32.2) = 0.253$ ft

From 2 to tailwater (given): $\Sigma h_L = 0.15V_2^2/2g + V_3^2/2g = 0.15(2.18) + 0.253 = 0.580$ ft

Energy equation from 2 to tailwater: $z_2 + p_2/\gamma + V_2^2/2g - \Sigma h_L = 0$

$11.0 + p_2/\gamma + 2.18 - 0.580 = 0$; $p_2/\gamma = -12.60$ ft ; $p_2 = -12.60(62.4/144) = -5.46$ psi ◀

or (from inside covers) $p_2 = -5.46(29.92/14.70) = 11.11$ inHg vacuum ◀

(*b*) Straight tube: $h_f = 0.020[18/(24.5/12)]2.18 = 0.385$ ft

From 2 to tailwater: $\Sigma h_L = h_f + h_d' = 0.385 + V_2^2/2g = 0.385 + 2.18 = 2.57$ ft

Energy Eq. from 2 to tailwater: $11.0 + p_2/\gamma + 2.18 - 2.57 = 0$

$p_2/\gamma = -10.62$ ft ; $p_2 = -10.62(62.4/144) = -4.60$ psi ◀

or (from inside covers) $p_2 = -4.60(29.92/14.70) = 9.36$ inHg vacuum ◀

(*c*) Head saved by diverging tube $= \Sigma h_L(b) - \Sigma h_L(a) = 2.57 - 0.580 = 1.986$ ft ◀

Sec. 16.9: Efficiency of Turbines – *Problem 16.20*

16.20 *A model turbine (one-twentieth of prototype size) has a maximum hydraulic efficiency of 86.2%. Estimate the efficiency of the prototype utilizing the Moody step-up formula.*

N

Eq. 16.15: $(1 - 0.862)/(1 - \eta_p) = (20/1)^{1/5} = 1.821$; $\eta_p = 0.924 = 92.4\%$ ◀

Sec. 16.10: Similarity Laws for Reaction Turbines -- *Exercises (3)*

16.10.1 *At a maximum efficiency a turbine runs at 150 rpm, discharges 10.0 m³/s, and develops a shaft power of 2600 kW under a net head of 30 m. (a) What is its efficiency? (b) What would be the revolutions per minute, the flow rate, and brake power of this same turbine under a net head of 48 m for homologous conditions? (c) How many poles would be required for 60-cycle electricity if the net head is 48 m?*

SI

(*a*) Eq. 16.14: $\eta = \dfrac{\text{shaft power}}{\text{water power}} = \dfrac{2600}{\gamma Qh} = \dfrac{2600(1000)}{9810(10)30} = 0.883 = 88.3\%$ ◄

(*b*) Eq. 16.16: $\dfrac{n_2}{n_1} = \left(\dfrac{h_2}{h_1}\right)^{1/2}\left(\dfrac{D_1}{D_2}\right) = \left(\dfrac{48}{30}\right)^{1/2}\left(\dfrac{D}{D}\right) = 1.265$

$n_2 = 1.265n_1 = 1.263(150) = 189.7$ rpm ◄

Eq. 16.17: $\dfrac{Q_2}{Q_1} = \left(\dfrac{h_2}{h_1}\right)^{1/2}\left(\dfrac{D_2}{D_1}\right)^2 = \left(\dfrac{48}{30}\right)^{1/2}\left(\dfrac{D}{D}\right)^2 = 1.265$; $Q_2 = 1.265(10) = 12.65$ m³/s ◄

Eq. 16.18: $\dfrac{P_2}{P_1} = \left(\dfrac{h_2}{h_1}\right)^{3/2}\left(\dfrac{D_2}{D_1}\right)^2 = \left(\dfrac{48}{30}\right)^{3/2} = 2.02$; $P_2 = 2.02(2600) = 5260$ kW ◄

(*c*) Sec. 16.1: Number of poles $= 7200/n_2 = 7200/189.7 = 37.9$. Use 38 poles ◄

16.10.2 *At a maximum efficiency of 93% a turbine runs at 300 rpm and discharges 310 cfs under a net head of 64 ft. This turbine drives a generator that is developing 50-cycle electricity. (a) How many poles are required for this generator? (b) What is the brake horsepower?*

BG

(*a*) Sec. 16.11: Number of poles = 6000/300 = 20 ◄

(*b*) From Eqs. 16.14, 5.40: $P = \eta\gamma Qh/550 = (0.93)62.4(310)64/550 = 2093$ hp ◄

16.10.3 *We wish to operate the turbine of Exer. 16.10.2 under homologous conditions and a net head as close as possible to 48 ft. Find the speed, net head, flow rate, and brake power for the homologous conditions. Be sure to use an even number of poles. Hint: It may not be possible to have perfectly homologous conditions.*

Exer. 16.10.2: Turbine (50-cycle generator), n = 300 rpm, Q = 310 cfs, η = 93%, net h = 64 ft, P = 2093 bhp.

BG

Eq. 16.16: $n_2/n_1 = (h_2/h_1)^{1/2}(D_1/D_2) = (48/64)^{1/2}(1) = 0.866$; $n_2 = n_1(0.866) = 300(0.866) = 260$ rpm

Sec. 16.1 for 50 cycles: $N_2 = 6000/n_2 = 6000/260 = 23.1$ poles (impossible)

N must be even so use 22 or 24 poles. These will result in conditions that are very close to being homologous to the original conditions.

For a given turbine, i.e. *D* = constant, with 22 poles: $n_2 = 6000/22 = 273$ rpm ◄

Eq. 16.16: $n \propto h^{1/2}$ so $h \propto n^2$; $h = 64(273/300)^2 = 52.9$ ft ◄

Eq. 16.17: $Q \propto h^{1/2}$ so $Q \propto n$; $Q = 310(273/300) = 282$ cfs ◄

Eq. 16.18: $P \propto h^{3/2}$ so $P \propto n^3$; $P = 2093(273/300)^3 = 1573$ hp ◄

Likewise for 24-pole generator: $n = 250$ rpm, $h = 44.4$ ft, $Q = 258$ cfs, $P = 1209$ hp ◄◄

Sec. 16.10: Similarity Laws for Reaction Turbines -- ***Problems*** *16.21–16.25*

16.21 *A turbine runs at 150 rpm, discharges 200 cfs, and develops 1600 bhp under a net head of 81 ft. (a) What is its efficiency? (b) What would be the revolutions per minute, Q, and brake horsepower of the same turbine under a net head of 162 ft for homologous conditions?*

BG

(*a*) Eq. 16.14: $\eta = T\omega/(\gamma Qh) = 1600(550)/[62.4(200)81] = 0.87 = 87\%$ ◄

(*b*) For homologous conditions with a fixed D:

Eq. 16.16: $n \propto h^{1/2}$; $n = 150(162/81)^{1/2} = 212$ rpm ◄

Eq. 16.17: $Q \propto h^{1/2}$; $Q = 200(162/81)^{1/2} = 283$ cfs ◄

Eq. 16.18: $P \propto h^{3/2}$; $P = 1600(162/81)^{3/2} = 4530$ bhp ◄

16.22 *If a turbine homologous to that of Prob. 16.21 has a runner of twice the diameter, what would be the revolutions per minute, Q, and brake horsepower under the same head of 81 ft?*

Prob. 16.21: n = 150 rpm, Q = 200 cfs, P = 1600 bhp, net h = 81 ft.

BG

For homologous conditions with a fixed h:

Eq. 16.16: $n \propto 1/D$; $n = 150(1/2) = 75$ rpm ◄

Eq. 16.17: $Q \propto D^2$; $Q = 200(2)^2 = 800$ cfs ◄

Eq. 16.18: $P \propto D^2$; $P = 1600(2)^2 = 6400$ bhp ◄

16.23 *A 2.6-m-diameter reaction turbine is to be operated at 200 rpm under a net head of 30 m. A 1:10 model of this turbine is built and tested in the laboratory. If the model is operated at 600 rpm, under what net head should it be tested to simulate normal operating conditions in the prototype?*

SI

Given: $D_m/D_p = 1/10$, and $n_m/n_p = 600/200 = 3$

Eq. 16.16: $n \propto h^{1/2}/D$ or $h \propto n^2D^2$; $h_m = h_p(3)^2(1/10)^2 = 30(0.09) = 2.70$ ◄

16.24 *A small Francis turbine ($N_s = 30$, $D = 2$ ft) is tested and found to have an efficiency of 0.893 when operating under optimum conditions. Approximately what would be the maximum efficiency of a homologous runner ($N_s = 30$) with a diameter of 6 ft?*

BG

Eq. 16.15: $(1 - 0.893)/(1 - \eta_p) = (6/2)^{1/5} = 1.246$; $\eta_p = 0.914$ ◄

16.25 *A 12-ft-diameter reaction turbine is to be operated at 100 rpm under a net head of 96 ft. A 1:8 model of this turbine is built and tested in the laboratory. If the model is operated at 450 rpm, under what net head should it be tested to simulate normal operating conditions in the prototype?*

BG

Given: $D_m/D_p = 1/8$, $n_m/n_p = 450/100 = 4.5$

Eq. 16.16: $n \propto h^{1/2}/D$ or $h \propto n^2D^2$; $h_m = h_p(4.5)^2(1/8)^2 = 96(0.316) = 30.4$ ft ◄

Sec. 16.11: Peripheral Velocity Factor and Specific Speed of Turbines -- *Exercises (3)*

16.11.1 *Find the specific speed of an impulse wheel operating at a maximum efficiency of 92.6% under a net head of 2314 ft at 360 rpm if the flow rate from the nozzle is 131.5 cfs. Estimate the diameter of the runner. Note: This is the turbine of Sample Prob. 16.4.*

BG

From Eq. 16.8: bhp = (0.926)62.4(131.5)2314/550 = 31,970

Eq. 16.21: $N_s = 360\sqrt{31{,}970}/2314^{5/4} = 4.01$ ◄

Fig. 16.14 for $N_s = 4.1$: $\phi_e \approx 0.43$; Eq. 16.20: $D \approx 153.3(0.43)\sqrt{2314}/360 = 8.8$ ft ◄

16.11.2 *Find the specific speed of a turbine that operates at a maximum efficiency of 90% at 150 rpm under a net head of 81 ft with a flow rate of 200 cfs. Estimate the diameter of the runner. What type of turbine is this?*

BG

From Eq. 16.8: bhp = (0.90)62.4(200)81/550 = 1654

Eq. 16.21: $N_s = 150\sqrt{1654}/81^{5/4} = 25.1$ ◄ Fig. 14.14 for $N_s = 25.1$: Francis turbine ◄

Fig. 16.14 for $N_s = 25.1$: $\phi_e \approx 0.70$; Eq. 16.20: $D \approx 153.3(0.70)\sqrt{81}/150 = 6.4$ ft ◄

16.11.3 *Find the specific speed of a turbine that operates at a maximum efficiency of 83% at 450 rpm under a net head of 20 ft with a flow rate of 100 cfs. Estimate the diameter of the runner. What type of turbine is this?*

BG

Eq. 16.8: bhp = (0.83)62.4(100)20/550 = 188.3

Eq. 16.21: $N_s = 450\sqrt{188.3}/20^{5/4} = 146.0$ ◄ Fig. 16.14 for $N_s = 146$: Propeller turbine ◄

Fig. 16.14 for $N_s = 146$: $\phi_e \approx 1.78$; Eq. 16.20: $D \approx 153.3(1.78)\sqrt{20}/450 = 2.71$ ft ◄

Sec. 16.11: Peripheral Velocity Factor and Specific Speed of Turbines -- *Problems 16.26–16.34*

16.26 *A 1:8 model of a 12-ft-diameter turbine is operated at 600 rpm under a net head of 54.0 ft. Under this mode of operation the bhp and Q of the model were observed to be 332 hp and 62 cfs, respectively. From the above data compute (a) the specific speed of the model and the value of ϕ_e, (b) the efficiency and shaft torque of the model, (c) the efficiency of the prototype, and (d) the flow rate and horsepower of the prototype if it is operated at 450 rpm under a net head of 200 ft.*

BG

(*a*) $(N_s)_m = n_e\sqrt{\text{bhp}}/h^{5/4} = 600\sqrt{332}/54^{5/4} = 74.7$ ◄ Fig. 16.14 for $N_s = 74.7$: $\phi_e \approx 0.74$ ◄

(*b*) Eqs. 5.38 and 5.40: bhp = $T\omega/550 = T(2\pi 600/60)/550 = 332$; $T = 2910$ lb·ft ◄

Eq. 16.8: $\eta = T\omega/(\gamma Q h) = 332(550)/[62.4(62)54] = 0.874$ ◄

(*c*) Eq. 16.15: $(1 - 0.874)/(1 - \eta_p) = (8/1)^{1/5} = 1.516$; $\eta_p = 0.917$ ◄

(*d*) Eq. 16.16 for model: $600 = (K_n)_m(54)^{1/2}/(12/8)$; $(K_n)_m = 122.5$

For prototype: $450 = (K_n)_p 200^{1/2}/12$; $(K_n)_p = 450(12)/200^{1/2} = 382$

Thus $K_m \neq K_p$ and homologous conditions do not occur with the given data. So predictions can not be made using similarity laws. ◄

16.27 *Calculate the specific speed of the turbine whose runner is shown in Fig. 16.2 and estimate the runner diameter.*
Fig. 16.2: Net h = 2200 ft, n = 250 rpm, D = 162 in.

BG

Eq. 16.21: $N_s = n_e\sqrt{bhp}/h^{5/4}$; Cannot solve this equation, as brake horsepower is not given. ◀

However, Eq. 16.20: $D = 162/12 = 153.3\phi_e\sqrt{2200}/250$: $\phi_e = 0.469$

From Fig. 16.14 with $\phi_e = 0.469$ it must be an impulse wheel, which agrees with Fig. 16.2

16.28 *Find the specific speed of the turbine shown in Fig. 16.7 and estimate the runner diameter.*
Fig. 16.7: P = 128 000 kW, n = 150 rpm, h = 87.6 m.

B

$h = (87.6 \text{ m})3.28 = 287$ ft; $P = (128\,000 \text{ kW})1.341 = 171{,}700$ hp

Eq. 16.21: $N_s = n_e\sqrt{\text{bhp}}/h^{5/4} = 150\sqrt{171{,}700}/287^{5/4} = 52.5$ ◀

Fig. 16.14 with $N_s = 52.5$: This is a Francis turbine, $\phi_e \approx 0.72$

Eq. 16.20: $D = 153.3\phi_e\sqrt{h}/n \approx 153.3(0.72)\sqrt{287}/150 = 12.5$ ft $= 3.80$ m ◀

This checks with Fig. 16.7 (see the man in the photo).

16.29 *Calculate the specific speed of the turbine shown in Fig. 16.9 and estimate the runner diameter.*
Fig. 16.9: P = 42,000 hp, n = 94.7 rpm, h = 52 ft.

BG

Eq. 16.21: $N_s = n_e\sqrt{\text{bhp}}/h^{5/4} = 94.7\sqrt{42{,}000}/52^{5/4} = 139.0$ ◀

Fig. 16.14 with $N_s = 139$: This is a propeller turbine, $\phi_e \approx 1.71$

Eq. 16.20: $D = 153.3\phi_e\sqrt{h}/n \approx 153.3(1.71)\sqrt{52}/94.7 = 20.0$ ft ◀

This checks with Fig. 16.9 (see the man in the photo).

16.30 *Find the specific speed of the propeller turbine mentioned in Exer. 16.7.1. Express the answer in BG units.*
Exer. 16.7.1: Kaplan turbine, n = 327.3 rpm, shaft P = 300 kW, Q = 5.5 m³/s.

B

Fig. 16.13 for Kaplan turbine: Assume $\eta = 0.92$

bhp $= (300 \text{ kW})1.341 = 402.3$ hp ; $Q = (5.5 \text{ m}^3/\text{s})35.31 = 194.2$ cfs

From Eq. 16.8: bhp $= \eta\gamma Qh/550 = (0.92)62.4(194.2)h/550 = 402.3$; $h = 19.84$ ft

Eq. 16.21: $N_s = 327.3\sqrt{402.3}/19.84^{5/4} = 156.7$ ◀

16.31 *Find the specific speed of a turbine that runs at a maximum efficiency of 90% at 300 rpm under a net head of 81 ft with a flow rate of 50 cfs. Estimate the runner diameter.*

BG

From Eq. 16.8: bhp $= \eta\gamma Qh/550 = 0.90(62.4)50(81)/550 = 413.5$

Eq. 16.21: $N_s = n_e\sqrt{\text{bhp}}/h^{5/4} = 300\sqrt{413.5}/81^{5/4} = 25.1$ ◀

Fig. 16.14 with $N_s = 25.1$: $\phi_e \approx 0.70$; Eq. 16.20: $D \approx 153.3(0.70)\sqrt{81}/300 = 3.2$ ft $= 39$ in ◀

16.32 *In Sec. 16.15 a double-overhung impulse turbine at Dixence, Switzerland is mentioned. Each wheel of the two-wheel unit develops 25,000 hp at 500 rpm under a head of 5330 ft. (a) What is the specific speed of these wheels? (b) Estimate their diameter and compare your answer with their actual diameter of 10.89 ft.*

BG

(*a*) Eq. 16.21: $N_s = n_e\sqrt{\text{bhp}}/5330^{5/4} = 500\sqrt{25{,}000}/5330^{5/4} = 1.736$ ◄

(*b*) Fig. 16.14 with $N_s = 1.736$: $\phi_e \approx 0.46$; Eq. 16.20: $D \approx 153.3(0.46)\sqrt{5330}/500 = 10.3$ ft ◄

This compares well with the actual diameter of 10.89 ft (5.4% difference). ◄

16.33 *In Sec. 16.15 a Francis turbine on the Susquehanna River is mentioned. This turbine develops 54,000 hp at 81.8 rpm under a head of 89 ft. (a) What is the specific speed of this turbine? (b) Estimate the runner diameter and compare your answer with the actual diameter of 18 ft.*

BG

(*a*) Eq. 16.21: $N_s = 81.8\sqrt{54{,}000}/89^{5/4} = 69.5$ ◄

(*b*) Fig. 16.14 with $N_s = 69.5$: $\phi_e \approx 0.74$; Eq. 16.20: $D \approx 153.3(0.74)\sqrt{89}/81.8 = 13.1$ ft ◄

This does not compare well with the actual diameter of 18 ft (27.2% difference). ◄

16.34 *A Kaplan turbine at Rock River, Illinois develops 800 hp at 80 rpm under a head of 7 ft. (a) What is the specific speed of this turbine? (b) Estimate the runner diameter and compare your answer with the actual diameter of 136 in.*

BG

(*a*) Eq. 16.21: $N_s = 80\sqrt{800}/7^{5/4} = 198.7$ ◄

(*b*) Fig. 16.14 with $N_s = 198$: $\phi_e \approx 2.39$; Eq. 16.20: $D \approx 153.3(2.39)\sqrt{7}/80 = 12.1$ ft ◄

This $D \approx 12.1$ ft = 145 in compares well with the actual $D = 136$ in (6.9% difference). ◄

Sec. 16.12: Cavitation in Turbines – *Exercises (3)*

16.12.1 *Using Eq.(16.24) and Fig. 16.17 for σ_c calculate the maximum permissible head under which a Francis turbine ($N_s = 40$) can operate if it is set 12 ft above tailwater. The installation is at sea level and the water temperature is 80°F. How does your answer compare to Fig. 16.16?*

BG

Fig. 16.17 for $N_s = 40$: $\sigma_c \approx 0.132$; Table A.1 for water at 80°F: $p_v/\gamma = 1.17$ ft abs

Inside cover for standard atmosphere: $p/\gamma = 33.91$ ft water abs.

Eq. 16.24: $z_B = p_{atm}/\gamma - p_v/\gamma - \sigma_c h$; $12 \approx 33.91 - 1.17 - 0.132h$; $h \approx 157$ ft ◄

Fig. 16.16 for $N_s = 40$ and 12 ft draft head: max $h \approx 205$ ft.

The above result (157 ft) compares fairly well with 205 ft from Fig. 16.16 ◄

16.12.2 *(a) Find the maximum permissible head under which an axial-flow turbine ($N_s = 160$) can operate if it is set 5 ft below tailwater. The installation is at elevation 3150 ft and the water temperature is 65°F. (b) What would be the maximum permissible head if the turbine had been set 5 ft above tailwater?*

BG

Table A.3 at 3150 ft, by interpolation: $p_{atm} = 13.14$ psia

Table A.1 for water at 65°F, by interpolation: $p_v/\gamma = 0.715$ ft abs

Fig. 16.17 for $N_s = 160$: $\sigma_c \approx 0.815$

Eq. 16.24: $z_B \approx 13.14(144)/62.4 - 0.715 - 0.815h = 29.6 - 0.815h$

(*a*) For $z_B = -5$ ft: $-5 \approx 29.6 - 0.815h$; $h \approx 42.5$ ft ◄

(*b*) For $z_B = +5$ ft: $5 \approx 29.6 - 0.815h$; $h \approx 30.2$ ft ◄

16.12.3 *Repeat Exer. 16.12.1 for the case where the turbine is at elevation 10,000 ft rather than at sea level. In this case one cannot compare the result with the curves presented in Fig. 16.16, because of the change in p_{atm}.*

Exer. 16.12.1: N_s = 40, water T = 80°F, z_B = 12 ft. Find the maximum permissible head.

BG

Fig. 16.17 for $N_s = 40$: $\sigma \approx 0.132$

Table A.3 at elev 10,000 ft: $p_{atm} = 10.11$ psia ; Table A.1 for water at 80°F: $p_v/\gamma = 1.17$ ft abs

Eq. 16.24: $z_B = p_{atm}/\gamma - p_v/\gamma - \sigma_c h$; $12 \approx 10.11(144/62.4) - 1.17 - 0.132h$; $h \approx 77$ ft ◄

Sec. 16.12: Cavitation in Turbines -- *Problems* *16.35–16.40*

16.35 *Consider the case of a Francis turbine having a specific speed of 40 that is set 10 ft above tailwater elevation. Perform calculations using Fig. 16.17 and Eq. (16.22) to find the maximum permissible head under which this turbine should operate in order to be safe from cavitation. Check your calculated result against the information shown on Fig. 16.16. Do they agree?*

BG

Assume sea-level conditions with T = 80°F ; Table A.3 at sea level: $p_{atm} = 14.70$ psia

Table A.1 for water at 80°F: $p_v/\gamma = 1.17$ ft abs ; Fig. 16.17 for $N_s = 40$: $\sigma_c \approx 0.132$

Eq. 16.23: $h \approx [14.70(144/62.4) - 1.17 - 10]/0.132 = 172$ ft ◄

Fig. 16.16 for $N_s = 40$ and $z_B = 10$ ft: max $h \approx 230$ ft.

The above result (172 ft) compares fairly well with Fig. 16.16 ◄

16.36 *Consider the case of a propeller turbine having a specific speed of 150 that is set 5 ft below tailwater. Perform calculations using Fig. 16.17 and Eq. (16.22) to find the maximum permissible head under which this turbine should operate in order to be safe from cavitation. Check your calculated result against the information shown in Fig. 16.16. Do they agree?*

BG

Assume sea-level conditions with T = 80°F ; Table A.3 at sea level: $p_{atm} = 14.70$ psia

Table A.1 for water at 80°F: $p_v/\gamma = 1.17$ ft abs ; Fig. 16.17 for $N_s = 150$: $\sigma_c \approx 0.71$

Eq. 16.23: $h \approx [14.70(144/62.4) - 1.17 + 5]/0.71 = 53$ ft ◄

Fig. 16.16 for $N_s = 150$ and $z_B = -5$ ft: max $h \approx 56$ ft. The above (53 ft) checks well with this. ◄

16.37 *At its maximum efficiency of 93% a turbine delivers 3000 hp to the shaft under a head of 72 ft when operating at 300 rpm. For water at 80°F, find the following: (a) the flow rate through the turbine; (b) the specific speed of the turbine, (c) the approximate diameter of the turbine runner; (d) how high above tailwater the turbine should be set to be safe against cavitation if it is installed at sea level.*

BG

(*a*) From Eq. 16.8: HP to shaft = $\gamma Q h \eta/550 = 62.4Q72(0.93)/550 = 3000$ hp ; $Q = 395$ cfs ◄

(*b*) Eq. 16.21: $N_s = n_e\sqrt{\text{bhp}}/h^{5/4} = 300\sqrt{3000}/72^{5/4} = 78.3$ ◄

(*c*) Fig. 16.14 for $N_s = 78.3$: $\phi_e \approx 0.74$; Eq. 16.20: $D \approx 153.3(0.74)\sqrt{72}/300 = 3.2$ ft ◄

(*d*) Fig. 16.17 for $N_s = 78.3$: $\sigma_c \approx 0.38$; Table A.3 at sea level: $p_{atm} = 14.70$ psia

Table A.1 for water at 80°F: $p_v/\gamma = 1.17$ ft abs

Eq. 16.24: $z_B = p_{atm}/\gamma - p_v/\gamma - \sigma_c h \approx 14.70(144/62.4) - 1.17 - 0.38(72) = 5.4$ ft ◄

Best to limit draft head to zero, though turbine should operate satisfactorily with a draft head of 5 ft.

16.38 *A propeller turbine operates at a maximum efficiency of 92% under a head of 30 ft at 450 rpm and develops a shaft power of 620 hp. For water at 50°F, find (a) the flow rate through the turbine, (b) the specific speed of the turbine, (c) the approximate diameter of the turbine runner, and (d) how far above trailwater elevation the turbine can be set and still be safe from cavitation. Assume the turbine is at an elevation of 100 ft.*

BG

(*a*) From Eq. 16.8: HP to shaft $= \gamma Q h \eta/550 = 62.4Q30(0.92)/550 = 620$ hp ; $Q = 198.0$ cfs ◄

(*b*) Eq. 16.21: $N_s = n_e\sqrt{\text{bhp}}/h^{5/4} = 450\sqrt{620}/30^{5/4} = 159.6$ ◄

(*c*) Fig. 16.14 for $N_s = 160$: $\phi_e \approx 1.94$; Eq. 16.20: $D \approx 153.3(1.94)\sqrt{30}/450 = 3.62$ ft $= 43.4$ in ◄

(*d*) Fig. 16.17 for $N_s = 160$: $\sigma_c \approx 0.81$; Table A.3 at 100 ft by interpolation: $p_{atm} = 14.65$ psia

Table A.1 for water at 50°F: $p_v/\gamma = 0.41$ ft abs

Eq. 16.24: $z_B = p_{atm}/\gamma - p_v/\gamma - \sigma_c h \approx 14.65(144/62.4) - 0.41 - 0.81(30) = 9.1$ ft

The turbine should be set no more than 9.1 ft above tailwater. ◄

16.39 *Repeat part (d) of Prob. 16.38 for the case where the turbine is installed at elevation 5000 ft.*
Prob. 16.38(d): A propeller turbine at maximum η has $n = 450$ rpm, $P = 620$ bhp, $h = 30$ ft, $T =$ 50°F. Find z_B.

BG

Eq. 16.21: $N_s = n_e\sqrt{\text{bhp}}/h^{5/4} = 450\sqrt{620}/30^{5/4} = 159.6$

(*d*) Fig. 16.17 for $N_s = 160$: $\sigma_c \approx 0.81$; Table A.3 at 5000 ft: $p_{atm} = 12.23$ psia

Table A.1 for water at 50°F: $p_v/\gamma = 0.41$ ft abs

Eq. 16.24: $z_B = p_{atm}/\gamma - p_v/\gamma - \sigma_c h \approx 12.23(144/62.4) - 0.41 - 0.81(30) = 3.5$ ft

The turbine should be set no more than 3.5 ft above tailwater. ◄

16.40 *A turbine whose specific speed is 80 is to operate under a head of 50 ft at an elevation where the atmospheric pressure is 12.8 psia. The water temperature is 50°F. (a) If this turbine is set 6 ft above tailwater, will it be safe from cavitation? (b) What is the highest permissible elevation of this turbine with respect to tailwater when operating under a head of 60 ft?*

BG

(*a*) Fig. 16.17 for $N_s = 80$: $\sigma_c \approx 0.395$; Table A.1 for water at 50°F: $p_v/\gamma = 0.41$ ft abs

Eq. 16.24: $z_B = p_{atm}/\gamma - p_v/\gamma - \sigma_c h \approx 12.8(144/62.4) - 0.41 - 0.395(50) = 9.4$ ft

The turbine can be set as high as 9.4 ft above tailwater.

So at 6 ft above tailwater the turbine will be safe from cavitation. ◄

(*b*) For h = 60 ft: $z_B \approx 29.54 - 0.41 - 0.395(60) = 5.4$ ft

The turbine can be set as much as 5.4 ft above tailwater. ◄

Sec. 16.13: Selection of Turbines – *Exercises (4)*

16.13.1 *(a) If Q = 40 cfs and h = 70 ft, would it be possible to use a single impulse wheel with a specific speed less than 7.0 at a site where 60-Hz current is to be generated? Assume a turbine efficiency of 90% and that the generator is to have no more than 96 poles. (b) If a single impulse wheel can be used under these conditions, calculate the approximate diameter of the turbine runner for the range of possible speeds of operation.*

BG

(*a*) bhp = (0.90)62.4(40)70/550 = 286 hp

For a (max) 96-pole generator, min n = 7200/96 = 75 rpm,

min $N_s = n\sqrt{\text{bhp}}/h^{5/4} = 75\sqrt{286}/70^{5/4} = 6.25$

Fig. 16.16 for N_s = 6.25 (with 80°F water at sea level): for no cavitation, max $h \approx 950$ ft.

Since $N_s = 6.25 < 7$ and h = 70 ft < 950 ft, Yes, a single impulse wheel ($N_s < 7$) would be possible. ◄

(*b*) Fig. 16.14 for N_s = 6.25: $\phi_e \approx 0.40$; Eq. 16.20: $D \approx 153.3(0.40)\sqrt{70}/75 = 6.8$ ft

Consider alternative = min number of poles (max N_s).

If $N_s = 7.0 = n\sqrt{\text{bhp}}/h^{5/4} = n\sqrt{286}/(70)^{5/4}$, n = 83.8 rpm and N = 7200/83.8 = 85.9 poles

So use 86 poles. Then n = 7200/86 = 83.7 rpm and $N_s = 83.7\sqrt{286}/70^{5/4} = 6.99$

Fig. 16.14 for N_s = 7: $\phi_e \approx 0.395$; $D \approx 153.3(0.395)\sqrt{70}/83.7 = 6.1$ ft

Resulting ranges:

N_s	N poles	n rpm	D ft
6.25	96 (max)	75 (min)	6.8
6.99 (max)	86	83.7 (max)	6.1
▲		▲	▲

16.13.2 *(a) If $Q = 40$ cfs and $h = 70$ ft, would it be possible to use a single Francis turbine with a specific speed of at least 20 at a site where 60-Hz current is to be generated? Assume a turbine efficiency of 90% and assume the generator is to have no less than 12 poles. (b) If a single Francis turbine can satisfy these conditions, calculate the approximate diameter of the turbine runner for the range of possible speeds of operation.*

BG

(*a*) From Eq. 16.4: bhp = (0.90)62.4(40)70/550 = 286 hp

For a (min) 12-pole generator, max n = 7200/12 = 600 rpm,

max $N_s = n\sqrt{\text{bhp}}/h^{5/4} = 600\sqrt{286}/70^{5/4} = 50.1$

Fig. 16.16 for $N_s = 50.1$ (with 80°F water at sea level): for no cavitation, max $h \approx 130$ ft (with draft head $z_B \leq 15$ ft).

Since $N_s = 50.1 > 20$ and $N_s = 50.1 < 100$ = max for a Francis turbine, and $h = 70$ ft < 130 ft, yes a single Francis turbine with $N_s > 20$ would be possible. ◄

(*b*) Fig. 16.14 for $N_s = 50.1$: $\phi_e \approx 0.72$; Eq. 16.20: $D \approx 153.3(0.72)\sqrt{70}/600 = 1.54$ ft

Consider alternative = max number of poles (min N_s)

If $N_s = 20 = n\sqrt{286}/(70)^{5/4}$, $n = 239$ rpm and $N = 7200/239 = 30.1$ poles

So use 30 poles; then $n = 7200/30 = 240$ rpm and $N_s = 240\sqrt{286}/(70)^{5/4} = 20.05$

Fig. 16.14 for $N_s = 20$: $\phi_e \approx 0.70$; $D \approx 153.3(0.70)\sqrt{70}/240 = 3.7$ ft

Resulting ranges:

N_s	N poles	n rpm	D ft
50.1	12 (min)	600 (max)	1.54
20.05 (min)	30	240 (min)	3.7
▲		▲	▲

16.13.3 *Specify the specific speed and the approximate runner diameter at a site where $Q = 2350$ cfs and $h = 480$ ft. Use a 40-pole generator for 60-cycle current. Assume a turbine efficiency of 90%. These are the specifications for some of the turbines at Hoover Dam.*

BG

From Eq. 16.14: bhp = (0.90)62.4(2350)400/550 = 115,200

$n = 7200/40 = 180$ rpm ; Eq. 16.21: $N_s = n\sqrt{\text{bhp}}/h^{5/4} = 180\sqrt{115{,}200}/480^{5/4} = 27.2$ ◄

Fig. 16.14 for $N_s = 27.2$: $\phi_e \approx 0.70$; Eq. 16.20: $D \approx 153.3(0.70)\sqrt{480}/180 = 13.1$ ft ◄

16.13.4 *(a) What is the maximum head for which a propeller-type turbine can be used if the flowrate is 40 cfs? Assume a turbine efficiency of 90% with specific speeds for propeller-type turbines varying from 100 to 200. Also, assume a 12-pole generator with 50-Hz current. (b) Find the maximum head for which a 20-pole generator could have been used with the same turbine efficiency and 50-Hz current. Neglect the possibility of cavitation.*

BG

From Eq. 16.14: bhp = (0.90)62.4(40)h/550 = 4.08h

(*a*) $n = 6000/12 = 500$ rpm ; $N_s = n\sqrt{\text{bhp}}/h^{5/4} = 500\sqrt{4.08h}/h^{5/4} = 1010/h^{3/4}$, so $h^{3/4} = 1010/N_s$

h will be largest when N_s is smallest, so use $N_s = 100$; $h = (1010/100)^{4/3} = 21.8$ ft ◄

(*b*) $n = 6000/20 = 300$ rpm ; $N_s = n\sqrt{\text{bhp}}/h^{5/4} = 300\sqrt{4.08h}/h^{5/4} = 606/h^{3/4}$, so $h^{3/4} = 606/N_s$

For min $N_s = 100$, max $h = (606/100)^{4/3} = 11.06$ ft ◄

Sec. 16.13: Selection of Turbines – ***Problems*** *16.41–16.53*

16.41 *(a) A turbine is to be installed at a point where the available head is 175 ft and the available flow will average 1000 cfs. What type of turbine would you recommend? Specify the operating speed and the number of generator poles for 60-cycle electricity if a turbine is selected with the highest specific speed that is safe against cavitation. Assume a draft head of 10 ft and 90% turbine efficiency. Approximately what size of turbine runner is required? (b) For the same conditions, select a set of two identical turbines to be operated in parallel. Specify the speed and size of the units.*

BG

(*a*) Assume sea-level conditions at 59°F, for single turbine

Table A.3 at sea level: p_{atm} = 14.70 psia

Table A.1 at 59°F by interpolation: $\gamma = 62.37$ pcf; $p_v/\gamma = 0.57$ ft

Eq. 16.22: $\sigma_c = [14.70(144/62.37) - 0.57 - 10]/175 = 0.1335$

From Fig. 16.17 for $\sigma_c = 0.1335$: $N_s \approx 41$, machine = Francis turbine ◀

From Eq. 16.8: $P = \gamma Qh\eta/550 = 62.37(1000)175(0.90)/550 = 17{,}860$ hp ◀

Eq. 16.21: $N_s = n_e\sqrt{\text{bhp}}/h^{5/4}$ so $41 \approx n_e\sqrt{17{,}860}/175^{5/4}$, $n_e \approx 195$ rpm

$N = 7200/n_e \approx 7200/195.3 = 36.9$ poles. So use $N = 36$ poles ◀

Then $N_e = 7200/36 = 200$ rpm ◀ and $N_s = 200\sqrt{17{,}860}/175^{5/4} = 42.0$

Fig. 16.14 for $N_s = 42$: $\phi_e \approx 0.71$; Eq. 16.20: $D \approx 153.3(0.71)\sqrt{175}/200 = 7.2$ ft ◀

(*b*) Use two turbines in parallel

$Q = 1000/2 = 500$ cfs per turbine, $h = 175$ ft, $N_s = 41$ from (*a*) above.

For each turbine, $P = 17{,}860/2 = 8930$ hp

Eq. 16.21: $N_s = n_e\sqrt{\text{bhp}}/h^{5/4}$; $41 = n_e\sqrt{8930}/175^{5/4}$ so $n_e = 276$ rpm and $N = 7200/276 = 26.1$

Choose 26 poles, so $n_e = 7200/26 = 277$ rpm ◀ and $N_s = 277\sqrt{8930}/175^{5/4} = 41.1$

Fig. 16.14 for $N_s = 41.1$: $\phi_e \approx 0.71$; Eq. 16.20: $D \approx 153.3(0.71)\sqrt{175}/277 = 5.2$ ft ◀

16.42 *What is the least number of identical turbines that can be used at a powerhouse where the available head is 1200 ft and Q = 1650 cfs? Assume turbine efficiency is 90% and speed of operation is 138.5 rpm. Specify the size and specific speed of the units.*

BG

From Eq. 16.8: Shaft power = bhp = $\gamma Qh\eta/550 = 62.4(1650)1200(0.90)/550 = 202{,}000$

Fig. 16.16 for $h = 1200$ ft: To be safe from cavitation the specific speed must be less than 5.4.

Let x = no. of identical units: Eq. 16.21: $N_s = n_e\sqrt{\text{bhp}}/h^{5/4} = 138.5\sqrt{202{,}000/x}/1200^{5/4}$

If $x = 1$, $N_s = 8.81$ (too large, cavitation). If $x = 2$, $N_s = 6.23$ (too large, cavitation).

If $x = 3$, $N_s = 5.09$, OK, no cavitation (and maximum efficiency of impulse wheel is at $N_s \approx 5$, see Sec. 16.9).

Thus, use three identical impulse turbines with $N_s = 5.1$ ◀

Fig. 16.14 for $N_s = 5.09$: $\phi_e \approx 0.43$; Eq. 16.20: $D \approx 153.3(0.43)\sqrt{1200}/138.5 = 16.5$ ft ◀

16.43 *A six-jet impulse turbine operating at 300 rpm develops 60,000 hp under a net head of 1060 ft. The runner has a diameter of 6.0 ft. (a) How large a homologous runner would be needed for a single-jet machine operating under the same head and developing the same horsepower? (b) Check to confirm that this turbine will be free of cavitation.*

BG

(*a*) Eq. 16.21: $N_s = n_e\sqrt{\text{bhp}}/h^{5/4} = 300\sqrt{(60{,}000/6)}/(1060)^{5/4} = 4.96$

Single nozzle $N_s = 4.96 = n_e\sqrt{60{,}000}/1060^{5/4}$; $n = 122.5$ rpm

Assume 60-cycle electricity. $N = \dfrac{7200}{122.4} = 58.8$ poles. Use 58 poles, so $n = \dfrac{7200}{58} = 124.1$ rpm

Eq. 16.16: $n \propto h^{1/2}/D$, $h = 1060$ ft = const, hence $D \propto 1/n$

$D_1 = D_6(n_6/n_1) = 6.0(300/124.1) = 14.50$ ft ◄

(*b*) Fig. 16.16 for $N_s = 4.96$, $h = 1060$ ft: There is no problem with cavitation. ◄

16.44 *An impulse turbine ($N_s \approx 4$) develops 100,000 hp under a head of 2000 ft. (a) For 60-cycle electricity calculate the turbine speed (rpm), wheel diameter (ft), and number of poles in the generator. (b) Solve the problem for a six-nozzle unit using the same N_s, bhp, and head. In both instances assume $\phi_e = 0.45$. Neglect cavitation.*

BG

(*a*) Eq. 16.21: $4 \approx N_s = n_e\sqrt{100{,}000}/2000^{5/4}$; $n_e \approx 169.2$ rpm, $N \approx 7200/169.2 = 42.6$ poles

Use 42 poles ◄ $n = 7200/42 = 171.4$ rpm ◄ $N_s = 171.4\sqrt{100{,}000}/2000^{5/4} = 4.05$

Eq. 16.20: $D = 153.3(0.45)\sqrt{2000}/171.4 = 18.00$ ft ◄

(*b*) Eq. 16.21: $N_s = 4 = n_e\sqrt{(100{,}000/6)}/2000^{5/4}$; $n = 414.4$ rpm

$N = 7200/414.4 = 17.37$, use 18 poles ◄ $n = 7200/18 = 400$ rpm ◄

$N_s = 400\sqrt{(100{,}000/6)}/2000^{5/4} = 3.86$; $D = 153.3(0.45)\sqrt{2000}/400 = 7.71$ ft ◄

Note: Fig. 16.16 indicates (for $N_s = 4$, $h = 2000$ ft) marginally no problem with cavitation.

16.45 *A multinozzle impulse turbine is to be designed to develop 60,000 hp at 300 rpm under a head of 1200 ft. How many nozzles should this turbine have? Specify the approximate wheel diameter for this design. How large would a single-jet machine have to be to satisfy these requirements?*

BG

For multi-nozzle turbine with x nozzles, Eq. 16.21: $N_s = n_e\sqrt{\text{bhp}/x}/h^{5/4} = 300\sqrt{(60{,}000/x)}/1200^{5/4}$

$\therefore$ $N_s = 10.4, 7.35, 6.0, 5.2, 4.65, 4.25$, for x values of 1, 2, 3, 4, 5, 6.

Fig. 16.16 indicates that for a head of 1200 ft, N_s should be less than about 5.4.

Footnote page 742 and Fig. 16.14 indicate maximum efficiency occurs at $N_s \approx 5$.

Hence use 4 or 5 nozzles. ◄ (5 will be safer against cavitation.)

For 4 or 5 nozzles, $N_s \approx 5$; Fig. 16.14 for $N_s = 5$: $\phi_e \approx 0.43$

Eq. 16.20: $D \approx 153.3(0.43)\sqrt{1200}/300 = 7.6$ ft ◄

For single nozzle, $N_s = 10.4$; per footnote p. 742 the N_s of impulse wheels ranges from 2 to 8, so a single-nozzle machine will not work. (Also, if it would work it would experience cavitation, as noted above.) ◄

16.46 *It is desired to develop 15,000 bhp under a head of 1000 ft. Make any necessary assumptions and estimate the diameter of the wheel required and the rotative speed.*

BG

Fig. 16.16 indicates that an impulse wheel with $N_s < 6$ or a Francis turbine with $N_s < 23$ (zero draft head) would be safe against cavitation. Hence, try (*i*) an impulse wheel with $N_s \approx 5$ (see footnote page 742) or (*ii*) a Francis turbine with $N_s = 20$.

Impulse wheel: Eq. 16.21: $N_s = 5 = n_e\sqrt{\text{bhp}}/h^{5/4} = n_e\sqrt{15{,}000}/1000^{5/4}$; thus $n_e = 229.6$ rpm

Assume 60-cycle electricity: $N = 7200/229.6 = 31.4$; Use 32 poles, $n = 7200/32 = 225$ rpm ◄

Fig. 16.14 for $N_s = 5$: $\phi_e \approx 0.43$. Eq. 16.20: $D \approx 153.3(0.43)\sqrt{1000}/225 = 9.3$ ft ◄

Francis turbine: $N_s = 20 = n_e\sqrt{15{,}000}/1000^{5/4}$; $n_e = 918$ rpm

$N = 7200/918 = 7.8$ (use 8 poles). But most generators have at least 12 poles (see page 721).

If an 8-poles generator is not available, cannot use a Francis turbine.

Another alternative: Use a double-overhung impulse wheel (Fig. 16.3), 7500 hp/wheel.

$N_s = 5 = n_e\sqrt{(15{,}000/2)}/h^{5/4}$; $n_e = 324.7$ rpm

$N = 7200/324.7 = 22.2$ poles, use 22 poles. Then $n = 7200/22 = 327.3$ rpm ◄

Eq. 16.20: $D = 153.3\phi_e\sqrt{h}/n \approx 153.3(0.43)\sqrt{1000}/327.3 = 6.7$ ft ◄

Advantage of double-overhung installation: It equalizes the thrust on the bearings.

16.47 *A single hydraulic turbine is to be selected for a power site with a net head of 100 ft. The turbine is to produce 25,000 hp at maximum efficiency. What speed (rpm) and diameter should this turbine have if (a) a Francis turbine is selected; (b) a propeller turbine is selected? What are the highest "settings" (above or below tailwater) that should be recommended for each of these machines for them to run cavitation-free at their points of maximum efficiency?*

BG

Fig. 16.16 indicates two possibilities:

(*a*) With $h = 100$ ft a Francis turbine can be used with $N_s \le 70$. If $N_s = 70$, use a draft head of zero, if $N_s = 60$, use a draft head of up to 10 ft, i.e., can set the turbine as much as 10 ft above tailwater.

(*b*) With $h = 100$ ft a propeller turbine can be used with $N_s = 100$ ft and a draft head of -8 ft (i.e. with runner 8 ft or more below water surface elevation at discharge).

Francis turbine with $N_s = 70$:

Eq. 16.21: $N_s = 70 = n_e\sqrt{\text{bhp}}/h^{5/4} = n_e\sqrt{25{,}000}/100^{5/4}$; $n_e = 140$ rpm

Assume 60-Hz electricity, $N = 7200/140 = 51.4$, use 52 poles. $n_e = 7200/52 = 138.5$ rpm ◀

$N_s = 138.5\sqrt{25{,}000}/100^{5/4} = 69.2$

Fig. 16.14 when $N_s = 69.2$: $\phi_e \approx 0.74$; Eq. 16.20: $D \approx 153.3(0.74)\sqrt{100}/138.5 = 8.2$ ft ◀

Recommend zero draft head for $N_s = 70$.

Francis turbine with $N_s = 60$:

Following the same procedure, $n_e = 120$ rpm, $N = 60$ poles, $\phi_e \approx 0.72$, and $D \approx 9.2$ ft. This turbine can be set as much as 10 ft above water level at discharge.

Propeller turbine with $N_s = 100$:

Eq. 16.21: $N_s = 100 = n_e\sqrt{25{,}000}/100^{5/4}$; $n_e = 200$ rpm

$N = 7200/200 = 36$, use 36 poles, $n_e = 200$ rpm

Fig. 16.14 for $N_s = 100$: $\phi_e \approx 1.27$; Eq. 16.20: $D \approx 153.3(1.27)\sqrt{100}/200 = 9.7$ ft

This turbine needs a draft head of -8 ft. Hence set the runner at least 8 ft below tailwater.

Resulting selections:

	N_s	N poles	max z_B, ft	n, rpm	D, ft
Francis turbine	69.2	52	0	138.5	8.2
Francis turbine	60	60	10	120	9.2
Propeller turbine	100	36	−8	200	9.7
			▲	▲	▲

16.48 *For 50-cycle electricity how many poles would you recommend for a generator that is connected to a turbine operating under a design head of 3000 ft with a flow of 80 cfs? Assume turbine efficiencies as given in Fig. 16.14 and be sure the turbine is free of cavitation.*

BG

Fig. 16.16 for $h = 3000$ ft: use impulse wheel with $N_s < 2.9$ to avoid cavitation.

Fig. 16.14 for $N_s = 2.9$: $\eta_{max} \approx 0.87$; From Eq. 16.8: bhp $\approx 62.4(80)3000(0.87)/550 = 23{,}700$ hp

Eq. 16.21: $N_s = 2.9 = n_e\sqrt{23{,}700}/3000^{5/4}$; $n_e = 418$ rpm

Sec. 16.1, for 50-cycle: $N = 6000/418 = 14.34$

Try 14 poles: $n = 6000/14 = 429$ rpm, $N_s = 2.9(429/418) = 2.97$ (not OK, > 2.9)

Try 16 poles: $n = 6000/16 = 375$ rpm, $N_s = 2.9(375/418) = 2.60$ (OK, < 2.9). Use 16 poles. ◀

16.49 *(a) Specify the type, speed, and size of a single turbine to be installed at a site with an effective head of 48 m, a maximum draft head of 2 m, and a flow rate of 5 m³/s. (b) How would your recommendation change if the available flow was 50 m³/s?*

B

(*a*) h = 48 m = 157.5 ft, z_B = 2 m = 6.56 ft, Q = 5 m³/s = 176.6 cfs

Fig. 16.16 for h = 157.5 ft, z_B = 6.56 ft: To be safe from cavitation, select Francis turbine with $N_s \le 53$

Fig. 16.14 for $N_s \approx 53$: η = 0.950

From Eq. 16.8: bhp = $\gamma Q h \eta/550 = 62.4(176.6)157.5(0.950)/550 = 3000$

Eq. 16.21: $N_s = n_e\sqrt{3000}/157.5^{5/4} \le 53$; $n_e \le 540$ rpm

Assume 60-cycle electricity, $N \ge 7200/540 = 13.33$, Use 14 poles

$n = 7200/14 = 514$ rpm ; $N_s = 514\sqrt{3000}/157.5^{5/4} = 50.5$

Fig. 16.14 for N_s = 50.5: $\phi_e \approx 0.72$; Eq. 16.20: $D \approx 153.3(0.72)\sqrt{157.5}/514 = 2.7$ ft

To be safer from cavitation (per Fig. 16.16), use a lower value of N_s such as N_s = 40, for example.

Fig. 16.14 for N_s = 40: η = 0.948. ∴ bhp = 2990 hp

$N_s = n_e\sqrt{2990}/157.5^{5/4} = 40$; n_e = 408 rpm , $N = 7200/408 = 17.65$

Use 18 poles: $n = 7200/18 = 400$ rpm , $N_s = 40(400/408) = 39.2$

Fig. 16.14 for $N_s \approx 40$: $\phi_e \approx 0.71$; Eq. 16.20: $D \approx 153.3(0.71)\sqrt{157.5}/400 = 3.4$ ft

Possible solutions include:

N poles	*n* rpm	$(N_s)_{BG}$	*D* ft	*D* m	Turbine
14 (min)	514	50.5 (max)	2.7	0.82	Francis
16	450	43.0	3.2	0.98	Francis
18	400	39.2	3.4	1.04	Francis
20	360	34.4	4.0	1.20	Francis
24	300	28.6	7.2	2.19	Francis
	▲		▲	▲	▲

(*b*) Case where Q = 50 m³/s = 1766 cfs

bhp = 30,000; $N_s = n_e\sqrt{30{,}000}/157.5^{5/4} \le 53$; $n_e \le 170.8$ rpm

With 60-cycle electricity, $N \ge 7200/170.8 = 42.2$, use 44 poles

$n = 7200/44 = 163.6$ rpm ; $N_s = 163.6\sqrt{30{,}000}/157.5^{5/4} = 50.8$

$D \approx 153.3(0.72)\sqrt{157.5}/163.6 = 8.5$ ft

Some possible solutions are:

N poles	*n* rpm	$(N_s)_{BG}$	*D* ft	*D* m	Turbine	
44 (min)	136.6	50.8 (max)	8.5	2.58	Francis	Close to cavitation
46	156.5	47.1	9.2	2.80	Francis	Best
52	138.5	41.7	10.4	3.17	Francis	Perhaps best
64	112.5	33.9	12.8	3.90	Francis	*D* is too large
	▲		▲	▲		

16.50 *It is desired to develop 300,000 hp under a head of 49 ft and to operate at 60 rpm. (a) If turbines with a specific speed of approximately 150 are to be used, how many units would be required? (b) If Francis turbines with a specific speed of 80 were to be used, how many units would be required?*

BG

Assume 60-cycle electricity

(*a*) Fig. 16.14 for $N_s \approx 150$: machine is a propeller turbine.

Eq. 16.21: $N_s = n_e\sqrt{\text{bhp}}/h^{5/4} = 60\sqrt{300{,}000}/49^{5/4} = 253 > 150$

Hence, it is impossible to satisfy the given conditions with a single propeller turbine.

Try two identical turbines: $N_s = 60\sqrt{(300{,}000/2)}/49^{5/4} = 179.2 > 150$

Try three: $N_s = 60\sqrt{(300{,}000/3)}/49^{5/4} = 146.4 \approx 150$

So use three identical propeller turbines with $N_s = 146.4$. ◀

(*b*) Fig. 16.14 for $N_s \approx 80$: machine is a Francis turbine.

Eq. 16.21: $N_s = n_e\sqrt{\text{bhp}}/h^{5/4} = 60\sqrt{300{,}000}/49^{5/4} = 253.5 > 80$

Try ten identical turbines: $N_s = 60\sqrt{30{,}000}/49^{5/4} = 80.2$

So use ten identical Francis turbines with $N_s = 80.2$ ◀

16.51 *Select two, four, and six identical turbines for an installation where $h = 400$ ft and total $Q = 300$ cfs. Develop 60-cycle electricity using either 36- or 72-pole generators. Be sure your selection is free of cavitation. Assume the turbine efficiency is 90%.*

BG

From Eq. 16.8: Total bhp $= \gamma Q h \eta/550 = 62.4(300)400(0.90)/550 = 12{,}250$ hp

2 units with 36 poles ($n = 7200/36 = 200$ rpm):

Eq. 16.21: $N_s = n_e\sqrt{\text{bhp}}/h^{5/4} = 200\sqrt{(12{,}250/2)}/400^{5/4} = 8.75$

Footnote on p. 748: N_s for impulse wheels are normally ≤ 8, $\therefore$ 8.75 is too high.

Fig. 16.14 for $N_s = 8.75$: The efficiency would be low.

Fig. 16.16 for $N_s = 8.75$: max $h = 300$ ft. $\therefore$ these machines with $h = 400$ ft will not be safe against cavitation.

4 units with 36 poles ($n = 200$ rpm):

$N_s = 200\sqrt{(12{,}250/4)}/400^{5/4} = 6.19$, OK; safe against cavitation (Fig. 16.16).

Fig. 16.14 for $N_s = 6.19$: $\phi_e \approx 0.42$; Eq. 16.20: $D \approx 153.3(0.42)\sqrt{400}/200 = 6.4$ ft ◀

6 units with 36 poles ($n = 200$ rpm):

$N_s = 200\sqrt{2042}/400^{5/4} = 5.05$, OK, cavitation safe; $\phi_e \approx 0.43$; $D \approx 6.6$ ft.

Note: The units for the 4-unit and 6-unit installations are of similar size, but their specific speeds are different.

2 units with 72 poles ($n = 7200/72 = 100$ rpm):

$N_s = 8.75/2 = 4.38$, OK, safe against cavitation (Fig. 16.16).

Fig. 16.14 for $N_s = 4.38$: $\phi_e \approx 0.43$; Eq. 16.20: $D \approx 153.3(0.43)\sqrt{400}/100 = 13.2$ ft

4 units with 72 poles ($n = 100$ rpm):

$N_s = 6.19/2 = 3.09$ (cavitation safe); $\phi_e \approx 0.43$; $D \approx 13.2$ ft

6 units with 72 poles ($n = 100$ rpm):

$N_s = 5.05/2 = 2.53$ (cavitation safe), on the low side, but OK; $\phi_e \approx 0.44$; $D \approx 13.5$ ft

Selections:

N poles	Units	N_s	Cavitation	*D* ft	Machine
36	2	8.75	Not safe	–	–
36	4	6.19	Safe	6.4	Impulse
36	6	5.05	Safe	6.6	Impulse
72	2	4.38	Safe	13.2	Impulse
72	4	3.09	Safe	13.2	Impulse
72	6	2.53	Safe	13.5	Impulse (low η?)
		▲	▲	▲	▲

16.52 *A turbine is to be installed at a point where the net available head is 35 m and the available flow will average 23 m^3/s. What type of turbine would you recommend? Specify the operating speed and number of generator poles for 60-cycle electricity if a turbine is selected with the highest specific speed that is safe against cavitation. Assume a draft head of 3 m and 90% turbine efficiency. Approximately what size of turbine runner is required?*

B

Assume sea-level conditions at 15°C. Table A.3 at sea level: p_{atm} = 101.3 kPa abs = 101.3 kN/m^2 abs

Table A.1 for water at 15°C: γ = 9.80 kN/m^3, p_v/γ = 0.17 m abs

Eq. 16.24: $z_B = p_{atm}/\gamma - p_v/\gamma - \sigma_c h$; 3 = (101.3/9.80) − 0.17 − σ_c(35) ; σ_c = 0.205

Fig. 16.17 for σ_c = 0.205: $(N_s)_{BG} \le 54$ which is for a Francis turbine ◄

(Note that N_s is closely confirmed by Fig. 16.16.)

From Eq. 16.8: $P = \gamma Q h \eta$ = 9.80(23)35(0.90) = 7100 kW

Eq. 16.21: $(N_s)_{BG} = n_e\sqrt{\text{bhp}}/h^{5/4}$ so $54 \ge n_e\sqrt{7100(1.341)}/(35\times 3.28)^{5/4}$; $n_e \le 208$ rpm

$N \ge 7200/208 = 34.6$, ∴ choose N = 36 poles ◄ so n_e = 7200/36 = 200 rpm ◄

$(N_s)_{BG}$ = 54(200/208) = 51.9 ; Fig. 16.14 for N_s = 51.9: $\phi_e \approx 0.72$

Eq. 16.20: $D \approx 153.3(0.72)\sqrt{35(3.28)}/200$ = 5.9 ft = 1.80 m ◄

16.53 *A turbine is to be installed where the net available head is 185 ft, and the available flow will average 900 cfs. What type of turbine would you recommend? Specify the operating speed and number of generator poles for 60-cycle electricity if a turbine is selected with the highest specific speed that is safe against cavitation. Assume the turbine is set 5 ft above tailwater, and its efficiency is 90%. Approximately what size of runner is required?*

BG

Assume sea-level conditions at 59°F. Table A.3 at sea level: p_{atm} = 14.70 psia

Table A.1 for water at 59°F by interpolation: γ = 62.4 pcf, p_v/γ = 0.57 ft abs

Eq. 16.24: 5 = 14.70(144/62.4) − 0.57 − σ_c(185) ; σ_c = 0.153

Fig. 16.17 for σ_c = 0.153: $N_s \le 45$ which is for a Francis turbine ◄

(Note: Fig. 16.16 indicates that a 10 ft draft head is tolerable for a turbine with N_s = 45.)

$P = \gamma Q h \eta/550$ = 62.4(900)185(0.90)/550 = 17,000 hp

Eq. 16.21: $N_s = n_e\sqrt{17{,}000}/185^{5/4} \le 45$ from which $n_e \le 235$ rpm

$N = 7200/n_e \ge 7200/235 = 30.6$, ∴ choose N = 32 poles ◄ so n_e = 7200/32 = 225 rpm ◄

N_s = 54(225/235) = 51.6 ; Fig. 16.14 for N_s = 51.6: $\phi_e \approx 0.72$

Eq. 16.20: $D \approx 153.3(0.72)\sqrt{185}/225$ = 6.7 ft = 80 in ◄